Choosing r objects from n objects

selection order \ replacement	w/o	w
matters	$_nP_r$	n^r
doesn't matter	$_nC_r = \binom{n}{r}$	$_{n-1+r}C_r$

$_nP_n = \frac{n!}{(n-n)!} = \frac{n!}{0!} = n!$

$_nP_1 = n$

$_nC_n = 1$

$_nC_1 = n = \frac{n!}{1!\,(n-1)!} = n$

$(x+y)^n = \sum_{r=0}^{n} \binom{n}{r} x^r y^{n-r}$

$\binom{n}{r} = \binom{n}{n-r}$

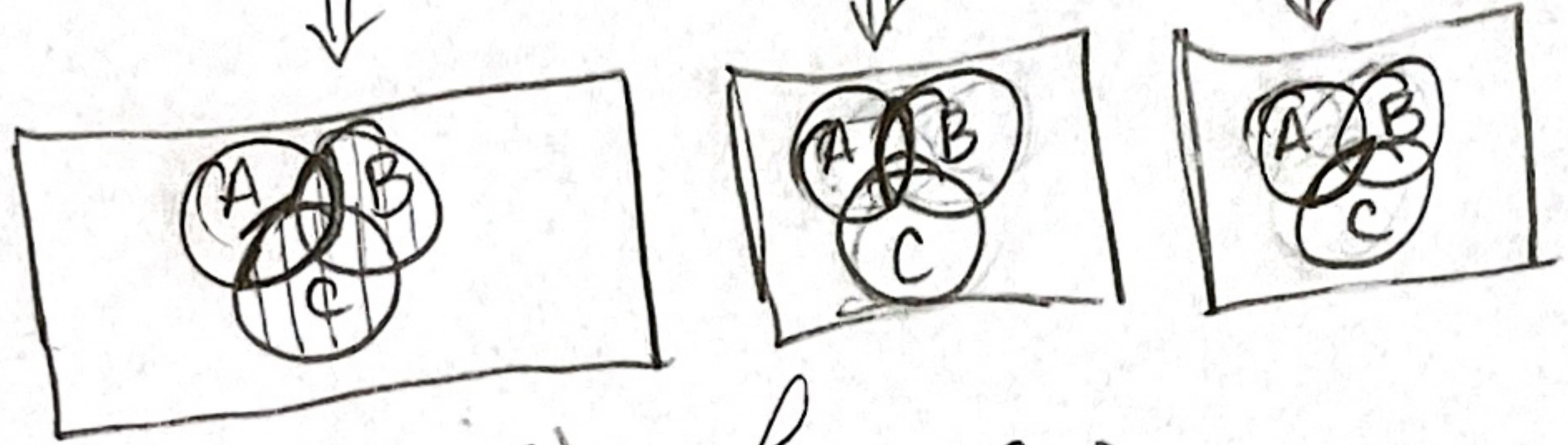
$_nC_r = {_nP_r} / r!$

$_nP_r = {_nC_r} \cdot r!$

Ex arrange 9 ppl in line $= n!$

Distributive

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$



De Morgan's Law $(A^c)^c = A$

- $(A \cap B)^c = A^c \cup B^c$
- $(A \cup B)^c = A^c \cap B^c$

distinguishable perm

n objects of 2 types

e.g. $\underbrace{A_1, A_2 \ldots A_r}_{r}\ \underbrace{B_1, B_2 \ldots B_{n-r}}_{n-r}$

$\frac{n!}{\underbrace{r!}_{A's}\,\underbrace{(n-r)!}_{B's}} = \binom{n}{r}$

n objects of k types

52 chocolates 19W 10Y 3G

eg. $\underbrace{a_1\, a_1 \ldots a_1}_{n_1}\ \underbrace{a_2 \ldots a_2}_{n_2} \ldots \underbrace{a_k\, a_k \ldots a_k}_{n_k}$

$\frac{n!}{n_1!\,n_2! \ldots n_k!} = \binom{n}{n_1, n_2 \ldots n_k}$

Multiplication Principle

ex 3 crust 2 cheese 12 toppings

$3 \times 2 \times 12$

$n(E_1) = n_1 \quad n(E_2) = n_2$

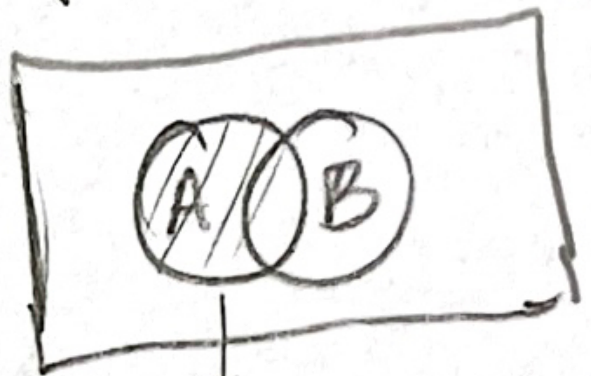
$n(E_1 E_2) = n_1 n_2$

partition?

1. $A \cap B \neq \emptyset$ m.e.
2. $A \cup B = S$ exhaustive

$a_1, a_2 \ldots a_k$ m.e. if $a_i \cap a_j \neq \emptyset \ \forall i \neq j$

pairwise disjoint / m.e.



$A \cap B^c$

$P(A) = ?$

(a) $P(A) \geq 0$

(b) $P(S) = 1$

(c) $a_1, \ldots, a_k$ m.e.

$P(A) = 1 - P(A^c)$

$P(\emptyset) = 0$

$A \subset B,\ P(A) \leq P(B)$

$P(A) \leq 1$

$P(A) = \frac{n(A)}{n(S)}$

Inclusion Exclusion Principle

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

if $A \cap B \neq \emptyset$, $P(A \cup B) = P(A) + P(B)$

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

if m.e.

$P(A \cup B \cup C) = P(A) + P(B) + P(C)$