Prob and Stat Inference

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Chapter 4

4.1 Bivariate Distributions of the Discrete Type

Definition 4.1-1

Let X and Y be two random variables defined on a discrete sample space. Let S denote the corresponding two-dimensional space of X and Y, the two random variables of the discrete type. The probability that X=x and Y=y is denoted by $f(x, y) = P(X = x, Y = y)$. The function $f(x, y)$ is called the *joint probability mass function* (joint pmf) of X and Y and has the following properties: (a) $0 \leq f(x, y \geq 1)$ (b)$\sum\sum_{(x,y \in S)} f(x, y) = 1$ (c)$P[(X, Y) \in A] = \sum\sum_{(x,y \in A)} f(x, y)$

Definition 4.1-2

Let X and Y have the joint probability mass function f(x,y), with Space S. The probability mass function of X alone, which is called the *marginal probability mass function of X*, is defined by

$f_x(x) = \sum_y f(x, y) = P(X = x)$

where the summation is taken over all prossible y values for each given x in the x space S_x. That is, the summation is over all (x,y) in S with a given x value. Similarly, the *marginal probability mass function of Y* is defined by

$f_y(y) = \sum_x f(x, y) = P(Y = y), y \in S_Y,$

where the summation is taken over all possible x values for each given y in the y space S_Y. The random variables X and Y are independent if and only if

$P(X = x, Y = y) = P(X = x)P(Y = y)$

or, equivalently,

$f(x, y) = f_x(x)f_y(y), x \in S_X, y \in S_Y;$

otherwise, X and Y are said to be dependent.

mean

variance

trinomial pmf

4.2 The correlation coefficient

covariance

correlation coefficient$\ldots\rho$

least squares regression line

4.3 Conditional Distributions

Definition 4.3-1

The conditional probability mass function of X, given that Y=y, is defined by

$g(x|y) = \frac{f(x,y)}{f_Y(y)}$, provided that $f_Y(y) > 0$.

Similarly, the conditional probability mass function of Y, given that X=x, is defined by

$h(y|x) = \frac{f(x.y)}{f_X(x)}$, provided that $f_X(x) > 0$.

Law of Total Probability for Expectation

Theorem 4.3-1

Let X and Y be random variables of discrete type such that E(Y) exists. Then $E[E(Y|X)] = E(Y)$.

Proof

$E[E(Y|X)] = \sum_{S_X}[\sum_{S_Y} y \cdot h(y|x)]f_X(x)$

$= \sum_{S_X}\sum_{S_Y}[y\frac{f(x,y)}{f_X(x)}] \cdot f_X(x)$

$= \sum_{S_X}\sum_{S_Y} y \cdot f(x,y) = \sum_{S_Y} y \sum_{S_X} f(x,y)$

$= \sum_{S_X} y \cdot f_Y(y) = E(Y)$

Law of Total Probaility of Variance

If X and Y are random variable of discrete type, then

$E[Var(Y|X)] + Var[E(Y|X)] = Var(Y)$

provided that all of the expectations and variances exist.

Proof Using the linearity of mathematical expectation and the Law of Total Probability for Expectation, we have

$E[Var(Y|X)] = E\{E(Y^2|X) - [E(Y|X)]^2\}$

$= E[E(Y^2|X)] - E\{[E(Y|X)]^2\}$

$= E(Y^2) - E\{[E(Y|X)]^2\}$.

By the same token, we have

$Var[E(Y|X)] = E\{[E(Y|X)]^2\} - \{E[E(Y|X)]\}^2$

$= E\{[E(Y|X)]^2\} - [E(Y)]^2$

Adding the equations, we find that

$E[Var(Y|X)] + Var[E(Y|X)] = E(Y^2) - [E(Y)]^2 = Var(Y)$

4.4 Bivariate Distributions of the Continuous Type

joint probability density function

(a)$f(x,y) \geq 0$ where f(x,y)=0 when (x,y) is not in the support (space) S of X and Y.

(b)$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)dxdy = 1$

(c)$P[(X,Y) \in A] = \iint_A f(x,y)dxdy, where\{(X,Y) \in A\}$ is an event defined in the plane.

marginal pdfs

$f_X(x) = \int_{-\infty}^{\infty} f(x,y)dy, x \in S_X$

$f_Y(y) = \int_{-\infty}^{\infty} f(x, y)dx, y \in S_Y$