Bayes Thm

given $\{B_1, B_2, \ldots, B_n\}$, partition of S

$$P(B_k \mid A) = \frac{P(A \mid B_k)\,P(B_k)}{\sum_{i=1}^{n} P(A \mid B_n)\,P(B_n)}$$

Law of Total Probability

also given $\{B_1, B_2, \ldots, B_n\}$, partition of S, for event A

$$P(A) = \sum_{i=1}^{n} P(A \cap B_n)$$

$$= P(A \cap B_1) + P(A \cap B_2) + \ldots + P(A \cap B_n)$$

$$= \sum P(A \mid B_n)\,P(B_n)$$

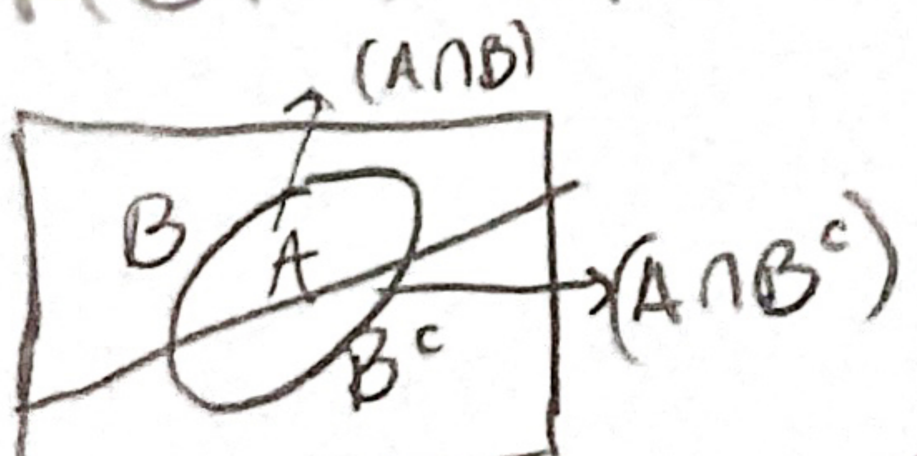
$$P(A) = P(A \mid B_1)P(B_1) + P(A \mid B_2)P(B_2) + \ldots P(A \mid B_n)P(B_n)$$

where $P(B_i) \neq 0$

Bayes Thm, special case

2 sets B & B^c

$$P(B \mid A) = \frac{P(A \mid B)\,P(B)}{P(A \mid B)P(B) + P(A \mid B^c)P(B^c)}$$



mutually independent

A, B, C m.d $\iff$ ① pairwise indep ② $P(A \cap B \cap C) = P(A)P(B)P(C)$

A, B, C, D m.d $\iff$ ① pairwise indep ② triplewise indep ③ $P(A \cap B \cap C \cap D) = P(A)P(B)P(C)P(D)$

Multiplication Rule

$\to P(A \cap B) = P(B)P(A \mid B)$ where $P(B) \neq 0$
$= P(A)P(B \mid A)$ where $P(A) \neq 0$

$$P(A_1 \cap A_2 \cap \ldots \cap A_n)$$

$$= P(A_1)P(A_2 \mid A_1)P(A_3 \mid A_1 \cap A_2) \ldots P(A_n \mid A_1 \cap A_2 \cap \ldots \cap A_{n-1})$$

ex. $n = 3$, A_1, A_2, A_3

$$P(A_1 \cap A_2 \cap A_3) = P(A_1)P(A_2 \mid A_1)P(A_3 \mid A_1 \cap A_2)$$

$$= \cancel{P(A_1)} \cdot \frac{P(A_2 \cap \cancel{A_1})}{\cancel{P(A_1)}} \cdot \frac{P(A_3 \cap A_2 \cap A_1)}{\cancel{P(A_2 \cap A_1)}}$$

$$P(A_1 \cap A_2 \cap A_3) = P(A_3 \cap A_2 \cap A_1)$$

Conditional prob

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

multiply both sides by denominator to get multiplication rule.

Same for

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)}$$

$P(\cdot \mid B)$

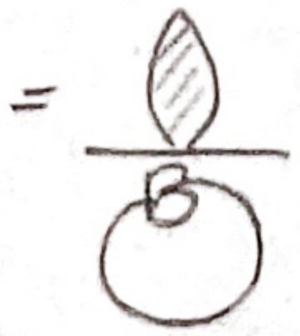
given is dwindle intersection by set on right side of line

Properties of cond. prob

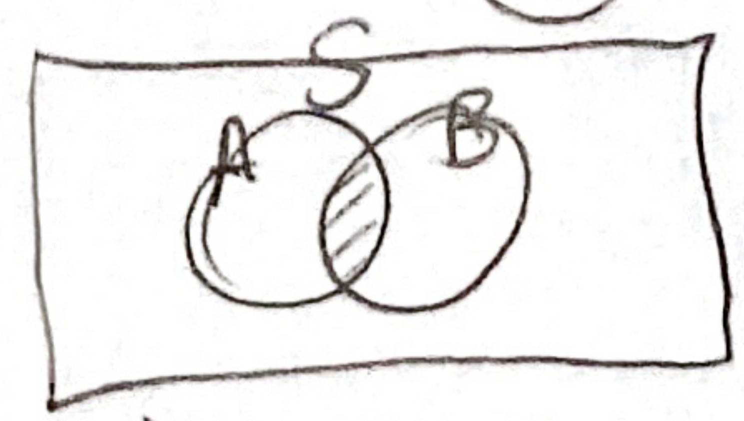
① $P(A \mid B) = 1 - P(A^c \mid B)$
② $P(\emptyset \mid B) = 0$
③ if $A_1 \subset A_2$, $P(A_1 \mid B) \leq P(A_2 \mid B)$
④ $0 \leq P(A \mid B) \leq 1$
⑤ $\mathcal{A} \in P$

$P(A_1 \cup A_2 \mid B) = P(A_1 \mid B) + P(A_2 \mid B) - P(A_1 \cap A_2 \mid B)$

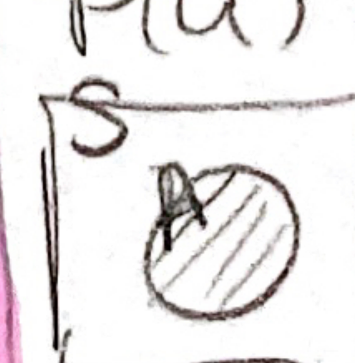
$0 \leq P(A \mid B) \leq 1$
$P(B \mid B) = 1$

$P(A \mid B) =$  $= \frac{P(A \cap B)}{P(B)}$

conditional



$P(A)$ unconditional



$P(A) = \frac{\text{◯}}{\square} = P(A)/P(S)$

note $P(S) = 1$

Events A & B are indep if & only if ($\iff$) $P(A \cap B) = P(A)P(B)$, otherwise dep

A & B indep $\iff P(A \cap B) = P(A)P(B)$

A & B dep $\iff P(A \cap B) \neq P(A)P(B)$

note: if A & B are indep

$P(A \mid B) = P(A)$, $P(B \mid A) = P(B)$

why? $P(A \mid B) \overset{\text{def}}{=} \frac{P(A \cap B)}{P(B)} \overset{\text{indep}}{=} \frac{P(A)P(B)}{P(B)}$

if A & B are indep, we have

$P(A \cap B) = P(A)P(B)$
$P(A \cap B^c) = P(A)P(B^c) \leftarrow A$ & B^c indep
$P(A^c \cap B^c) = P(A^c)P(B^c) \leftarrow A^c$ & B^c indep
$P(A^c \cap B) = P(A^c)P(B) \leftarrow A^c$ & B indep