Electric Machine Control

Chapter 3

d,q Models for Solid State Power Converters

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3.1 Introduction

- Combining machine-converter modelling by the dq reference frame
  - VSI dq models in a stationary reference frame
    - Six-step operation
    - PWM operation
  - CSI dq models in a stationary reference frame
  - VSI dq models in a synchronous reference frame
  - Example
    - Stationary/ synchronous RF dq models of a VSI/CSI
    - Simplified stationary/ synchronous RF dq models of a VSI/CSI
    - Including the effects of the dc link filter
    - Neglecting the effect of the inverter harmonics
    - Dc link variables referred to the induction motor stator

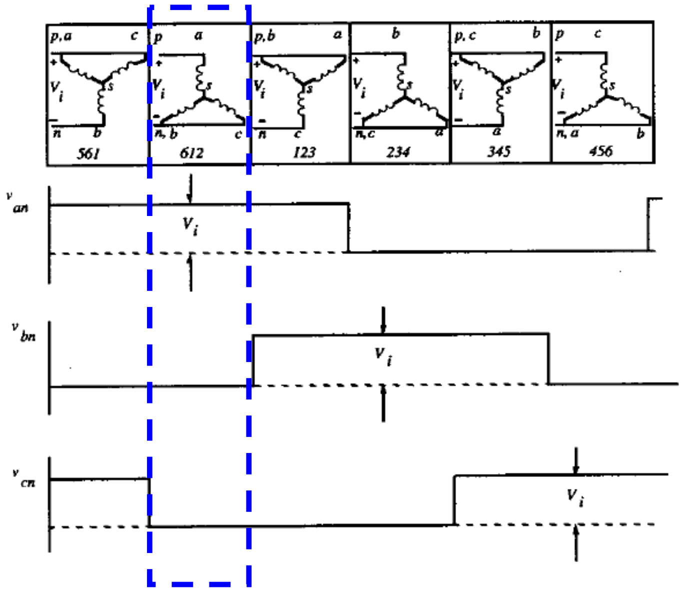
3.2 d,q Model for VSI

- Six step operation
  - Consider first the connection 612

$$\begin{cases} v_{ab} = -v_{ca} = v_i & (3.2\text{–}4) \\ v_{bc} = 0 & (3.2\text{–}5) \\ i_{as} = i_i & (3.2\text{–}6) \\ \begin{cases} i_{as} + i_{bs} + i_{cs} = 0 & (3.2\text{–}7) \\ v_{as} + v_{bs} + v_{cs} = 0 & (3.2\text{–}8) \end{cases} \end{cases}$$

$$\Downarrow$$

$$\begin{cases} v_{as} = \frac{1}{3}(v_{ab} - v_{ca}) = \frac{2}{3}v_i & (3.2\text{–}9) \\ v_{bs} = \frac{1}{3}(v_{bc} - v_{ab}) = -\frac{1}{3}v_i & (3.2\text{–}10) \\ v_{cs} = \frac{1}{3}(v_{ca} - v_{bc}) = -\frac{1}{3}v_i & (3.2\text{–}11) \end{cases}$$



3.2 d,q Model for VSI

$$f_{qs}^s = \frac{2}{3}f_{as} - \frac{1}{3}f_{bs} - \frac{1}{3}f_{cs} \quad (3.2\text{–}1)$$

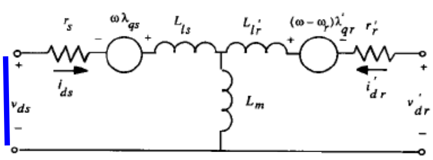
$$f_{ds}^s = \frac{1}{\sqrt{3}}f_{cs} - \frac{1}{\sqrt{3}}f_{bs} \quad (3.2\text{–}2)$$

$$f_{0s}^s = \frac{1}{3}(f_{as} + f_{bs} + f_{cs}) \quad (3.2\text{–}3)$$

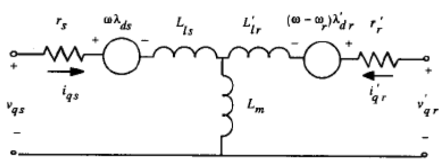
- Six step operation
  - Consider first the connection 612

$$\begin{cases} v_{as} = \frac{1}{3}(v_{ab} - v_{ca}) = \frac{2}{3}v_i & (3.2\text{–}9) \\ v_{bs} = \frac{1}{3}(v_{bc} - v_{ab}) = -\frac{1}{3}v_i & (3.2\text{–}10) \\ v_{cs} = \frac{1}{3}(v_{ca} - v_{bc}) = -\frac{1}{3}v_i & (3.2\text{–}11) \end{cases} \Rightarrow \begin{cases} v_{qs}^s = \frac{2}{3}v_i & (3.2\text{–}12) \\ v_{ds}^s = 0 & (3.2\text{–}13) \\ v_{0s}^s = 0 & (3.2\text{–}14) \end{cases}$$

$$\begin{cases} i_{as} = i_i & (3.2\text{–}6) \\ i_{as} + i_{bs} + i_{cs} = 0 & (3.2\text{–}7) \end{cases} \Rightarrow \begin{cases} i_{qs}^s = i_i & (3.2\text{–}15) \\ i_{ds}^s = \frac{1}{\sqrt{3}}(i_{cs} - i_{bs}) & (3.2\text{–}16) \\ i_{0s}^s = 0 & (3.2\text{–}17) \end{cases}$$



$d-$axis equivalent circuit



$q-$axis equivalent circuit

表示什么意思？

Six step operation

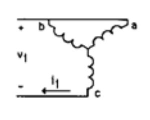
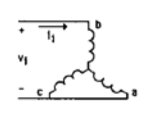
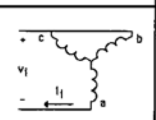
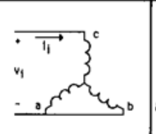
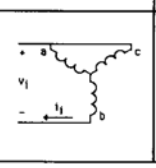
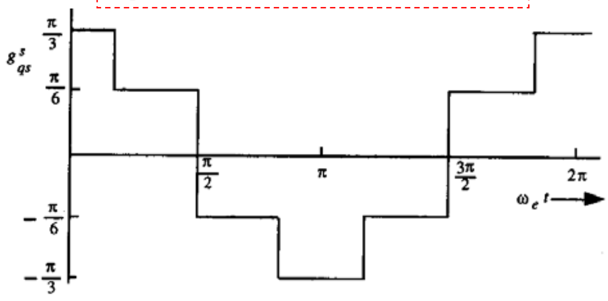
1	$-\frac{\pi}{6} < \omega_e t < \frac{\pi}{6}$		$v_{qs}^s = \frac{2}{3}v_i$ $v_{ds}^s = 0$ $i_i = i_{qs}^s$	$\underline{v}_{qds}^s = \frac{2}{3}v_i e^{j0}$
2	$\frac{\pi}{6} < \omega_e t < \frac{\pi}{2}$		$v_{qs}^s = \frac{v_i}{3}$ $v_{ds}^s = -\frac{v_i}{\sqrt{3}}$ $i_i = \frac{1}{2}i_{qs}^s - \frac{\sqrt{3}}{2}i_{ds}^s$	$\underline{v}_{qds}^s = \frac{2}{3}v_i e^{j\frac{\pi}{3}}$
3	$\frac{\pi}{2} < \omega_e t < \frac{5\pi}{6}$		$v_{qs}^s = -\frac{v_i}{3}$ $v_{ds}^s = -\frac{v_i}{\sqrt{3}}$ $i_i = -\frac{1}{2}i_{qs}^s - \frac{\sqrt{3}}{2}i_{ds}^s$	$\underline{v}_{qds}^s = \frac{2}{3}v_i e^{j\frac{2\pi}{3}}$
4	$\frac{5\pi}{6} < \omega_e t < \frac{7\pi}{6}$		$v_{qs}^s = -\frac{2}{3}v_i$ $v_{ds}^s = 0$ $i_i = -i_{qs}^s$	$\underline{v}_{qds}^s = \frac{2}{3}v_i e^{j\pi}$
5	$\frac{7\pi}{6} < \omega_e t < \frac{3\pi}{2}$		$v_{qs}^s = -\frac{v_i}{3}$ $v_{ds}^s = \frac{v_i}{\sqrt{3}}$ $i_i = -\frac{1}{2}i_{qs}^s + \frac{\sqrt{3}}{2}i_{ds}^s$	$\underline{v}_{qds}^s = \frac{2}{3}v_i e^{j\frac{4\pi}{3}}$
6	$\frac{3\pi}{2} < \omega_e t < \frac{11\pi}{6}$		$v_{qs}^s = \frac{v_i}{3}$ $v_{ds}^s = \frac{v_i}{\sqrt{3}}$ $i_i = \frac{1}{2}i_{qs}^s + \frac{\sqrt{3}}{2}i_{ds}^s$	$\underline{v}_{qds}^s = \frac{2}{3}v_i e^{j\frac{5\pi}{3}}$

Figure 3.1 *d,q* equations for the six modes of a VSI

$$v_{qs}^s = \frac{2}{\pi}v_i g_{qs}^s \qquad (3.2\text{–}18)$$

$$v_{ds}^s = \frac{2}{\pi}v_i g_{ds}^s \qquad (3.2\text{–}19)$$

$$\frac{\pi}{3}i_i = i_{qs}^s g_{qs}^s + i_{ds}^s g_{ds}^s \qquad (3.2\text{–}20)$$



switching function

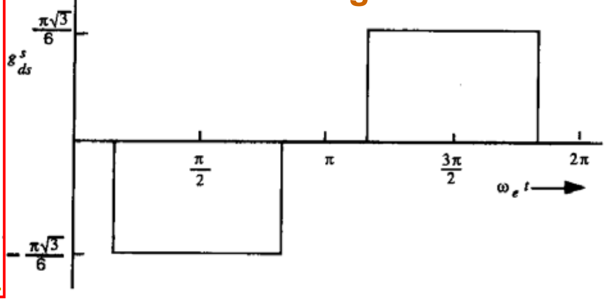
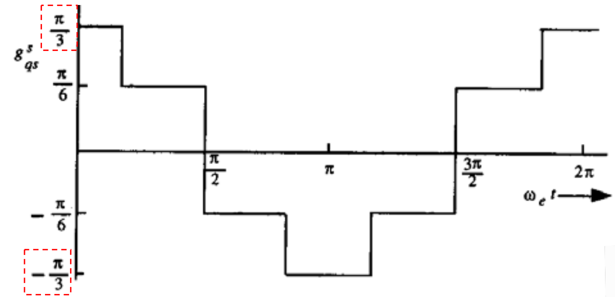


Figure 3.2 The VSI switching functions g_{qs}^s and g_{ds}^s

Switching function



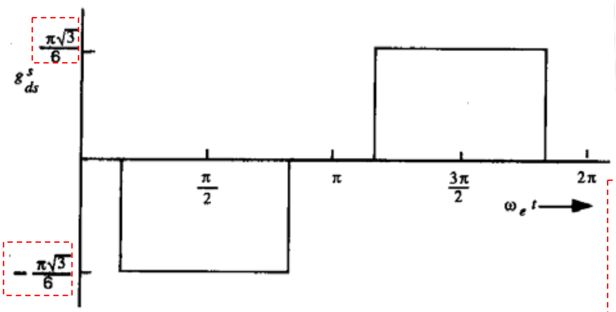


Figure 3.2 The VSI switching functions g_{qs}^s and g_{ds}^s

$$v_{qs}^s = \frac{2}{\pi}v_i g_{qs}^s \qquad (3.2\text{–}18)$$

$$v_{ds}^s = \frac{2}{\pi}v_i g_{ds}^s \qquad (3.2\text{–}19)$$

$$\frac{\pi}{3}i_i = i_{qs}^s g_{qs}^s + i_{ds}^s g_{ds}^s \qquad (3.2\text{–}20)$$

Fourier series of sw fun.

$$g_{qs}^s = \boxed{\cos\theta_e} + \frac{1}{5}\cos 5\theta_e - \frac{1}{7}\cos 7\theta_e - \ldots \qquad (3.2\text{–}21)$$

$$g_{ds}^s = \boxed{-\sin\theta_e} + \frac{1}{5}\sin 5\theta_e + \frac{1}{7}\sin 7\theta_e - .. \qquad (3.2\text{–}22)$$

$$\underline{v}_{qds}^s = v_{qs}^s - jv_{ds}^s = \frac{2}{\pi}v_i(g_{qs}^s - jg_{ds}^s) = \frac{2}{\pi}v_i \underline{g}_{qds}^s \qquad (3.2\text{–}23)$$

$$\frac{\pi}{3}i_i = Re[\underline{i}_{qds}^s(\underline{g}_{qds}^s)^\dagger] \qquad (3.2\text{–}25)$$

Six step operation

$$\underline{v}^s_{qds} = v^s_{qs} - jv^s_{ds} = \frac{2}{\pi}v_i\left(g^s_{qs} - jg^s_{ds}\right) = \frac{2}{\pi}v_i\,\underline{g}^s_{qds} \qquad (3.2\text{–}23)$$

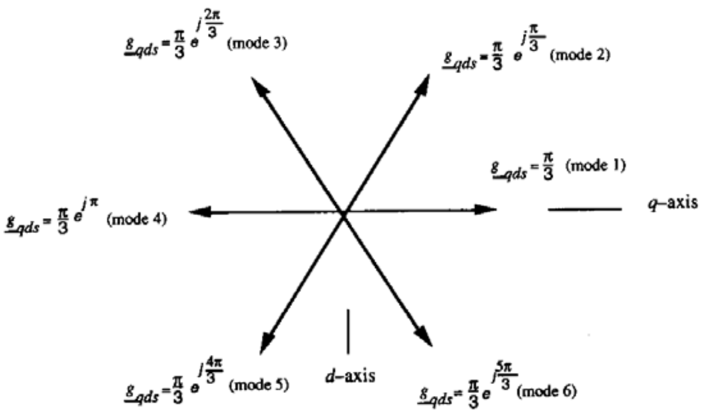


Figure 3.3 The six voltage vectors characterizing VSI operation

(mode 1)

$$\underline{v}^s_{qds} = \frac{2}{3}v_i e^{j0} \qquad (3.2\text{–}26)$$

(mode 2)

$$\underline{v}^s_{qds} = \frac{2}{3}v_i\left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) = \frac{2}{3}v_i e^{j\pi/3} \qquad (3.2\text{–}27)$$

Mode 1…6

$$\underline{v}^s_{qds} = \frac{2}{3}v_i e^{j(k-1)\pi/3} \qquad k = 1,2,\ldots,6 \qquad (3.2\text{–}28)$$

3.3 d,q Model for PWM Operation

- Additional states: zero states

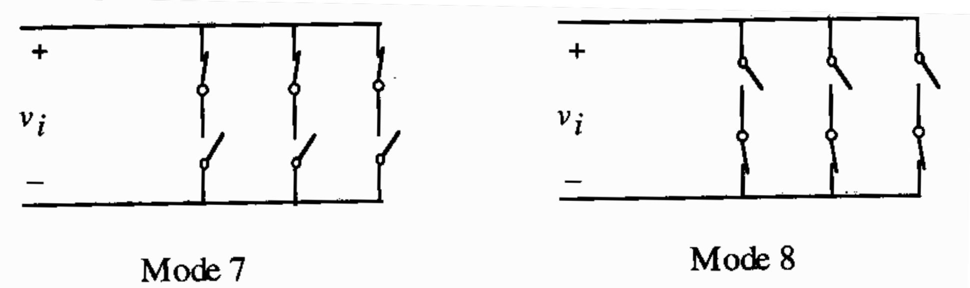


Figure 3.4 Switch conditions corresponding to the zero voltage vector state for a PWM inverter

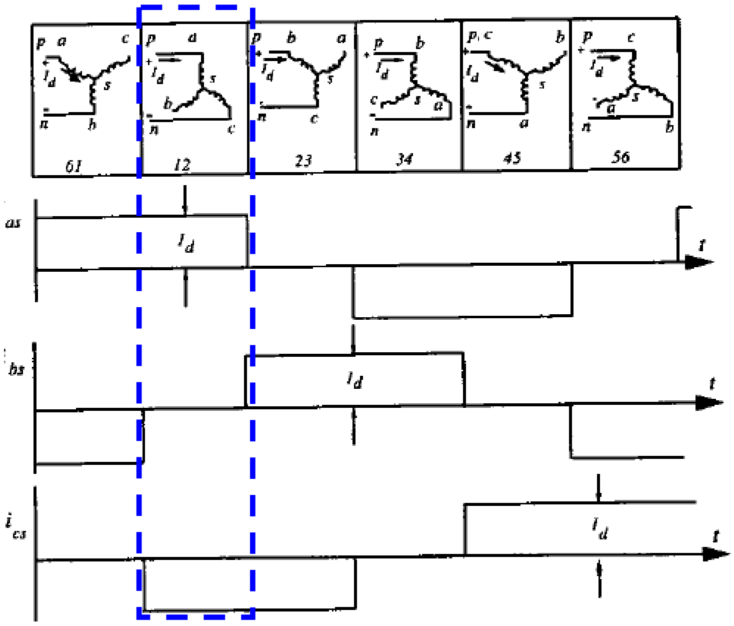
3.4 d,q Model for CSI

- Six step operation
  - Consider first the connection 12

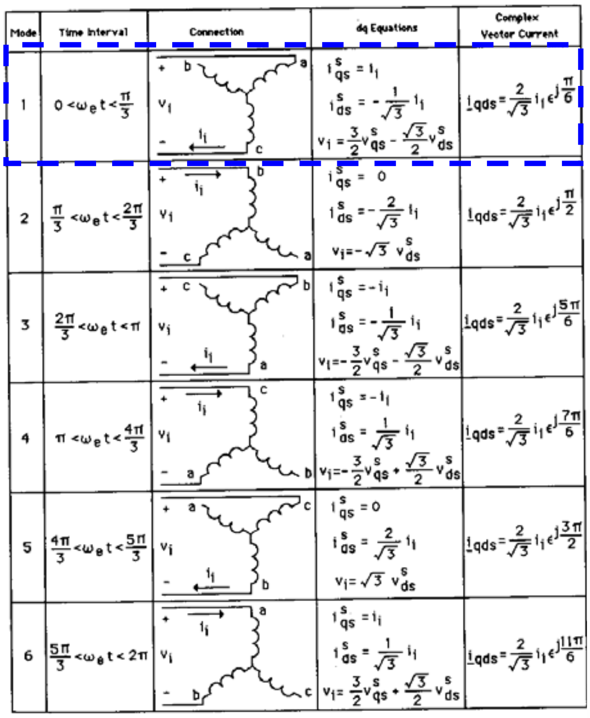
$$
\left\{
\begin{aligned}
&\boxed{\begin{aligned} i_{as} &= i_i \\ i_{bs} &= 0 \\ i_{cs} &= -i_i \end{aligned}} \\
&\left\{ \begin{aligned} v_i &= v_{as} - v_{cs} \\ v_{as} + v_{bs} + v_{cs} &= 0 \end{aligned} \right.
\end{aligned}
\right.
$$

$\Downarrow$

$$
\left\{
\begin{aligned}
i_{qs}^s &= i_i \\
i_{ds}^s &= -\frac{1}{\sqrt{3}} i_i \\
v_i &= \frac{3}{2} v_{qs}^s - \frac{\sqrt{3}}{2} v_{ds}^s
\end{aligned}
\right.
$$



Six step operation

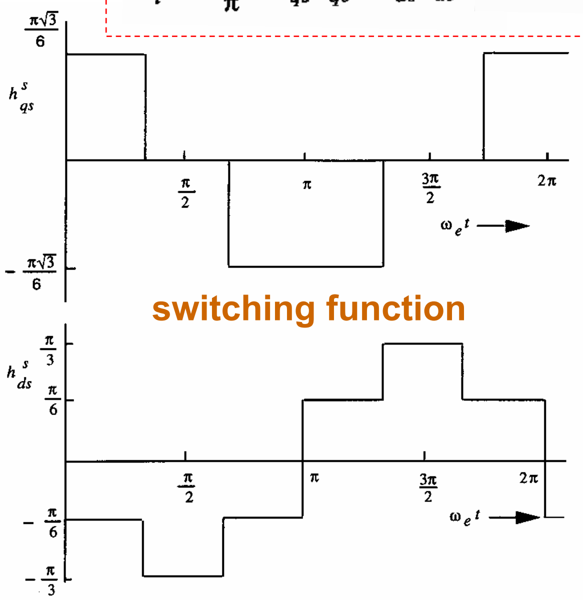


Mode	Time Interval	Connection	dq Equations	Complex Vector Current
1	$0 < \omega_e t < \frac{\pi}{3}$	+ b, a; v_i; − c; i_i	$i_{qs}^s = i_i$ $i_{ds}^s = -\frac{1}{\sqrt{3}} i_i$ $v_i = \frac{3}{2} v_{qs}^s - \frac{\sqrt{3}}{2} v_{ds}^s$	$\underline{i}_{qds} = \frac{2}{\sqrt{3}} i_i \epsilon^{j\frac{\pi}{6}}$
2	$\frac{\pi}{3} < \omega_e t < \frac{2\pi}{3}$	+ i_i, b; v_i; − c, a	$i_{qs}^s = 0$ $i_{ds}^s = -\frac{2}{\sqrt{3}} i_i$ $v_i = -\sqrt{3}\, v_{ds}^s$	$\underline{i}_{qds} = \frac{2}{\sqrt{3}} i_i \epsilon^{j\frac{\pi}{2}}$
3	$\frac{2\pi}{3} < \omega_e t < \pi$	+ c, b; v_i; − a; i_i	$i_{qs}^s = -i_i$ $i_{ds}^s = -\frac{1}{\sqrt{3}} i_i$ $v_i = -\frac{3}{2} v_{qs}^s - \frac{\sqrt{3}}{2} v_{ds}^s$	$\underline{i}_{qds} = \frac{2}{\sqrt{3}} i_i \epsilon^{j\frac{5\pi}{6}}$
4	$\pi < \omega_e t < \frac{4\pi}{3}$	+ i_i, c; v_i; − a, b	$i_{qs}^s = -i_i$ $i_{ds}^s = \frac{1}{\sqrt{3}} i_i$ $v_i = -\frac{3}{2} v_{qs}^s + \frac{\sqrt{3}}{2} v_{ds}^s$	$\underline{i}_{qds} = \frac{2}{\sqrt{3}} i_i \epsilon^{j\frac{7\pi}{6}}$
5	$\frac{4\pi}{3} < \omega_e t < \frac{5\pi}{3}$	+ a, c; v_i; − b; i_i	$i_{qs}^s = 0$ $i_{ds}^s = \frac{2}{\sqrt{3}} i_i$ $v_i = \sqrt{3}\, v_{ds}^s$	$\underline{i}_{qds} = \frac{2}{\sqrt{3}} i_i \epsilon^{j\frac{3\pi}{2}}$
6	$\frac{5\pi}{3} < \omega_e t < 2\pi$	+ i_i, a; v_i; − b, c	$i_{qs}^s = i_i$ $i_{ds}^s = \frac{1}{\sqrt{3}} i_i$ $v_i = \frac{3}{2} v_{qs}^s + \frac{\sqrt{3}}{2} v_{ds}^s$	$\underline{i}_{qds} = \frac{2}{\sqrt{3}} i_i \epsilon^{j\frac{11\pi}{6}}$

$$i_{qs}^s = \frac{2\sqrt{3}}{\pi} h_{qs}^s i_i \qquad (3.4\text{–}1)$$

$$i_{ds}^s = \frac{2\sqrt{3}}{\pi} h_{ds}^s i_i \qquad (3.4\text{–}2)$$

$$v_i = \frac{3\sqrt{3}}{\pi} \left(v_{qs}^s h_{qs}^s + v_{ds}^s h_{ds}^s \right) \qquad (3.4\text{–}3)$$



switching function

Figure 3.6 The CSI switching functions h_{qs}^s and h_{ds}^s

Switching Function

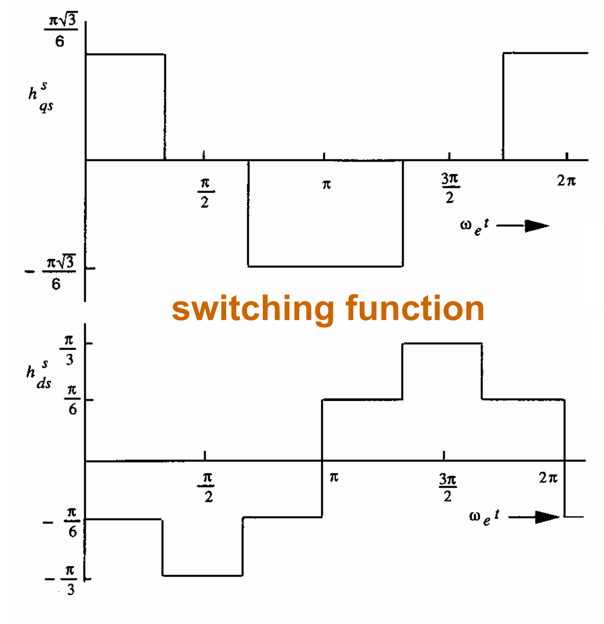


Figure 3.6 The CSI switching functions h_{qs}^s and h_{ds}^s

$$i_{qs}^s = \frac{2\sqrt{3}}{\pi} h_{qs}^s i_i \qquad (3.4\text{–}1)$$

$$i_{ds}^s = \frac{2\sqrt{3}}{\pi} h_{ds}^s i_i \qquad (3.4\text{–}2)$$

$$v_i = \frac{3\sqrt{3}}{\pi} (v_{qs}^s h_{qs}^s + v_{ds}^s h_{ds}^s) \qquad (3.4\text{–}3)$$

Fourier series of sw fun.

$$h_{qs}^s = \boxed{\cos\omega_e t} - \frac{1}{5}\cos 5\omega_e t + \frac{1}{7}\cos 7\omega_e t - .. \quad (3.4\text{–}4)$$

$$h_{ds}^s = \boxed{\sin\omega_e t} - \frac{1}{5}\sin 5\omega_e t + \frac{1}{7}\sin 7\omega_e t - ... \quad (3.4\text{–}5)$$

$$\underline{i}_{qds}^s = \frac{2\sqrt{3}}{\pi} i_i (h_{qs}^s - j h_{ds}^s) = \frac{2\sqrt{3}}{\pi} i_i \underline{h}_{qds}^s \quad (3.4\text{–}6)$$

$$v_i = \frac{3\sqrt{3}}{\pi} Re[\underline{v}_{qds}^s \underline{h}_{qds}^{s\dagger}] \qquad (3.4\text{–}7)$$

Six step operation

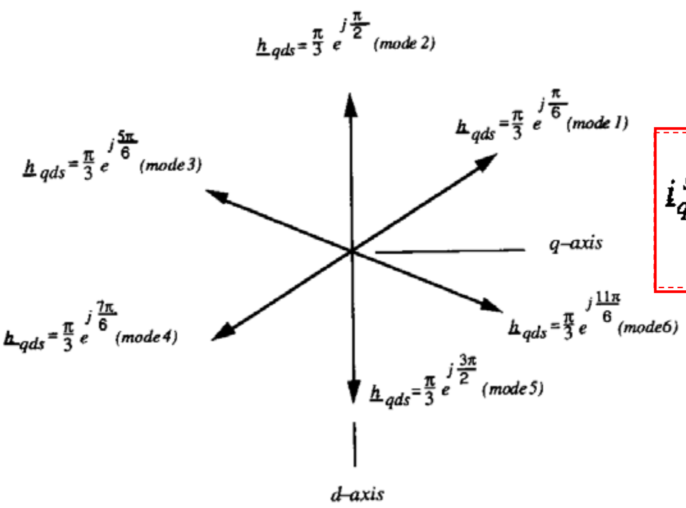


Figure 3.7 The six current vectors characterizing CSI operation

Mode 1…6

$$\underline{i}_{qds}^s = \frac{2}{\sqrt{3}} i_i e^{j[\pi/6 + (k-1)(\pi/3)]}$$

$k = 1,2,\ldots,6$

(3.4–9)

3.5 Inverter d,q Models in a Syn.R.F.

- Transform stationary RF to synchronous RF (for VSI)

$$\begin{cases} \underline{v}^e_{qds} = \underline{v}^s_{qds} e^{-j\theta_e} \\ \quad = \frac{2}{\pi} v_i \underline{g}^s_{qds} e^{-j\theta_e} \\ \quad = \frac{2}{\pi} v_i \underline{g}^e_{qds} \quad (3.5\text{–}5) \\ \frac{\pi}{3} i_i = Re[\underline{i}^e_{qds} \underline{g}^{e\dagger}_{qds}] \quad (3.5\text{–}6) \end{cases} \Rightarrow \begin{cases} v^e_{qs} = \frac{2}{\pi} v_i g^e_{qs} \quad (3.5\text{–}8) \\ v^e_{ds} = \frac{2}{\pi} v_i g^e_{ds} \quad (3.5\text{–}9) \\ \frac{\pi}{3} i_i = i^e_{qs} g^e_{qs} + i^e_{ds} g^e_{ds} \quad (3.5\text{–}10) \end{cases}$$

$$\underline{g}^s_{qds} = e^{j\omega_e t} + \frac{1}{5} e^{-j5\omega_e t} - \frac{1}{7} e^{j7\omega_e t} - \ldots \quad (3.2\text{–}24)$$

$$\Rightarrow \underline{g}^e_{qds} = e^{j(\omega_e t - \theta_e)} + \frac{1}{5} e^{-j(5\omega_e t + \theta_e)} - \frac{1}{7} e^{j(7\omega_e t - \theta_e)} - \ldots \quad (3.5\text{–}11)$$

- Transform stationary RF to synchronous RF (for CSI)

$$\begin{cases} \underline{i}^e_{qds} = \underline{i}^s_{qds} e^{-j\theta_e} \\ \quad = \frac{2\sqrt{3}}{\pi} i_i \underline{h}^s_{qds} e^{-j\theta_e} \quad (3.5\text{–}14) \\ \quad = \frac{2\sqrt{3}}{\pi} i_i \underline{h}^e_{qds} \\ v_i = \frac{3\sqrt{3}}{\pi} Re[\underline{v}^e_{qds} \underline{h}^{e\dagger}_{qds}] \quad (3.5\text{–}15) \end{cases}$$

$$\underline{h}^s_{qds} = e^{j\omega_e t} - \frac{1}{5} e^{-j5\omega_e t} + \frac{1}{7} e^{j7\omega_e t} - \ldots \quad (3.4\text{–}8)$$

$$\Rightarrow \underline{h}^e_{qds} = e^{j(\omega_e t - \theta_e)} - \frac{1}{5} e^{-j(5\omega_e t + \theta_e)} + \frac{1}{7} e^{j(7\omega_e t - \theta_e)} \quad (3.5\text{–}16)$$

Switching functions of the VSI and CSI

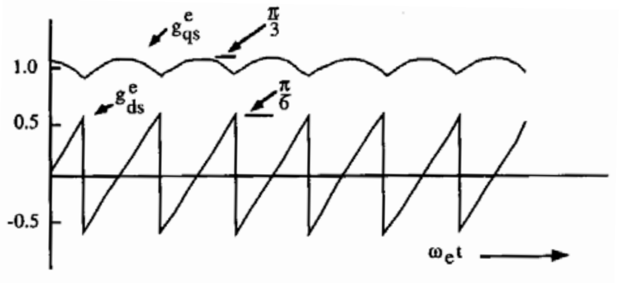


Figure 3.8 The synchronous frame VSI switching functions

$$g^e_{qs} = 1 + \frac{2}{35}\cos 6\omega_e t - \frac{2}{143}\cos 12\omega_e t + \ldots \quad (3.5\text{–}12)$$

$$g^e_{ds} = \frac{12}{35}\sin 6\omega_e t - \frac{24}{143}\sin 12\omega_e t + \ldots \quad (3.5\text{–}13)$$

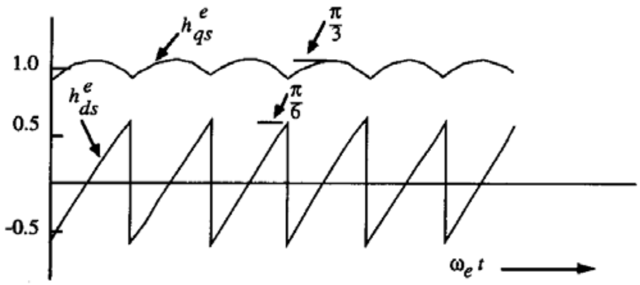


Figure 3.9 The synchronous frame CSI switching functions

$$h^e_{qs} = 1 - \frac{2}{35}\cos 6\omega_e t - \frac{2}{143}\cos 12\omega_e t + \ldots \quad (3.5\text{–}17)$$

$$h^e_{ds} = -\frac{12}{35}\sin 6\omega_e t - \frac{24}{143}\sin 12\omega_e t + \ldots \quad (3.5\text{–}18)$$

3.6 Example of Inverter-Induction Motor Models

怎么理解？

- Stationary RF dq model of a VSI driven IM

$$v^s_{qs} = \frac{2}{\pi} v_i g^s_{qs} \quad (3.2\text{–}18)$$

$$v^s_{ds} = \frac{2}{\pi} v_i g^s_{ds} \quad (3.2\text{–}19)$$

$$\frac{\pi}{3} i_i = i^s_{qs} g^s_{qs} + i^s_{ds} g^s_{ds} \quad (3.2\text{–}20)$$

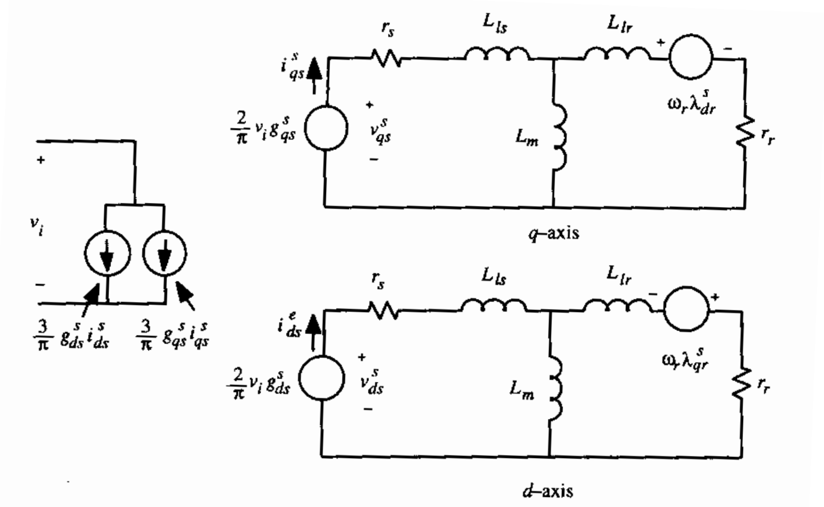


Figure 3.10 Stationary frame d,q model of a VSI driven induction machine

Synchronous RF dq model of a **VSI** driven IM

$$v_{qs}^{e} = \frac{2}{\pi} v_i g_{qs}^{e} \qquad (3.5\text{–}8)$$

$$v_{ds}^{e} = \frac{2}{\pi} v_i g_{ds}^{e} \qquad (3.5\text{–}9)$$

$$\frac{\pi}{3} i_i = i_{qs}^{e} g_{qs}^{e} + i_{ds}^{e} g_{ds}^{e} \qquad (3.5\text{–}10)$$

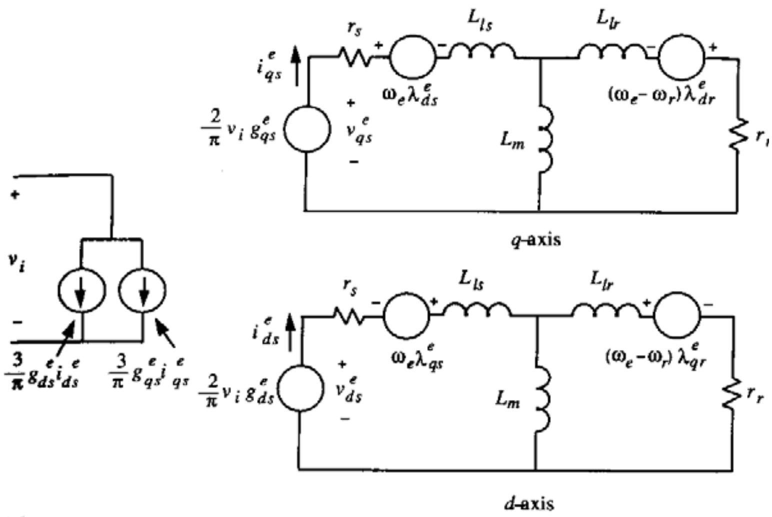


Figure 3.11 Synchronous frame *d,q* model of a VSI driven induction machine

Stationary RF dq model of a **CSI** driven IM

$$i_{qs}^{s} = \frac{2\sqrt{3}}{\pi} h_{qs}^{s} i_i \qquad (3.4\text{–}1)$$

$$i_{ds}^{s} = \frac{2\sqrt{3}}{\pi} h_{ds}^{s} i_i \qquad (3.4\text{–}2)$$

$$v_i = \frac{3\sqrt{3}}{\pi} (v_{qs}^{s} h_{qs}^{s} + v_{ds}^{s} h_{ds}^{s}) \qquad (3.4\text{–}3)$$

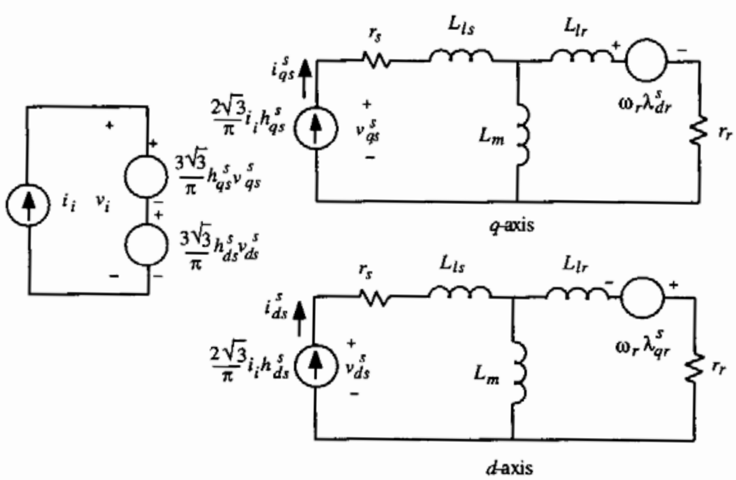


Figure 3.12 Stationary frame *d,q* model of a CSI driven induction machine

Synchronous RF dq model of a **CSI** driven IM

$$\underline{i}^e_{qds} = \underline{i}^s_{qds} e^{-j\theta_e} = \frac{2\sqrt{3}}{\pi} i_i \underline{h}^s_{qds} e^{-j\theta_e} = \frac{2\sqrt{3}}{\pi} i_i \underline{h}^e_{qds} \qquad (3.5\text{–}14)$$

$$v_i = \frac{3\sqrt{3}}{\pi} Re\left[\underline{v}^e_{qds} \underline{h}^{e\dagger}_{qds}\right] \qquad (3.5\text{–}15)$$

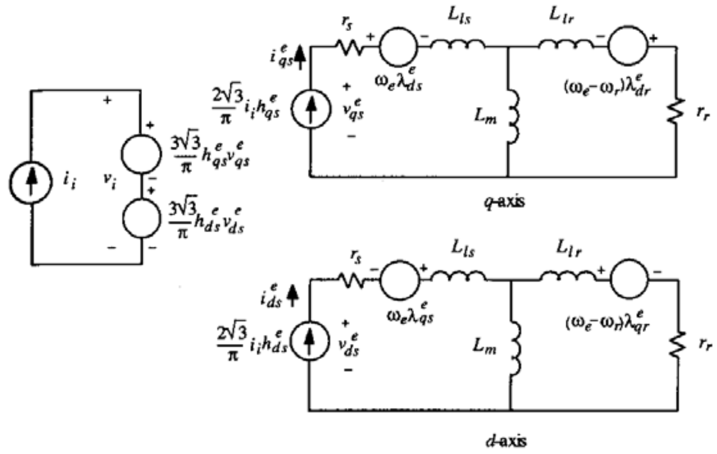


Figure 3.13 Synchronous frame d,q model of a CSI driven induction machine

Simplified Synchronous RF dq model of a **VSI** driven IM **(neglect harmonics)**

$$v^e_{qs} = \frac{2}{\pi} v_i g^e_{qs} \qquad (3.5\text{–}8)$$

$$v^e_{ds} = \frac{2}{\pi} v_i g^e_{ds} \qquad (3.5\text{–}9)$$

$$\frac{\pi}{3} i_i = i^e_{qs} g^e_{qs} + i^e_{ds} g^e_{ds} \qquad (3.5\text{–}10)$$

w/o harmonics:

$$g^e_{qs} \cong 1$$

$$g^e_{ds} \cong 0$$

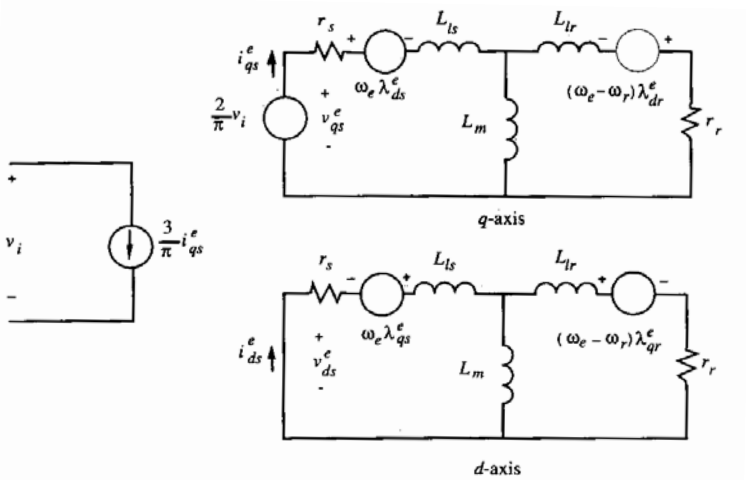


Figure 3.14 Simplified synchronous frame d,q model of VSI driven induction machine

Simplified Synchronous RF dq model of a **CSI** driven IM **(neglect harmonics)**

$$\underline{i}^e_{qds} = \underline{i}^s_{qds} e^{-j\theta_e}$$
$$= \frac{2\sqrt{3}}{\pi} i_i \underline{h}^s_{qds} e^{-j\theta_e}$$
$$= \frac{2\sqrt{3}}{\pi} i_i \underline{h}^e_{qds} \qquad (3.5\text{–}14)$$

$$v_i = \frac{3\sqrt{3}}{\pi} Re\,[\underline{v}^e_{qds} \underline{h}^{e\dagger}_{qds}] \qquad (3.5\text{–}15)$$

w/o harmonics:

$$h^e_{qs} \cong 1$$

$$h^e_{ds} \cong 0$$

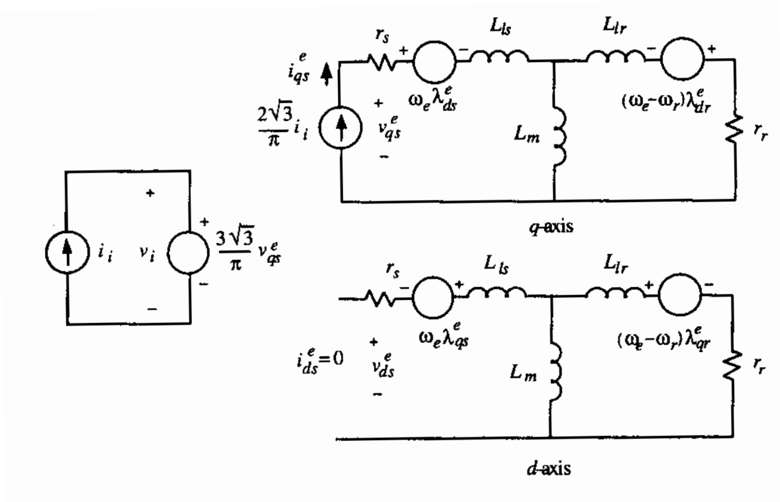


Figure 3.15 Simplified synchronous frame d,q model of CSI driven induction machine

Synchronous RF dq model of a **VSI** driven IM **(Consider the effects of the dc link filter)**

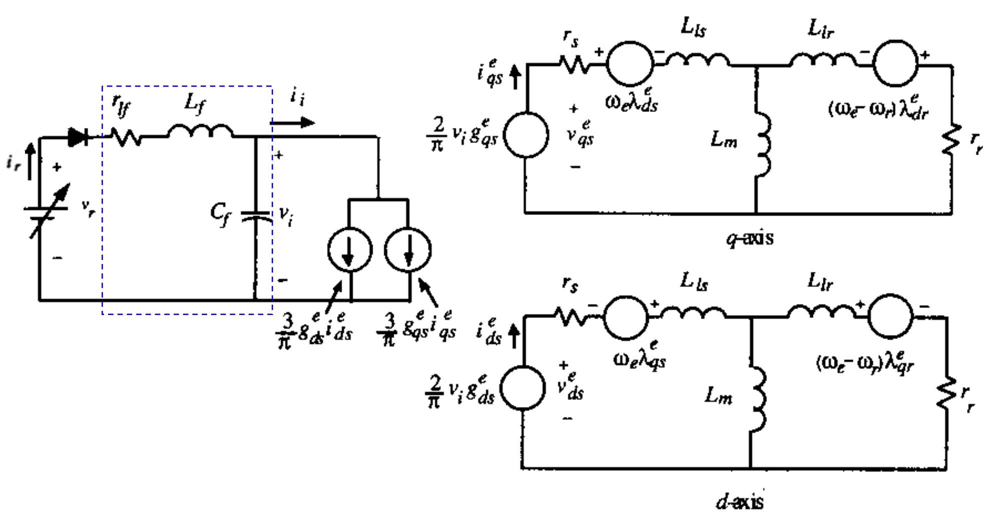


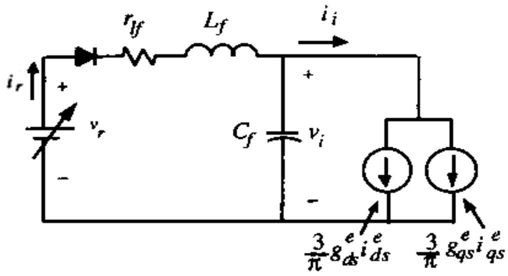
Figure 3.19 Synchronous frame d,q model of VSI driven induction machine including the effects of the dc link filter

Synchronous RF dq model of a **VSI** driven IM **(Consider the effects of the dc link filter)**

$$v_r = v_i + (r_{lf} + pL_f)\, i_r \qquad i_r > 0 \qquad (3.6\text{–}6)$$

$$v_r = v_i \qquad i_r = 0 \qquad (3.6\text{–}7)$$

$$v_i = \frac{1}{p}\frac{1}{C_f}(i_r - i_i) \qquad (3.6\text{–}8)$$



$$\frac{2}{\pi}v_r = \frac{2}{\pi}v_i + \frac{6}{\pi^2}(r_{lf} + pL_f)\left(\frac{\pi}{3}i_r\right) \qquad i_r > 0 \qquad (3.6\text{–}9)$$

$$\frac{2}{\pi}v_r = \frac{2}{\pi}v_i \qquad i_r = 0 \qquad (3.6\text{–}10)$$

$$\frac{2}{\pi}v_i = \frac{1}{p}\left(\frac{6}{\pi^2}\frac{1}{C_f}\right)\left(\frac{\pi}{3}i_r - \frac{\pi}{3}i_i\right) \qquad (3.6\text{–}11)$$

Simplified Synchronous RF dq model of a **VSI** driven IM **(Considering the effects of the dc link filter but neglecting the effect of the inverter harmonics)**

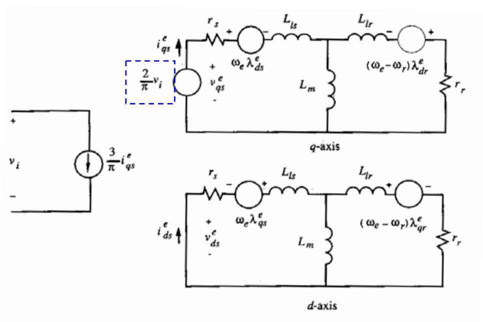


Figure 3.14 Simplified synchronous frame d,q model of VSI driven induction machine

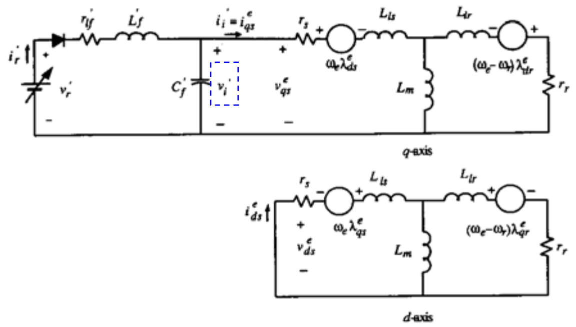


Figure 3.20 Simplified synchronous frame d,q model incorporating the effects of the dc link but neglecting the effect of the inverter harmonics

$$v_{qs}^e = \frac{2}{\pi}v_i g_{qs}^e \qquad (3.5\text{–}8)$$

$$v_{ds}^e = \frac{2}{\pi}v_i g_{ds}^e \qquad (3.5\text{–}9)$$

$$\frac{\pi}{3}i_i = i_{qs}^e g_{qs}^e + i_{ds}^e g_{ds}^e \qquad (3.5\text{–}10)$$

w/o harmonics: $g_{qs}^e \cong 1$, $g_{ds}^e \cong 0$

$$\overset{v_r'}{\boxed{\frac{2}{\pi}v_r}} = \overset{v_i'}{\boxed{\frac{2}{\pi}v_i}} + \frac{6}{\pi^2}(r_{lf} + pL_f)\overset{i_r'}{\boxed{\left(\frac{\pi}{3}i_r\right)}} \qquad i_r > 0 \qquad (3.6\text{–}9)$$

$$\frac{2}{\pi}v_r = \frac{2}{\pi}v_i \qquad i_r = 0 \qquad (3.6\text{–}10)$$

$$\frac{2}{\pi}v_i = \frac{1}{p}\left(\frac{6}{\pi^2}\frac{1}{C_f}\right)\left(\frac{\pi}{3}i_r - \frac{\pi}{3}i_i\right) \qquad (3.6\text{–}11)$$

Simplified Synchronous RF dq model of a **CSI** driven IM (Considering the effects of the dc link filter)

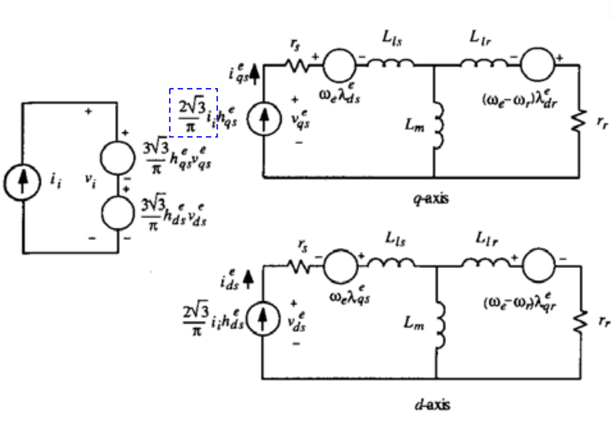


Figure 3.13 Synchronous frame d,q model of a CSI driven induction machine

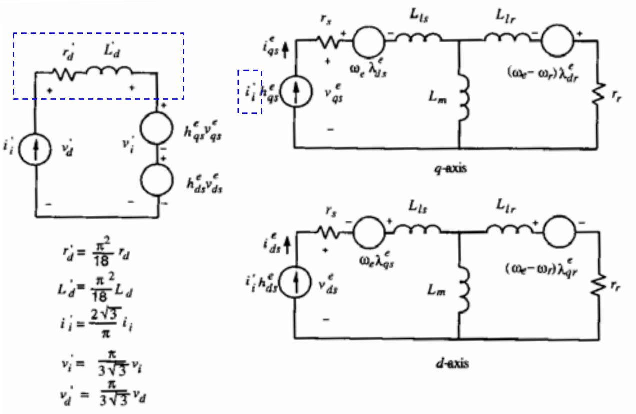


Figure 3.22 Synchronous frame d,q equivalent circuit of CSI fed induction motor incorporating the effect of the dc link inductor. Link variables referred to the induction motor stator

$$\underline{i}^e_{qds} = \underline{i}^s_{qds} e^{-j\theta_e}$$

$$= \frac{2\sqrt{3}}{\pi} i_i \underline{h}^s_{qds} e^{-j\theta_e}$$

$$= \frac{2\sqrt{3}}{\pi} i_i \underline{h}^e_{qds} \qquad (3.5\text{–}14)$$

$$v_i = \frac{3\sqrt{3}}{\pi} Re\,[\underline{v}^e_{qds} \underline{h}^{e\dagger}_{qds}] \qquad (3.5\text{–}15)$$

Simplified Synchronous RF dq model of a **CSI** driven IM (Considering the effects of the dc link filter but neglecting the effect of the inverter harmonics)

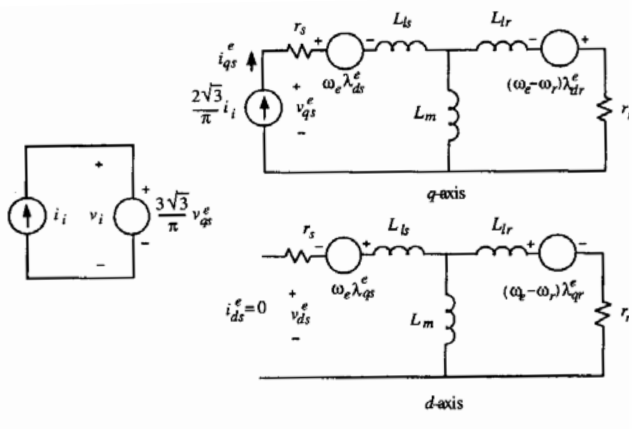


Figure 3.15 Simplified synchronous frame d,q model of CSI drive induction machine

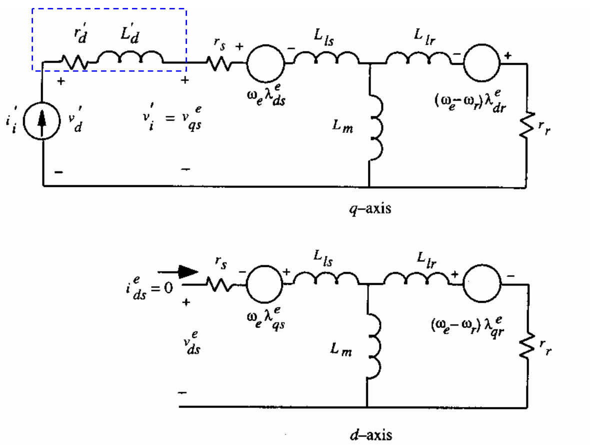
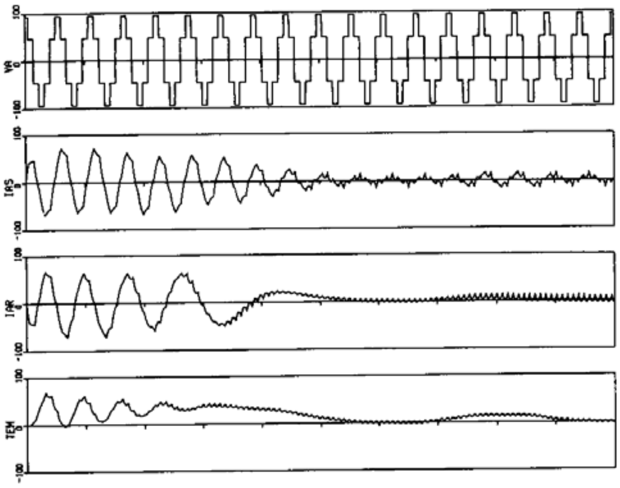


Figure 3.23 Simplified synchronous frame d,q model of CSI fed induction motor incorporating the effects of the dc link but neglecting the effect of the inverter harmonics

Stationary RF dq model of a VSI driven IM



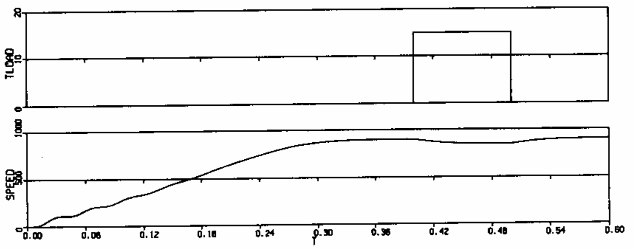
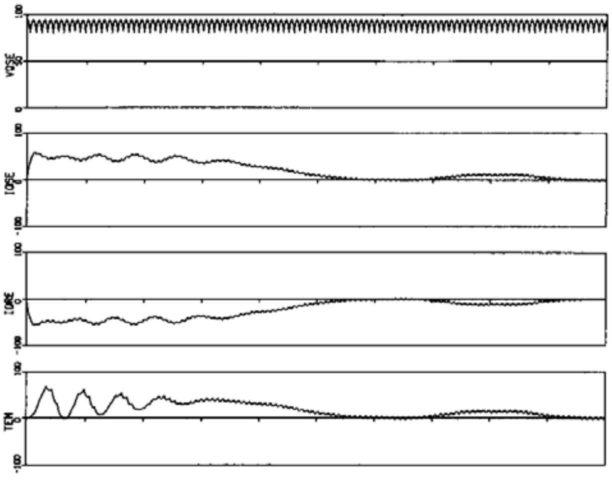
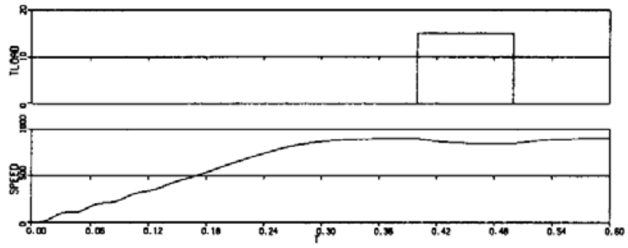


Figure 3.16 Simulation of an induction machine accelerating from rest using a VSI supply with the system represented in the stationary reference frame. The inverter frequency is fixed at 30 Hz. VA = v_{as} = v_{qs}^{s} (V), IAS = i_{as} = i_{qs}^{s} (A), IAR = i_{ar}' = $i_{qr}^{r\prime}$ (A), TEM = T_e (N–m), TLOAD = Load Torque (N–m), SPEED = ω_{mech} = $2\omega_r/P$ (rad/s)

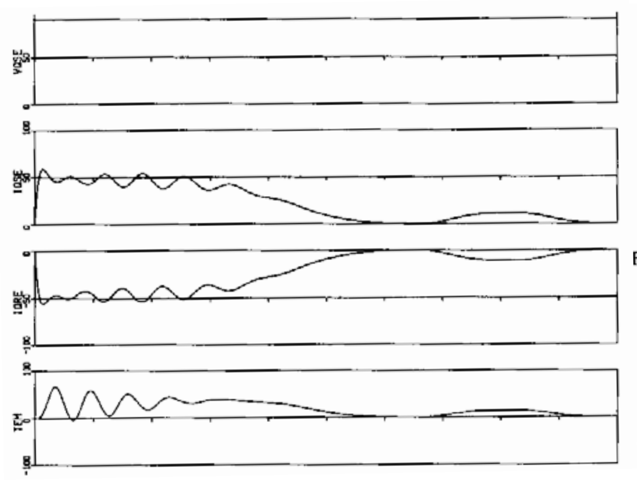
Synchronous RF dq model of a VSI driven IM





ure 3.17 Simulation of an induction machine accelerating from rest with a VSI supply with the system represented in the synchronous reference frame. The inverter frequency is fixed at 30 Hz. VQSE = v_{qs}^{e} (V), IDSE = i_{ds}^{e} (A), IQRE = $i_{dr}^{e\prime}$ (A), TEM = T_e, (N–m),TLOAD = T_{load} (N–m), SPEED = $(2\omega_r)/P$ rad/s

Synchronous RF dq model of a VSI driven IM
(Neglecting the inverter harmonics)



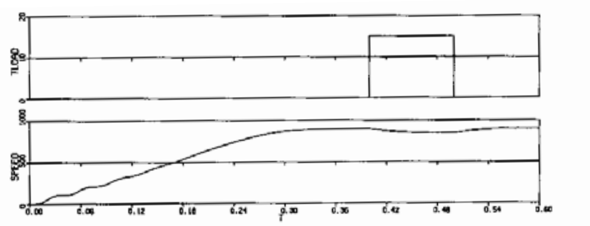
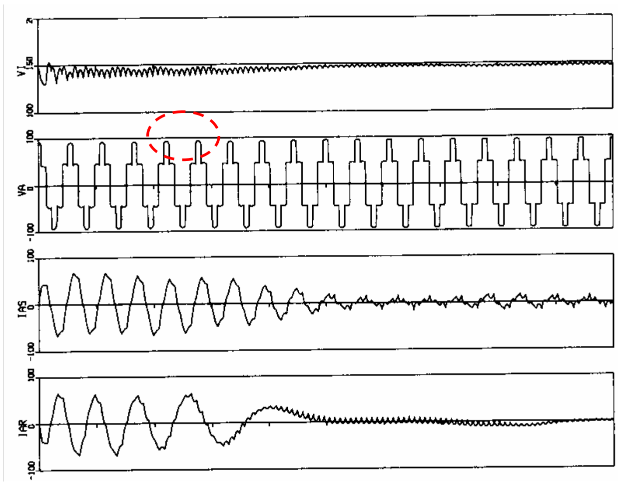


Figure 3.18 Simulation of an induction machine accelerating from rest with a VSI supply with the system represented in the synchronous reference frame neglecting the effect of the inverter harmonics. The inverter frequency is fixed at 30 Hz. VDSE = v_{ds}^{e} (V), VQSE = v_{qs}^{e} (V), IDSE = i_{ds}^{e} (A), IQSE = i_{qs}^{e} (A), IQRE = $i_{qr}^{e\,\prime}$ (A), TEM = T_e (N–m), SPEED = $(2\omega_r)/P$ rad/s

Synchronous RF dq model of a VSI driven IM
(Incorporating the dc link filter)



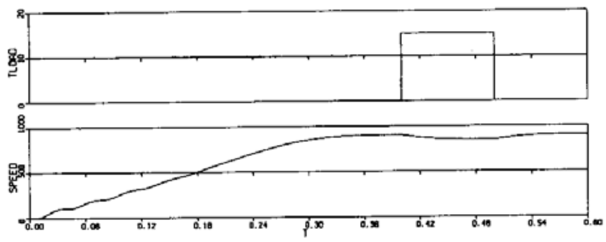
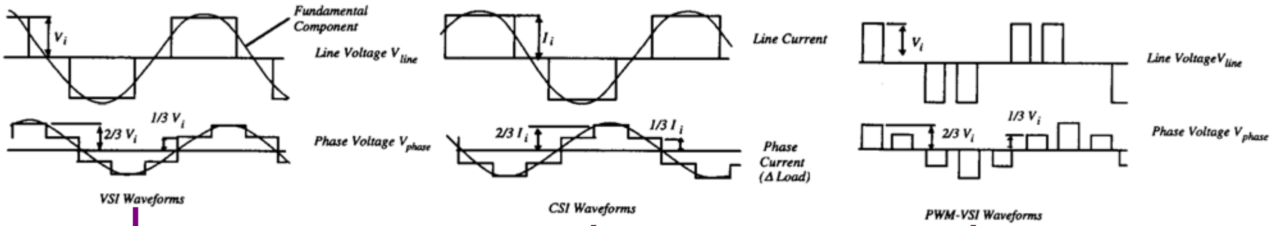


Figure 3.21 Simulation of an induction machine accelerating from rest with an voltage source inverter supply. Simulation incorporates the effect of the dc link filter. The inverter frequency is fixed at 30 Hz. VI = V_i (V), VA = v_{as} = v_{qs}^{s} (V), IA = i_{as} = i_{qs}^{s} (A), IAR = i_{ar}' (A), TEM = T_e (N–m), TLOAD = T_{load} (N–m), SPEED = $(2\omega_r)/P$ (rad/s)

3.7 Fundamental Component Approx. for Steady-State Operation

Fig. 3.24 Output waveforms for basic inverters with constant dc input



$$v_{line}(=v_{bc}) = \frac{2\sqrt{3}}{\pi}V_i\left[\cos\theta - \frac{1}{5}\cos 5\theta - \frac{1}{7}\cos 7\theta + \frac{1}{11}\cos 11\theta + \dots\right] \quad (3.7\text{–}1)$$

$$v_{phase}(=v_{as}) = \frac{2}{\pi}V_i\left[\sin\theta + \frac{1}{5}\sin 5\theta - \frac{1}{7}\sin 7\theta - \frac{1}{11}\sin 11\theta + \dots\right] \quad (3.7\text{–}2)$$

$$i_{line}(=i_{as}) = \frac{2\sqrt{3}}{\pi}I_i\left[\cos\theta - \frac{1}{5}\cos 5\theta - \frac{1}{7}\cos 7\theta + \frac{1}{11}\cos 11\theta + \dots\right] \quad (3.7\text{–}3)$$

$$i_{phase}(=i_{bc}) = \frac{2}{\pi}I_i\left[\sin\theta + \frac{1}{5}\sin 5\theta - \frac{1}{7}\sin 7\theta - \frac{1}{11}\sin 11\theta + \dots\right] \quad (3.7\text{–}4)$$

ai: depends on the *mi*

$$v_{phase} = \frac{2}{\pi}V_i\left[a_1\sin\theta_1 + \frac{a_5}{5}\sin\theta_5 - \frac{a_7}{7}\sin\theta_7 + \frac{a_{11}}{11}\sin\theta_{11} + \dots\right] \quad (3.7\text{–}5)$$

Input-output relations of a inverter

$$V_iI_i = 3V_{phase,rms}I_{line,rms}\cos\phi, \quad \phi = \text{power factor angle} \quad (3.7\text{–}12)$$

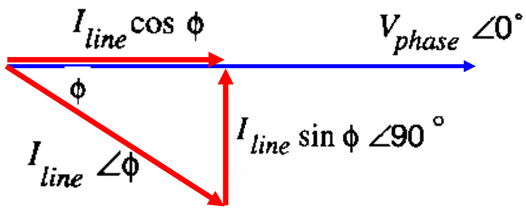
Six step VSI	*Six step CSI*	*PWM-VSI*
$V_{phase,rms} = \frac{\sqrt{2}}{\pi}V_i$ (3.7–7)	$I_{line,rms} = \frac{\sqrt{6}}{\pi}I_i$ (3.7–8)	$V_{phase,rms} = \frac{\sqrt{2}}{\pi}a_1V_i$ (3.7–11)
$I_i = \frac{3\sqrt{2}}{\pi}I_{line,rms}\cos\phi$ (3.7–13)	$V_i = \frac{3\sqrt{6}}{\pi}V_{phase,rms}\cos\phi$ (3.7–14)	$I_i = \frac{3\sqrt{2}}{\pi}a_1I_{line,rms}\cos\phi$ (3.7–15)

Equivalent circuit of a six step VSI

$$V_{phase,\,rms} = \frac{\sqrt{2}}{\pi} V_i \qquad (3.7\text{–}7)$$

$$I_i = \frac{3\sqrt{2}}{\pi} I_{line,\,rms} \cos\phi \qquad (3.7\text{–}13)$$

$$I_{line,\,rms} \cos\phi = \frac{\pi}{3\sqrt{2}} I_i \qquad (3.7\text{–}17)$$



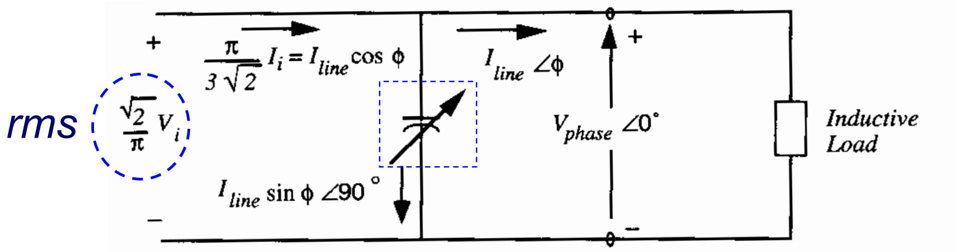


Figure 3.25 Partial per phase fundamental component equivalent circuit of three phase six step VSI

Equivalent circuit of a six step VSI

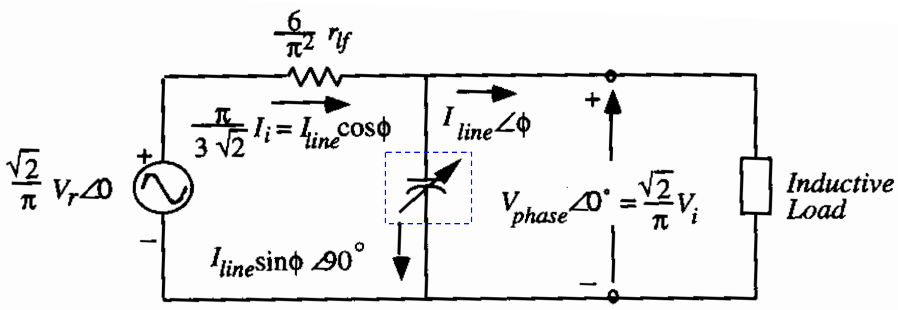


Figure 3.26 Per phase fundamental component equivalent circuit of three phase six step VSI

Equivalent circuit of a six step CSI

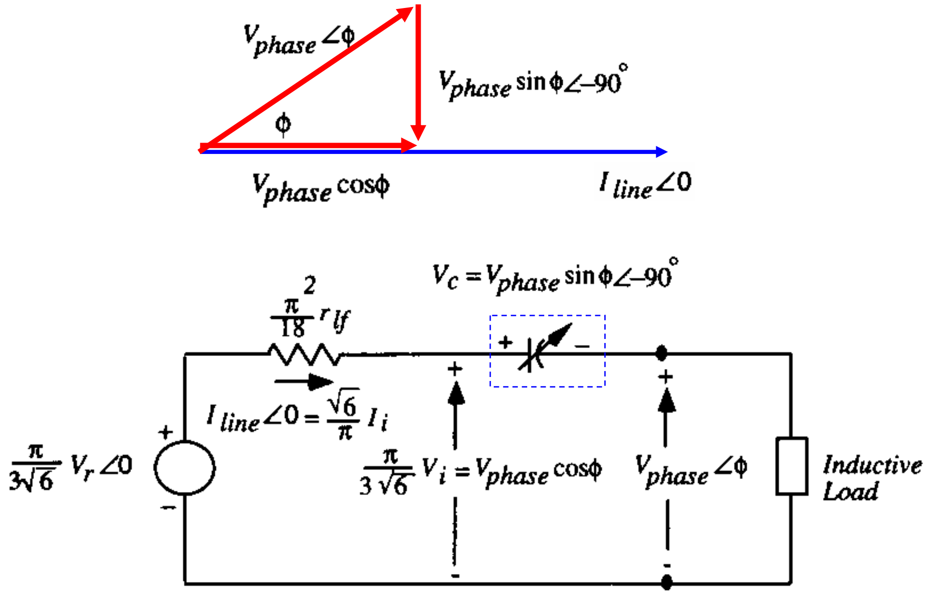


Figure 3.27 Per phase fundamental component equivalent circuit of three phase six step CSI

Equivalent circuit of a PWM-VSI

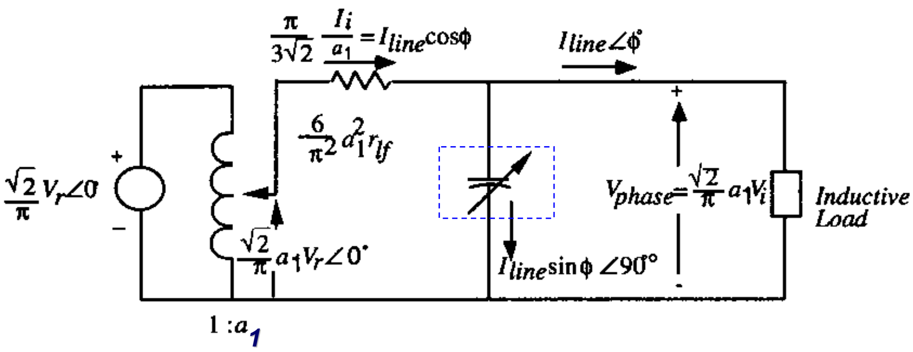


Figure 3.28 Per phase fundamental component equivalent circuit of the three phase PWM–VSI inverter

3.8 Duality of VSI and CSI Systems

Table 3.1 Duality of VSI and CSI Systems

VSI	CSI
1) Output is constrained voltage	1) Output is constrained current
2) dc bus dominated by shunt capacitor	2) dc bus dominated by series inductor
3) dc bus current proportional to motor power and hence dependent on motor power factor	3) dc bus voltage proportional to motor power and hence dependent on motor power factor
4) Output contains voltage harmonics varying inversely as harmonic order	4) Output contains current harmonics varying inversely as harmonic order
5) Prefers motors with larger leakage reactance	5) Prefers motors with lower leakage reactance
6) Can handle motors smaller than inverter rating	6) Can handle motors larger than inverter rating
7) dc bus current reverses in regeneration	7) dc bus voltage reverses in regeneration
8) Immune to open circuits	8) Immune to short circuits