Electric Machine Control

Chapter 2

d-q Modelling of Induction and Synchronous Machines

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2.2 Winding Inductances

- Assumptions
  - Uniform air gap induction machine (constant air gap length : g)
  - Stator with sinusoidally wound three phase windings
  - Neglect the slotting effects
    - Neglect harmonic components of flux arising from the placement of the actual conductors in the discrete slot
  - The permeability of the stator and rotor iron is assumed to be Infinite
  - Two poles construction

- The flux density is sinusoidally distributed spatially in the air gap
  - The induced rotor currents are always sinusoidally distributed spatially
  - Actual winding distribution (an n phase concentrated winding) can be replaced by an equivalent 3Φ sinusoidally distributed winding

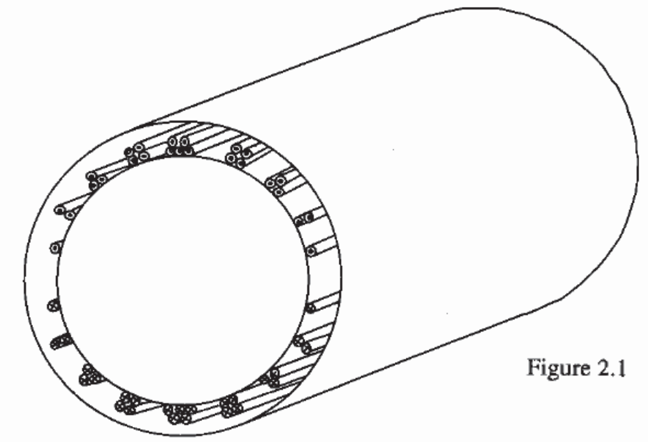


Figure 2.1 Placement of conductors of one stator or rotor phase around the air gap of a two pole machine. The details of the slot shape are not shown for clarity.

Winding density distribution

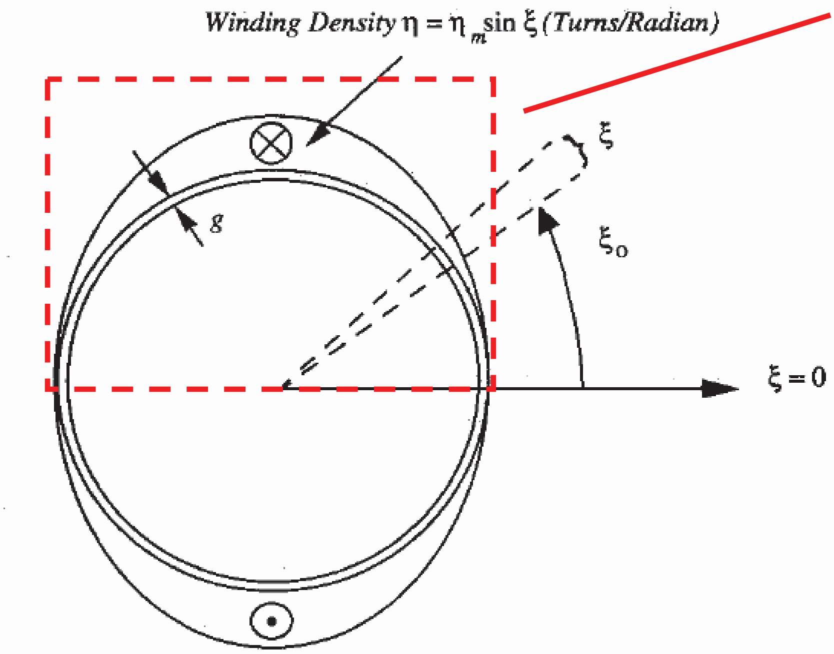


Figure 2.2 Idealized induction machine illustrating sinusoidal distribution of one phase winding

Nx: effective number of the turns

$$\int_0^{\pi} \eta_m \sin\xi d\xi = N_x \quad (2.2\text{-}2)$$

$$\eta_m = \frac{N_x}{2} \quad (2.2\text{-}3)$$

$$N_x = k_p k_d k_s N_t$$

Nt: actual number of the turns

Ampere's Law

$$dN(\xi_o) = \frac{N_x}{2}\sin\xi_o d\xi \qquad (2.2\text{-}5)$$

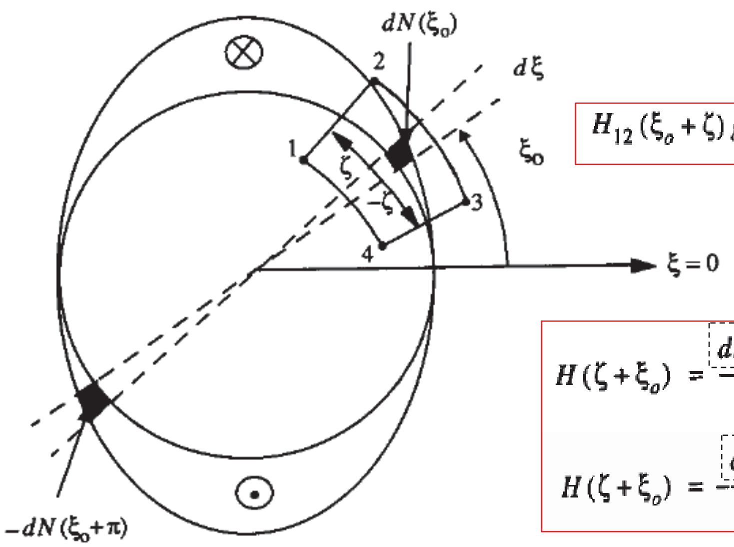
Nt: actual number of the turns

(2.2-3)

$$\oint \bar{H}\cdot d\bar{l} = \int_1^2 \bar{H}_{12}\cdot d\bar{l}_{12} + \cancel{\int_2^3 \bar{H}_{23}\cdot d\bar{l}_{23}} + \int_3^4 \bar{H}_{34}\cdot d\bar{l}_{34} + \cancel{\int_4^1 \bar{H}_{41}\cdot d\bar{l}_{41}} = I_x dN(\xi_o) \qquad (2.2\text{–}6)$$

$$H_{iron} = \frac{B_{iron}}{\mu_{iron}} << H_{air} = \frac{B_{air}}{\mu_{air}}$$

$$\boxed{H_{12}(\xi_o+\zeta)g + H_{34}(\xi_o-\zeta)g = dN(\xi_o)I_x \qquad (2.2\text{–}8)}$$



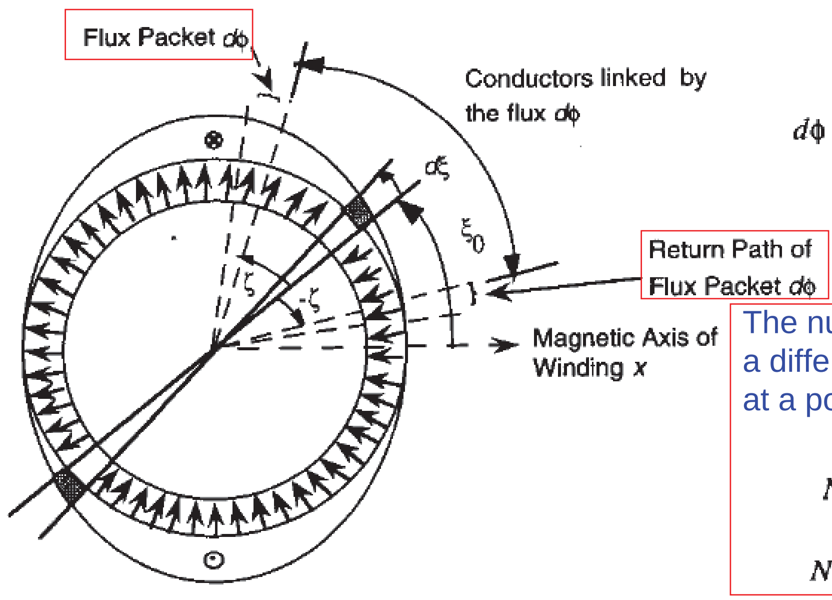
$$H(\zeta+\xi_o) = \frac{dN(\xi_o)I_x}{2g} \quad \text{for} \quad 0<\zeta<\pi \qquad (2.2\text{–}9)$$

$$H(\zeta+\xi_o) = -\frac{dN(\xi_o)I_x}{2g} \quad \text{for} \quad \pi<\zeta<2\pi \qquad (2.2\text{–}10)$$

Figure 2.3 Differential number of winding turns of winding x

Flux (weber)

$$d\phi = \bar{B}\cdot d\bar{A}$$
$$= \mu_o \bar{H}\cdot d\bar{A} \qquad (2.2\text{-}11)$$
$$= \mu_o H dl\,(r d\zeta)$$



$$d\phi = \mu_o N_x I_x\left(\frac{r}{4g}\right)\sin\xi_o d\xi\,(d\zeta dl)$$

$$\text{for} \quad 0<\zeta<\pi \qquad (2.2\text{–}12)$$

The number of turns linked by a differential packed of flux located at a position ξ relative to ξ_o is

$$N = \int_{\xi_o-\zeta}^{\xi_o+\zeta} \frac{N_x}{2}\sin u\,du \qquad (2.2\text{–}13)$$

$$N = N_x\sin\xi_o\sin\zeta \qquad (2.2\text{–}14)$$

Figure 2.4 Showing winding portion linked by an arbitrary flux packet located at $\zeta + \xi_0$

Flux linkage (weber turns)

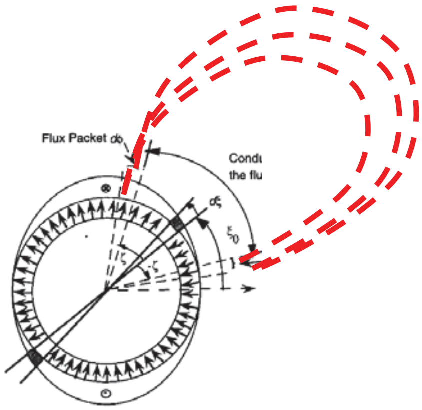
$$d\phi = \mu_o N_x I_x \left(\frac{r}{4g}\right)\sin\xi_o d\xi \,(d\zeta dl)$$

$$\text{for} \quad 0<\zeta<\pi \qquad (2.2\text{–}12)$$

The number of turns linked by a differential packed of flux located at a position ξ relative to ξ_o is

$$N = \int_{\xi_o-\zeta}^{\xi_o+\zeta} \frac{N_x}{2}\sin u\, du \qquad (2.2\text{–}13)$$

$$N = N_x \sin\xi_o \sin\zeta \qquad (2.2\text{–}14)$$



The total number of flux linkages linked by all such packets of flux d φ is

$$d\lambda_{xm} = \mu_o \frac{N_x I_x}{4g} r \sin^2\xi_o d\xi \boxed{\int_0^l dl \int_0^\pi N_x \sin\zeta d\zeta} \qquad (2.2\text{–}15)$$

$$= \mu_o \frac{N_x^2 I_x}{2g} rl \sin^2\xi_o d\xi \qquad (2.2\text{–}16)$$

Magnetizing inductance

The total number of flux linkages linked by all such packets of flux d φ is

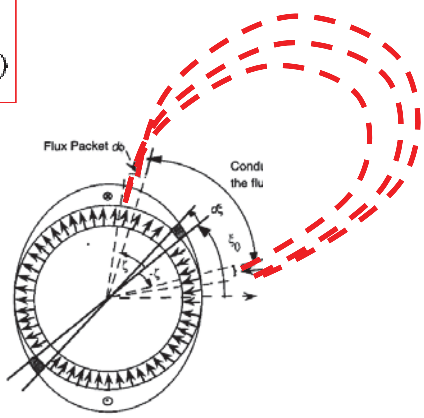
$$d\lambda_{xm} = \mu_o \frac{N_x I_x}{4g} r \sin^2\xi_o d\xi \boxed{\int_0^l dl \int_0^\pi N_x \sin\zeta d\zeta} \qquad (2.2\text{–}15)$$

$$= \mu_o \frac{N_x^2 I_x}{2g} rl \sin^2\xi_o d\xi \qquad (2.2\text{–}16)$$

The total flux linkages is the sum of the linkages from such differential windings

$$\lambda_{xm} = \int_0^\pi \mu_o \frac{N_x^2 I_x}{2g}(rl)\sin^2\xi d\xi \qquad (2.2\text{–}17)$$

$$= \mu_o N_x^2 I_x \left(\frac{rl}{g}\right)\left(\frac{\pi}{4}\right) \qquad (2.2\text{–}18)$$



The magnetizing inductance associated with this winding is

$$L_m = \frac{\lambda_{xm}}{I_x} = \mu_o N_x^2 \left(\frac{rl}{g}\right)\left(\frac{\pi}{4}\right) \qquad (2.2\text{–}19)$$

Mutual inductance

The number of turns N linked by the flux dφ is given by the modification of eq. (2.2.14)

$$N = N_y \sin(\xi_o - \alpha)\sin\zeta \qquad (2.2\text{–}20)$$

The number of flux linkages of winding y linked by the flux dφ located at $\xi_o + \zeta$ is

$$d\lambda_{xy} = \mu_o \frac{N_x I_x}{4g} r l \sin\xi_o d\xi \int_0^{\pi} N_y \sin(\xi_o - \alpha)\sin\zeta d\zeta \qquad (2.2\text{–}21)$$

$$= \mu_o \frac{N_x N_y I_x}{2g} r l \sin\xi_o \sin(\xi_o - \alpha)\, d\xi \qquad (2.2\text{–}22)$$

The number of flux linkages of winding y due to a current in winding 'x' is

$$\lambda_{xy} = \int_0^{\pi} \mu_o \frac{N_x N_y I_x}{2g} r l \sin\xi \sin(\xi - \alpha)\, d\xi$$

$$= \mu_o N_x N_y I_x \left(\frac{rl}{g}\right)\left(\frac{\pi}{4}\right)\cos \qquad (2.2\text{–}23)$$

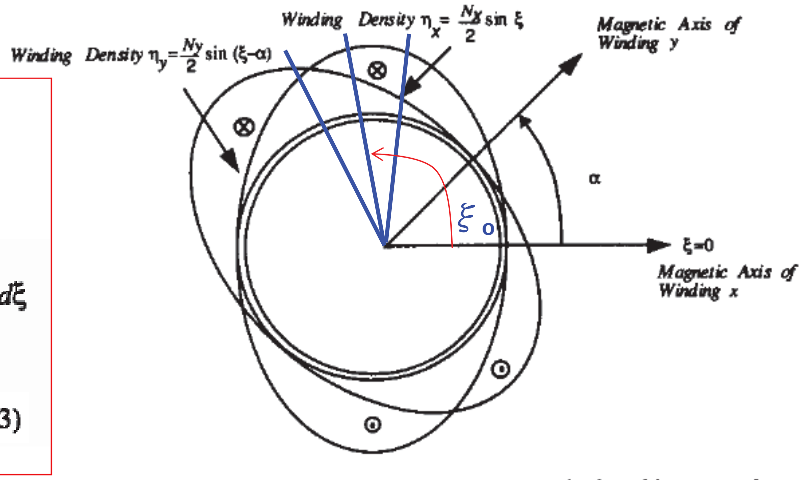


Figure 2.5 Sinusoidal current sheets for two windings displaced by an angle α

Mutual inductance

$$\lambda_{xy} = \int_0^{\pi} \mu_o \frac{N_x N_y I_x}{2g} r l \sin\xi \sin(\xi - \alpha)\, d\xi$$

$$= \mu_o N_x N_y I_x \left(\frac{rl}{g}\right)\left(\frac{\pi}{4}\right)\cos\alpha \qquad (2.2\text{–}23)$$

The mutual inductance between winding x and winding y

$$L_{xy} = \frac{\lambda_{xy}}{I_x} = \mu_o N_x N_y \left(\frac{rl}{g}\right)\left(\frac{\pi}{4}\right)\cos\alpha \qquad (2.2\text{–}24)$$

$$L_{yx} = \frac{\lambda_{yx}}{I_y} = L_{xy} \qquad (2.2\text{–}25)$$

2.3 System Equations in Stationary a,b,c R.F.

The voltage equations describing the stator and rotor circuits can be written conveniently in matrix form as

$$v_{abcs} = r_s i_{abcs} + p\lambda_{abcs} \qquad (2.3\text{–}1)$$

$$v_{abcr} = r_r i_{abcr} + p\lambda_{abcr} \qquad (2.3\text{–}2)$$

where p represents the operator d/dt and v_{abcs}, i_{abcs} and λ_{abcs} are 3x1 vectors defined by

$$v_{abcs} = \begin{bmatrix} v_{as} \\ v_{bs} \\ v_{cs} \end{bmatrix}; \qquad i_{abcs} = \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix}; \qquad \lambda_{abcs} = \begin{bmatrix} \lambda_{as} \\ \lambda_{bs} \\ \lambda_{cs} \end{bmatrix} \qquad (2.3\text{–}3)$$

Coupling

$$\lambda_{abcs} = \lambda_{abcs(s)} + \lambda_{abcs(r)} \qquad (2.3\text{–}4)$$

$$\lambda_{abcr} = \lambda_{abcr(s)} + \lambda_{abcr(r)} \qquad (2.3\text{–}5)$$

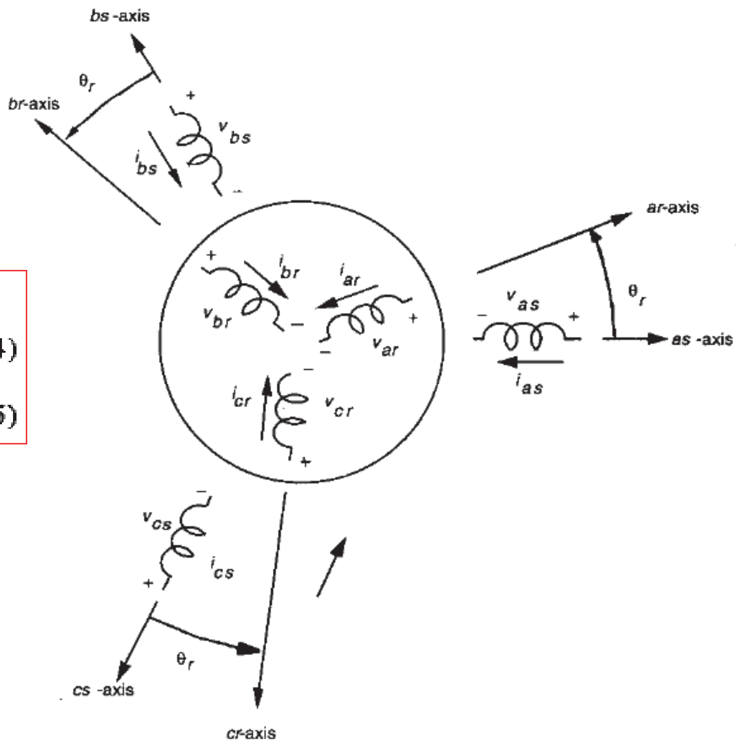


Figure 2.6 Magnetic axes of a three phase induction machine

Coupling

$$\boldsymbol{\lambda}_{abcs(s)} = \begin{bmatrix} L_{as} & L_{abs} & L_{acs} \\ L_{abs} & L_{bs} & L_{bcs} \\ L_{acs} & L_{bcs} & L_{cs} \end{bmatrix} \mathbf{i}_{abcs} \qquad (2.3\text{–}6)$$

$$\boldsymbol{\lambda}_{abcs(r)} = \begin{bmatrix} L_{as,ar} & L_{as,br} & L_{as,cr} \\ L_{bs,ar} & L_{bs,br} & L_{bs,cr} \\ L_{cs,ar} & L_{cs,br} & L_{cs,cr} \end{bmatrix} \mathbf{i}_{abcr} \qquad (2.3\text{–}7)$$

$$\boldsymbol{\lambda}_{abcr(s)} = \begin{bmatrix} L_{ar,as} & L_{ar,bs} & L_{ar,cs} \\ L_{br,as} & L_{br,bs} & L_{br,cs} \\ L_{cr,as} & L_{cr,bs} & L_{cr,cs} \end{bmatrix} \mathbf{i}_{abcs} \qquad (2.3\text{–}8)$$

$$\boldsymbol{\lambda}_{abcr(r)} = \begin{bmatrix} L_{ar} & L_{abr} & L_{acr} \\ L_{abr} & L_{br} & L_{bcr} \\ L_{acr} & L_{bcr} & L_{cr} \end{bmatrix} \mathbf{i}_{abcr} \qquad (2.3\text{–}9)$$

2.4 Determination of Induction Machine Inductances

The mutual inductance between winding x and winding y

$$\boxed{L_{xy} = \frac{\lambda_{xy}}{I_x} = \mu_o N_x N_y \left(\frac{rl}{g}\right)\left(\frac{\pi}{4}\right)\cos\alpha \qquad (2.2\text{–}24)}$$

- Self inductance

$$L_{as} = L_{ls} + L_{am} \qquad (2.4\text{–}2)$$

$$L_{am} = \mu_o N_s^2 \left(\frac{rl}{g}\right)\left(\frac{\pi}{4}\right) \qquad (2.4\text{–}1)$$

magnetizing inductance (α =0, Nx=Ny)

leakage inductance: flux lines don not cross the air gap

- Slot leakage : cross close with the stator slot itself
- Harmonic leakage : in the air gap
- End winding leakage

Stator self inductance

$$L_{as} = L_{ls} + L_{am} \qquad (2.4\text{–}2)$$

$$L_{bs} = L_{ls} + L_{bm} \qquad (2.4\text{–}3)$$

$$L_{cs} = L_{ls} + L_{cm} \qquad (2.4\text{–}4)$$

$$L_{ms} = \mu_o N_s^2 \frac{rl}{g}\left(\frac{\pi}{4}\right) \qquad (2.4\text{–}5)$$

$$\boxed{L_{as} = L_{bs} = L_{cs} = L_{ls} + L_{ms} \qquad (2.4\text{–}6)}$$

Stator mutual inductance

The mutual inductance between winding x and winding y

$$\boxed{L_{xy} = \frac{\lambda_{xy}}{I_x} = \mu_o N_x N_y \left(\frac{rl}{g}\right)\left(\frac{\pi}{4}\right)\cos\alpha \qquad (2.2\text{–}24)}$$

mutual inductance ($\alpha = 2\pi/3$, Nx=Ny=Ns)

$$L_{abs} = L_{bcs} = L_{cas} = -\mu_o N_s^2 \frac{rl}{g}\left(\frac{\pi}{8}\right) \qquad (2.4\text{–}7)$$

$$\boxed{L_{abs} = L_{bcs} = L_{cas} = -\frac{L_{ms}}{2} \qquad (2.4\text{–}8)}$$

Stator flux linkages resulting from stator currents

$$\lambda_{abcs} = \boxed{\lambda_{abcs(s)}} + \lambda_{abcs(r)} \qquad (2.3\text{–}4)$$

$$\lambda_{abcr} = \lambda_{abcr(s)} + \lambda_{abcr(r)} \qquad (2.3\text{–}5)$$

$$\lambda_{abcs(s)} = \begin{bmatrix} L_{as} & L_{abs} & L_{acs} \\ L_{abs} & L_{bs} & L_{bcs} \\ L_{acs} & L_{bcs} & L_{cs} \end{bmatrix} i_{abcs} \qquad (2.3\text{–}6)$$

$$\lambda_{abcs(s)} = \begin{bmatrix} L_{ls}+L_{ms} & -\frac{L_{ms}}{2} & -\frac{L_{ms}}{2} \\ -\frac{L_{ms}}{2} & L_{ls}+L_{ms} & -\frac{L_{ms}}{2} \\ -\frac{L_{ms}}{2} & -\frac{L_{ms}}{2} & L_{ls}+L_{ms} \end{bmatrix} \cdot i_{abcs} \qquad (2.4\text{–}9)$$

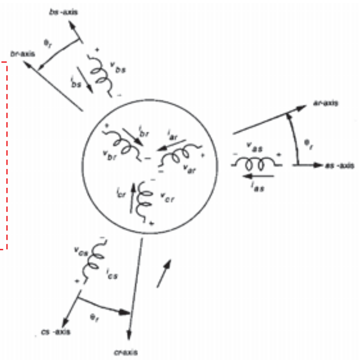
Mutual coupling between the stator and the rotor windings

The mutual inductance between winding x and winding y

$$L_{xy} = \frac{\lambda_{xy}}{I_x} = \mu_o N_x N_y \left(\frac{rl}{g}\right)\left(\frac{\pi}{4}\right)\cos\alpha \qquad (2.2\text{–}24)$$

- $\alpha = \theta_r$, Nx=Ns, Ny=Nr

$$L_{as,ar} = L_{bs,br} = L_{cs,cr} = \mu_o N_s N_r \frac{rl}{g}\left(\frac{\pi}{4}\right)\cos\theta_r$$
$$= \frac{N_r}{N_s} L_{ms}\cos\theta_r \qquad (2.4\text{–}10)$$



$$L_{as,br} = L_{bs,cr} = L_{cs,ar} = \frac{N_r}{N_s} L_{ms}\cos(\theta_r + 2\pi/3) \qquad (2.4\text{–}11)$$

$$L_{as,cr} = L_{bs,ar} = L_{cs,br} = \frac{N_r}{N_s} L_{ms}\cos(\theta_r - 2\pi/3) \qquad (2.4\text{–}12)$$

Stator flux linkages resulting from rotor currents

$$\lambda_{abcs} = \lambda_{abcs(s)} + \boxed{\lambda_{abcs(r)}} \quad (2.3\text{–}4)$$

$$\lambda_{abcr} = \lambda_{abcr(s)} + \lambda_{abcr(r)} \quad (2.3\text{–}5)$$

$$\lambda_{abcs(r)} = \begin{bmatrix} L_{as,ar} & L_{as,br} & L_{as,cr} \\ L_{bs,ar} & L_{bs,br} & L_{bs,cr} \\ L_{cs,ar} & L_{cs,br} & L_{cs,cr} \end{bmatrix} \mathbf{i}_{abcr}$$

$$\lambda_{abcs(r)} = \frac{N_r}{N_s} L_{ms} \begin{bmatrix} \cos\theta_r & \cos(\theta_r + 2\pi/3) & \cos(\theta_r - 2\pi/3) \\ \cos(\theta_r - 2\pi/3) & \cos\theta_r & \cos(\theta_r + 2\pi/3) \\ \cos(\theta_r + 2\pi/3) & \cos(\theta_r - 2\pi/3) & \cos\theta_r \end{bmatrix} \begin{bmatrix} i_{ar} \\ i_{br} \\ i_{cr} \end{bmatrix} \quad (2.4\text{–}13)$$

Rotor flux linkages resulting from stator currents

$$\lambda_{abcs} = \lambda_{abcs(s)} + \lambda_{abcs(r)} \quad (2.3\text{–}4)$$

$$\lambda_{abcr} = \boxed{\lambda_{abcr(s)}} + \lambda_{abcr(r)} \quad (2.3\text{–}5)$$

$$\lambda_{abcr(s)} = \begin{bmatrix} L_{ar,as} & L_{ar,bs} & L_{ar,cs} \\ L_{br,as} & L_{br,bs} & L_{br,cs} \\ L_{cr,as} & L_{cr,bs} & L_{cr,cs} \end{bmatrix} \mathbf{i}_{abcs}$$

$$\lambda_{abcr(s)} = \frac{N_r}{N_s} L_{ms} \begin{bmatrix} \cos\theta_r & \cos(\theta_r - 2\pi/3) & \cos(\theta_r + 2\pi/3) \\ \cos(\theta_r + 2\pi/3) & \cos\theta_r & \cos(\theta_r - 2\pi/3) \\ \cos(\theta_r - 2\pi/3) & \cos(\theta_r + 2\pi/3) & \cos\theta_r \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix} \quad (2.4\text{–}16)$$

Rotor self inductance

The mutual inductance between winding x and winding y

$$L_{xy} = \frac{\lambda_{xy}}{I_x} = \mu_o N_x N_y \left(\frac{rl}{g}\right)\left(\frac{\pi}{4}\right)\cos\alpha \qquad (2.2\text{–}24)$$

- Stator magnetizing inductance

$$L_{ms} = \mu_o N_s^2 \frac{rl}{g}\left(\frac{\pi}{4}\right) \qquad (2.4\text{–}5)$$

magnetizing inductance (α =0, Nx=Ny)

- Rotor magnetizing inductance

$$L_{am} = \mu_o N_r^2 \left(\frac{rl}{g}\right)\left(\frac{\pi}{4}\right) = \left(\frac{N_r}{N_s}\right)^2 L_{ms}$$

Rotor flux linkages resulting from rotor currents

$$\lambda_{abcs} = \lambda_{abcs(s)} + \lambda_{abcs(r)} \qquad (2.3\text{–}4)$$

$$\lambda_{abcr} = \lambda_{abcr(s)} + \boxed{\lambda_{abcr(r)}} \qquad (2.3\text{–}5)$$

$$\lambda_{abcr(r)} = \begin{bmatrix} L_{lr} + \left(\frac{N_r}{N_s}\right)^2 L_{ms} & -\frac{1}{2}\left(\frac{N_r}{N_s}\right)^2 L_{ms} & -\frac{1}{2}\left(\frac{N_r}{N_s}\right)^2 L_{ms} \\ -\frac{1}{2}\left(\frac{N_r}{N_s}\right)^2 L_{ms} & L_{lr} + \left(\frac{N_r}{N_s}\right)^2 L_{ms} & -\frac{1}{2}\left(\frac{N_r}{N_s}\right)^2 L_{ms} \\ -\frac{1}{2}\left(\frac{N_r}{N_s}\right)^2 L_{ms} & -\frac{1}{2}\left(\frac{N_r}{N_s}\right)^2 L_{ms} & L_{lr} + \left(\frac{N_r}{N_s}\right)^2 L_{ms} \end{bmatrix} \begin{bmatrix} i_{ar} \\ i_{br} \\ i_{cr} \end{bmatrix} \qquad (2.4\text{–}15)$$

2.5 Complex Vector Representation of 3-P Variables

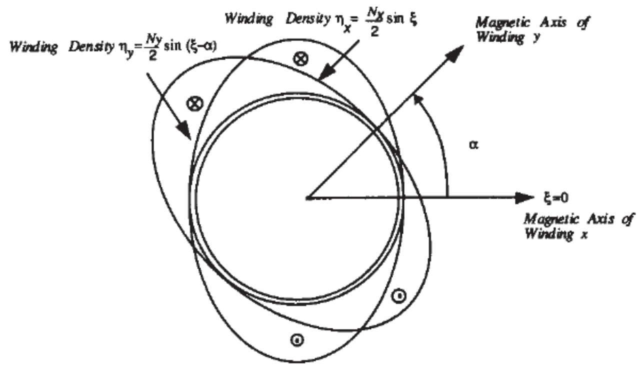
- Consider the equation describing the instantaneous position of the stator air gap MMF
  - If phase as is sinusoidally distributed with a max. value located at $\xi = \alpha$, then the MMF in the air gap resulting from current flowing in phase as is

$$F_{as} = \frac{N_s}{2} i_{as} \cos(\beta) \qquad (2.5\text{–}1)$$

where $\beta = \xi - \alpha$

$$F_{bs} = \frac{N_s}{2} i_{bs} \cos(\beta - 2\pi/3) \qquad (2.5\text{–}2)$$

$$F_{cs} = \frac{N_s}{2} i_{cs} \cos(\beta + 2\pi/3) \qquad (2.5\text{–}3)$$



- Total air gap MMF resulting from three phase currents:

$$F_{abcs} = \frac{N_s}{2}\left[i_{as}\cos\beta + i_{bs}\cos(\beta - 2\pi/3) + i_{cs}\cos(\beta + 2\pi/3)\right] \qquad (2.5\text{–}4)$$

Complex vector representation

$$e^{j\theta} = \cos\theta + j\sin\theta \qquad (2.5\text{–}5)$$

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2} \qquad (2.5\text{–}6)$$

$$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j} \qquad (2.5\text{–}7)$$

$$F_{abcs} = \frac{N_s}{2}\left[i_{as}\cos\beta + i_{bs}\cos(\beta - 2\pi/3) + i_{cs}\cos(\beta + 2\pi/3)\right] \qquad (2.5\text{–}4)$$

$$F_{abcs} = \frac{N_s}{2}\left[i_{as}\frac{(e^{j\beta} + e^{-j\beta})}{2} + i_{bs}\frac{(e^{j(\beta - 2\pi/3)} + e^{-j(\beta - 2\pi/3)})}{2} + i_{cs}\frac{(e^{j(\beta + 2\pi/3)} + e^{-j(\beta + 2\pi/3)})}{2}\right] \qquad (2.5\text{–}8)$$

Complex vector representation

$$F_{abcs} = \frac{N_s}{2}\left[i_{as}\frac{(e^{j\beta}+e^{-j\beta})}{2} + i_{bs}\frac{(e^{j(\beta-2\pi/3)}+e^{-j(\beta-2\pi/3)})}{2} + i_{cs}\frac{(e^{j(\beta+2\pi/3)}+e^{-j(\beta+2\pi/3)})}{2}\right] \qquad (2.5\text{–}8)$$

$$\Rightarrow F_{abcs} = \frac{N_s}{4}\{ [i_{as} + i_{bs}e^{-j2\pi/3} + i_{cs}e^{j2\pi/3}]\, e^{j\beta} + [i_{as} + i_{bs}e^{j2\pi/3} + i_{cs}e^{-j2\pi/3}]\, e^{-j\beta}\} \qquad (2.5\text{–}9)$$

- Defining $\begin{cases} \underline{a} = e^{j2\pi/3} \\ \underline{a}^2 = \underline{a}^{-1} = e^{j4\pi/3} = e^{-j2\pi/3} = \underline{a}^\dagger \end{cases}$

† denotes the complex conjugate

$$\Rightarrow \boxed{F_{abcs} = \frac{N_s}{4}\{ [i_{as} + \underline{a}^2 i_{bs} + \underline{a} i_{cs}]\, e^{j\beta} + [i_{as} + \underline{a} i_{bs} + \underline{a}^2 i_{cs}]\, e^{-j\beta}\}} \qquad (2.5\text{–}12)$$

Complex vector representation

$$F_{abcs} = \frac{N_s}{4}\{ \boxed{[i_{as} + \underline{a}^2 i_{bs} + \underline{a} i_{cs}]\, e^{j\beta}} + \boxed{[i_{as} + \underline{a} i_{bs} + \underline{a}^2 i_{cs}]\, e^{-j\beta}}\} \qquad (2.5\text{–}12)$$

$$\underline{i}^\dagger_{abcs} = \frac{2}{3}(i_{as} + \underline{a}^2 i_{bs} + \underline{a} i_{cs}) \qquad (2.5\text{–}14)$$

$$\underline{i}_{abcs} = \frac{2}{3}(i_{as} + \underline{a} i_{bs} + \underline{a}^2 i_{cs}) \qquad (2.5\text{–}13)$$

complex space vector

$$F_{abcs} = \frac{3}{2}\frac{N_s}{4}(\underline{i}_{abcs}e^{-j\beta} + \underline{i}^\dagger_{abcs}e^{j\beta}) \qquad (2.5\text{–}15)$$

Notation of complex space vector : $\underline{f}_{abc}$

Complex space vector

$$\begin{cases} \underline{a} = e^{j2\pi/3} \boxed{= -\frac{1}{2} + j\frac{\sqrt{3}}{2}} \\ \underline{a}^2 = \underline{a}^{-1} = e^{j4\pi/3} = e^{-j2\pi/3} = \underline{a}^{\dagger} \end{cases}$$

$$1 - \frac{\underline{a}}{2} - \frac{\underline{a}^2}{2} = \frac{3}{2}$$

$$1 + \underline{a} + \underline{a}^2 = 0$$

$$\underline{f}_{abc} = \frac{2}{3}\left(f_a + \underline{a} f_b + \underline{a}^2 f_c \right)$$

$$f_a = \mathrm{Re}\left(\underline{f}_{abc} \right) \qquad (2.5\text{-}17)$$

$$f_b = \mathrm{Re}\left(\underline{a}^2 \underline{f}_{abc} \right) \qquad (2.5\text{-}19)$$

$$f_c = \mathrm{Re}\left(\underline{a} \underline{f}_{abc} \right) \qquad (2.5\text{-}20)$$

2.6 Complex Variables Model of 3-P IM

Stator voltage equations:

$$\boxed{\underline{v}_{abcs} = r_s \underline{i}_{abcs} + p\underline{\lambda}_{abcs}} \qquad (2.6\text{–}1)$$

where

$$\underline{v}_{abcs} = \frac{2}{3}(v_{as} + \underline{a} v_{bs} + \underline{a}^2 v_{cs}) \qquad (2.6\text{–}2)$$

$$\boxed{\underline{v}_{abcr} = r_r \underline{i}_{abcr} + p\underline{\lambda}_{abcr}} \qquad (2.6\text{–}3)$$

Stator flux linkage equations:

$$\underline{\lambda}_{abcs(s)} = \lambda_{as(s)} + \underline{a}\lambda_{bs(s)} + \underline{a}^2\lambda_{cs(s)}$$

$$= \left[L_{ls} + L_{ms}\underbrace{\left(1 - \frac{\underline{a}}{2} - \frac{\underline{a}^2}{2}\right)} \right](i_{as} + \underline{a} i_{bs} + \underline{a}^2 i_{cs}) \qquad (2.6\text{–}4)$$

$$= \left(L_{ls} + \boxed{\frac{3}{2}} L_{ms} \right) \underline{i}_{abcs} \qquad (2.6\text{–}6)$$

Stator flux linkage arises from rotor current

$$\lambda_{abcs(r)} = \frac{N_r}{N_s}L_{ms}\begin{bmatrix} \cos\theta_r & \cos(\theta_r + 2\pi/3) & \cos(\theta_r - 2\pi/3) \\ \cos(\theta_r - 2\pi/3) & \cos\theta_r & \cos(\theta_r + 2\pi/3) \\ \cos(\theta_r + 2\pi/3) & \cos(\theta_r - 2\pi/3) & \cos\theta_r \end{bmatrix}\begin{bmatrix} i_{ar} \\ i_{br} \\ i_{cr} \end{bmatrix}$$

$$\begin{bmatrix} \lambda_{as(r)} \\ \lambda_{bs(r)} \\ \lambda_{cs(r)} \end{bmatrix} = \frac{N_r}{N_s}\frac{L_{ms}}{2}\left\{\begin{bmatrix} e^{j\theta_r} & \underline{a}e^{j\theta_r} & \underline{a}^2e^{j\theta_r} \\ \underline{a}^2e^{j\theta_r} & e^{j\theta_r} & \underline{a}e^{j\theta_r} \\ \underline{a}e^{j\theta_r} & \underline{a}^2e^{j\theta_r} & e^{j\theta_r} \end{bmatrix} + \begin{bmatrix} e^{-j\theta_r} & \underline{a}^2e^{-j\theta_r} & \underline{a}e^{-j\theta_r} \\ \underline{a}e^{-j\theta_r} & e^{-j\theta_r} & \underline{a}^2e^{-j\theta_r} \\ \underline{a}^2e^{-j\theta_r} & \underline{a}e^{-j\theta_r} & e^{-j\theta_r} \end{bmatrix}\right\} \cdot \begin{bmatrix} i_{ar} \\ i_{br} \\ i_{cr} \end{bmatrix} \quad (2.6\text{–}7)$$

$$\underline{\lambda}_{abcs(r)} = \frac{2}{3}\underbrace{(\lambda_{as(r)} + \underline{a}\lambda_{bs(r)} + \underline{a}^2\lambda_{cs(r)})} \quad (2.6\text{–}11)$$

$$= \frac{N_r}{N_s}\frac{L_{ms}}{2}[3(i_{ar} + \underline{a}i_{br} + \underline{a}^2 i_{cr})e^{j\theta_r} + (1 + \underline{a} + \underline{a}^2)(i_{ar} + i_{br} + i_{cr})e^{-j\theta_r}] \quad (2.6\text{–}8)$$

$$= \frac{N_r}{N_s}\frac{3L_{ms}}{2}\underline{i}_{abcr}e^{j\theta_r}$$

Total flux linkages in complex space vector form

Total stator flux linkages

$$\underline{\lambda}_{abcs} = \left(L_{ls} + \frac{3}{2}L_{ms}\right)\underline{i}_{abcs} + \frac{N_r}{N_s}\frac{3L_{ms}}{2}\underline{i}_{abcr}e^{j\theta_r} \quad (2.6\text{–}13)$$

$$\underline{\lambda}_{abcs(s)} = \left(L_{ls} + \frac{3}{2}L_{ms}\right)\underline{i}_{abcs} \quad (2.6\text{–}6)$$

$$\underline{\lambda}_{abcs(r)} = \frac{N_r}{N_s}\frac{3L_{ms}}{2}\underline{i}_{abcr}e^{j\theta_r} \quad (2.6\text{–}10)$$

Total rotor flux linkages

$$\underline{\lambda}_{abcr} = \left[L_{lr} + \frac{3}{2}\left(\frac{N_r}{N_s}\right)^2 L_{ms}\right]\underline{i}_{abcr} + \frac{N_r}{N_s}\frac{3L_{ms}}{2}\underline{i}_{abcs}e^{-j\theta_r} \quad (2.6\text{–}14)$$

2.7 Turns Ration Transformation

- Refer the rotor circuits to the stator

$$\begin{cases} \underline{v}_{abcr} = r_r \underline{i}_{abcr} + p\underline{\lambda}_{abcr} & (2.6\text{–}3) \\ \left(\frac{N_s}{N_r}\right)\underline{v}_{abcr} = \left(\frac{N_s}{N_r}\right)^2 r_r \left[\left(\frac{N_r}{N_s}\right)\underline{i}_{abcr}\right] + p\left[\left(\frac{N_s}{N_r}\right)\underline{\lambda}_{abcr}\right] & (2.7\text{–}1) \end{cases}$$

$$\begin{cases} \underline{\lambda}_{abcr} = \left[L_{lr} + \frac{3}{2}\left(\frac{N_r}{N_s}\right)^2 L_{ms}\right]\underline{i}_{abcr} + \frac{N_r}{N_s}\frac{3L_{ms}}{2}\underline{i}_{abcs}e^{-j\theta_r} & (2.6\text{–}14) \\ \left(\frac{N_s}{N_r}\right)\underline{\lambda}_{abcr} = \left[\left(\frac{N_s}{N_r}\right)^2 L_{lr} + \frac{3}{2}L_{ms}\right]\left[\left(\frac{N_r}{N_s}\right)\underline{i}_{abcr}\right] + \frac{3L_{ms}}{2}\underline{i}_{abcs}e^{-j\theta_r} & (2.7\text{–}3) \end{cases}$$

$$\begin{cases} \underline{\lambda}_{abcs} = \left(L_{ls} + \frac{3}{2}L_{ms}\right)\underline{i}_{abcs} + \frac{N_r}{N_s}\frac{3L_{ms}}{2}\underline{i}_{abcr}e^{j\theta_r} & (2.6\text{–}13) \\ \underline{\lambda}_{abcs} = \left(L_{ls} + \frac{3}{2}L_{ms}\right)\underline{i}_{abcs} + \frac{3L_{ms}}{2}\left[\frac{N_r}{N_s}\underline{i}_{abcr}\right]e^{j\theta_r} & (2.7\text{–}2) \end{cases}$$

Refer the rotor circuits to the stator

$$\left(\frac{N_s}{N_r}\right)\underline{v}_{abcr} = \underline{v}'_{abcr} \qquad (2.7\text{–}4)$$

$$\left(\frac{N_s}{N_r}\right)\underline{\lambda}_{abcr} = \underline{\lambda}'_{abcr} \qquad (2.7\text{–}5)$$

$$\left(\frac{N_r}{N_s}\right)\underline{i}_{abcr} = \underline{i}'_{abcr} \qquad (2.7\text{–}6)$$

$$\left(\frac{N_s}{N_r}\right)^2 r_r = r'_r \qquad (2.7\text{–}7)$$

$$\left(\frac{N_s}{N_r}\right)^2 L_{lr} = L'_{lr} \qquad (2.7\text{–}8)$$

Refer the rotor circuits to the stator

$$\boxed{L_m = \frac{3}{2}L_{ms}} = \frac{3}{2}N_s^2\left(\mu_o \frac{rl}{g}\right)\frac{\pi}{4} \qquad (2.7\text{–}9)$$

$$\begin{cases} \underline{v}_{abcs} = r_s \underline{i}_{abcs} + (L_{ls} + L_m)\, p\underline{i}_{abcs} + \boxed{L_m\, p\,(\underline{i}'_{abcr} e^{j\theta_r})} & (2.7\text{–}10) \\ \underline{v}_{abcs} = r_s \underline{i}_{abcs} + (L_{ls} + L_m)\, p\underline{i}_{abcs} + \boxed{L_m\,(p\underline{i}'_{abcr})\, e^{j\theta_r} + j\omega_r L_m \underline{i}'_{abcr}\, e^{j\theta_r}} & (2.7\text{–}12) \end{cases}$$

$$\begin{cases} \underline{v}'_{abcr} = r'_r \underline{i}'_{abcr} + (L'_{lr} + L_m)\, p\underline{i}'_{abcr} + L_m \boxed{p\,(\underline{i}_{abcs} e^{-j\theta_r})} & (2.7\text{–}11) \\ \underline{v}'_{abcr} = r'_r \underline{i}'_{abcr} + (L'_{lr} + L_m)\, p\underline{i}'_{abcr} + \boxed{L_m\,(p\underline{i}_{abcs})\, e^{-j\theta_r} - j\omega_r L_m \underline{i}_{abcs} e^{-j\theta_r}} & (2.7\text{–}13) \end{cases}$$

where $\omega_r = p\theta_r = \frac{d\theta_r}{dt}$.

2.8 Transformation to Rotating Reference Frame

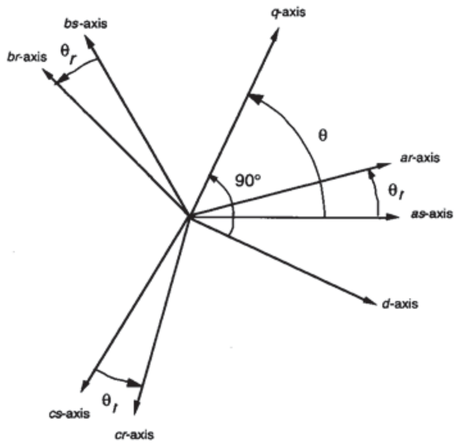


Figure 2.7 Location of rotating d,q axes relative to the magnetic axes of the stator and rotor phases

$$\underbrace{\begin{bmatrix} f_{qs} \\ f_{ds} \\ f_{0s} \end{bmatrix} = \frac{2}{3}\begin{bmatrix} \cos\theta & \cos\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta + \frac{2\pi}{3}\right) \\ \sin\theta & \sin\left(\theta - \frac{2\pi}{3}\right) & \sin\left(\theta + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} f_{as} \\ f_{bs} \\ f_{cs} \end{bmatrix}}_{\boxed{\underline{f}_{qds} = f_{qs} - \mathrm{j}\cdot f_{ds}}}$$

$$\begin{aligned} \boxed{\underline{f}_{qds} = f_{qs} - jf_{ds}} &= \frac{2}{3}[f_{as}e^{-j\theta} + f_{bs}e^{-j(\theta - 2\pi/3)} + f_{cs}e^{-j(\theta + 2\pi/3)}] \\ &= \frac{2}{3}e^{-j\theta}[f_{as} + \underline{a}f_{bs} + \underline{a}^2 f_{cs}] && (2.8\text{–}4) \\ &= e^{-j\theta}\underline{f}_{abcs} && (2.8\text{–}5) \end{aligned}$$

Transformation matrix

$$\begin{bmatrix} f_{qs} \\ f_{ds} \\ f_{0s} \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \cos\theta & \cos\left(\theta - 2\pi/3\right) & \cos\left(\theta + 2\pi/3\right) \\ \sin\theta & \sin\left(\theta - 2\pi/3\right) & \sin\left(\theta + 2\pi/3\right) \\ 1/2 & 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} f_{as} \\ f_{bs} \\ f_{cs} \end{bmatrix}$$

$$\begin{bmatrix} f_{as} \\ f_{bs} \\ f_{cs} \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 1 \\ \cos\left(\theta - 2\pi/3\right) & \sin\left(\theta - 2\pi/3\right) & 1 \\ \cos\left(\theta + 2\pi/3\right) & \sin\left(\theta + 2\pi/3\right) & 1 \end{bmatrix} \begin{bmatrix} f_{qs} \\ f_{ds} \\ f_{0s} \end{bmatrix}$$

Rotating Reference Frame

stator variables

$$\underline{f}_{qds} = f_{qs} - jf_{ds} = \frac{2}{3}[f_{as}e^{-j\theta} + f_{bs}e^{-j(\theta - 2\pi/3)} + f_{cs}e^{-j(\theta + 2\pi/3)}]$$

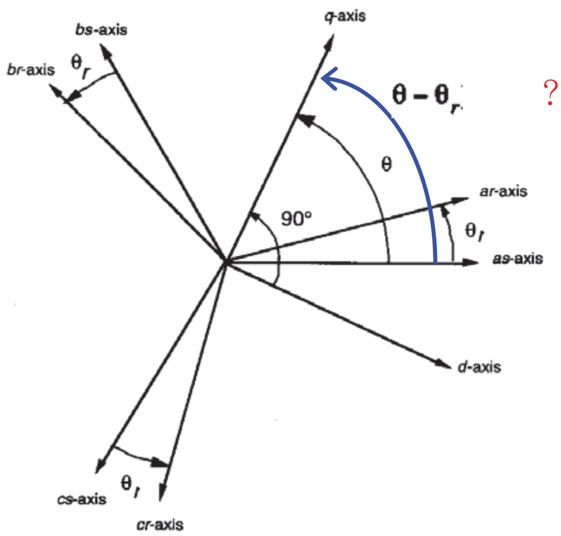
$$= \frac{2}{3}e^{-j\theta}[f_{as} + \underline{a}f_{bs} + \underline{a}^2 f_{cs}] \qquad (2.8\text{–}4)$$

$$\boxed{= e^{-j\theta}\underline{f}_{abcs}} \qquad (2.8\text{–}5)$$

rotor variables

$$\underline{f}_{qdr} = \frac{2}{3}e^{-j(\theta-\theta_r)}[f_{ar} + \underline{a}f_{br} + \underline{a}^2 f_{cr}]$$

$$\boxed{= e^{-j(\theta-\theta_r)}\underline{f}_{abcr}} \qquad (2.8\text{–}6)$$



Voltage Equations in Rotating Reference Frame

$$e^{-j\theta}\underline{v}_{abcs} = r_s e^{-j\theta}\underline{i}_{abcs} + (L_{ls}+L_m)\boxed{e^{-j\theta}p\underline{i}_{abcs}} + L_m\boxed{e^{-j\theta}p\,(\underline{i}'_{abcr}\,e^{j\theta_r})} \quad (2.8\text{–}7)$$

$$= r_s e^{-j\theta}\underline{i}_{abcs} + (L_{ls}+L_m)\boxed{p\,(e^{-j\theta}\underline{i}_{abcs})} + L_m\boxed{p\,[\underline{i}'_{abcr}\,e^{-j(\theta-\theta_r)}]}$$

$$+\, j\omega\,[\,\boxed{(L_{ls}+L_m)\,e^{-j\theta}\underline{i}_{abcs}} + \boxed{L_m\,\underline{i}'_{abcr}\,e^{-j(\theta-\theta_r)}}\,] \quad (2.8\text{–}8)$$

$$\underline{v}_{qds} = r_s\underline{i}_{qds} + (L_{ls}+L_m)\,p\underbrace{\underline{i}_{qds}}_{e^{-j\theta}\underline{i}_{abcs}} + L_m p\underbrace{\underline{i}'_{qdr}}_{\underline{i}'_{abcr}\,e^{-j(\theta-\theta_r)}} + j\omega\,[\,(L_{ls}+L_m)\underbrace{\underline{i}_{qds}}_{e^{-j\theta}\underline{i}_{abcs}} + L_m\underbrace{\underline{i}'_{qdr}}_{\underline{i}'_{abcr}\,e^{-j(\theta-\theta_r)}}\,] \quad (2.8\text{–}10)$$

$$\underline{v}'_{qdr} = r'_r\,\underline{i}'_{qdr} + (L'_{lr}+L_m)\,p\underline{i}'_{qdr} + L_m p\underline{i}_{qds} + j(\omega-\omega_r)\,[\,(L'_{lr}+L_m)\,\underline{i}'_{qdr} + L_m\underline{i}_{qds}\,] \quad (2.8\text{–}11)$$

Voltage Equations in Rotating Reference Frame

$$\overbrace{v_{as}+v_{bs}+v_{cs}}^{v_{0s}} = (r_s + pL_{ls})\,\overbrace{(i_{as}+i_{bs}+i_{cs})}^{i_{0s}}$$

$$v_{0s} = (r_s + p\boxed{L_{ls}})\,i_{0s} \qquad (2.8\text{–}12)$$

$$v'_{0r} = (r'_r + p\boxed{L'_{lr}})\,i'_{0r} \qquad (2.8\text{–}13)$$

The zero sequence inductance is typically less than the per phase leakage inductance being roughly 0.8 to 0.95 of the value

Voltage Equations in Rotating Reference Frame

$$\begin{cases} v_{ds} = r_s i_{ds} + \dfrac{d\lambda_{ds}}{dt} - \omega\lambda_{qs} \\ v_{qs} = r_s i_{qs} + \dfrac{d\lambda_{qs}}{dt} + \omega\lambda_{ds} \\ v_{0s} = r_s i_{0s} + \dfrac{d\lambda_{0s}}{dt} \end{cases}$$

where

$$\lambda_{ds} = L_{ls} i_{ds} + L_m (i_{ds} + i'_{dr})$$

$$\lambda_{qs} = L_{ls} i_{qs} + L_m (i_{qs} + i'_{qr})$$

$$\lambda_{0s} = L_{ls} i_{0s}$$

$$\begin{cases} v'_{dr} = r'_r i'_{dr} + \dfrac{d\lambda'_{dr}}{dt} - (\omega - \omega_r)\lambda'_{qr} \\ v'_{qr} = r'_r i'_{qr} + \dfrac{d\lambda'_{qr}}{dt} + (\omega - \omega_r)\lambda'_{dr} \\ v'_{0r} = r'_r i'_{0r} + \dfrac{d\lambda'_{0r}}{dt} \end{cases}$$

where

$$\lambda'_{dr} = L'_{lr} i'_{dr} + L_m (i_{ds} + i'_{dr})$$

$$\lambda'_{qr} = L'_{lr} i'_{qr} + L_m (i_{qs} + i'_{qr})$$

$$\lambda'_{0r} = L'_{lr} i'_{0r}$$

Voltage Equations in Rotating Reference Frame

$$\begin{bmatrix} v_{ds} \\ v_{qs} \\ v'_{dr} \\ v'_{qr} \end{bmatrix} = \begin{bmatrix} r_s + L_s p & -\omega L_s & L_m p & -\omega L_m \\ \omega L_s & r_s + L_s p & \omega L_m & L_m p \\ L_m p & -(\omega - \omega_r) L_m & r'_r + L'_r p & -(\omega - \omega_r) L'_r \\ (\omega - \omega_r) L_m & L_m p & (\omega - \omega_r) L'_r & r'_r + L'_r p \end{bmatrix} \cdot \begin{bmatrix} i_{ds} \\ i_{qs} \\ i'_{dr} \\ i'_{qr} \end{bmatrix} \quad (2.8\text{–}26)$$

where $\boxed{L'_r = L'_{lr} + L_m}$ and $\boxed{L_s = L_{ls} + L_m}$

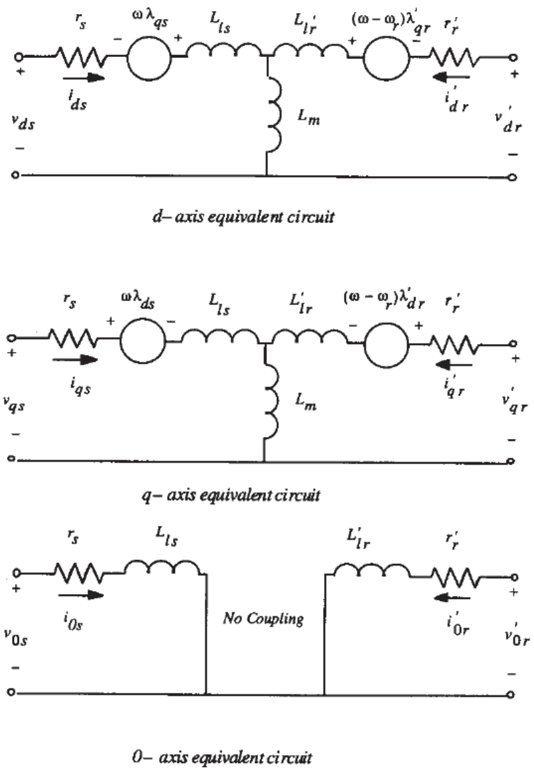


Figure 2.8 The d,q,0 equivalent circuits of three phase induction machine

2.9 Interpretation of Complex Vectors

- Complex space vector (abc axes)

$$\underline{f}_{abc} = \frac{2}{3}\left(f_a + \underline{a} f_b + \underline{a}^2 f_c\right)$$

$$f_a = \mathrm{Re}\left(\underline{f}_{abc}\right) \quad (2.5\text{-}17)$$

$$f_b = \mathrm{Re}\left(\underline{a}^2 \underline{f}_{abc}\right) \quad (2.5\text{-}19)$$

$$f_c = \mathrm{Re}\left(\underline{a}\underline{f}_{abc}\right) \quad (2.5\text{-}20)$$

- Complex space vector (dq axes)

$$\underline{f}_{qds} = \frac{2}{3} e^{-j\theta}\left(f_{as} + \underline{a} f_{bs} + \underline{a}^2 f_{cs}\right)$$

- Stationary reference frame

$$\theta = 0$$

$$\underline{f}_{qds}^{s} = \frac{2}{3}\left(f_{as} + \underline{a} f_{bs} + \underline{a}^2 f_{cs}\right) \quad (2.9\text{-}2)$$

- Synchronous reference frame

$$\theta = \theta_e = \omega_e t$$

$$\underline{f}_{qds}^{e} = \underline{f}_{qds}^{s} e^{-j\theta_e} \quad (2.9\text{-}3)$$

- Rotor reference frame

$$\theta = \theta_r = \omega_r t$$

$$\underline{f}_{qds}^{r} = \underline{f}_{qds}^{s} e^{-j\theta_r} \quad (2.9\text{-}12)$$

$$\theta = 0$$

$$\underline{f}_{qds}^{s} = \frac{2}{3}\left(f_{as} + \underline{a} f_{bs} + \underline{a}^{2} f_{cs} \right) \quad \text{(2.9-2)}$$

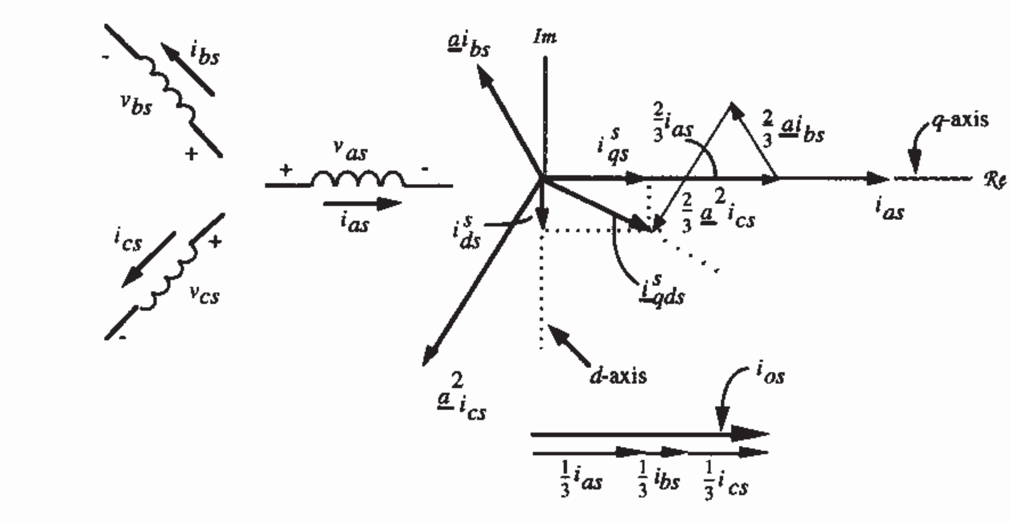
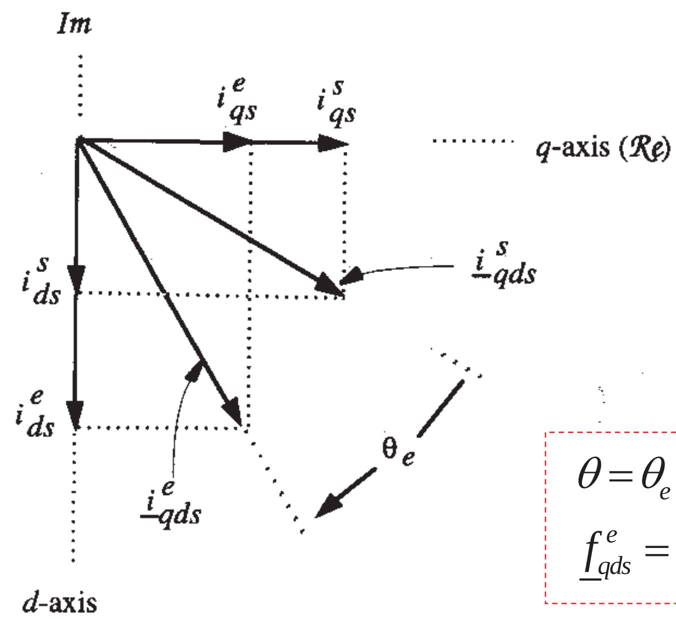


Figure 2.9 Transformation from three phase stator to complex vector voltage



$$\theta = \theta_e = \omega_e t$$

$$\underline{f}_{qds}^{e} = \underline{f}_{qds}^{s} e^{-j\theta_e} \quad \text{(2.9-3)}$$

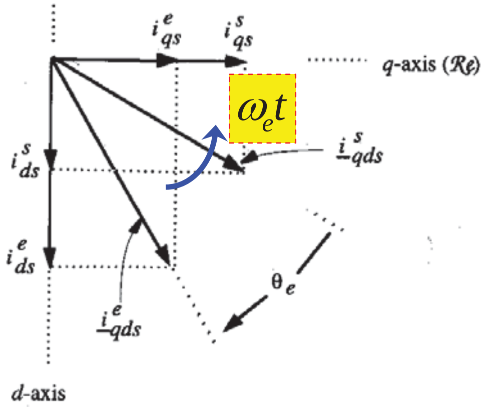
Figure 2.10 Transformation from d,q variables in a stationary axes to rotating d,q variables, (pure rotation)

Example

$$i_{as} = I_s \cos \omega_e t \qquad (2.9\text{–}4)$$

$$i_{bs} = I_s \cos\left(\omega_e t - \frac{2\pi}{3} \right) \qquad (2.9\text{–}5)$$

$$i_{cs} = I_s \cos\left(\omega_e t + \frac{2\pi}{3} \right) \qquad (2.9\text{–}6)$$



$$\underline{i}_{qds}^{s} = \frac{2}{3}\frac{I_s}{2}\left[e^{j\omega_e t} + e^{-j\omega_e t} + \underline{a}\left(e^{j(\omega_e t - 2\pi/3)} + e^{-j(\omega_e t - 2\pi/3)}\right)\right.$$

$$\left. + \underline{a}^2\left(e^{j(\omega_e t + 2\pi/3)} + e^{-j(\omega_e t + 2\pi/3)}\right)\right] \qquad (2.9\text{–}7)$$

$$= \frac{I_s}{3}\left[3e^{j\omega_e t} + e^{-j\omega_e t}(1 + \underline{a} + \underline{a}^2)\right]$$

$$= I_s e^{j\omega_e t} \qquad (2.9\text{–}8)$$

$$\theta = \theta_e = \omega_e t$$

$$\underline{f}_{qds}^{e} = \underline{f}_{qds}^{s} e^{-j\theta_e} \qquad (2.9\text{-}3)$$

Inverse of complex vector quantity

$$\underline{f}_{qdx}^{x} = \frac{2}{3}\left[f_{ax} + \underline{a} f_{bx} + \underline{a}^2 f_{cx}\right]$$

$$\underline{f}_{qdx}^{x} = \frac{2}{3}\left[f_{ax} + \left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)f_{bx} + \left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)f_{cx}\right]$$

$$= \frac{2}{3}\left[\frac{3}{2}f_{ax} - \frac{1}{2}(f_{ax} + f_{bx} + f_{cx}) + j\frac{\sqrt{3}}{2}(f_{bx} - f_{cx})\right] \qquad (2.9\text{–}14)$$

$$f_{ax} = Re\left[\underline{f}_{qdx}^{x}\right] + f_{0x} \qquad (2.9\text{–}15)$$

$$f_{bx} = Re\left[\underline{a}^2 \underline{f}_{qdx}^{x}\right] + f_{0x} \qquad (2.9\text{–}16)$$

$$f_{cx} = Re\left[\underline{a} \underline{f}_{qdx}^{x}\right] + f_{0x} \qquad (2.9\text{–}17)$$

$$f_{ax} = |f| \cos\phi + f_{0x} \qquad (2.9\text{–}19)$$

$$f_{bx} = |f| \cos\left(\frac{2\pi}{3} - \phi\right) + f_{0x} \qquad (2.9\text{–}20)$$

$$f_{cx} = |f| \cos\left(\frac{2\pi}{3} + \phi\right) + f_{0x} \qquad (2.9\text{–}21)$$

Inverse of complex vector quantity

$$f_{ax} = Re\,[\underline{f}^{\,x}_{qdx}] + f_{0x}$$
$$f_{bx} = Re\,[\underline{a}^2\underline{f}^{\,x}_{qdx}] + f_{0x}$$
$$f_{cx} = Re\,[\underline{a}\underline{f}^{\,x}_{qdx}] + f_{0x}$$

$$\underline{f}^{\,x}_{qdx} = |f|e^{j\phi}$$

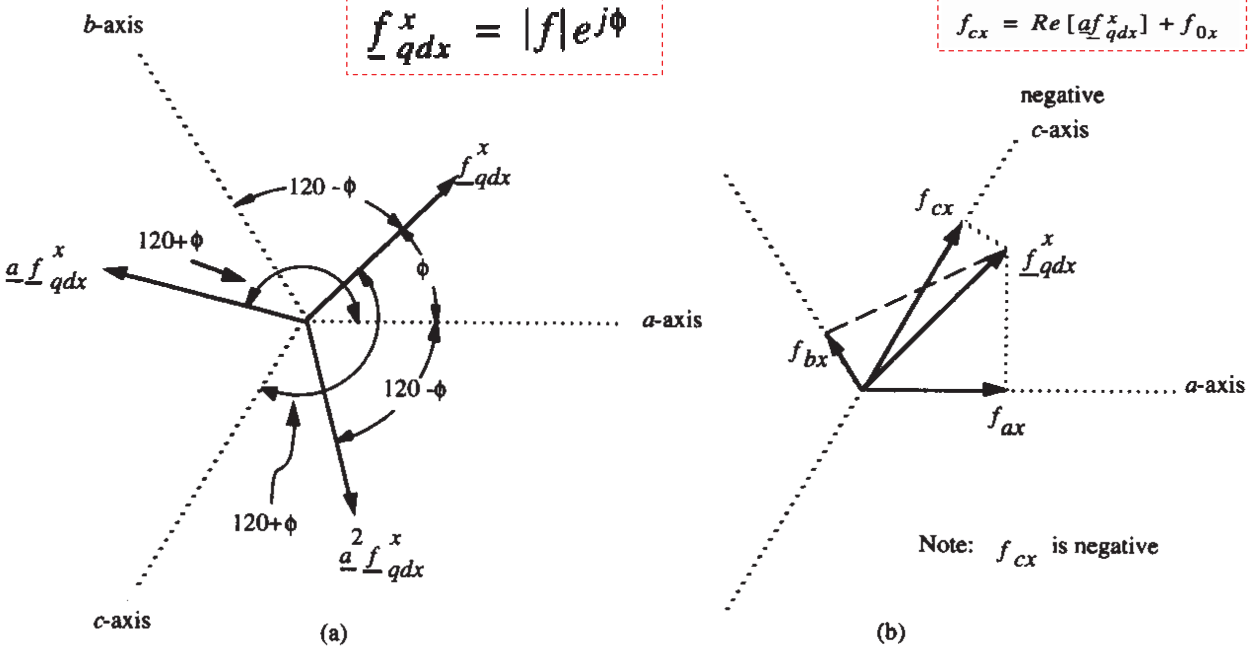


Figure 2.11 Graphical inverse of complex vector quantity

2.10 Power Flow in the d,q Equivalent Circuit

$$\underline{v}_{abcs}\underline{i}^{\dagger}_{abcs} = \frac{2}{3}(v_{as} + \underline{a}v_{bs} + \underline{a}^2 v_{cs})\boxed{\frac{2}{3}(i_{as} + \underline{a}^2 i_{bs} + \underline{a}\,i_{cs})}$$ **conjugate**

$$= \frac{4}{9}[v_{as}i_{as} + \underline{a}(v_{bs}i_{as} + v_{as}i_{cs}) + \underline{a}^2(v_{as}i_{bs} + v_{cs}i_{as} + v_{bs}i_{cs})$$
$$+ \underline{a}^3(v_{bs}i_{bs} + v_{cs}i_{cs}) + \underline{a}^4(v_{cs}i_{bs})\;]$$
$$= \frac{4}{9}[v_{as}i_{as} + v_{bs}i_{bs} + v_{cs}i_{cs} + \underline{a}(v_{bs}i_{as} + v_{as}i_{cs} + v_{cs}i_{bs})$$
$$+ \underline{a}^2(v_{as}i_{bs} + v_{cs}i_{as} + v_{bs}i_{cs})\;]$$

$$Re\,[\underline{v}_{abcs}\underline{i}^{\dagger}_{abcs}] = \frac{4}{9}\{v_{as}i_{as} + v_{bs}i_{bs} + v_{cs}i_{cs}$$
$$-\frac{1}{2}[v_{as}(i_{bs} + i_{cs}) + v_{bs}(i_{as} + i_{cs}) + v_{cs}(i_{as} + i_{bs})]\;\}$$

Assume ia+ib+ic=0
$$\boxed{= \frac{2}{3}\{v_{as}i_{as} + v_{bs}i_{bs} + v_{cs}i_{cs}\}} \qquad (2.10\text{–}2)$$

rms, phase voltage

Real power representation in dq plane

Assume three phase machine without a neural return (ia+ib+ic=0)

$$P_e = \frac{3}{2}\{Re[\underline{v}_{abcs}\underline{i}^{\dagger}_{abcs}] + Re[\underline{v}'_{abcr}\,\underline{i}'^{\dagger}_{abcr}]\} \qquad (2.10\text{–}3)$$

$$\boxed{\underline{f}_{abcs} = e^{j\theta}\underline{f}_{qds} \quad (2.10\text{–}5)}$$

$$\boxed{\underline{f}'_{abcr} = e^{j(\theta-\theta_r)}\underline{f}'_{qdr} \quad (2.10\text{–}6)}$$

$$P_e = \frac{3}{2}Re[(e^{j\theta}\underline{v}_{qds})(e^{-j\theta}\underline{i}^{\dagger}_{qds}) + (e^{j(\theta-\theta_r)}\underline{v}'_{qdr})(e^{-j(\theta-\theta_r)}\underline{i}'^{\dagger}_{qdr})] \quad (2.10\text{–}7)$$

$$P_e = \frac{3}{2}Re[\underline{v}_{qds}\underline{i}^{\dagger}_{qds} + \underline{v}'_{qdr}\,\underline{i}'^{\dagger}_{qdr}] \qquad (2.10\text{–}8)$$

In scalar form:

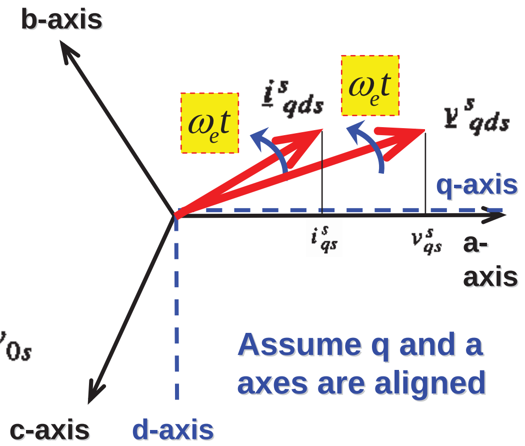
$$\boxed{P_e = \frac{3}{2}(v_{ds}i_{ds} + v_{qs}i_{qs} + v'_{dr}\,i'_{dr} + v'_{qr}\,i'_{qr})} \qquad (2.10\text{–}9)$$

2.11 Example: Stator abc and rotor dq model

$$\begin{bmatrix} v^s_{ds} \\ v^s_{qs} \\ v^s_{dr} \\ v^s_{qr} \end{bmatrix} = \begin{bmatrix} r_s + L_s p & -\omega L_s & L_m p & -\omega L_m \\ \omega L_s & r_s + L_s p & \omega L_m & L_m p \\ L_m p & (\omega-\omega_r)L_m & r_r + L_r p & -(\omega-\omega_r)L_r \\ (\omega-\omega_r)L_m & L_m p & (\omega-\omega_r)L_r & r_r + L_r p \end{bmatrix} \begin{bmatrix} i^s_{ds} \\ i^s_{qs} \\ i^s_{dr} \\ i^s_{qr} \end{bmatrix}$$

$$\begin{bmatrix} v_{as} \\ v_{bs} \\ v_{cs} \\ v^s_{qr} \\ v^s_{dr} \end{bmatrix} = \begin{bmatrix} & & & & \\ & & & & \\ & & & & \\ & & & r_r + L_r p & -(\omega-\omega_r)L_r \\ & & & (\omega-\omega_r)L_r & r_r + L_r p \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \\ i^s_{qr} \\ i^s_{dr} \end{bmatrix}$$

$$v_{qs}^{s} = \mathrm{Re}\left(\underline{v}_{qds}^{s}\right) = \mathrm{Re}\left[\frac{2}{3}\left(v_{as} + \underline{a}v_{bs} + \underline{a}^{2}v_{cs}\right)\right]$$
$$= \frac{2}{3}v_{as} - \frac{1}{3}v_{bs} - \frac{1}{3}v_{cs} = v_{as} - \frac{1}{3}\left(v_{as} + v_{bs} + v_{cs}\right)$$
$$= v_{as} - v_{0s}$$



$$v_{as} = v_{qs}^{s} + v_{0s} = (r_s + L_s p)\, i_{qs}^{s} + L_m p i_{qr}^{s} + v_{0s}$$

$$i_{qs}^{s} = i_{as} - i_{0s}$$

$$v_{as} = (r_s + L_s p)\, i_{as} + L_m p\, i_{qr}^{s} + v_{0s} - (r_s + L_s p)\, i_{0s}$$

$$v_{0s} = [r_s + (L_s - L_m)\, p]\, i_{0s} \qquad i_{0s} = \frac{1}{3}(i_{as} + i_{bs} + i_{cs})$$

$$\boxed{v_{as} = \left[r_s + \left(L_s - \frac{L_m}{3}\right)p\right] i_{as} - \frac{L_m}{3} p i_{bs} - \frac{L_m}{3} p i_{cs} + L_m p i_{qr}^{s}}$$

Stator voltage equations

$$\boxed{v_{as} = \left[r_s + \left(L_s - \frac{L_m}{3}\right)p\right] i_{as} - \frac{L_m}{3} p i_{bs} - \frac{L_m}{3} p i_{cs} + L_m p i_{qr}^{s}}$$

$$\boxed{v_{bs} = \left[r_s + \left(L_s - \frac{L_m}{3}\right)p\right] i_{bs} - \frac{L_m}{3} p i_{as} - \frac{L_m}{3} p i_{cs} - \frac{L_m}{2} p i_{qr}^{s} - \frac{\sqrt{3}}{2} L_m p i_{dr}^{s}}$$

$$\boxed{v_{cs} = \left[r_s + \left(L_s - \frac{L_m}{3}\right)p\right] i_{cs} - \frac{L_m}{3} p i_{as} - \frac{L_m}{3} p i_{bs} - \frac{L_m}{2} p i_{qr}^{s} + \frac{\sqrt{3}}{2} L_m p i_{dr}^{s}}$$

Rotor voltage equations

$$\underline{v}^{s}_{qdr} = (r_r + L_r(p - j\omega_r))\, \underline{i}^{s}_{qdr} + L_m(p - j\omega_r)\, \boxed{\underline{i}^{s}_{qds}}$$

$$\boxed{\begin{aligned} \underline{i}^{s}_{qds} &= \frac{2}{3}(i_{as} + a i_{bs} + a^2 i_{cs}) \\ &= \frac{2}{3}i_{as} - \frac{1}{3}i_{bs} - \frac{1}{3}i_{cs} + j\frac{1}{\sqrt{3}}(i_{bs} - i_{cs}) \end{aligned}}$$

$$\underline{v}^{s}_{qdr} = [r_r + L_r(p - j\omega_r)]\, \underline{i}^{s}_{qdr} + L_m(p - j\omega_r)\left[\frac{2}{3}i_{as} - \frac{1}{3}i_{bs} - \frac{1}{3}i_{cs} + j\frac{1}{\sqrt{3}}(i_{bs} - i_{cs})\right]$$

re. part $$\boxed{v^{s}_{qr} = (r_r + L_r p)\, i^{s}_{qr} - \omega_r L_r i^{s}_{dr} + L_m p\left(\frac{2}{3}i_{as} - \frac{1}{3}i_{bs} - \frac{1}{3}i_{cs}\right) + \frac{\omega_r L_m}{\sqrt{3}}(i_{bs} - i_{cs})}$$

Im. part $$\boxed{v^{s}_{dr} = (r_r + L_r p)\, i^{s}_{dr} + \omega_r L_r i^{s}_{qr} + L_m p\frac{1}{\sqrt{3}}(i_{cs} - i_{bs}) + \omega_r L_m\left(\frac{2}{3}i_{as} - \frac{1}{3}i_{bs} - \frac{1}{3}i_{cs}\right)}$$

$$v^{s}_{qr} = (r_r + L_r p)\, i^{s}_{qr} - \omega_r L_r i^{s}_{dr} + \boxed{L_m p\left(\frac{2}{3}i_{as} - \frac{1}{3}i_{bs} - \frac{1}{3}i_{cs}\right) + \frac{\omega_r L_m}{\sqrt{3}}(i_{bs} - i_{cs})}$$

$$v^{s}_{dr} = (r_r + L_r p)\, i^{s}_{dr} + \omega_r L_r i^{s}_{qr} + \boxed{L_m p\frac{1}{\sqrt{3}}(i_{cs} - i_{bs}) + \omega_r L_m\left(\frac{2}{3}i_{as} - \frac{1}{3}i_{bs} - \frac{1}{3}i_{cs}\right)}$$

$$\begin{bmatrix} v_{as} \\ v_{bs} \\ v_{cs} \\ v^{s}_{qr} \\ v^{s}_{dr} \end{bmatrix} = \begin{bmatrix} r_s + \left(L_s - \frac{1}{3}L_m\right)p & -\frac{1}{3}L_m p & -\frac{1}{3}L_m p & L_m p & 0 \\ -\frac{1}{3}L_m p & r_s + \left(L_s - \frac{1}{3}L_m\right)p & -\frac{1}{3}L_m p & -\frac{1}{2}L_m p & -\frac{\sqrt{3}}{2}L_m p \\ -\frac{1}{3}L_m p & -\frac{1}{3}L_m p & r_s + \left(L_s - \frac{1}{3}L_m\right)p & -\frac{1}{2}L_m p & \frac{\sqrt{3}}{2}L_m p \\ \frac{2}{3}L_m p & -\frac{L_m}{3}(p - \sqrt{3}\omega_r) & -\frac{L_m}{3}(p + \sqrt{3}\omega_r) & r_r + L_r p & -\omega_r L_r \\ \frac{2}{3}L_m \omega_r & -\frac{L_m}{3}(\omega_r + \sqrt{3}p) & -\frac{L_m}{3}(\omega_r - \sqrt{3}p) & \omega_r L_r & r_r + L_r p \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \\ i^{s}_{qr} \\ i^{s}_{dr} \end{bmatrix} \quad (2.11\text{–}1)$$

$$L_{0s} = L_s - L_m = L_{ls}, \text{ and } L_m = \frac{3}{2}L_{ms}$$

$$L_s - \frac{1}{3}L_m = \frac{2}{3}L_m + L_{ls} = L_{ms} + L_{ls}$$

2.12 The Electromagnetic Torque

$$P_e = \frac{3}{2}Re[\underline{v}_{qds}\underline{i}^{\dagger}_{qds} + \underline{v}'_{qdr}\,\underline{i}'^{\dagger}_{qdr}] \qquad (2.10\text{–}8)$$

$$\begin{aligned} P_e = \frac{3}{2}Re\Big\{ & [r_s\,\underline{i}_{qds} + \boxed{(L_{ls}+L_m)\,p\underline{i}_{qds}} + L_m\,p\underline{i}'_{qdr} \\ & + j\omega[(L_{ls}+L_m)\,\underline{i}_{qds} + L_m\underline{i}'_{qdr}]\,]\,\boxed{\underline{i}^{\dagger}_{qds}}\Big\} \\ + \frac{3}{2}Re\Big\{ & [r'_r\,\underline{i}'_{qdr} + (L'_{lr}+L_m)\,p\underline{i}'_{qdr} + L_m\,p\underline{i}_{qds} \\ & + j(\omega-\omega_r)[(L'_{lr}+L_m)\underline{i}'_{qdr} + L_m\underline{i}_{qds}]\,]\;\underline{i}'^{\dagger}_{qdr}\Big\} \end{aligned} \qquad (2.12\text{–}1)$$

$$\begin{aligned} &= \mathrm{Re}\left[(L_{ls}+L_m)\,\underline{i}^{\dagger}_{qds}\cdot p\,\underline{i}_{qds}\right] \\ &= \mathrm{Re}\left[(L_{ls}+L_m)\cdot(i_{qs}+ji_{ds})\cdot(pi_{qs}-jpi_{ds})\right] \\ &= \mathrm{Re}\left[(L_{ls}+L_m)\cdot(i_{qs}\cdot pi_{qs}+i_{ds}\cdot pi_{ds})\right] \\ &= \tfrac{1}{2}\mathrm{Re}\left[(L_{ls}+L_m)\cdot p\left(i_{qs}^2+i_{ds}^2\right)\right] \\ &= \tfrac{1}{2}\mathrm{Re}\left[(L_{ls}+L_m)\cdot p\left(\underline{i}_{qds}\right)^2\right] \end{aligned}$$

$$P_e = \overbrace{\frac{3}{2}r_s|\underline{i}_{qds}|^2 + \frac{3}{2}r'_r|\underline{i}'_{qdr}|^2}^{\text{Power lost in Conductors}} + \overbrace{\frac{3}{2}p\left[\boxed{\frac{L_{ls}}{2}|\underline{i}_{qds}|^2} + \frac{L'_{lr}}{2}|\underline{i}'_{qdr}|^2 + L_m|\underline{i}_{qds}+\underline{i}'_{qdr}|^2\right]}^{\text{Time Rate of Change of Stored Energy}}$$

$$\left.\begin{aligned} &+\frac{3}{2}Re\{j\,\omega[(L_{ls}+L_m)|\underline{i}_{qds}|^2 + L_m\underline{i}'_{qdr}\,\underline{i}^{\dagger}_{qds}] \\ &+ j(\omega-\omega_r)\,[(L_{lr}'+L_m)|\underline{i}'_{qdr}|] + L_m\underline{i}_{qds}\underline{i}'^{\dagger}_{qdr}\;\} \end{aligned}\right\}\ \text{Energy Conversion Term}$$

Power transfer to mechanical load

$$P_e = \overbrace{\frac{3}{2}r_s|\underline{i}_{qds}|^2 + \frac{3}{2}r'_r|\underline{i}'_{qdr}|^2}^{\text{Power lost in Conductors}} + \overbrace{\frac{3}{2}p\left[\frac{L_{ls}}{2}|\underline{i}_{qds}|^2 + \frac{L'_{lr}}{2}|\underline{i}'_{qdr}|^2 + L_m|\underline{i}_{qds}+\underline{i}'_{qdr}|^2\right]}^{\text{Time Rate of Change of Stored Energy}}$$

$$\left.\begin{aligned} &+\frac{3}{2}Re\{j\,\omega[(L_{ls}+L_m)|\underline{i}_{qds}|^2 + L_m\underline{i}'_{qdr}\,\underline{i}^{\dagger}_{qds}] \\ &+ j(\omega-\omega_r)\,[(L_{lr}'+L_m)|\underline{i}'_{qdr}|] + L_m\underline{i}_{qds}\underline{i}'^{\dagger}_{qdr}\;\} \end{aligned}\right\}\ \text{Energy Conversion Term}$$

$$P_{em} = \frac{3}{2}Re\{j\omega L_m\,\underline{i}'_{qdr}\,\underline{i}^{\dagger}_{qds} + j(\omega-\omega_r)\,L_m\,\underline{i}_{qds}\,\underline{i}'^{\dagger}_{qdr}\} \qquad (2.12\text{–}2)$$

$$= \frac{3}{2}Re\{j\omega L_m\underbrace{(\underline{i}'_{qdr}\,\underline{i}^{\dagger}_{qds} + \underline{i}_{qds}\,\underline{i}'^{\dagger}_{qdr})}_{\underline{a}\underline{b}^{\dagger}+\underline{a}^{\dagger}\underline{b} = 2(a_r b_r + a_i b_i)} - j\omega_r L_m\,\underline{i}_{qds}\,\underline{i}'^{\dagger}_{qdr}\}$$

…pure real number

$$= -\frac{3}{2}Re\{j\omega_r L_m\,\underline{i}_{qds}\,\underline{i}'^{\dagger}_{qdr}\} \qquad (2.12\text{–}3)$$

Electromagnetic torque

$$P_{em} = -\frac{3}{2} Re\{ j\omega_r L_m \, i_{qds} \, i'^{\dagger}_{qdr} \} \qquad (2.12\text{–}3)$$

$$P_{em} = \frac{3}{2} Im\{ \omega_r L_m \, i_{qds} \, i'^{\dagger}_{qdr} \} \qquad (2.12\text{–}4)$$

$$P_{em} = \frac{3}{2} \omega_r L_m (i_{qs} i'_{dr} - i_{ds} i'_{qr}) \qquad (2.12\text{–}5)$$

$$\omega_r = \frac{P}{2} \omega_{rm}$$

Type-I

$$\boxed{\begin{aligned} T_e &= \frac{3}{2}\frac{P}{2} L_m Im\{ i_{qds} \, i'^{\dagger}_{qdr} \} \\ &= \frac{3}{2}\frac{P}{2} L_m (i_{qs} i'_{dr} - i_{ds} i'_{qr}) \end{aligned}} \qquad (2.12\text{–}7)$$

Electromagnetic torque

Type-I

$$\boxed{T_e = \frac{3}{2}\frac{P}{2} L_m Im\{ i_{qds} \, \boxed{i'^{\dagger}_{qdr}} \}} \qquad (2.12\text{–}7)$$

$$\lambda_{qds} = (L_{ls} + L_m) \, i_{qds} + L_m \boxed{i'_{qdr}} \qquad (2.12\text{–}9)$$

$$T_e = \frac{3}{2}\frac{P}{2} Im[-\cancel{(L_{ls} + L_m)} \, i_{qds} \, i^{\dagger}_{qds} + i_{qds} \boxed{\lambda^{\dagger}_{qds}}] \qquad (2.12\text{–}10)$$

Type-II

$$\boxed{\begin{aligned} T_e &= \frac{3}{2}\frac{P}{2} Im(i_{qds} \cdot \lambda^{\dagger}_{qds}) && (2.12\text{–}11) \\ &= \frac{3}{2}\frac{P}{2} (\lambda_{ds} i_{qs} - \lambda_{qs} i_{ds}) && (2.12\text{–}12) \end{aligned}}$$

Electromagnetic torque

Type-I

$$T_e = \frac{3}{2}\frac{P}{2} L_m Im\{\boxed{i_{qds}}\, i'^{\dagger}_{qdr}\} \quad (2.12\text{–}7)$$

$$\lambda'_{qdr} = (L'_{lr} + L_m)\, i'_{qdr} + L_m \boxed{i_{qds}}$$

Type-III

$$\boxed{T_e = \frac{3}{2}\frac{P}{2} Im(\lambda'_{qdr}\, i'^{\dagger}_{qdr})} \quad (2.12\text{–}14)$$

Electromagnetic torque

Type-I

$$T_e = \frac{3}{2}\frac{P}{2} L_m Im\{i_{qds}\, \boxed{i'^{\dagger}_{qdr}}\} \quad (2.12\text{–}7)$$

$$\lambda_{qdm} = L_m (i_{qds} + \boxed{i'_{qdr}}) \quad (2.12\text{–}15)$$

Type-IV

$$\boxed{T_e = \frac{3}{2}\frac{P}{2} Im(i_{qds}\, \lambda^{\dagger}_{qdm})} \quad (2.12\text{–}16)$$

Electromagnetic torque

Type-I

$$T_e = \frac{3}{2}\frac{P}{2} L_m Im\{ i_{qds} \, i'^{\dagger}_{qdr} \} \quad (2.12\text{–}7)$$

$$\begin{cases} \lambda'_{dr} = L'_{lr} i'_{dr} + L_m (i_{ds} + i'_{dr}) & (2.8\text{–}23) \\ \lambda'_{qr} = L'_{lr} i'_{qr} + L_m (i_{qs} + i'_{qr}) & (2.8\text{–}24) \end{cases}$$

$$i'_{qdr} = \frac{1}{L'_{lr} + L_m} \lambda'_{qdr} - \frac{L_m}{L'_{lr} + L_m} i_{qds} \quad (2.12\text{–}17)$$

Type-V

$$T_e = \frac{3}{2}\frac{P}{2}\frac{L_m}{L'_r} Im(i_{qds} \lambda'^{\dagger}_{qdr}) \quad (2.12\text{–}18)$$

where $L'_r = L'_{lr} + L_m$

$$\lambda_{qds} = (L_{ls} + L_m) i_{qds} + L_m i'_{qdr} \quad (2.12\text{–}9)$$

$$\lambda_{qdm} = L_m (i_{qds} + i'_{qdr}) \quad (2.12\text{–}15)$$

$$\lambda'_{qdr} = (L'_{lr} + L_m) i'_{qdr} + L_m i_{qds}$$

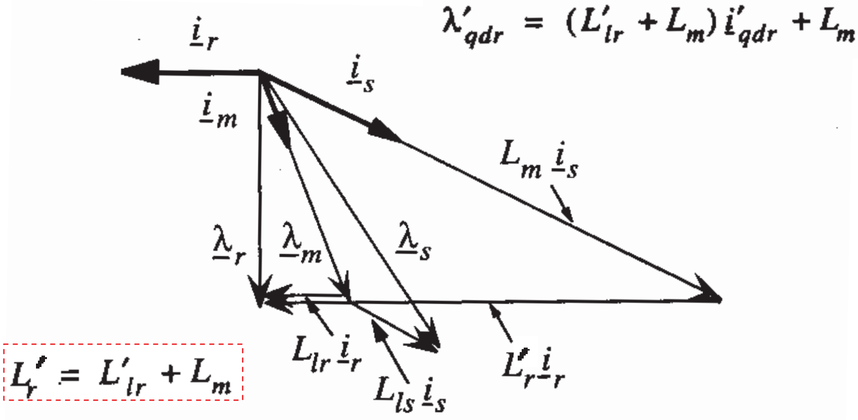


Figure 2.12 Illustrating torque production by vectors on the d,q plane

TABLE 2.1 Complex Vector Expressions for Electromagnetic Torque. ‡ denotes variable used to calculate torque

	i_{qds}	i'_{qdr}	λ_{qds}	λ'_{qdr}	λ_{qdm}	Torque Expression
1	‡	‡				$T_e = \frac{3}{2}\frac{P}{2} L_m Im\{i'^{\dagger}_{qdr} i_{qds}\}$
2	‡				‡	$T_e = \frac{3}{2}\frac{P}{2} Im\{\lambda^{\dagger}_{qdm} i_{qds}\}$
3	‡		‡			$T_e = \frac{3}{2}\frac{P}{2} Im\{\lambda^{\dagger}_{qds} i_{qds}\}$
4	‡			‡		$T_e = \frac{3}{2}\frac{P}{2}\frac{L_m}{L'_r} Im\{i_{qds} \lambda'^{\dagger}_{qdr}\}$
5		‡			‡	$T_e = \frac{3}{2}\frac{P}{2} Im\{i'^{\dagger}_{qdr} \lambda_{qdm}\}$
6		‡		‡		$T_e = \frac{3}{2}\frac{P}{2} Im\{i'^{\dagger}_{qdr} \lambda'_{qdr}\}$
7		‡	‡			$T_e = \frac{3}{2}\frac{P}{2}\frac{L_m}{L_s} Im\{i'^{\dagger}_{qdr} \lambda_{qds}\}$
8			‡	‡		$T_e = \frac{3}{2}\frac{P}{2}\frac{L_m}{\sigma L_s L'_r} Im\{\lambda'^{\dagger}_{qdr} \lambda_{qds}\}$

$\sigma = 1 - L_m^2 / L'_r L_s$

Electromagnetic torque

$$T_e = J\frac{d\omega_{rm}}{dt} + T_l \qquad (2.12\text{–}20)$$

$$T_e = \frac{2J}{P}\frac{d\omega_r}{dt} + T_l \qquad (2.12\text{–}21)$$

where $\omega_r = 2\omega_{rm}/P$.

2.13 Analysis of IM Starting Performance Using d,q,0 Variables

- System parameters

$r_s = 0.531\ \Omega$ $\quad r_r' = 0.408\ \Omega$ $\quad J = 0.1$ kg–m^2

$L_{ls} = L_{lr}' = 2.52$ mH $\quad L_m = 84.7$ mH

- Source voltages

$$v_{as} = \sqrt{\frac{2}{3}}230\cos(377t) \tag{2.13–1}$$

$$v_{bs} = \sqrt{\frac{2}{3}}230\cos(377t - 2\pi/3) \tag{2.13–2}$$

$$v_{cs} = \sqrt{\frac{2}{3}}230\cos(377t + 2\pi/3) \tag{2.13–3}$$

Moter is modelled in abc axes

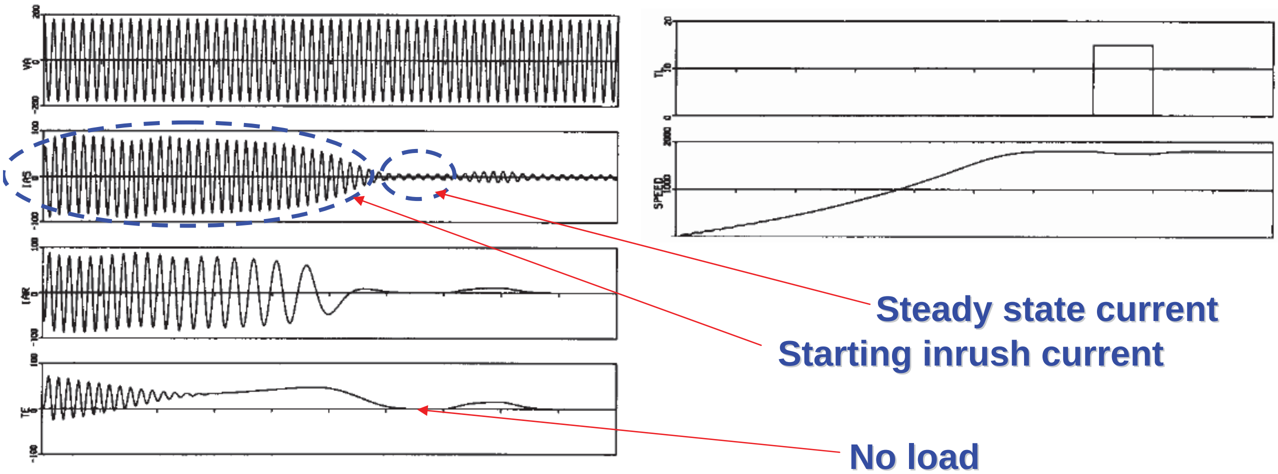


Figure 2.13 Starting performance of 220 V, 5 hp squirrel cage induction motor with a balanced sinusoidal supply showing physical variables. After reaching rated speed the motor is loaded to 0.83 times rated torque. Traces from top to bottom: v_{as}, phase a line to neutral voltage, i_{as}, line current of stator phase as, i'_{ar}, phase current of rotor phase ar (referred to stator turns), T_e, electromagnetic torque, T_l, load torque, rotor speed in RPM; time axis 0.1 s/div.

Motor is modelled in stationary reference frame

$$v_{ds}^s = -\frac{2}{3}Im\,[v_{as} + \underline{a}v_{bs} + \underline{a}^2 v_{cs}] \qquad v_{0s} = 0 \qquad (2.13\text{–}6)$$

$$= -\frac{2}{3}\frac{\sqrt{3}}{2}(v_{bs} - v_{cs})$$

$$= \sqrt{\frac{2}{3}}\cdot\frac{\sqrt{3}}{2}\left[\sqrt{\frac{2}{3}}220\cos(377t + 2\pi/3) - \sqrt{\frac{2}{3}}220\cos(377t - 2\pi/3)\right]$$

$$= -\sqrt{\frac{2}{3}}230\sin(377t) \qquad (2.13\text{–}4)$$

$$v_{qs}^s = \frac{2}{3}Re\,[v_{as} + \underline{a}v_{bs} + \underline{a}^2 v_{cs}]$$

$$= \frac{2}{3}\left[\sqrt{\frac{2}{3}}220\cos(377t) - \frac{1}{2}\sqrt{\frac{2}{3}}220\cos(377t - 2\pi/3)\right.$$

$$\left. - \frac{1}{2}\sqrt{\frac{2}{3}}\cos(377t + 2\pi/3)\right]$$

$$= \sqrt{\frac{2}{3}}220\cos(377t) \qquad (2.13\text{–}5)$$

Motor is modelled in stationary reference frame

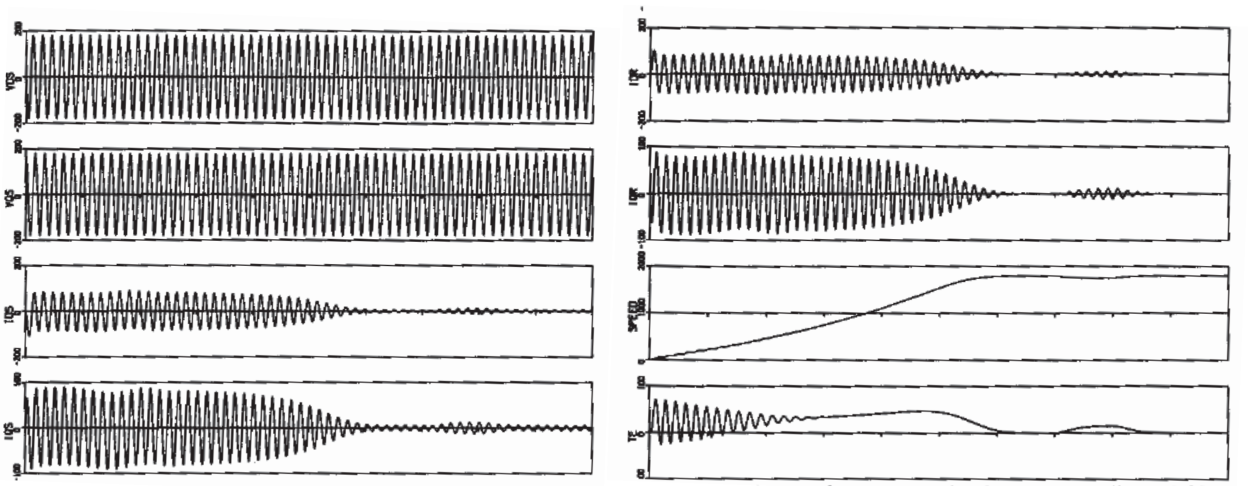


Figure 2.14 Starting performance and subsequent loading of 220 V, 5 hp squirrel cage induction motor with a balanced 60 Hz sinusoidal supply. Motor is modelled in stationary d,q axes. (Stationary reference frame). Traces from top to bottom – v_{ds}^s, v_{qs}^s, i_{ds}^s, i_{qs}^s, i'^s_{dr}, i'^s_{qr}, rotor speed in RPM, T_e.; time axis 0.1 s/div.

Motor is modelled in rotor reference frame

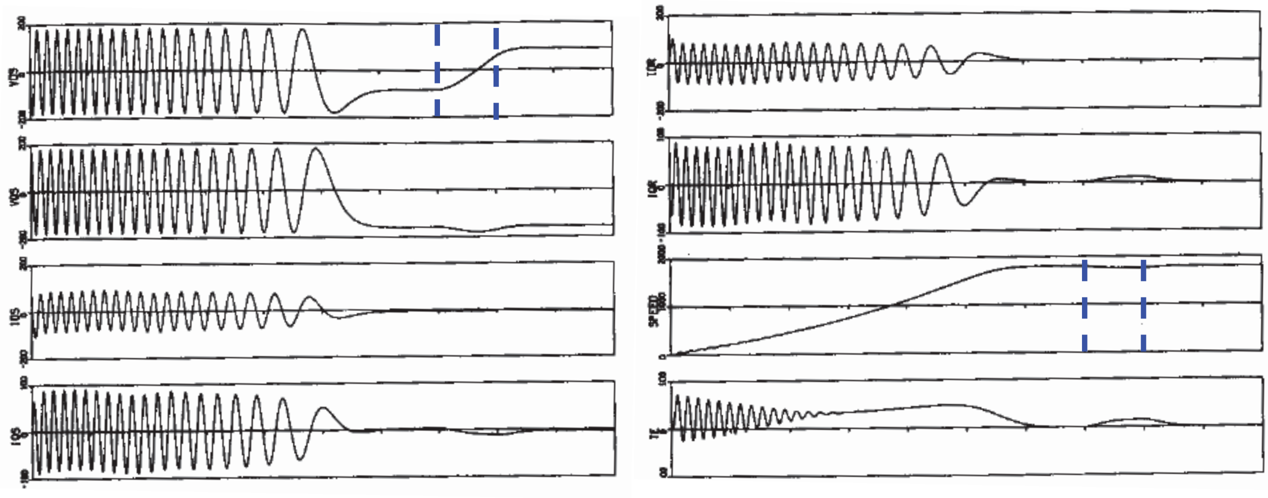


Figure 2.15 Starting performance and subsequent loading of 220 V, 5 hp squirrel cage induction motor with a balanced sinusoidal supply. Motor is modelled in d,q axes rotating at rotor speed. (Rotor reference frame). Traces from top to bottom – v_{ds}^r, v_{qs}^r, i_{ds}^r, i_{qs}^r, $i_{dr}'^r$, $i_{qr}'^r$, rotor speed in RPM, torque T_e.; time axis 0.1 s/div.

Motor is modelled in synchronous reference frame

$$\omega = 377 \quad rad/s \qquad (2.13\text{–}10)$$

$$\theta(0) = 0 \qquad (2.13\text{–}11)$$

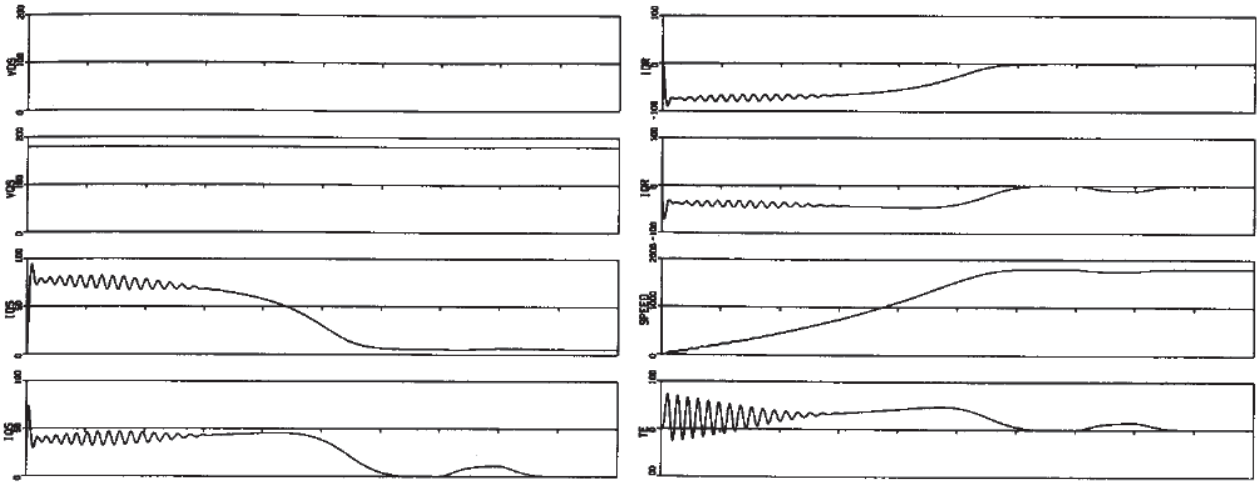
$$v_{ds}^{sv} = 0 \qquad (2.13\text{–}12)$$

$$v_{qs}^{sv} = \sqrt{\frac{2}{3}}\,220\ V \qquad (2.13\text{–}13)$$

$$v_{0s}^{sv} = 0 \qquad (2.13\text{–}14)$$

Stator voltage (synch.) reference frame

Motor is modelled in synchronous reference frame



2.16 Starting performance and subsequent loading of 5 hp, 220 V squirrel cage induction motor with a balanced sinusoidal supply. Motor is modelled in d,q axes synchronously rotating with the applied voltage vector. (Synchronous reference frame). Traces from the top: v^{sv}_{ds}, v^{sv}_{qs}, i^{sv}_{ds}, i^{sv}_{qs}, i'^{sv}_{dr}, i'^{sv}_{qr}, rotor speed in RPM, electromagnetic torque T_e; time axis 0.1 s/div.

Motor is modelled in rotor flux reference frame

$$v^{rf}_{ds} = -\sqrt{\frac{2}{3}}220\sin(377t - \theta_{rf}) \qquad (2.13\text{–}15)$$

$$v^{rf}_{qs} = \sqrt{\frac{2}{3}}220\cos(377t - \theta_{rf}) \qquad (2.13\text{–}16)$$

$$v^{rf}_{0s} = 0 \qquad (2.13\text{–}17)$$

θ_{rf} is the instantaneous position of the rotor flux vector

- d-axis is continuously aligned with the rotor flux vector so that the q-axis rotor flux component is always identically zero.

$$T_e = \frac{3}{2}\frac{P}{2}\frac{L_m}{L'_r}Im(i_{qds}\lambda'^{\dagger}_{qdr}) \quad (2.12\text{–}18) \Rightarrow T_e = \frac{3}{2}\frac{P}{2}\frac{L_m}{L_r}\lambda^{rf}_{dr} i^{rf}_{qs} \quad (2.13\text{–}18)$$

Motor is modelled in rotor flux reference frame

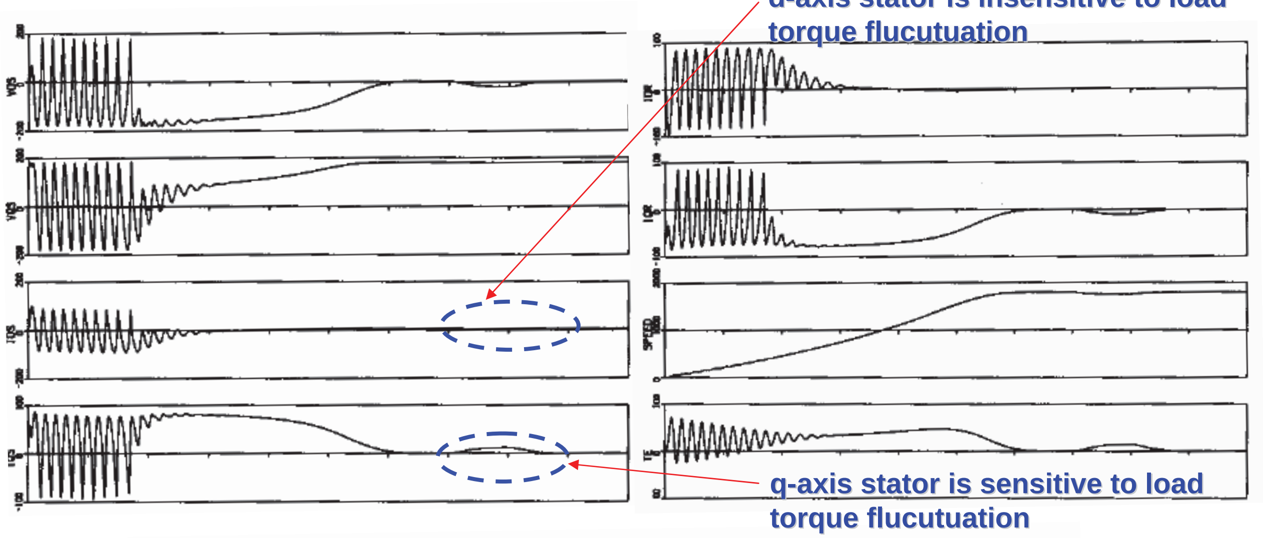


Figure 2.17 Starting performance and subsequent loading of 220 V, 5 hp squirrel cage induction motor with a balanced sinusoidal supply. Motor is modelled in d,q axes rotating synchronously with the rotor flux. (Rotor flux reference frame). Traces from the top: v_{ds}^{rf}, v_{qs}^{rf}, i_{ds}^{rf}, i_{qs}^{rf}, $i_{dr}^{\prime rf}$, $i_{qr}^{\prime rf}$, rotor speed in RPM, electromagnetic torque T_e; time axis 0.1 s/div.

2.14 Extension of d,q,0 Theory to Analysis of Salient Pole Synchronous Machines

- Salient pole synchronous machines
  - Non-uniform air gap between stator and rotor
    - Stator self inductances vary with rotor position

➢ **Self inductance of any winding must pulsate once each time the rotor moves one pole pitch**

➢ **There exists a second harmonic component in addition to the constant component represented by (2.4-5)**

- **Self inductance of phase a is**

$$L_{as,as} = L_{ls} + L_{0s} - L_{2s}\cos 2\theta_r \quad (2.14\text{–}1)$$

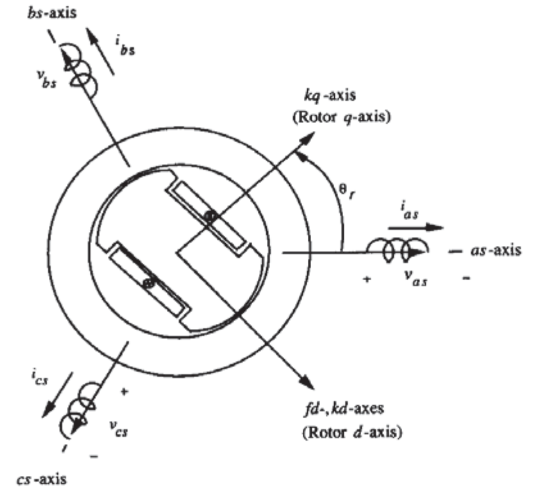


Figure 2.18 Magnetic axes of a salient pole synchronous machine

- Self inductance of phase a is

$$L_{as,as} = L_{ls} + L_{0s} - L_{2s}\cos 2\theta_r \qquad (2.14\text{–}1)$$

$$L_{0s} = \mu_0 r l N_s^2 \left(\frac{\pi}{8}\right)\left(\frac{1}{g_{min}} + \frac{1}{g_{max}}\right) \qquad (2.14\text{–}2)$$

$$L_{2s} = \mu_0 r l N_s^2 \left(\frac{\pi}{8}\right)\left(\frac{1}{g_{min}} - \frac{1}{g_{max}}\right) \qquad (2.14\text{–}3)$$

- Self inductance of phase b and c is

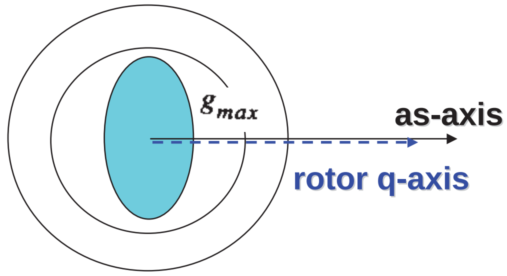
$$L_{bs,bs} = L_{ls} + L_{0s} - L_{2s}\cos(2\theta_r + 2\pi/3)$$

$$L_{cs,cs} = L_{ls} + L_{0s} - L_{2s}\cos(2\theta_r - 2\pi/3)$$

$\theta_r = 0^\circ$

smallest

$L_{as,as}$

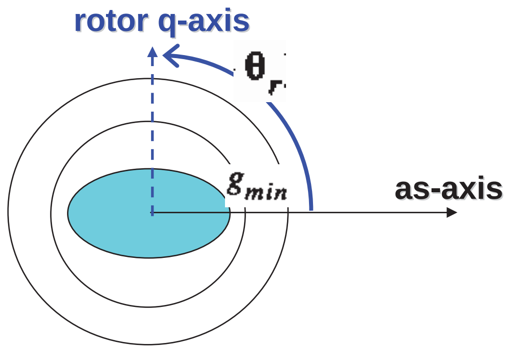


$$L_{as,as} = L_{ls} + \mu_0 r l N_s^2 \left(\frac{\pi}{8}\right)\frac{2}{g_{max}}$$

$\theta_r = 90^\circ$

largest

$L_{as,as}$



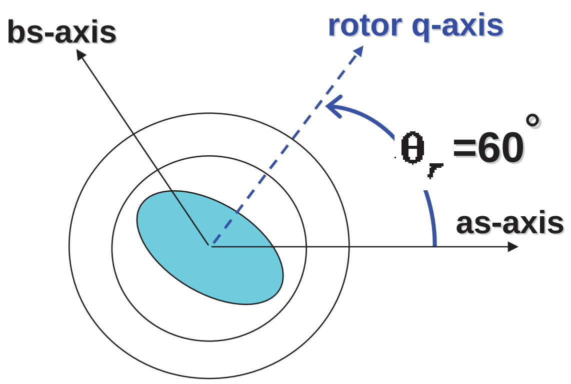
$$L_{as,as} = L_{ls} + \mu_0 r l N_s^2 \left(\frac{\pi}{8}\right)\frac{2}{g_{min}}$$

Mutual inductances between stator phases

$$L_{as,bs} = L_{bs,as} = -\frac{1}{2}L_{0s} - L_{2s}\cos(2\theta_r - 2\pi/3) \qquad (2.14\text{–}6)$$

$$L_{as,cs} = L_{cs,as} = -\frac{1}{2}L_{0s} - L_{2s}\cos(2\theta_r + 2\pi/3) \qquad (2.14\text{–}7)$$

$$L_{bs,cs} = L_{cs,bs} = -\frac{1}{2}L_{0s} - L_{2s}\cos 2\theta_r \qquad (2.14\text{–}8)$$



largest $L_{as,bs}$

Mutual inductances between field winding and stator phases

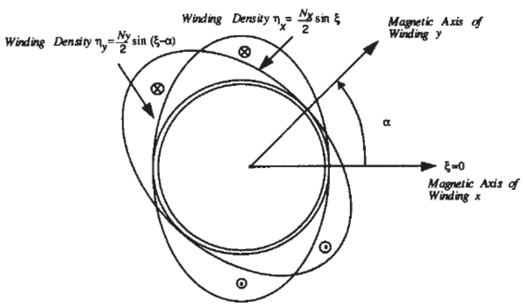
$$L_{as,fd} = L_{fd,as} = L_{sfd}\cos\theta_r \qquad (2.14\text{–}9)$$

$$L_{bs,fd} = L_{fd,bs} = L_{sfd}\cos(\theta_r - 2\pi/3) \qquad (2.14\text{–}10)$$

$$L_{cs,fd} = L_{fd,cs} = L_{sfd}\cos(\theta_r + 2\pi/3) \qquad (2.14\text{–}11)$$

where

$$L_{sfd} = \mu_0 r l N_s N_{fd}\left(\frac{\pi}{4}\right)\frac{1}{g_{min}} \qquad (2.14\text{–}12)$$



$$\boxed{L_{xy} = \frac{\lambda_{xy}}{I_x} = \mu_o N_x N_y\left(\frac{rl}{g}\right)\left(\frac{\pi}{4}\right)\cos\alpha \qquad (2.2\text{–}24)}$$

Mutual inductances between d-axis damper winding and stator phases

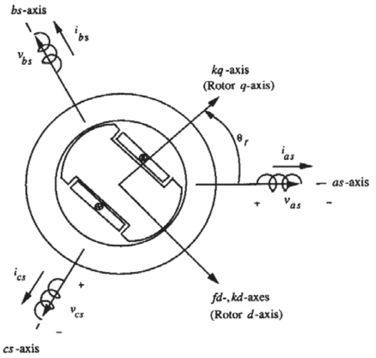
$$L_{as,kd} = L_{kd,as} = L_{skd}\cos\theta_r \qquad (2.14\text{–}13)$$

$$L_{bs,kd} = L_{kd,bs} = L_{skd}\cos(\theta_r - 2\pi/3) \qquad (2.14\text{–}14)$$

$$L_{cs,kd} = L_{kd,cs} = L_{skd}\cos(\theta_r + 2\pi/3) \qquad (2.14\text{–}15)$$

where

$$L_{skd} = \mu_0 r l N_s N_{kd}\left(\frac{\pi}{4}\right)\frac{1}{g_{min}} \qquad (2.14\text{–}16)$$



Mutual inductances between q-axis damper winding and stator phases

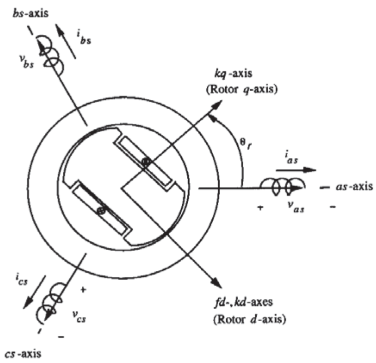
$$L_{as,kq} = L_{kq,as} = -L_{skq}\boxed{\sin\theta_r} \tag{2.14–17}$$

$$L_{bs,kq} = L_{kq,bs} = -L_{skq}\sin(\theta_r - 2\pi/3) \tag{2.14–18}$$

$$L_{cs,kq} = L_{kq,cs} = -L_{skq}\sin(\theta_r + 2\pi/3) \tag{2.14–19}$$

where

$$L_{skq} = \mu_0 r l N_s N_{kq}\left(\frac{\pi}{4}\right)\frac{1}{\boxed{g_{max}}} \tag{2.14–20}$$



Stator voltage equations

$$\begin{bmatrix} v_{as} \\ v_{bs} \\ v_{cs} \end{bmatrix} = r_s \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix} + p\boxed{\begin{bmatrix} \lambda_{as} \\ \lambda_{bs} \\ \lambda_{cs} \end{bmatrix}} \tag{2.14–21}$$

$$\boxed{\begin{bmatrix} \lambda_{as} \\ \lambda_{bs} \\ \lambda_{cs} \end{bmatrix} = \begin{bmatrix} \quad & \quad & \quad \\ & & \\ & & \end{bmatrix}\begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix} + \begin{bmatrix} \quad & \quad & \quad \\ & & \\ & & \end{bmatrix}\cdot\begin{bmatrix} i_{fd} \\ i_{kd} \\ i_{kq} \end{bmatrix}} \tag{2.14–22}$$

Voltage equation for field, d-axis damper and q-axis damper windings

$$v_{fd} = r_{fd} i_{fd} + p\lambda_{fd} \qquad (2.14\text{–}23)$$

$$v_{kd} = r_{kd} i_{kd} + p\lambda_{kd} \qquad (2.14\text{–}24)$$

$$v_{kq} = r_{kq} i_{kq} + p\lambda_{kq} \qquad (2.14\text{–}25)$$

$$\begin{bmatrix}\lambda_{fd}\\ \lambda_{kd}\\ \lambda_{kq}\end{bmatrix} = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}\begin{bmatrix}i_{as}\\ i_{bs}\\ i_{cs}\end{bmatrix} + \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}\cdot\begin{bmatrix}i_{fd}\\ i_{kd}\\ i_{kq}\end{bmatrix}$$

Voltage equation for field, d-axis damper and q-axis damper windings

$$\begin{bmatrix}\lambda_{fd}\\ \lambda_{kd}\\ \lambda_{kq}\end{bmatrix} = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}\begin{bmatrix}i_{as}\\ i_{bs}\\ i_{cs}\end{bmatrix} + \begin{bmatrix} & & 0\\ & & 0\\ 0 & 0 & X\end{bmatrix}\cdot\begin{bmatrix}i_{fd}\\ i_{kd}\\ i_{kq}\end{bmatrix}$$

$$\lambda_{fd} = (L_{lfd} + \boxed{L_{mfd}}) i_{fd} + \boxed{L_{fkd}} i_{kd} + L_{sfd}[i_{as}\cos\theta_r + i_{bs}\cos(\theta_r - 2\pi/3) + i_{cs}\cos(\theta_r + 2\pi/3)] \qquad (2.14\text{–}26)$$

$$\lambda_{kd} = (L_{lkd} + \boxed{L_{mkd}}) i_{kd} + \boxed{L_{fkd}} i_{fd} + L_{skd}[i_{as}\cos\theta_r + i_{bs}\cos(\theta_r - 2\pi/3) + i_{cs}\cos(\theta_r + 2\pi/3)] \qquad (2.14\text{–}27)$$

$$\lambda_{kq} = (L_{lkq} + \boxed{L_{mkq}}) i_{kq} - L_{skq}[i_{as}\sin\theta_r + i_{bs}\sin(\theta_r - 2\pi/3) + i_{cs}\sin(\theta_r + 2\pi/3)] \qquad (2.14\text{–}28)$$

Voltage equation for field, d-axis damper and q-axis damper windings

$$L_{mfd} = \mu_0 r l N_f^2 \left(\frac{\pi}{4}\right)\frac{1}{g_{min}} \qquad (2.14\text{–}29)$$

$$L_{kfd} = \mu_0 r l N_f N_{kd} \left(\frac{\pi}{4}\right)\frac{1}{g_{min}} \qquad (2.14\text{–}30)$$

$$L_{mkd} = \mu_0 r l N_{kd}^2 \left(\frac{\pi}{4}\right)\frac{1}{g_{min}} \qquad (2.14\text{–}31)$$

$$L_{mkq} = \mu_0 r l N_{kq}^2 \left(\frac{\pi}{4}\right)\frac{1}{g_{max}} \qquad (2.14\text{–}32)$$

Stator voltage in space vector form

$$\begin{bmatrix} v_{as} \\ v_{bs} \\ v_{cs} \end{bmatrix} = r_s \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix} + p \begin{bmatrix} \lambda_{as} \\ \lambda_{bs} \\ \lambda_{cs} \end{bmatrix} \Rightarrow \begin{bmatrix} \lambda_{as} \\ \lambda_{bs} \\ \lambda_{cs} \end{bmatrix} = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix} + \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \cdot \begin{bmatrix} i_{fd} \\ i_{kd} \\ i_{kq} \end{bmatrix}$$

$$\begin{bmatrix} L_{ls}+L_{0s} & -\frac{1}{2}L_{0s} & -\frac{1}{2}L_{0s} \\ -\frac{1}{2}L_{0s} & L_{ls}+L_{0s} & -\frac{1}{2}L_{0s} \\ -\frac{1}{2}L_{0s} & -\frac{1}{2}L_{0s} & L_{ls}+L_{0s} \end{bmatrix} - \begin{bmatrix} L_{2s}\cos 2\theta_r & L_{2s}\cos(2\theta_r - 2\pi/3) & L_{2s}\cos(2\theta_r + 2\pi/3) \\ L_{2s}\cos(2\theta_r - 2\pi/3) & L_{2s}\cos(2\theta_r + 2\pi/3) & L_{2s}\cos 2\theta_r \\ L_{2s}\cos(2\theta_r + 2\pi/3) & L_{2s}\cos 2\theta_r & L_{2s}\cos(2\theta_r - 2\pi/3) \end{bmatrix}$$

$$+ \begin{bmatrix} L_{sfd}\cos\theta_r & L_{skd}\cos\theta_r & -L_{skq}\sin\theta_r \\ L_{sfd}\cos(\theta_r - 2\pi/3) & L_{skd}\cos(\theta_r - 2\pi/3) & -L_{skq}\sin(\theta_r - 2\pi/3) \\ L_{sfd}\cos(\theta_r + 2\pi/3) & L_{skd}\cos(\theta_r + 2\pi/3) & -L_{skq}\sin(\theta_r + 2\pi/3) \end{bmatrix}$$

Stator voltage in space vector form

$$
\begin{bmatrix} \lambda_{as} \\ \lambda_{bs} \\ \lambda_{cs} \end{bmatrix} = \begin{bmatrix} L_{ls}+L_{0s} & -\frac{1}{2}L_{0s} & -\frac{1}{2}L_{0s} \\ -\frac{1}{2}L_{0s} & L_{ls}+L_{0s} & -\frac{1}{2}L_{0s} \\ -\frac{1}{2}L_{0s} & -\frac{1}{2}L_{0s} & L_{ls}+L_{0s} \end{bmatrix} \cdot \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix}
$$

$$
-\frac{L_{2s}}{2}\left\{ \begin{bmatrix} e^{2j\theta_r} & \underline{a}^2 e^{2j\theta_r} & \underline{a} e^{2j\theta_r} \\ \underline{a}^2 e^{2j\theta_r} & \underline{a} e^{2j\theta_r} & e^{2j\theta_r} \\ \underline{a} e^{2j\theta_r} & e^{2j\theta_r} & \underline{a}^2 e^{2j\theta_r} \end{bmatrix} + \begin{bmatrix} e^{-2j\theta_r} & \underline{a} e^{-2j\theta_r} & \underline{a}^2 e^{-2j\theta_r} \\ \underline{a} e^{-2j\theta_r} & \underline{a}^2 e^{-2j\theta_r} & e^{-2j\theta_r} \\ \underline{a}^2 e^{-2j\theta_r} & e^{-2j\theta_r} & \underline{a} e^{-2j\theta_r} \end{bmatrix} \right\} \cdot \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix}
$$

$$
+\frac{L_{sfd}}{2}\left\{ \begin{bmatrix} e^{j\theta_r} \\ \underline{a}^2 e^{j\theta_r} \\ \underline{a} e^{j\theta_r} \end{bmatrix} + \begin{bmatrix} e^{-j\theta_r} \\ \underline{a} e^{-j\theta_r} \\ \underline{a}^2 e^{-j\theta_r} \end{bmatrix} \right\} i_{fd} + \frac{L_{skd}}{2}\left\{ \begin{bmatrix} e^{j\theta_r} \\ \underline{a}^2 e^{j\theta_r} \\ \underline{a} e^{j\theta_r} \end{bmatrix} + \begin{bmatrix} e^{-j\theta_r} \\ \underline{a} e^{-j\theta_r} \\ \underline{a}^2 e^{-j\theta_r} \end{bmatrix} \right\} i_{kd}
$$

$$
-\frac{L_{skq}}{2j}\left\{ \begin{bmatrix} e^{j\theta_r} \\ \underline{a}^2 e^{j\theta_r} \\ \underline{a} e^{j\theta_r} \end{bmatrix} - \begin{bmatrix} e^{-j\theta_r} \\ \underline{a} e^{-j\theta_r} \\ \underline{a}^2 e^{-j\theta_r} \end{bmatrix} \right\} i_{kq} \qquad (2.14\text{–}34)
$$

Stator voltage in space vector form

$$
\lambda_{as} + \underline{a}\lambda_{bs} + \underline{a}^2\lambda_{cs} = \left(L_{ls} + \frac{3}{2}L_{0s}\right)(i_{as} + \underline{a}i_{bs} + \underline{a}^2 i_{cs})
$$

$$
-\frac{3}{2}L_{2s}(i_{as} + \underline{a}^2 i_{bs} + \underline{a}i_{cs})e^{j2\theta_r}
$$

$$
+\frac{3}{2}L_{sfd}i_{fd}e^{j\theta_r} + \frac{3}{2}L_{skd}i_{kd}e^{j\theta_r} - \frac{3}{2}L_{skq}i_{kq}e^{j\left(\theta_r - \frac{\pi}{2}\right)} \qquad (2.14\text{–}35)
$$

⇓

$$
\underline{\lambda}_{abcs} = \left(L_{ls} + \frac{3}{2}L_{0s}\right)\underline{i}_{abcs} - \frac{3}{2}L_{2s}\,\underline{i}^{\dagger}_{abcs}\,e^{j2\theta_r} + \frac{3}{2}L_{sfd}i_{fd}e^{j\theta_r}
$$

$$
+\frac{3}{2}L_{skd}i_{kd}e^{j\theta_r} - \frac{3}{2}L_{skq}i_{kq}e^{j\left(\theta_r - \frac{\pi}{2}\right)} \qquad (2.14\text{–}36)
$$

⇓

$$
\underline{v}_{abcs} = r_s\underline{i}_{abcs} + p\underline{\lambda}_{abcs} \qquad (2.14\text{–}37)
$$

Transform the complex vector eqs. to the rotor reference frame

$$\underline{v}_{abcs} = r_s \underline{i}_{abcs} + p\underline{\lambda}_{abcs} \qquad (2.14\text{–}37)$$

$$\underline{v}_{abcs} e^{-j\theta_r} = r_s \underline{i}_{abcs} e^{-j\theta_r} + \boxed{e^{-j\theta_r} p\underline{\lambda}_{abcs}} \qquad (2.14\text{–}38)$$

$$\boxed{\underline{\lambda}_{abcs} e^{-j\theta_r}} = \left(L_{ls} + \frac{3}{2}L_{0s}\right)\underline{i}_{abcs} e^{-j\theta_r} - \frac{3}{2}L_{2s}\underline{i}^{\dagger}_{abcs} e^{j\theta_r} + \frac{3}{2}L_{sfd} i_{fd} + \frac{3}{2}L_{skd} i_{kd} - \frac{3}{2}L_{skq} i_{kq} e^{-j\frac{\pi}{2}} \qquad (2.14\text{–}39)$$

$$= \frac{2}{3}\underline{\lambda}_{abcs} e^{-j\theta_r}$$

$$\boxed{\underline{\lambda}^r_{qds}} = \left(L_{ls} + \frac{3}{2}L_{0s}\right)\underline{i}^r_{qds} - \frac{3}{2}L_{2s}(\underline{i}^r_{qds})^{\dagger} + L_{sfd} i_{fd} + L_{skd} i_{kd} + jL_{skq} i_{kq} \qquad (2.14\text{–}44)$$

Transform the complex vector eqs. to the rotor reference frame

$$\underline{v}_{abcs} = r_s \underline{i}_{abcs} + \boxed{p\underline{\lambda}_{abcs}} \qquad (2.14\text{–}37)$$

$$\underline{v}^r_{qds} = r_s \underline{i}^r_{qds} + \boxed{p\underline{\lambda}^r_{qds} + j\omega_r \underline{\lambda}^r_{qds}} \qquad (2.14\text{–}43)$$

$$\underline{v}_{abcs} e^{-j\theta_r} = r_s \underline{i}_{abcs} e^{-j\theta_r} + e^{-j\theta_r} p\underline{\lambda}_{abcs} \qquad (2.14\text{–}38)$$

$$\underline{v}^r_{qds} = v^r_{qs} - jv^r_{ds} = \frac{2}{3}\underline{v}_{abcs} e^{-j\theta_r} \qquad (2.14\text{–}40)$$

$$\boxed{\underline{\lambda}^r_{qds}} = \lambda^r_{qs} - j\lambda^r_{ds} = \frac{2}{3}\underline{\lambda}_{abcs} e^{-j\theta_r} \qquad (2.14\text{–}41)$$

$$\underline{i}^r_{qds} = i^r_{qs} - ji^r_{ds} = \frac{2}{3}\underline{i}_{abcs} e^{-j\theta_r} \qquad (2.14\text{–}42)$$

Direct axis and quadrature axis inductance
--- specify that the rotor is not symmetrical

$$\underline{\lambda}^r_{qds} = \left(L_{ls} + \frac{3}{2}L_{0s}\right)\underline{i}^r_{qds} - \frac{3}{2}L_{2s}(\underline{i}^r_{qds})^{\dagger} + L_{sfd}i_{fd} + L_{skd}i_{kd} + jL_{skq}i_{kq} \tag{2.14-44}$$

$$\underline{\lambda}^r_{qds} = \left(L_{ls} + \frac{L_{md}+L_{mq}}{2}\right)\underline{i}^r_{qds} + \frac{2}{3}L_{md}(i'_{fd} + i'_{kd}) - \left(\frac{L_{md}-L_{mq}}{2}\right)(\underline{i}^r_{qds})^{\dagger} + \frac{2}{3}jL_{mq}i'_{kq} \tag{2.14-47}$$

where

direct axis ind. $$L_{md} = \frac{3}{2}(L_{0s}+L_{2s}) = \frac{3}{2}\frac{N_s}{N_{fd}}L_{sfd} = \frac{3}{2}\frac{N_s}{N_{kd}}L_{skd} \tag{2.14-45}$$

quad. axis ind. $$L_{mq} = \frac{3}{2}(L_{0s}-L_{2s}) = \frac{3}{2}\frac{N_s}{N_{kq}}L_{skq} \tag{2.14-46}$$

Direct axis and quadrature axis inductance
--- transform rotor var. to stator

$$\lambda_{fd} = (L_{lfd}+L_{mfd})\,i_{fd} + L_{fkd}i_{kd} + \frac{3L_{sfd}}{4}[\underline{i}^r_{qds} + (\underline{i}^r_{qds})^{\dagger}] \tag{2.14-48}$$

$$\lambda_{kd} = (L_{lkd}+L_{mkd})\,i_{kd} + L_{fkd}i_{fd} + \frac{3L_{skd}}{4}[\underline{i}^r_{qds} + (\underline{i}^r_{qds})^{\dagger}] \tag{2.14-49}$$

$$\lambda_{kq} = (L_{lkq}+L_{mkq})\,i_{kq} - j\frac{3L_{skq}}{4}[\underline{i}^r_{qds} - (\underline{i}^r_{qds})^{\dagger}] \tag{2.14-50}$$

$$\lambda'_{fd} = L'_{lfd}\,i'_{fd} + L_{md}\{i'_{fd} + i'_{kd} + \frac{1}{2}[\underline{i}^r_{qds} + (\underline{i}^r_{qds})^{\dagger}]\} \tag{2.14-53}$$

$$\lambda'_{kd} = L'_{lkd}\,i'_{kd} + L_{md}\{i'_{kd} + i'_{fd} + \frac{1}{2}[\underline{i}^r_{qds} + (\underline{i}^r_{qds})^{\dagger}]\} \tag{2.14-54}$$

$$\lambda'_{kq} = L'_{lkq}\,i'_{kq} + L_{mq}\{i'_{kq} - j\frac{1}{2}[\underline{i}^r_{qds} - (\underline{i}^r_{qds})^{\dagger}]\} \tag{2.14-55}$$

Direct axis and quadrature axis inductance
--- transform rotor var. to stator

$$L_{md} = \frac{3}{2}(L_{0s} + L_{2s}) = \frac{3}{2}\frac{N_s^2}{N_{fd}^2}L_{mfd} = \frac{3}{2}\frac{N_s^2}{N_{kd}^2}L_{mkd} = \frac{3}{2}\frac{N_s^2}{N_{fd}N_{kd}}L_{fkd}$$

$$= \frac{3}{2}\frac{N_s}{N_{kd}}L_{skd} = \frac{3}{2}\frac{N_s}{N_{fd}}L_{sfd} \qquad (2.14\text{–}51)$$

$$L_{mq} = \frac{3}{2}(L_{0s} - L_{2s}) = \frac{3}{2}\frac{N_s^2}{N_{fq}^2}L_{mkq} = \frac{3}{2}\frac{N_s}{N_{kq}}L_{skq} \qquad (2.14\text{–}52)$$

$i'_{kd} = \frac{2}{3}\frac{N_{kd}}{N_s}i_{kd}$ (2.14–56)	$L'_{lfd} = \frac{3}{2}L_{lfd}\frac{N_s^2}{N_{fd}^2}$ (2.14–59)	$\lambda'_{kd} = \frac{N_s}{N_{kd}}\lambda_{kd}$ (2.14–62)
$i'_{fd} = \frac{2}{3}\frac{N_{fd}}{N_s}i_{fd}$ (2.14–57)	$L'_{lkd} = \frac{3}{2}L_{lkd}\frac{N_s^2}{N_{kd}^2}$ (2.14–60)	$\lambda'_{fd} = \frac{N_s}{N_{fd}}\lambda_{fd}$ (2.14–63)
$i'_{kq} = \frac{2}{3}\frac{N_{kq}}{N_s}i_{kq}$ (2.14–58)	$L'_{lkq} = \frac{3}{2}L_{lkq}\frac{N_s^2}{N_{kq}^2}$ (2.14–61)	$\lambda'_{kq} = \frac{N_s}{N_{kq}}\lambda_{kq}$ (2.14–64)

Direct axis and quadrature axis inductance
--- transform rotor var. to stator

$$v_{ds}^r = r_s i_{ds}^r + p\lambda_{ds}^r - \omega_r \lambda_{qs}^r \qquad (2.14\text{–}65)$$

$$v_{qs}^r = r_s i_{qs}^r + p\lambda_{qs}^r + \omega_r \lambda_{ds}^r \qquad (2.14\text{–}66)$$

$$v'_{fd} = r'_{fd}\, i'_{fd} + p\lambda'_{fd} \qquad (2.14\text{–}67)$$

$$\boxed{v'_{kd}} = r'_{kd}\, i'_{kd} + p\lambda'_{kd} \qquad (2.14\text{–}68)$$

$$\boxed{v'_{kq}} = r'_{kq}\, i'_{kq} + p\lambda'_{kq} \qquad (2.14\text{–}69)$$

=0

$$\lambda_{ds}^r = L_{ls} i'_{ds} + \boxed{L_{md}(i_{ds}^r + i'_{fd} + i'_{kd})} \qquad (2.14\text{–}70)$$

$$\lambda_{qs}^r = L_{ls} i'_{qs} + \boxed{L_{mq}(i_{qs}^r + i'_{kq})} \qquad (2.14\text{–}71)$$

$$\lambda'_{fd} = L'_{lfd}\, i'_{fd} + \boxed{L_{md}(i'_{fd} + i'_{kd} + i_{ds}^r)} \qquad (2.14\text{–}72)$$

$$\lambda'_{kd} = L'_{lkd}\, i'_{kd} + \boxed{L_{md}(i'_{kd} + i'_{fd} + i_{ds}^r)} \qquad (2.14\text{–}73)$$

$$\lambda'_{kq} = L'_{lkq}\, i'_{kq} + \boxed{L_{mq}(i'_{kq} + i_{qs}^r)} \qquad (2.14\text{–}74)$$

$$r'_{kd} = \frac{2}{3}r_{kd}\ ;\ r'_{kq} = \frac{2}{3}r_{kq}\ ;\ \text{and}\ r'_{fd} = \frac{2}{3}r_{fd}$$

Power input and torque output equations

$$P_e = \frac{3}{2}[v_{ds}^{r} i_{ds}^{r} + v_{qs}^{r} i_{qs}^{r} + v'_{fd}\, i'_{fd}] \quad (2.14\text{–}75)$$

$$T_e = \frac{3}{2}\frac{P}{2}[\lambda_{ds}^{r} i_{qs}^{r} - \lambda_{qs}^{r} i_{ds}^{r}] \quad (2.14\text{–}76)$$

$$T_e = \frac{3}{2}\frac{P}{2}[\underbrace{(L_{ds} - L_{qs})\, i_{qs}^{r} i_{ds}^{r}}_{\text{Saliency torque}} + \underbrace{L_{md}\, i'_{fd}\, i_{qs}^{r}}_{\text{Excitation torque}} + \underbrace{L_{md}\, i'_{kd} i_{qs}^{r} - L_{mq} i'_{kq} i_{ds}^{r}}_{\text{Damping torque or induction motor torque}}] \quad (2.14\text{–}77)$$

Motor is modelled in rotor reference frame

--- a trace of acceleration of a SM

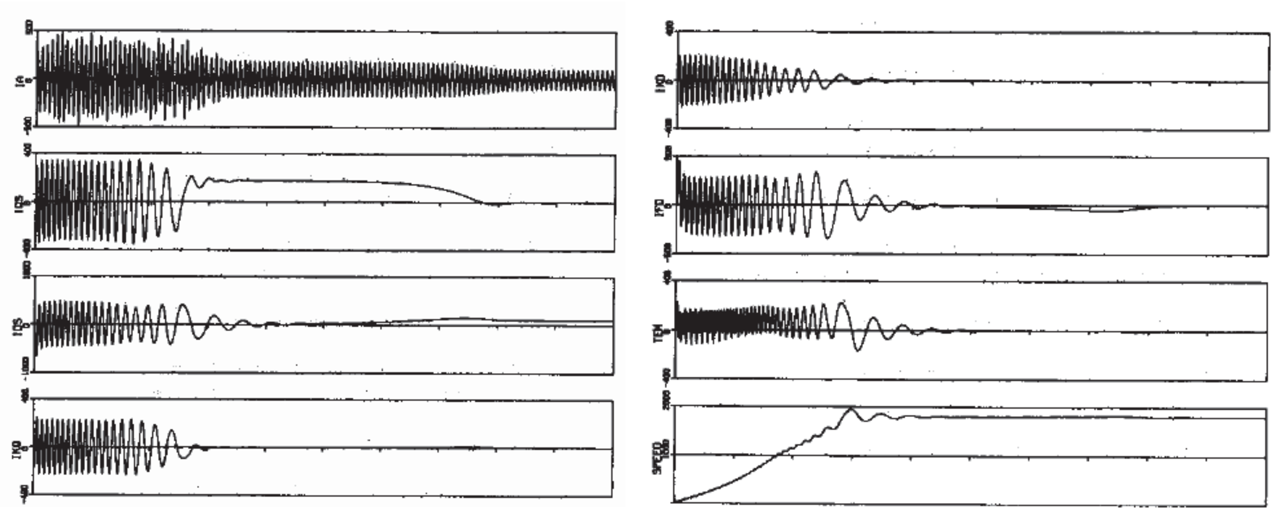


Figure 2.19 Acceleration of a 20 kVA, 230 V salient pole synchronous machine from rest with a shorted field winding. Traces from the top: i_{as}, i_{qs}^{r}, i_{ds}^{r}, i_{kq}, i_{kd}, i_{fd}, T_{em}, rotor speed in rpm; time axis – 0.1 s/div.

2.14 Extension of d,q,0 Theory to Analysis of Permanent Magnet Motor

- Assume constant field current

$$\boxed{\Lambda'_{mf} = L'_{md}\, i'_{fd} \qquad (2.15\text{–}1)}$$

$$v^r_{ds} = r_s i^r_{ds} + p\lambda^r_{ds} - \omega_r \lambda^r_{qs} \qquad (2.15\text{–}2)$$

$$v^r_{qs} = r_s i^r_{qs} + p\lambda^r_{qs} + \omega_r \lambda^r_{ds} \qquad (2.15\text{–}3)$$

$$v'_{kd} = r'_{kd}\, i'_{kd} + p\lambda'_{kd} \qquad (2.15\text{–}4)$$

$$v'_{kq} = r'_{kq}\, i'_{kq} + p\lambda'_{kq} \qquad (2.15\text{–}5)$$

$$\lambda^r_{ds} = L_{ls}\, i'_{ds} + L_{md}\,(i^r_{ds} + i'_{kd}) + \Lambda'_{mf} \qquad (2.15\text{–}6)$$

$$\lambda^r_{qs} = L_{ls}\, i'_{qs} + L_{mq}\,(i^r_{qs} + i'_{kq}) \qquad (2.15\text{–}7)$$

$$\lambda'_{kd} = L'_{lkd}\, i'_{kd} + L_{md}\,(i'_{kd} + i^r_{ds}) + \Lambda'_{mf} \qquad (2.15\text{–}8)$$

$$\lambda'_{kq} = L'_{lkq}\, i'_{kq} + L_{mq}\,(i'_{kq} + i^r_{qs}) \qquad (2.15\text{–}9)$$