Electric Machine Control

Chapter 6

Dynamics of Vector Control and Field Orientation

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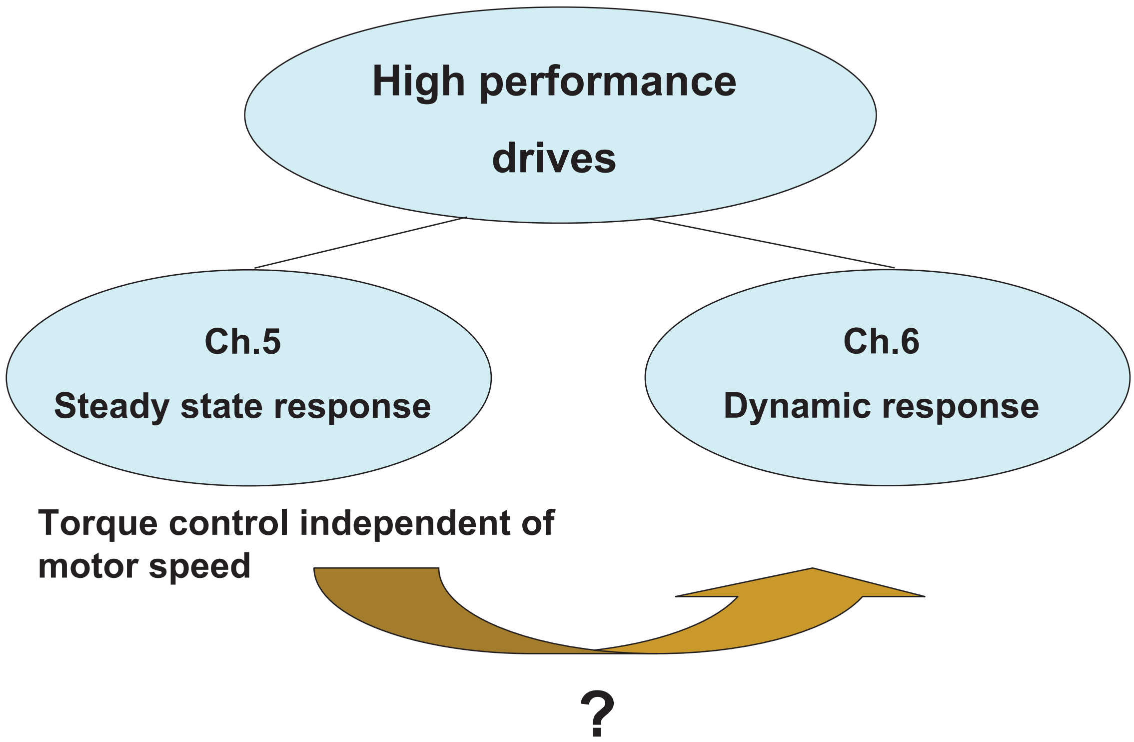
6.1 Introduction



6.2 Dynamics of IM Field Orientation

- Steady state concepts in Chapter 5 will now be examined using a full d,q variable, transient state model
  - The only essential difference between them is the existence of a **significant lag** in the response of the flux to a flux command

6.2.1 Induction machine d,q model with axis rotation at an angular velocity ω_e

$$v_{qs}^e = r_s i_{qs}^e + p\lambda_{qs}^e + \omega_e \lambda_{ds}^e \quad (6.2\text{–}1)$$

$$v_{ds}^e = r_s i_{ds}^e + p\lambda_{ds}^e - \omega_e \lambda_{qs}^e \quad (6.2\text{–}2)$$

$$0 = r_r i_{qr}^e + p\lambda_{qr}^e + (\omega_e - \omega_r)\lambda_{dr}^e \quad (6.2\text{–}3)$$

$$0 = r_r i_{dr}^e + p\lambda_{dr}^e - (\omega_e - \omega_r)\lambda_{qr}^e \quad (6.2\text{–}4)$$

$$T_e = \frac{3}{2}\frac{P}{2}\frac{L_m}{L_r}(\lambda_{dr}^e i_{qs}^e - \lambda_{qr}^e i_{ds}^e) \quad (6.2\text{–}5)$$

where

$$\lambda_{ds}^e = L_{ls} i_{ds}^e + L_m (i_{ds}^e + i_{dr}^e) \quad (6.2\text{–}6)$$

$$\lambda_{qs}^e = L_{ls} i_{qs}^e + L_m (i_{qs}^e + i_{qr}^e) \quad (6.2\text{–}7)$$

$$\lambda_{dr}^e = L_{lr} i_{dr}^e + L_m (i_{ds}^e + i_{dr}^e) \quad (6.2\text{–}8)$$

$$\lambda_{qr}^e = L_{lr} i_{qr}^e + L_m (i_{qs}^e + i_{qr}^e) \quad (6.2\text{–}9)$$

6.2.2 Rotor flux referred equations

Torque and flux control concepts

- Currents supplied to the machine should be oriented in phase and in quadrature to the rotor flux vector
  - Choosing ω_e to be the instantaneous speed of λ_{qdr}
  - Locking the phase of the reference system such that the rotor flux is entirely in the d-axis

$$\lambda_{qr}^e = 0 \qquad (6.2\text{–}10)$$

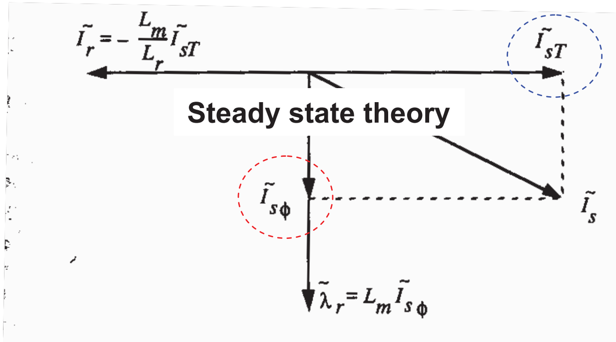


Figure 5.22 Phasor diagram showing torque and flux components of current

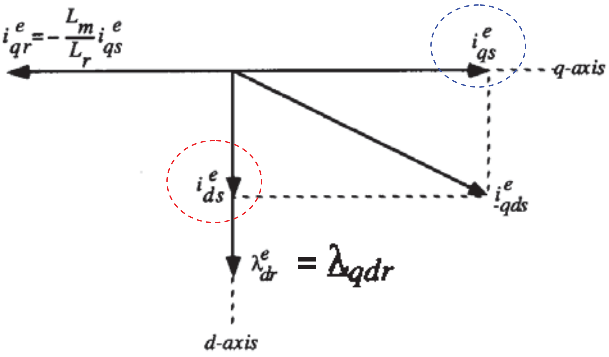


Figure 6.1 d,q currents with reference axes oriented to the rotor flux

- **Assume the machine is supplied from a CSI**
  - ✓**the stator equations can be omitted**
  - ✓**the d,q eqs. in the rotor flux oriented (field oriented) frame become:**

Dynamic response of a field oriented IM

$$0 = r_r i_{qr}^e + (\omega_e - \omega_r)\lambda_{dr}^e \qquad (6.2\text{–}11)$$

$$0 = r_r i_{dr}^e + p\lambda_{dr}^e \qquad (6.2\text{–}12)$$

$$T_e = \frac{3}{2}\frac{P}{2}\frac{L_m}{L_r}(\lambda_{dr}^e i_{qs}^e) \qquad (6.2\text{–}14)$$

$$\lambda_{qr}^e = L_m i_{qs}^e + L_r i_{qr}^e = 0 \qquad (6.2\text{–}13)$$

6.2.3 Dynamic response of field oriented IM

$$\boxed{\begin{aligned} 0 &= r_r i_{qr}^e + (\omega_e - \omega_r)\lambda_{dr}^e \quad (6.2\text{–}11) \\ 0 &= r_r i_{dr}^e + p\lambda_{dr}^e \quad (6.2\text{–}12) \end{aligned}}$$

$$T_e = \frac{3}{2}\frac{P}{2}\frac{L_m}{L_r}(\lambda_{dr}^e i_{qs}^e) \qquad (6.2\text{–}14) \quad \Rightarrow \quad \propto i_{qs}^e$$

$$\lambda_{qr}^e = L_m i_{qs}^e + L_r i_{qr}^e = 0 \quad (6.2\text{–}13) \quad \Rightarrow \quad \boxed{i_{qr}^e = -\frac{L_m}{L_r} i_{qs}^e} \qquad (6.2\text{–}15)$$

$$0 = r_r i_{qr}^e + (\omega_e - \omega_r)\lambda_{dr}^e \quad (6.2\text{–}11) \quad \Rightarrow \quad \omega_e - \omega_r = S\omega_e = -\frac{r_r i_{qr}^e}{\lambda_{dr}^e} \qquad (6.2\text{–}16)$$

$$\boxed{S\omega_e = \frac{r_r}{L_r}\frac{L_m i_{qs}^e}{\lambda_{dr}^e}} \qquad (6.2\text{–}17)$$

$$\lambda_{dr}^e = L_{lr} i_{dr}^e + L_m(i_{ds}^e + i_{dr}^e) \quad (6.2\text{–}8) \quad \Rightarrow \quad i_{dr}^e = \frac{\lambda_{dr}^e - L_m i_{ds}^e}{L_r} \quad (6.2\text{–}18)$$

$$0 = r_r i_{dr}^e + p\lambda_{dr}^e \quad (6.2\text{–}12)$$

$$\boxed{(r_r + L_r p)\, i_{dr}^e = -L_m p i_{ds}^e} \quad (6.2\text{–}20) \qquad \boxed{(r_r + L_r p)\,\lambda_{dr}^e = r_r L_m i_{ds}^e} \quad (6.2\text{–}19)$$

i_{dr}^e exists only when i_{ds}^e is changing

λ_{dr}^e rises only when i_{ds}^e is increasing

$$\lambda_{dr}^e = L_m i_{ds}^e \qquad \text{(steady state)} \qquad (6.2\text{–}21)$$

Time constant $\tau_r = \dfrac{L_r}{r_r}$ (6.2–22)

Inputs : i_{ds}^{e} / i_{qs}^{e}

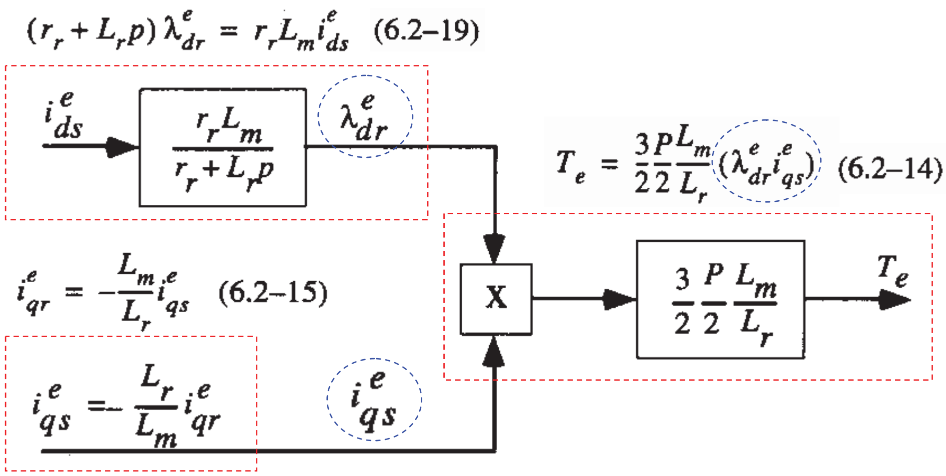


Figure 6.2 Torque production for field orientation in terms of rotor flux and q–axis stator current

Inputs : i_{ds}^{e} / i_{qs}^{e}

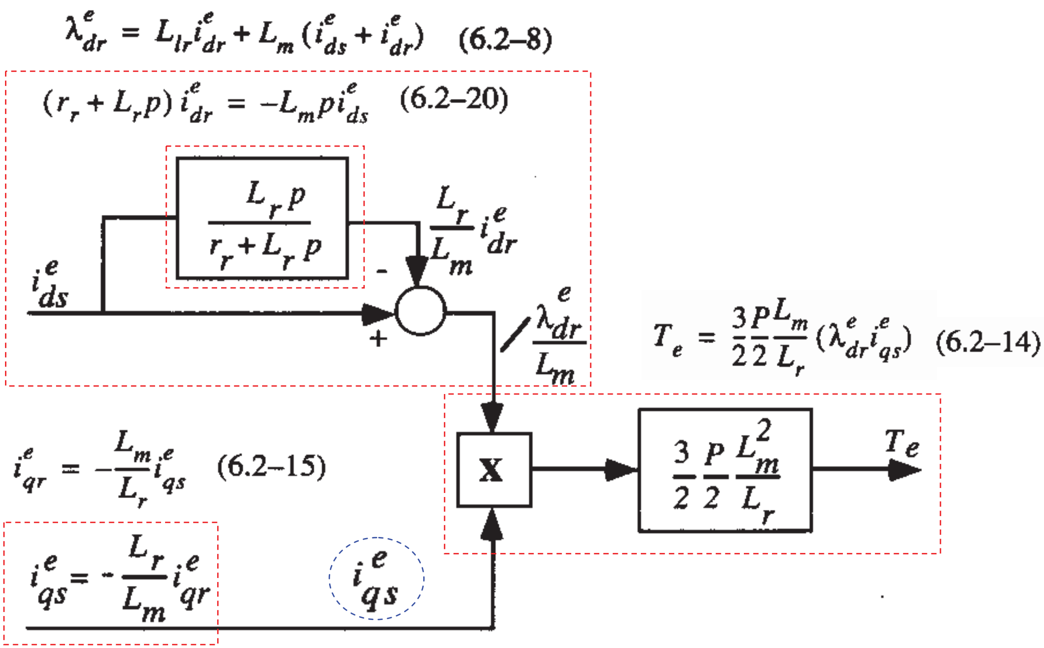


Figure 6.3 Torque production for field orientation in terms of currents

$$T_e = \frac{3}{2}\frac{P}{2}\frac{L_m}{L_r}(\lambda_{dr}^e i_{qs}^e) \quad (6.2\text{–}14)$$

(1) Change in torque command

$$\boxed{i_{qr}^e = -\frac{L_m}{L_r} i_{qs}^e} \quad (6.2\text{–}15)$$

(2) Change in flux command

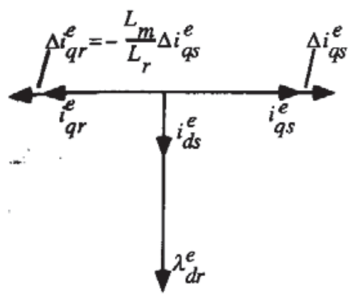
$$\boxed{(r_r + L_r p)\, i_{dr}^e = -L_m p i_{ds}^e} \quad (6.2\text{–}20)$$

i_{dr}^e exists only when i_{ds}^e is changing

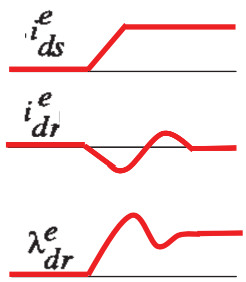
$$\boxed{(r_r + L_r p)\, \lambda_{dr}^e = r_r L_m i_{ds}^e} \quad (6.2\text{–}19)$$

λ_{dr}^e rises only when i_{ds}^e is increasing

λ_{dr}^e =const : $\Delta i_{qr}^e \rightarrow \Delta i_{qs}^e \rightarrow \Delta T_e$



$\left.\begin{matrix} i_{qr}^e \\ i_{qs}^e \end{matrix}\right\}$ =const : $\Delta i_{ds}^e \left\{\begin{matrix} \rightarrow \Delta i_{dr}^e & \text{decays to zero in } \tau_r \\ \rightarrow \Delta\lambda_{dr}^e & \text{rises in } \tau_r \end{matrix}\right.$



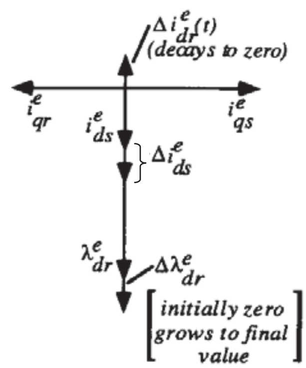
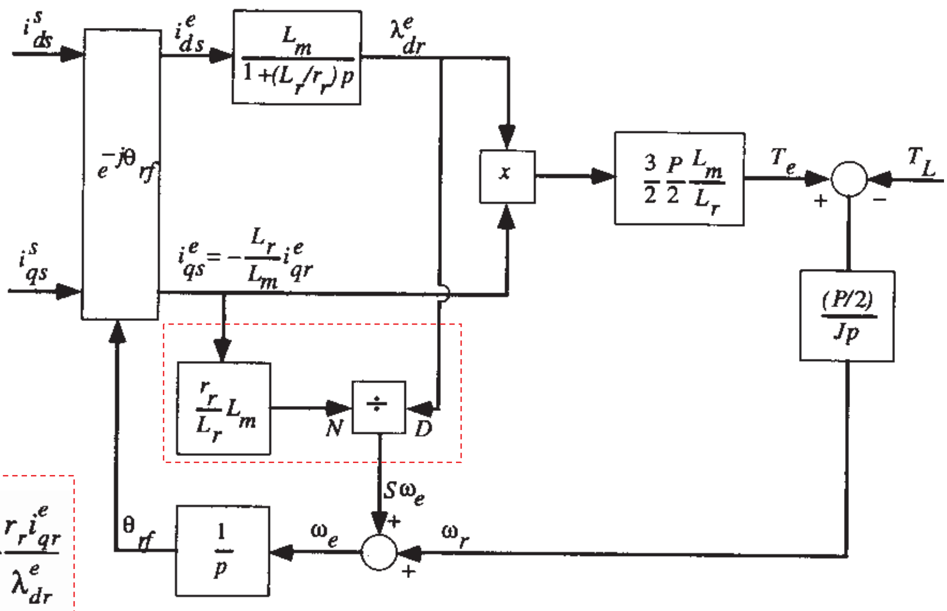


Figure 6.4 Illustration of response to step changes in torque command and flux command

6.2.4 Block diagram of field oriented IM



$$\omega_e - \omega_r = S\omega_e = -\frac{r_r i_{qr}^e}{\lambda_{dr}^e} \quad (6.2\text{–}16)$$

Figure 6.5 Complete block diagram of current fed induction machine in rotor flux oriented reference frame (field orientation)

6.3 Indirect Controllers for IM Field Orientation

- Indirect Controller
  - Based on steady state considerations
  - Field orientation will not be properly maintained during transients which involve changes in the flux level
- Goal : correctly handles flux variations

6.3.1 Indirect controller with i_{ds}^{e*} and i_{qs}^{e*} as inputs

- Modifies the slip calculator to maintain field orientation during flux changes

slip relation

$$\boxed{S\omega_e = \frac{r_r L_m i_{qs}^e}{L_r \lambda_{dr}^e}} \qquad (6.2\text{–}17)$$

Flux-current relation

$$\boxed{(r_r + L_r p)\lambda_{dr}^e = r_r L_m i_{ds}^e} \qquad (6.2\text{–}19)$$

lag

$$\lambda_{dr}^e = \frac{L_m}{1 + p\tau_r} i_{ds}^e \qquad (6.3\text{–}1)$$

$$S\omega_e^* = \frac{\left(\frac{1}{\hat{\tau}_r}\right) i_{qs}^{e*}}{\left(\frac{1}{1 + p\hat{\tau}_r}\right) i_{ds}^{e*}} \qquad (6.3\text{–}2)$$

- To maintain field orientation the **lag** in the flux response must be incorporated in a nonlinear slip calculator based on entirely on current commands

Uncompensated flux response controller (for CRPWM inverter)

- The flux response follows $\boxed{(r_r + L_r p)\lambda^e_{dr} = r_r L_m i^e_{ds}}$ (6.2–19)
- During the transient period following a change in i^{e*}_{ds}, the lag element delays the influence of the change in i^{e*}_{ds} to match the actual change in flux.

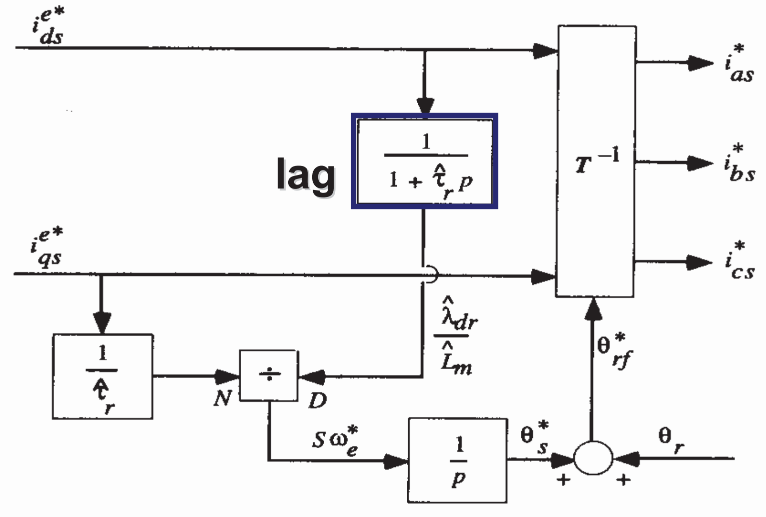


Figure 6.6 Indirect field orientation controller using input current commands (uncompensated flux response)

6.3.2 Indirect controller with λ^{e*}_{dr} and i^{e*}_{qs} as inputs

- Controller must calculate the correct value of i^{e*}_{ds} as well as the slip

Flux-current relation

$$\boxed{(r_r + L_r p)\lambda^e_{dr} = r_r L_m i^e_{ds}} \quad (6.2\text{–}19)$$

lead

$$i^{e*}_{ds} = \frac{1}{\hat{L}_m}(1 + p\hat{\tau}_r)\lambda^{e*}_{dr} \quad (6.3\text{–}3)$$

slip relation

$$\boxed{S\omega_e = \frac{r_r L_m i^e_{qs}}{L_r \lambda^e_{dr}}} \quad (6.2\text{–}17)$$

Compensated flux response controller (for CRPWM inverter)

- The changes in flux command immediately alter the slip and also give rise to a compensation component of i_{ds}^{e*}
- Feedforward the proper θ_{rf}^{*} to control λ_{dr}^{e} and the torque (via i_{ds}^{e*}) in the presence of a system disturbance

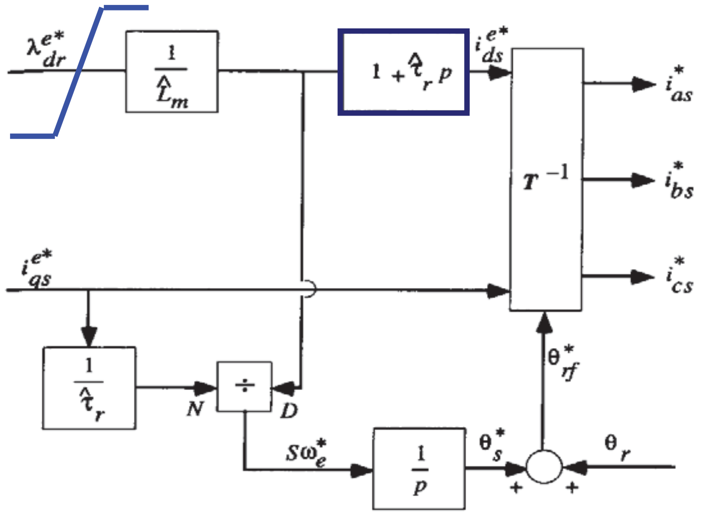


Figure 6.7 Indirect field orientation controller using flux and torque current commands (compensated flux response)

6.3.3 Other indirect controllers

- (ex-1) Patterns after Fig. 6.6
  - Employment of the **measured currents** in the slip calculator rather than the command currents
    - Extending the field orientation mode into the region where the current regulator is reaching its practical limits
- (ex-2) Insertion of a limiter after the lead compensator for i_{ds}^{e*} in Fig. 6.7
  - **a lag element** in the flux branch of the slip calculator is then required to maintain correct field orientation

Ex-2 (for CRPWM inverter)

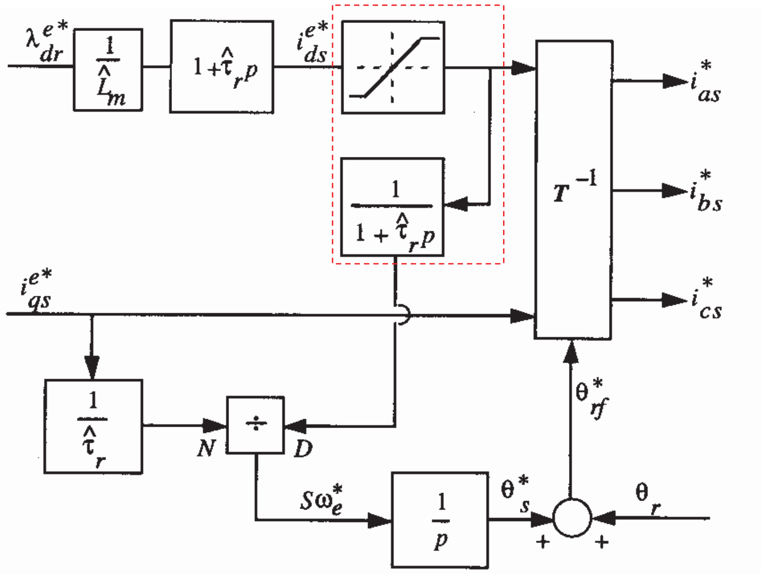
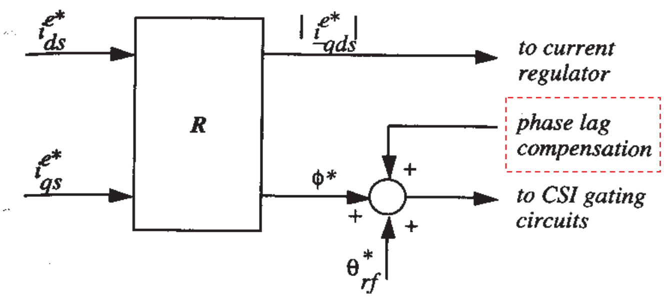


Figure 6.8 Indirect field orientation controller using flux and torque current commands with flux command current limiter

6.3.4 Indirect controller using a CSI



For commutation delay

well suited to digital implementation

Figure 6.9 CSI implementation of field oriented control (replaces T^{-1} block of CRPWM systems)

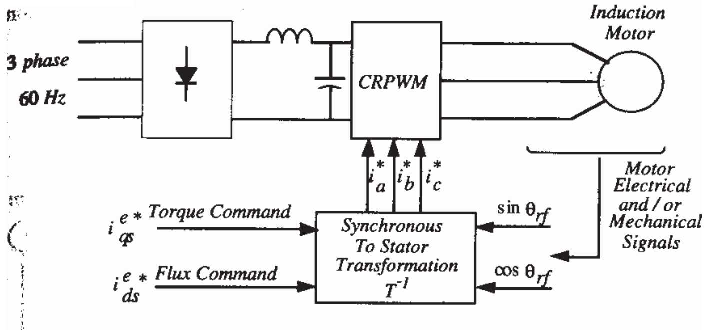


Figure 5.26 Basic induction motor field orientation system using a CPRWM

6.3.5 Indirect field orientation start up transient

- Issue of the start up transient
  - Buildup of the rotor flux during the start-up period
  - Correct field orientation with the indirect from a start with no initial rotor flux
- The error angle, $\Delta\theta_{rf}$, will become zero when the machine reach the correct field orientation
  - The **transient flux build up process** is independent of **rotor speed** and thus the analysis can be carried out at zero speed for simplicity

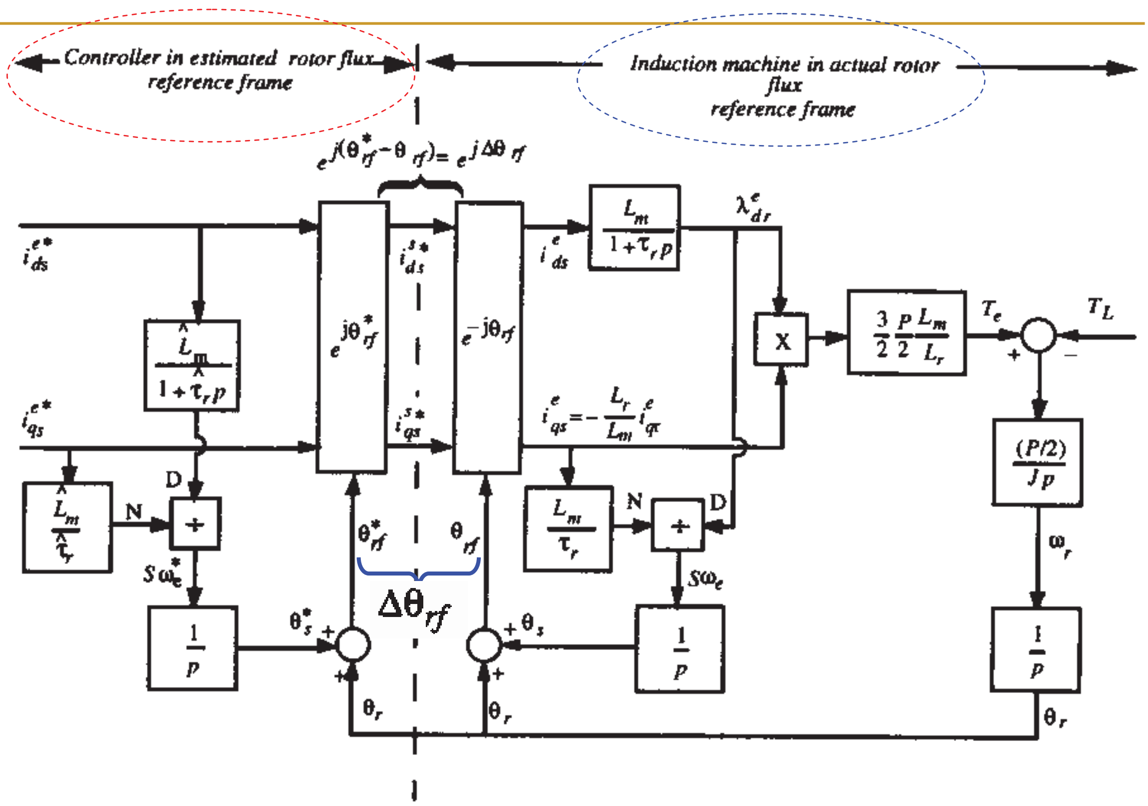


Figure 6.10 Induction machine indirect field orientation system for current source excitation showing error angle $\Delta\theta_{rf}$

Case 1: sudden application of flux command only, $\beta = 0$

define $\beta = i_{qs}^{e*} / i_{ds}^{e*}$ (6.3–4) $(i_{qs}^{e*} = 0)$

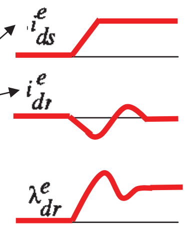
slip relation

$$\left\{\begin{array}{l} i_{qs}^{e*} = 0 \\ \lambda_{dr}^{e*} = 0 \end{array}\right. \Rightarrow \boxed{S\omega_e = \frac{r_r L_m i_{qs}^e}{L_r \lambda_{dr}^e} \quad (6.2\text{–}17)} \Rightarrow S\omega_e = 0/0$$

$$\left\{\begin{array}{l} i_{qs}^{e*} = 0 \\ \lambda_{dr}^{e*} \neq 0 \end{array}\right. \Rightarrow \boxed{S\omega_e = \frac{r_r L_m i_{qs}^e}{L_r \lambda_{dr}^e} \quad (6.2\text{–}17)} \Rightarrow S\omega_e = 0/\text{finite} = 0$$

Dc excitation

- With zero slip frequency the applied dc current remains on the d-axis and the rotor flux builds up according to the open ckt. time const and aligned with the flux command current.
- The build up of the flux is associated with the induced rotor current which continuous oppose the stator dc current and dies out as the flux grows.



Case 2: sudden application of flux and torque command, $\beta \neq 0$

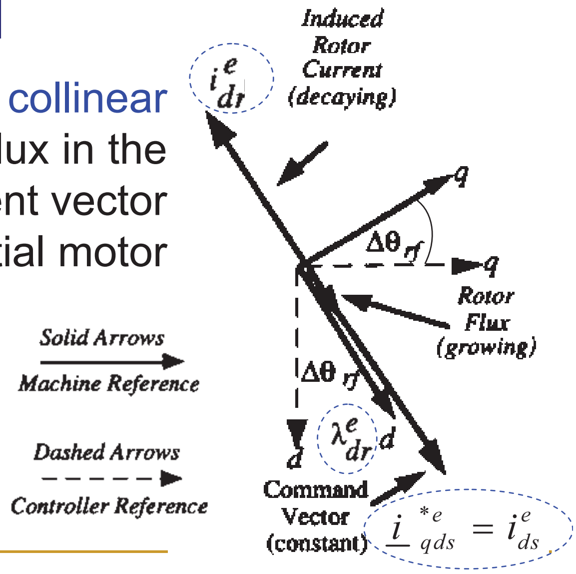
-- Step-1 --

- At the first instant the imposed stator current excites a nearly equal and opposite rotor current vector

$$\boxed{(r_r + L_r p)\, i_{dr}^e = -L_m p i_{ds}^e \qquad (6.2\text{–}20)}$$

- The difference between the two collinear current vectors creates a small flux in the initial direction of the stator current vector which therefore becomes the initial motor d-axis

$$\boxed{i_{dr}^e = \frac{\lambda_{dr}^e - L_m i_{ds}^e}{L_r} \qquad (6.2\text{–}18)}$$



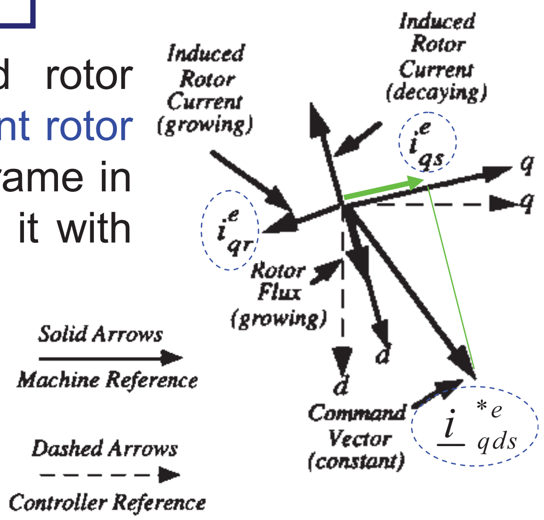
Case 2: sudden application of flux and torque command, $\beta \neq 0$

-- Step-2 --

- The established flux interacts with the commanded slip frequency to create a rotor voltage and an induced rotor current 90° away in space from this initial d-axis

$$\boxed{0 = r_r i_{qr}^e + (\omega_e - \omega_r)\lambda_{dr}^e \quad (6.2\text{–}11)}$$

- Stator current and the induced rotor current tends to rotate the resultant rotor flux and hence the machine ref. frame in the direction to more nearly align it with the controller ref. frame

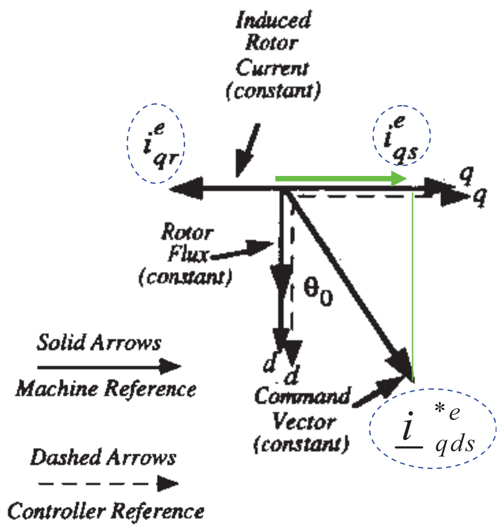


Case 2: sudden application of flux and torque command, $\beta \neq 0$

-- Step-3 --

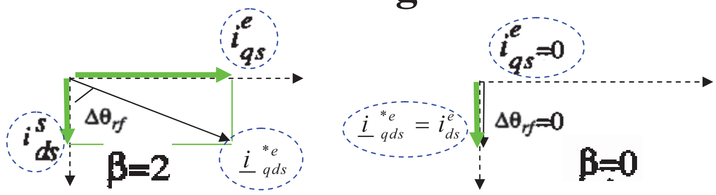
- At the end of the transient period the machine ref. frame is aligned with the controller ref. frame and only the q-axis rotor current remains

- The actual transient can be overdamped or underdamped depending on the value of β and the transient time is governed by the rotor open ckt. time const



Transient responses:

1. Varying ratios of i_{qs}/i_{ds} (β=0, 1/2, 1, 1.5 and 2) and with a const 1.0 pu flux command
2. The rotor speed was held at zero throughout the simulations



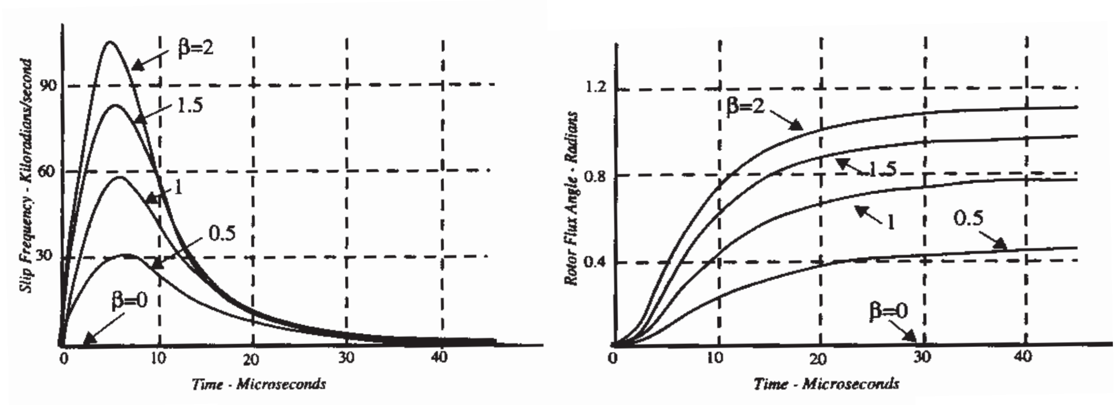


Figure 6.12 "Switching transient" associated with invalid initial conditions in simulation of field orientation start up transient, $\beta = i^{e\,*}_{qs} / i^{e\,*}_{ds}$

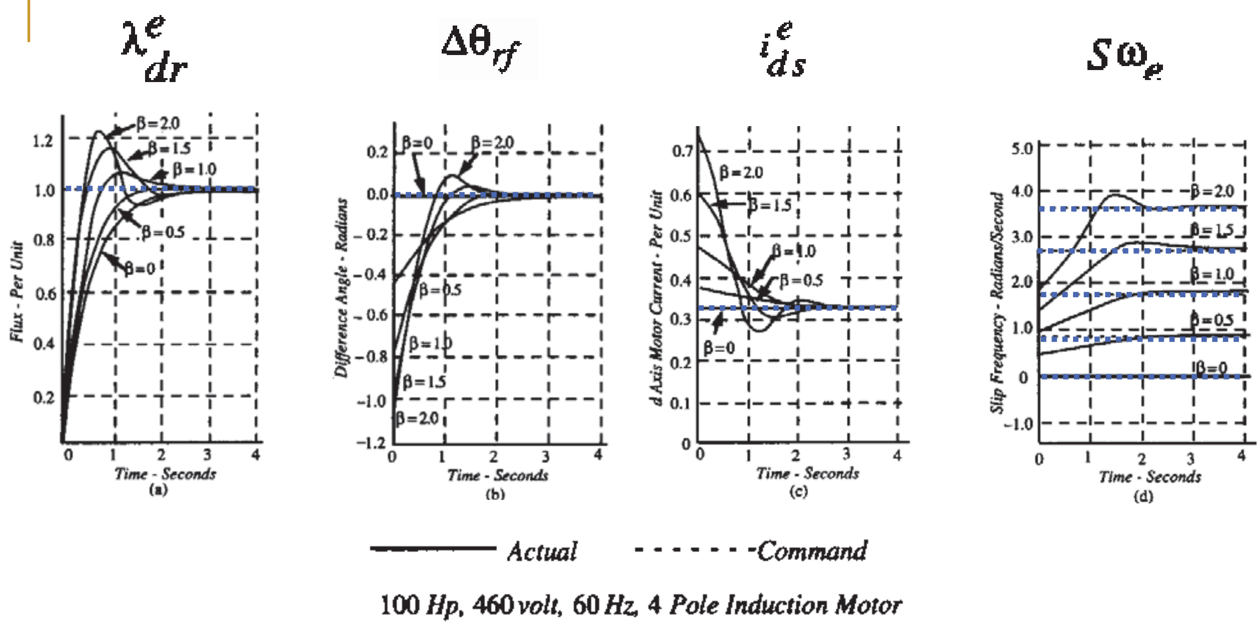


Figure 6.13 Indirect field orientation start up transient – 100 hp machine

6.4 Direct Controllers for IM Field Orientation

- Direct controller
  - Measurement or calculation of the **rotor flux angle** directly from machine electrical variables
    - The measurement of rotor speed or rotor position is replaced by other electrical quantities (slip relation is no longer directly employed)
      - Loss of the direct information of a significant disturbance

slip relation　　　　rotor position signal

- Limits on the scheme which required **only the voltage and current quantities**
  - At low speed
  - Parameters dependencies

6.4.1 Direct determination of rotor flux angle

➢Two steps required to measure the flux angle

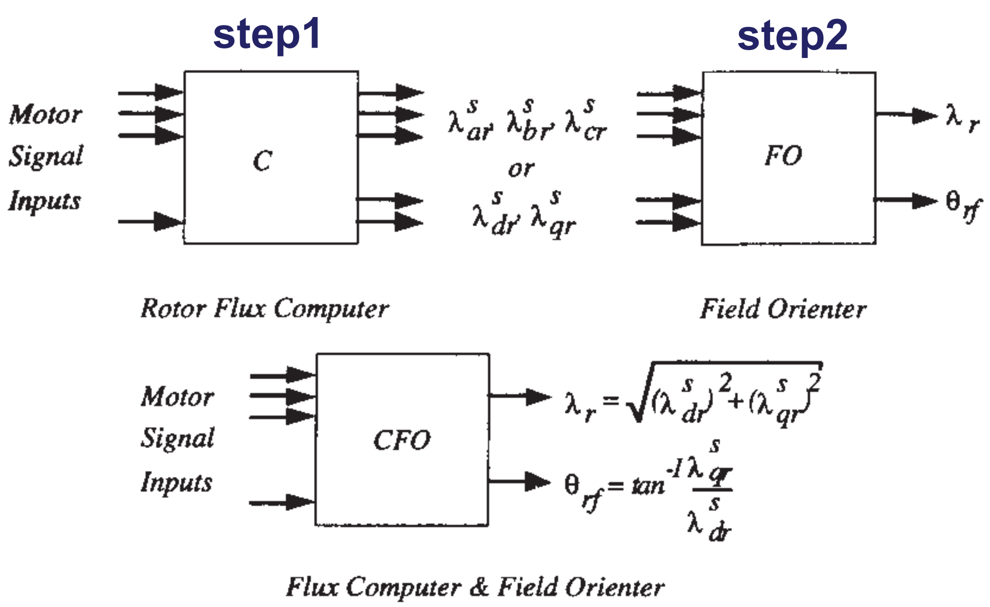


Figure 6.14 Rotor flux computer and field orientation

6.4.2 Measurement of air gap flux (sensing by flux sensing coils or Hall elements)

$$\lambda_{qdr}^{s} = L_m i_{qds}^{s} + L_r \boxed{i_{qdr}^{s}} \quad (6.4\text{–}1)$$

$$\lambda_{qdm} = L_m (i_{qds}^{s} + \boxed{i_{qdr}^{s}}) \quad (6.4\text{–}2)$$

$$\lambda_{qdr}^{s} = \frac{L_r}{L_m}\lambda_{qdm}^{s} - (L_r - L_m)\, i_{qds}^{s} \quad (6.4\text{–}3)$$

$$= \frac{L_r}{L_m}\lambda_{qdm}^{s} - L_{lr} i_{qds}^{s} \quad (6.4\text{–}4)$$

$$\lambda_{qdr}^{s} = \frac{L_r}{L_m}\lambda_{qdm}^{s} - L_{lr}\, i_{qds}^{s}$$

$$\lambda_r = |\lambda_{qdr}^{s}|$$

$$\theta_{rf} = Ang\,\lambda_{qdr}^{s}$$

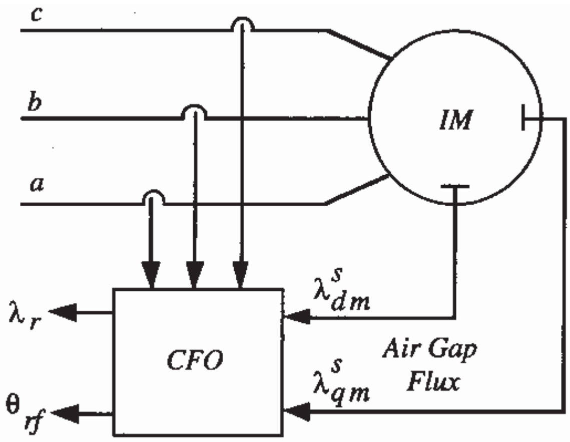


Figure 6.15 Field angle determination using flux sensors

6.4.2 Measurement of air gap flux (sensing by flux sensing coils or Hall elements)

$$\boxed{\begin{aligned}\lambda_{qdr}^{s} &= \frac{L_r}{L_m}\lambda_{qdm}^{s} - (L_r - L_m)\, i_{qds}^{s} \quad (6.4\text{–}3)\\ &= \frac{L_r}{L_m}\lambda_{qdm}^{s} - L_{lr} i_{qds}^{s} \quad (6.4\text{–}4)\end{aligned}}$$

- **Advantages:**
  - ✓Requiring only two motor parameters
  - ✓The rotor leakage inductance (except for closed slot rotor) is substantially a constant value independent of temperature
  - ✓Lr/Lm is only moderately affected by saturation of the main flux paths
- **Disadvantages:**
  - ✓Need for special sensing elements: integrating signals at low frequencies (near zero speed)… only Hall sensors can provide useful signals near zero speed
  - ✓Closed rotor slots: rotor leakage inductance is strongly dependent on rotor current, especially at low values of rotor current

6.4.3 Voltage and current sensing

$$\underline{\lambda}^s_{qds} = L_s \underline{i}^s_{qds} + L_m \boxed{\underline{i}^s_{qdr}} \quad (6.4\text{–}7) \quad \Rightarrow \quad \boxed{\underline{i}^s_{qdr}} = \frac{\underline{\lambda}^s_{qds} - L_s \underline{i}^s_{qds}}{L_m} \quad (6.4\text{–}8)$$

$$\underline{\lambda}^s_{qdr} = L_m \underline{i}^s_{qds} + L_r \boxed{\underline{i}^s_{qdr}} \quad (6.4\text{–}1)$$

$$\begin{aligned}
\underline{\lambda}^s_{qdr} &= L_m \underline{i}^s_{qds} + \frac{L_r}{L_m}(\underline{\lambda}^s_{qds} - L_s \underline{i}^s_{qds}) \\
&= \frac{L_r}{L_m}\left[\underline{\lambda}^s_{qds} - \left(L_s - \frac{L_m^2}{L_r}\right)\underline{i}^s_{qds}\right] \\
&= \frac{L_r}{L_m}(\underline{\lambda}^s_{qds} - L_s' \underline{i}^s_{qds}) \qquad (6.4\text{–}9)
\end{aligned}$$

$$\underline{v}^s_{qds} = r_s \underline{i}^s_{qds} + p\underline{\lambda}^s_{qds} \quad (6.4\text{–}5) \qquad \underline{\lambda}^s_{qds} = \frac{1}{p}(\underline{v}^s_{qds} - r_s \underline{i}^s_{qds}) \quad (6.4\text{–}6)$$

$$p\,\underline{\lambda}^s_{qds} = \underline{v}^s_{qds} - r_s \underline{i}^s_{qds}$$

$$\underline{\lambda}^s_{qdr} = \frac{L_r}{L_m}\left[\underline{\lambda}^s_{qds} - L_s' \underline{i}^s_{qds}\right]$$

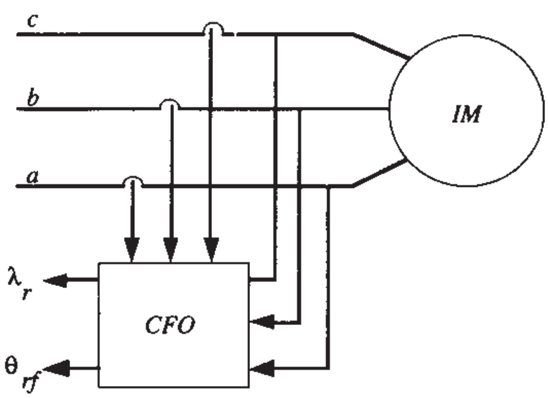


Figure 6.16 Field angle determination from terminal voltage and current

Difficulty for voltage and current sensing scheme

➢Difficulty : Need for three motor parameters : r_s, L_s', and L_r/L_m

✓r_s :
- temperature dependence
- stator IR drop becomes dominant at low speed (at low frequency).... Low speed limitations

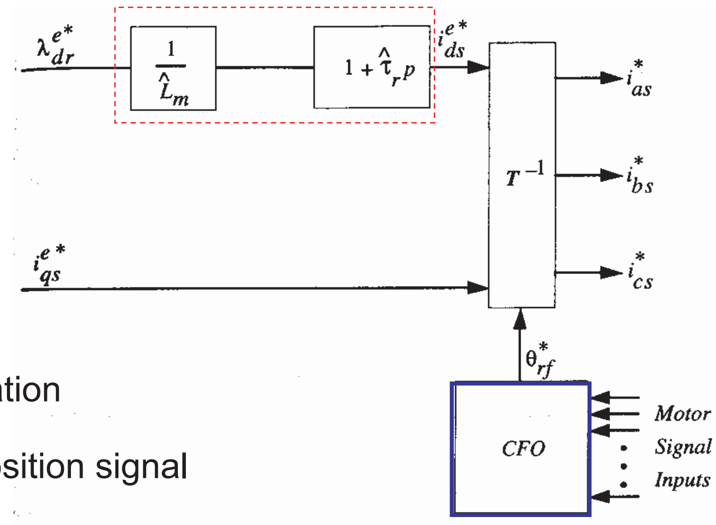
✓L_s' :
- becomes strongly dependent on rotor current for the closed slot rotor

✓L_r/L_m :
- only moderately affected by saturation of the main flux paths

6.4.4 Implementation of direct field orientation

Flux-current relation

$$\boxed{(r_r + L_r p)\lambda^e_{dr} = r_r L_m i^e_{ds}} \quad (6.2\text{–}19) \quad \Rightarrow \quad i^{e*}_{ds} = \frac{1}{\hat{L}_m}(1 + p\hat{\tau}_r)\lambda^{e*}_{dr} \quad (6.3\text{–}3)$$



✗slip relation

✗rotor position signal

Figure 6.18 Direct field orientation scheme

Omitting lead element in the flux compensator

If the CFO can provide **rotor flux amplitude**

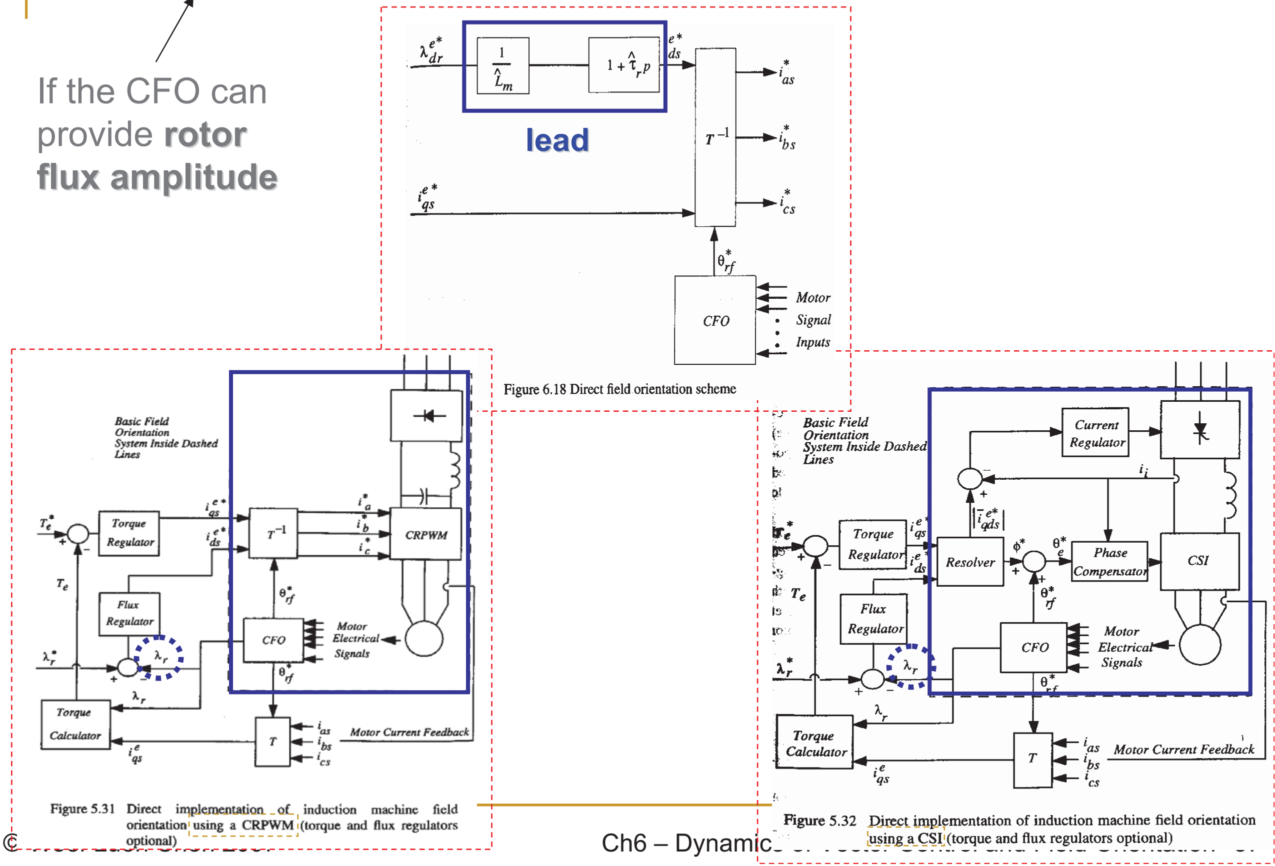


Figure 6.18 Direct field orientation scheme

Figure 5.31 Direct implementation of induction machine field orientation using a CRPWM (torque and flux regulators optional)

Figure 5.32 Direct implementation of induction machine field orientation using a CSI (torque and flux regulators optional)

Closed loop torque regulator
: combines the flux amplitude, angle with current feedback

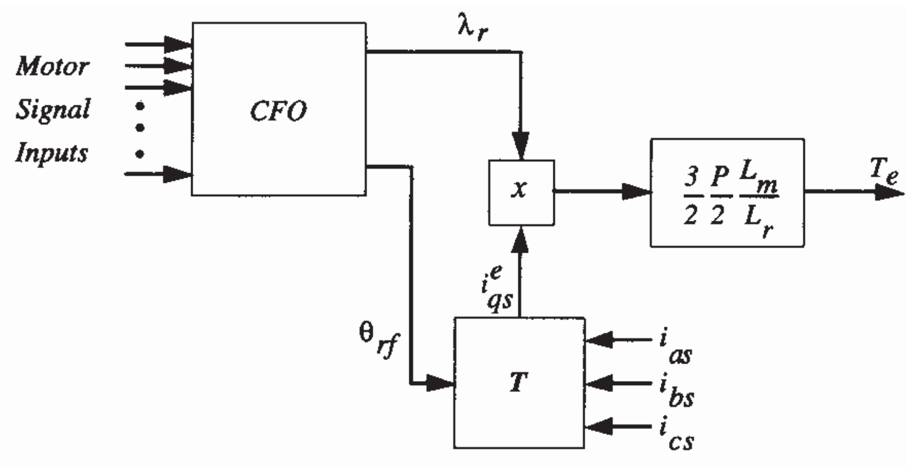


Figure 6.19 Torque computation using CFO output and current feedback

Direct field orientation controller

- Although Hall sensor can provide useful signals near zero speed
  - ✓Employing it in the air gap are considered to be unreliable
  - ✓Direct field orientation has not been widely applied for servo drives where zero operation is necessary
    - If operation near zero speed is not required, direct field orientation using voltage and current sensing is very attractive

6.5 IM Field Orientation Using Air Gap Flux

- Field orientation
  - Instantaneous torque control is attained by controlling the spatial angle between the rotor flux and torque command current
- Air gap flux based field orientation
  - Directly measurable (air gap flux)
  - Suitable for treating saturation effects
- Stator flux based field orientation
  - See section 6.8

6.5.1 Air gap flux referred d,q equations

$$\lambda_{ds}^e = L_{ls} i_{ds}^e + L_m (i_{ds}^e + i_{dr}^e) \quad (6.2\text{–}6)$$

$$\lambda_{qs}^e = L_{ls} i_{qs}^e + L_m (i_{qs}^e + i_{qr}^e) \quad (6.2\text{–}7)$$

$$\lambda_{dr}^e = L_{lr} i_{dr}^e + L_m (i_{ds}^e + i_{dr}^e) \quad (6.2\text{–}8)$$

$$\lambda_{qr}^e = L_{lr} i_{qr}^e + L_m (i_{qs}^e + i_{qr}^e) \quad (6.2\text{–}9)$$

defining:

$$\begin{cases} \lambda_{qm}^e = L_m (i_{qs}^e + i_{qr}^e) & (6.5\text{–}1) \\ \lambda_{dm}^e = L_m (i_{ds}^e + i_{dr}^e) & (6.5\text{–}2) \end{cases}$$

$$\Rightarrow \begin{cases} \lambda_{qds}^e = \lambda_{qdm}^e + L_{ls} i_{qds}^e & (6.5\text{–}3) \\ \lambda_{qdr}^e = \lambda_{qdm}^e + L_{lr} i_{qdr}^e & (6.5\text{–}4) \end{cases}$$

$$T_e = \frac{3}{2}\frac{P}{2}\frac{L_m}{L_r}(\lambda_{dr}^e i_{qs}^e - \lambda_{qr}^e i_{ds}^e) \quad (6.2\text{–}5)$$

$$\Rightarrow \quad T_e = \frac{3}{2}\frac{P}{2}(\lambda_{dm}^e i_{qs}^e - \lambda_{qm}^e i_{ds}^e) \quad (6.5\text{–}7)$$

Air gap flux based field orientation

$$T_e = \frac{3}{2}\frac{P}{2}(\lambda_{dm}^e i_{qs}^e - \lambda_{qm}^e i_{ds}^e) \quad (6.5\text{–}7)$$

$$\boxed{\lambda_{qm}^e = 0 \quad (6.5\text{–}8)}$$

(Air gap flux based field orientation)

- ***main*** **or air gap flux referred d,q equations**

$$0 = r_r i_{qr}^m + pL_{lr} i_{qr}^m + (\omega_e - \omega_r)(\lambda_{dm}^m + L_{lr} i_{dr}^m) \quad (6.5\text{–}9)$$

$$0 = r_r i_{dr}^m + p(\lambda_{dm}^m + L_{lr} i_{dr}^m) - (\omega_e - \omega_r) L_{lr} i_{qr}^m \quad (6.5\text{–}10)$$

$$\lambda_{qm}^m = L_m (i_{qs}^m + i_{qr}^m) = 0 \quad (6.5\text{–}11)$$

$$\lambda_{dm}^m = L_m (i_{ds}^m + i_{dr}^m) \quad (6.5\text{–}12)$$

$$T_e = \frac{3}{2}\frac{P}{2}\lambda_{dm}^m i_{qs}^m \quad (6.5\text{–}13)$$

$$i_{qr}^m = -i_{qs}^m \quad (6.5\text{–}14)$$

$$i_{dr}^m = \frac{\lambda_{dm}^m}{L_m} - i_{ds}^m \quad (6.5\text{–}15)$$

6.5.2 Dynamic response of air gap flux controlled IM

- *representation of slip relation in terms of stator currents*

$$\begin{cases} i_{qr}^{m} = -i_{qs}^{m} & (6.5\text{–}14) \\ i_{dr}^{m} = \dfrac{\lambda_{dm}^{m}}{L_m} - i_{ds}^{m} & (6.5\text{–}15) \\ 0 = r_r \boxed{i_{qr}^{m}} + pL_{lr}\boxed{i_{qr}^{m}} + (\omega_e - \omega_r)(\lambda_{dm}^{m} + L_{lr}\boxed{i_{dr}^{m}}) & (6.5\text{–}9) \end{cases}$$

$$\Rightarrow \omega_e - \omega_r = S\omega_e = \frac{(r_r + L_{lr}p)\, i_{qs}^{m}}{\lambda_{dm}^{m} + L_{lr}\left(\dfrac{\lambda_{dm}^{m}}{L_m} - i_{ds}^{m}\right)} \qquad (6.5\text{–}16)$$

$$= \frac{(r_r + L_{lr}p)\, i_{qs}^{m}}{\dfrac{L_r}{L_m}\lambda_{dm}^{m} - L_{lr} i_{ds}^{m}} \qquad (6.5\text{–}17)$$

Dynamic response of air gap flux controlled IM

- *representation of flux relation in terms of stator currents*

$$\begin{cases} i_{qr}^{m} = -i_{qs}^{m} & (6.5\text{–}14) \\ i_{dr}^{m} = \dfrac{\lambda_{dm}^{m}}{L_m} - i_{ds}^{m} & (6.5\text{–}15) \\ 0 = r_r \boxed{i_{dr}^{m}} + p(\lambda_{dm}^{m} + L_{lr}\boxed{i_{dr}^{m}}) - (\omega_e - \omega_r) L_{lr}\boxed{i_{qr}^{m}} & (6.5\text{–}10) \end{cases}$$

$$\boxed{p\lambda_{dm}^{m}} = -(r_r + L_{lr}p)\boxed{\left(\frac{\lambda_{dm}^{m}}{L_m}\right.} \left. - i_{ds}^{m}\right) - S\omega_e L_{lr} i_{qs}^{m} \qquad (6.5\text{–}18)$$

$$p\lambda_{dm}^{m} = -\frac{r_r}{L_r}\lambda_{dm}^{m} + \frac{L_m}{L_r}(r_r + L_{lr}p)\, i_{ds}^{m} - S\omega_e \frac{L_{lr}L_m}{L_r} i_{qs}^{m} \qquad (6.5\text{–}19)$$

Coupling between the slip relation and flux relation

- flux relation and slip relation for rotor flux oriented system are decoupled equs.

  In air gap flux oriented system, flux relation and slip relation are coupled equs.

rotor flux oriented system

$$\begin{cases} S\omega_e = \dfrac{r_r L_m i_{qs}^e}{L_r \lambda_{dr}^e} & (6.2\text{–}17) \\ (r_r + L_r p)\lambda_{dr}^e = r_r L_m i_{ds}^e & (6.2\text{–}19) \end{cases}$$

air gap flux oriented system

$$\begin{cases} S\omega_e = \dfrac{(r_r + L_{lr} p)\, i_{qs}^m}{\dfrac{L_r}{L_m}\lambda_{dm}^m - L_{lr} i_{ds}^m} & (6.5\text{–}17) \\ p\lambda_{dm}^m = -\dfrac{r_r}{L_r}\lambda_{dm}^m + \dfrac{L_m}{L_r}(r_r + L_{lr} p)\, i_{ds}^m - S\omega_e \dfrac{L_{lr} L_m}{L_r} i_{qs}^m & (6.5\text{–}19) \end{cases}$$

Coupling between the slip relation and flux relation

- Because of coupling, the torque production block diagram must include both slip and flux relations

  Rewriting (6.5.17)

$$\begin{cases} \lambda_{dm}^m = \dfrac{L_m}{1 + \dfrac{L_r}{r_r} p}\left[\left(1 + p\dfrac{L_{lr}}{r_r}\right) i_{ds}^m - S\omega_e \dfrac{L_{lr}}{r_r} i_{qs}^m\right] & (6.5\text{–}20) \\ S\omega_e = \dfrac{(r_r + L_{lr} p)\, i_{qs}^m}{\dfrac{L_r}{L_m}\lambda_{dm}^m - L_{lr} i_{ds}^m} & (6.5\text{–}17) \end{cases}$$

$(r_r + L_r p)\lambda_{dr}^e = r_r L_m i_{ds}^e$ (6.2–19)

$T_e = \frac{3}{2}\frac{P}{2}\frac{L_m}{L_r}(\lambda_{dr}^e i_{qs}^e)$ (6.2–14)

$i_{qr}^e = -\frac{L_m}{L_r} i_{qs}^e$ (6.2–15)

$i_{qs}^e = -\frac{L_r}{L_m} i_{qr}^e$

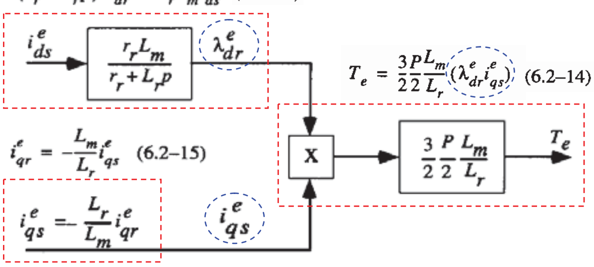


Figure 6.2 Torque production for field orientation in terms of rotor flux and q–axis stator current

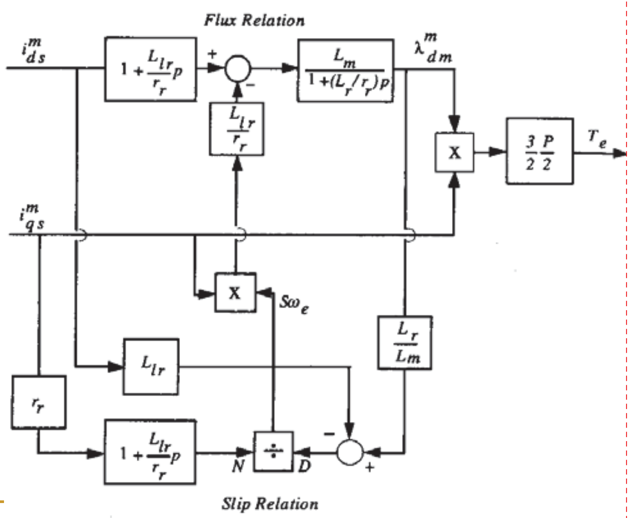


Figure 6.20 Torque production in air gap flux controlled induction machine

Coupling between the slip relation and flux relation

- Instantaneous torque control is still reachable for the air gap oriented system by **maintaining the air gap flux constant**

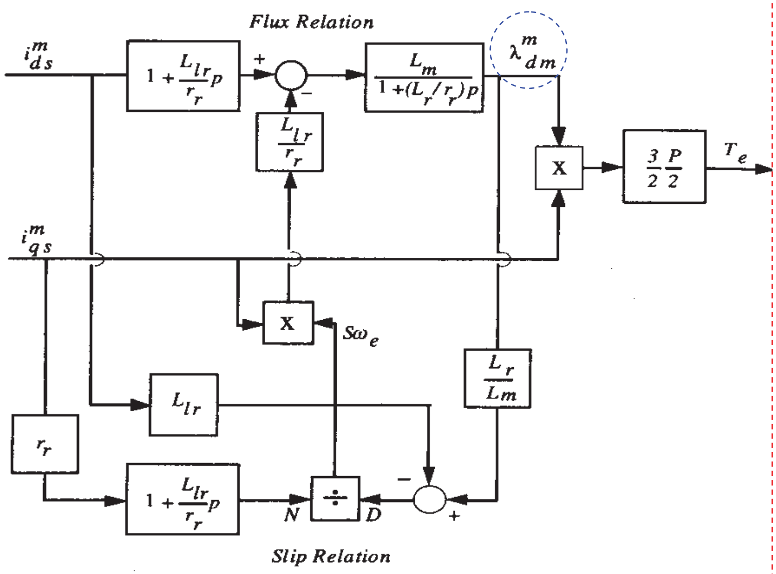


Figure 6.20 Torque production in air gap flux controlled induction machine

6.5.3 Steady state interpretation of air gap flux control

- Dynamic relations

$$S\omega_e = \frac{(r_r + L_{lr}p)\, i_{qs}^m}{\frac{L_r}{L_m}\lambda_{dm}^m - L_{lr} i_{ds}^m} \quad (6.5\text{–}17)$$

$$\lambda_{dm}^m = \frac{L_m}{1 + \frac{L_r}{r_r}p}\left[\left(1 + p\frac{L_{lr}}{r_r}\right) i_{ds}^m - S\omega_e \frac{L_{lr}}{r_r} i_{qs}^m\right] \quad (6.5\text{–}20)$$

- Steady state relations

$$S\omega_e = \frac{r_r I_{qs}^m}{\frac{L_r}{L_m}\lambda_{dm}^m - L_{lr} I_{ds}^m} \quad (6.5\text{–}21)$$

$$\lambda_{dm}^m = L_m I_{ds}^m - S\omega_e \frac{L_{lr}}{r_r} L_m I_{qs}^m \quad (6.5\text{–}22)$$

- Even in the steady state, the coupling persists

- The d-axis current I_{ds}^{m} does not independently control the flux as was the case for rotor flux control

…. the rotor current has a d-axis component because of the leakage inductance of the rotor as shown in Fig. 6.21

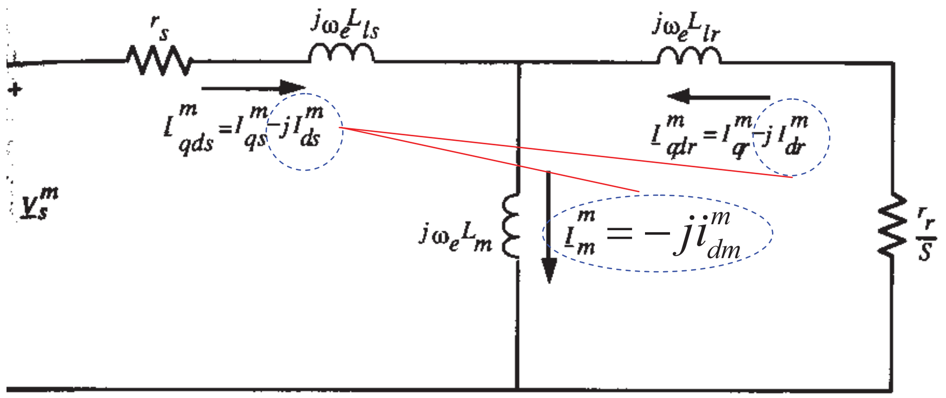


Figure 6.21 Steady state complex vector equivalent circuit for air gap flux control

- The d-axis rotor current increases as the slip frequency increases and hence the d-axis stator current <u>must</u> also increase if the magnetizing current (and air gap flux) is to remain constant

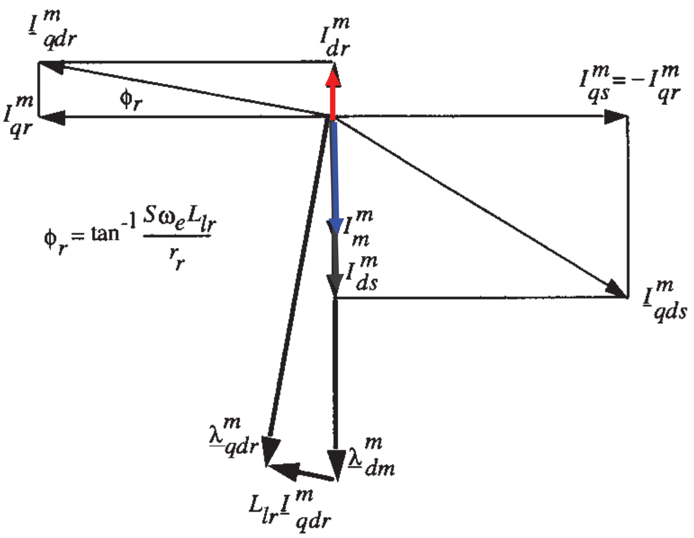


Figure 6.22 Complex vector diagram for air gap flux control

6.5.4 Indirect field orientation controller for air gap flux

- Implementation of indirect controller for air gap flux control:
  - Slip relation
  - Evaluating the d-axis stator current i^{m*}_{ds} to control the flux λ^m_{dm}
    - Considering λ^{m*}_{dm} as an input and solving the flux relation for the required i^{m*}_{ds}

$$
\begin{cases}
S\omega_e = \dfrac{(r_r + L_{lr}p)\, i^m_{qs}}{\dfrac{L_r}{L_m}\lambda^m_{dm} - L_{lr} i^m_{ds}} & (6.5\text{–}17) \\[2ex]
\boxed{\lambda^m_{dm} = \dfrac{L_m}{1+\dfrac{L_r}{r_r}p}\left[\left(1+p\dfrac{L_{lr}}{r_r}\right) \boxed{i^m_{ds}} - S\omega_e \dfrac{L_{lr}}{r_r} i^m_{qs}\right] \quad (6.5\text{–}20)}
\end{cases}
$$

$$
\boxed{i^{m*}_{ds} = \frac{1}{1+\dfrac{\hat{L}_{lr}}{\hat{r}_r}p}\left[\left(1+\frac{\hat{L}_r}{\hat{r}_r}p\right)\frac{\lambda^*_{dm}}{\hat{L}_m} + (S\omega_e)^* \frac{\hat{L}_{lr}}{\hat{r}_r} i^{m*}_{qs}\right] \quad (6.5\text{–}23)}
$$

- **Two additional parameters are required in comparison with rotor flux control :**
  1) leakage time constant
  2) rotor leakage inductance

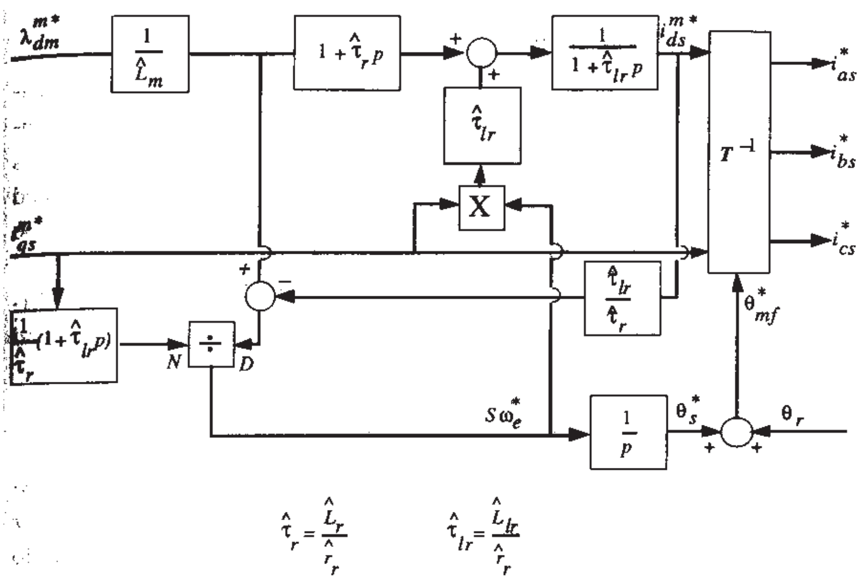


Figure 6.23 Indirect field orientation controller for air gap flux control

6.5.5 Direct field orientation controller for air gap flux

- Indirect controller for **rotor oriented** system

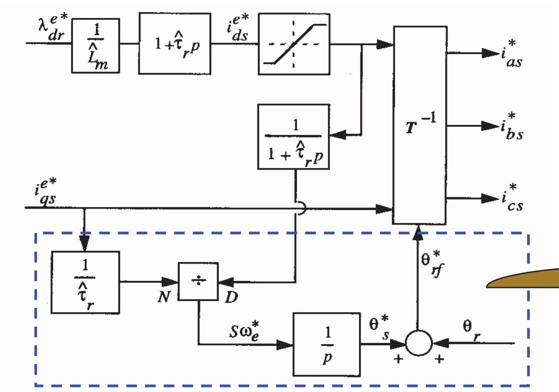


Figure 6.8 Indirect field orientation controller using flux and torque current commands with flux command current limiter

- direct controller for **rotor oriented** system

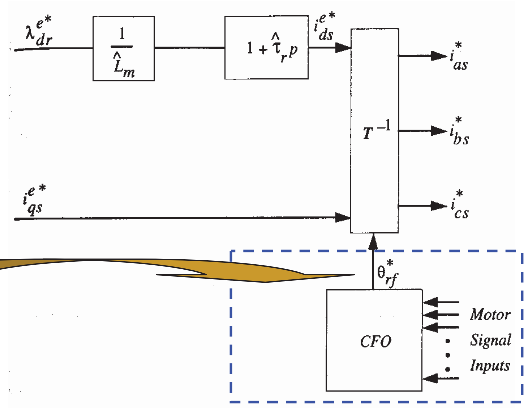


Figure 6.18 Direct field orientation scheme

Direct field orientation controller for air gap flux

- Indirect controller for **rotor oriented** system

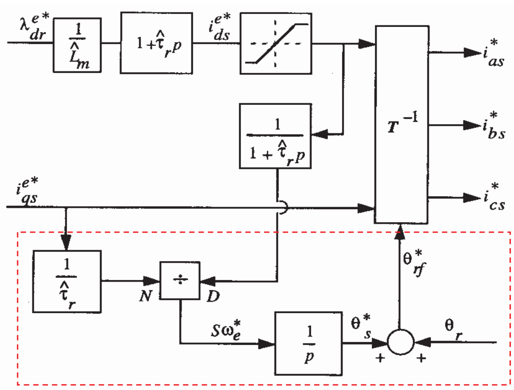
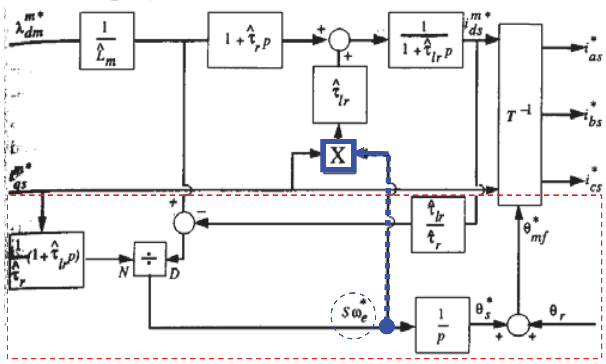


Figure 6.8 Indirect field orientation controller using flux and torque current commands with flux command current limiter

- Indirect controller for **air gap oriented** system



$\hat{\tau}_r = \frac{\hat{L}_r}{\hat{r}_r}$ $\quad \hat{\tau}_{lr} = \frac{\hat{L}_{lr}}{\hat{r}_r}$

Figure 6.23 Indirect field orientation controller for air gap flux control

Direct field orientation controller for air gap flux

- **Direct controller for air gap oriented system:**

  1) slip relation is still required (because of coupling)

  2) the CFO computation no longer requires the rotor leakage inductance

  3) In the case of Hall sensor, no calculation is required at all the air gap flux is obtained simply as

$$\underline{\lambda}_{qdm}^{s} = \underline{\lambda}_{qds}^{s} - L_{ls}\underline{i}_{qds}^{s} \quad (6.5\text{–}24)$$

  4) The rotor leakage inductance does enter slip relation and s-axis stator current calculation, but these computations are less important at light load (low rotor current)

  5) A closed loop flux regulator utilizing the flux magnitude from the CFO as the feedback signal can also reduce the sensitivity of rotor leakage inductance

6.6 Dynamics of Syn.M Vector Control and Field Orientation

- the dynamic response of vector controlled synchronous machines
- the influence on the systems which do not employ the 90° spatial relationship of the **field flux** and **stator MMF** used in field orientation

6.6.1 d,q model of synchronous machine (rotor R.F.)

$$
\left\{
\begin{aligned}
&\boxed{\begin{aligned}\lambda_{qs} &= L_{qs} i_{qs} + L_{mq} i_{qr} && (6.6\text{–}1)\\ \lambda_{ds} &= L_{ds} i_{ds} + L_{md} i_{dr} + L_{md} i_{fr} && (6.6\text{–}2)\end{aligned}}\\
&\lambda_{qr} = L_{qr} i_{qr} + L_{mq} i_{qs} \qquad (6.6\text{–}3)\\
&\lambda_{dr} = L_{dr} i_{dr} + L_{md} i_{ds} + L_{md} i_{fr} \qquad (6.6\text{–}4)\\
&\lambda_{fr} = L_{fr} i_{fr} + L_{md} i_{ds} + L_{md} i_{dr} \qquad (6.6\text{–}5)\\
&T_e = \frac{3}{2}\frac{P}{2}\left(\boxed{\lambda_{ds}} i_{qs} - \boxed{\lambda_{qs}} i_{ds}\right) \qquad (6.6\text{–}6)
\end{aligned}
\right.
$$

induction torque

$$
\Rightarrow \quad T_e = \frac{3}{2}\frac{P}{2}\left[\boxed{L_{md}(i_{fr}+i_{dr})\, i_{qs}} - \boxed{L_{mq} i_{qr} i_{ds}} + \boxed{(L_{ds}-L_{qs})\, i_{ds} i_{qs}}\right] \qquad (6.6\text{–}7)
$$

reluctance torque (saliency torque)

q–axis damper winding torque

field plus d–axis damper winding torque

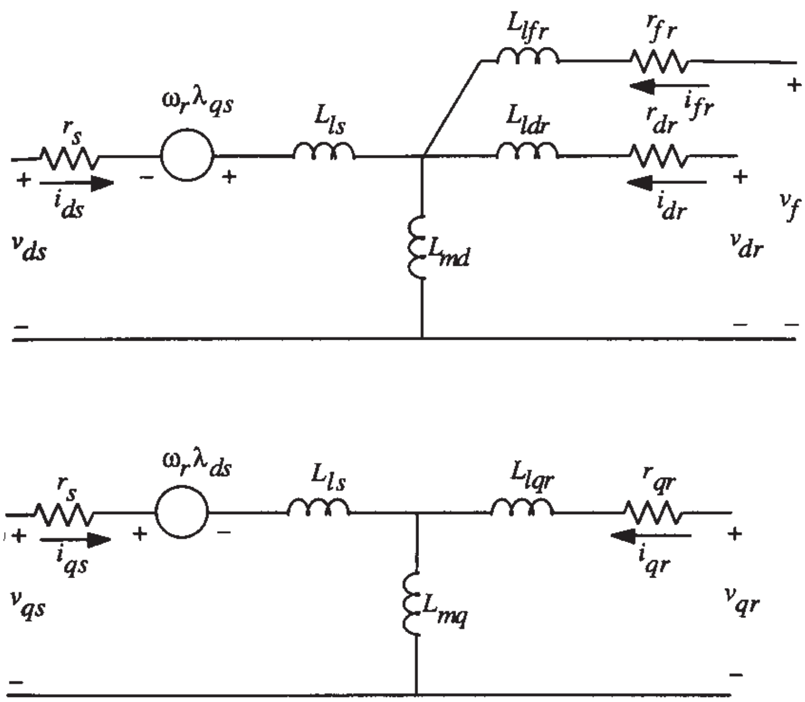


Figure 6.24 Equivalent circuits of three phase salient pole synchronous machine

6.6.2 Vector control and angle control

- Concept of **rotor position feedback** and **vector control**
  - Maintain the space angle between the field winding and stator MMF results
- Concept of **angle control** and field orientation
  - The controlled current supply to the machine maintains this condition for transient changes in machine speed as well as steady state conditions
  - Instantaneous control of **the phase of the stator current** to always maintain the same orientation of the stator MMF vector relative to the field winding in the d-axis

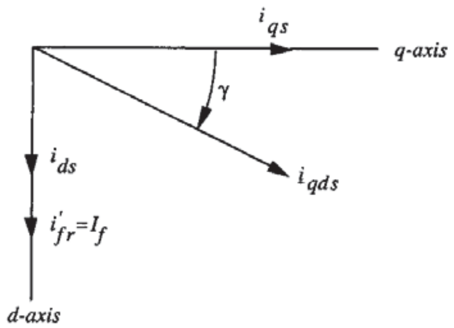


Figure 6.25 Synchronous machine currents in d,q axes

6.6.3 Dynamic of synchronous machine field orientation

- assume

1) the stator current is the independently controlled input variable, and

2) the orientation of the stator d,q currents is maintained for all speeds including transient changes.

- For the case $(\gamma = 0) \Longrightarrow i_{ds} = 0$

q–axis damper circuit

$$r_{qr} i_{qr} + p\lambda_{qr} = 0 \qquad \lambda_{qr} = L_{mq} i_{qs} + L_{qr} i_{qr} \qquad (6.6\text{–}8)$$

d–axis damper circuit

$$r_{dr} i_{dr} + p\lambda_{dr} = 0 \qquad \lambda_{dr} = L_{md} i_{fr} + L_{dr} i_{dr} \qquad (6.6\text{–}9)$$

field circuit

$$r_{fr} i_{fr} + p\lambda_{fr} = v_{fr} \qquad \lambda_{fr} = L_{md} i_{dr} + L_{fr} i_{fr} \qquad (6.6\text{–}10)$$

and the torque expression reduces to

$$T_e = \frac{3}{2}\frac{P}{2} L_{md} (i_{fr} + i_{dr})\, i_{qs} \qquad (6.6\text{–}11)$$

6.6.3.1 Constant field current operation (constant torque region)

• For the case $(\gamma = 0) \Longrightarrow i_{ds} = 0$

field circuit

$$r_{fr}\underbrace{i_{fr}}_{=\text{const}} + \overbrace{p\lambda_{fr}}^{=0} = \underbrace{v_{fr}}_{=\text{const}} \Rightarrow \lambda_{fr} = L_{md}\underbrace{i_{dr}}_{=\text{const}} + L_{fr} i_{fr} \qquad (6.6\text{–}10)$$

d–axis damper circuit

$$r_{dr}\underbrace{i_{dr}}_{=0} + p\lambda_{dr} = 0 \Leftarrow \underbrace{\lambda_{dr}}_{=\text{const}} = L_{md} i_{fr} + L_{dr} i_{dr} \qquad (6.6\text{–}9)$$

$$\Rightarrow i_{dr} = 0 \qquad (6.6\text{–}12)$$

$$i_{fr} = \frac{v_{fr}}{r_{fr}} = I_f \qquad (6.6\text{–}13)$$

$$r_{qr} i_{qr} + pL_{qr} i_{qr} = -pL_{mq} i_{qs} \qquad (6.6\text{–}14)$$

$$\Rightarrow i_{qr} = \frac{-L_{mq}p}{r_{qr} + L_{qr}p} i_{qs} \qquad (6.6\text{–}15)$$

• A change in q-axis stator current will induce a transient q-axis damper current with the initial value

$$\Delta i_{qr}(0) = -\frac{L_{mq}}{L_{qr}} \Delta i_{qs}(0) \qquad (6.6\text{–}16)$$

$$\therefore T_e = \frac{3}{2}\frac{P}{2} L_{md}(i_{fr} + \cancel{i_{dr}})\, i_{qs} \qquad (6.6\text{–}11)$$

$$\Rightarrow T_e = \frac{3}{2}\frac{P}{2} L_{md} I_f i_{qs} \qquad (6.6\text{–}17)$$

:Constant field excitation torque

6.6.3.1 Constant field current operation (constant torque region)

• For the case $(\gamma = 0) \Longrightarrow i_{ds} = 0$

- Constant field excitation torque
  - ☆ The torque response for field orientation is instantaneous and follows the commanded value of **q-axis stator current**

$$T_e = \frac{3}{2}\frac{P}{2} L_{md} I_f \boxed{i_{qs}} \qquad (6.6\text{–}17)$$

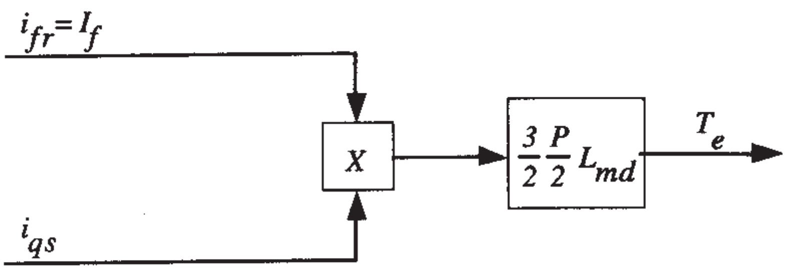


Figure 6.26 Torque production – field orientation $(\gamma = 0)$ with constant field excitation

6.6.3.2 Variable field excitation (field weakening region)

- **For the case** $(\gamma = 0) \Longrightarrow i_{ds} = 0$

- Variable field excitation
  ☆ induce an **d-axis damper current**

d–axis damper circuit

$$r_{dr} i_{dr} + p\lambda_{dr} = 0 \qquad \boxed{\lambda_{dr}} = L_{md} i_{fr} + L_{dr} i_{dr} \qquad (6.6\text{–}9)$$

$$\Longrightarrow \quad i_{dr} = \frac{-L_{md} p}{r_{dr} + L_{dr} p} i_{fr} \qquad (6.6\text{–}18)$$

$$T_e = \frac{3}{2}\frac{P}{2} L_{md} (i_{fr} + \boxed{i_{dr}})\, i_{qs}$$
(6.6–11)

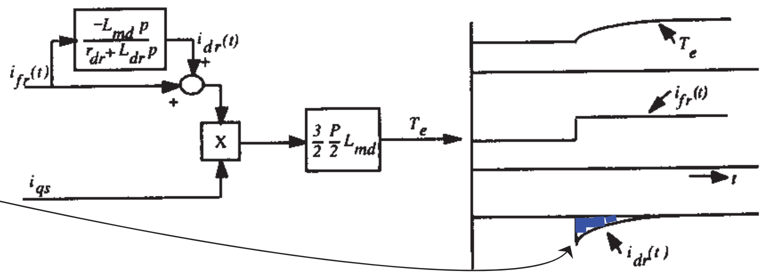


Figure 6.27 Torque production for a change in field current in a field oriented synchronous machine (γ=0,i_{ds}=0)

6.6.4 Dynamic response with $\gamma \neq 0$ (angle control)

$\searrow \; i_{ds} \neq 0$

- **(Case 1)**
  Consider the case with constant d-axis stator current

$$i_{ds} = I_{ds} \qquad (6.6\text{–}19)$$

$$(6.6\text{–}7) \Longrightarrow \quad T_e = \frac{3}{2}\frac{P}{2}\left[L_{md}(i_{fr} + i_{dr})\, i_{qs} - L_{mq} i_{qr} I_{ds} + (L_{ds} - L_{qs})\, I_{ds} i_{qs}\right]$$

- **Negative d-axis stator current (leading pf compensation):**
  : a lag and a reduction in the torque response resulting from an increase in q-axis stator current

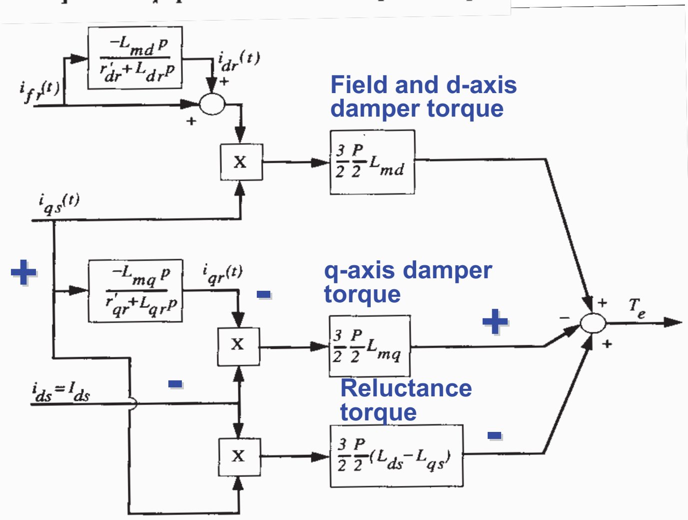


Figure 6.28 Torque production with $\gamma \neq 0$ for the case of constant I_{ds}

Dynamic response with $\gamma \neq 0$ (angle control)

$i_{ds} \neq 0$

- (Case 1)
  Consider the case with constant field and d-axis stator currents

$$T_e = \frac{3}{2}\frac{P}{2}\{[L_{md}I_{fr} + (L_{ds}-L_{qs})\boxed{I_{ds}}]\,i_{qs} - L_{mq}\boxed{I_{ds}}\,i_{qr}\} \qquad (6.6\text{–}21)$$

- Linear torque relation, except for the relatively small term involve q-axis rotor current
  (nearly instantaneous response for changes in q-axis stator current)

- The field orientation with $\gamma \neq 0$ is still reachable when d-axis currents (field and d-axis stator currents) are held constant

Dynamic response with $\gamma \neq 0$ (angle control)

$i_{ds} \neq 0$

- (Case 3) general case
  Consider the case with the non-constant d-axis currents

$$T_e = \frac{3}{2}\frac{P}{2}[L_{md}(i_{fr}+\boxed{i_{dr}})\,i_{qs} - L_{mq}\boxed{i_{qr}}\,I_{ds} + (L_{ds}-L_{qs})\,I_{ds}i_{qs}] \qquad (6.6\text{–}21)$$

$$i_{dr} = \frac{-L_{md}p}{r_{dr}+L_{dr}p}\,i_{fr}$$

(6.6–18)

$$i_{dr} = \frac{-L_{md}p}{r_{dr}+L_{dr}p}\,(i_{fr}+\boxed{i_{ds}})$$

(6.6–20)

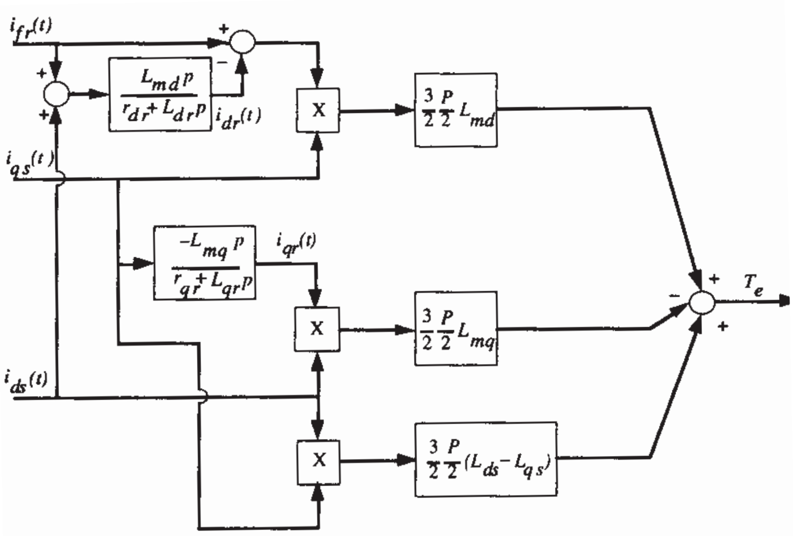


Figure 6.29 Torque production with $\gamma \neq 0$ for the general case

6.6.5 Example-synchronous machine field orientation

A 100 hp, 460 volt, 3 phase, 4 pole synchronous motor with the following per unit parameters

$X_{ls} = 0.1$	$X_{lf} = 0.2$	$r_s = 0.04$
$X_{md} = 1.1$	$X_{ldr} = 0.1$	$r_{dr} = 0.04$
$X_{mq} = 0.3$	$X_{lqr} = 0.15$	$r_{qr} = 0.08$
		$r_{fr} = 0.01$

is operated as a field orientation controlled machine. For this example assume an ideal current regulator.

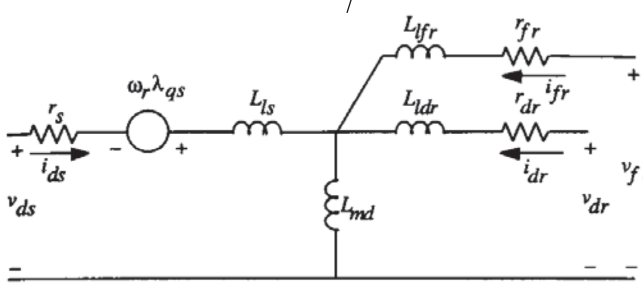
- a) For field orientation ($\gamma = 0$), find the rated current ($i^*_{qs} = I$), rated internal voltage E, and the terminal power factor for operation with rated terminal voltage, rated current, and rated frequency.
- b) Assume the encoder is incorrectly aligned such that $\gamma_0 = 20^\circ$. Find and plot T_e vs. I^*_{qs} from zero to rated value, also find the terminal voltage at rated $I = I^*_{as}$.
- c) Same as (b), but $\gamma_0 = -20^\circ$.
- d) Find the torque $T_e(t)$ following a step input of i^*_{qs} (rated value) for cases (a), (b), and (c). Express time in seconds.

At steady state:

$$\boxed{\lambda_{qs} = L_{qs} i_{qs} + L_{mq} \cancel{i_{qr}}} \qquad \boxed{\lambda_{ds} = L_{ds} i_{ds} + L_{md} \cancel{i_{dr}} + L_{md} i_{fr}}$$

$$V_{ds} = r_s I_{ds} - \omega_r \lambda_{qs}$$
$$= r_s I_{ds} - X_{qs} I_{qs}$$
$$V_{qs} = r_s I_{ds} + \omega_r \lambda_{ds}$$
$$= r_s I_{qs} + X_{ds} I_{ds} + \underbrace{X_{md} I_{fr}}_{E}$$
$$\underline{V} = V_{qs} - jV_{ds}$$
$$= r_s \left(I_{qs} - jI_{ds}\right) + X_{ds} I_{ds} + jX_{qs} I_{qs} + E$$

$$T_e = (\lambda_{ds} i_{qs} - \lambda_{qs} i_{ds})$$



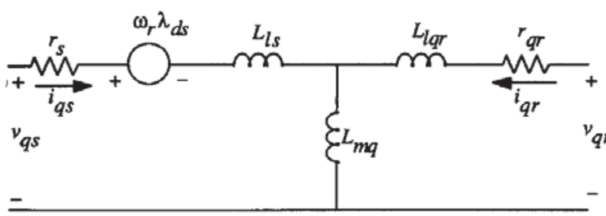
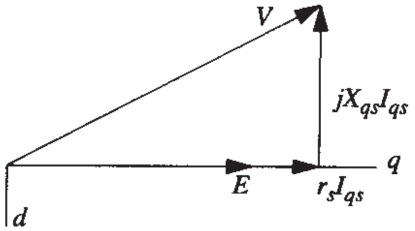


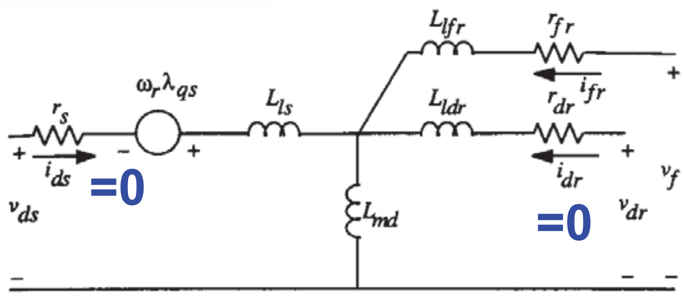
Figure 6.24 Equivalent circuits of three phase salient pole synchronous machine

At steady state: • For the case $(\gamma = 0) \Longrightarrow i_{ds} = 0$

$$\underline{V} = V_{qs} - jV_{ds}$$
$$= r_s I_{qs} + jX_{qs} I_{qs} + E$$

$$|V|^2 = (X_{qs}|I_{qs}|)^2 + (|E| + r_s|I_{qs}|)^2$$





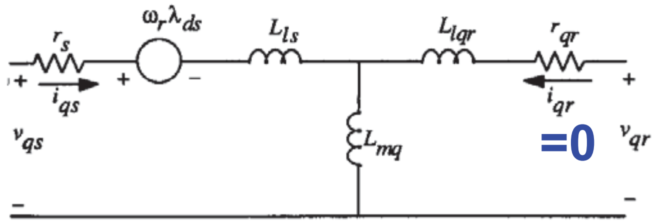


Figure 6.24 Equivalent circuits of three phase salient pole synchronous machine

$$\begin{cases} \lambda_{ds} = L_{ldr}\, i_{ds} + L_{md}\,(i_{ds} + i_{dr} + i_f) \\ \Rightarrow \lambda_{ds} = L_{md}\, I_f \text{(recall } I_{ds} = 0) \end{cases}$$

$$\begin{cases} T_e = (\lambda_{ds}\, i_{qs} - \lambda_{qs}\, i_{ds}) \\ \Rightarrow T_e = \underbrace{L_{md} I_f}_{E} I_{qs} = |E||I_{qs}| \end{cases}$$

since $\omega = 1.0$ pu

- a) For field orientation ($\gamma = 0$), find the rated current ($i^*_{qs} = I$), rated internal voltage E, and the terminal power factor for operation with rated terminal voltage, rated current, and rated frequency.

Given: $V = 1.0$, $T_e = 1.0$, $X_q = \omega(L_{mq} + L_{ls}) = 1\,(0.3 + 0.1) = 0.4$

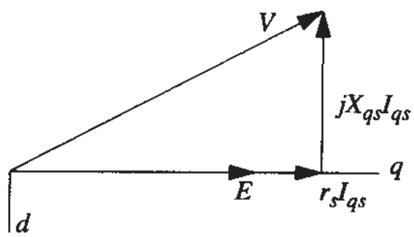
$$|V|^2 = (X_{qs}|I_{qs}|)^2 + \left(\frac{T_e}{|I_{qs}|} + r_s|I_{qs}|\right)^2 \Longrightarrow |V|^2 = (X_{qs}|I_{qs}|)^2 + (|E| + r_s|I_{qs}|)^2$$

$$1 = (0.4|I_{qs}|)^2 + \frac{1}{|I_{qs}|^2} + 2\,(0.04) + (0.04)^2|I_{qs}|^2$$

$$0 = 0.1616|I_{qs}|^4 + (-0.92)\,|I_{qs}|^2 + 1 \Longrightarrow |I_{qs}| = 1.22\,\text{pu}$$

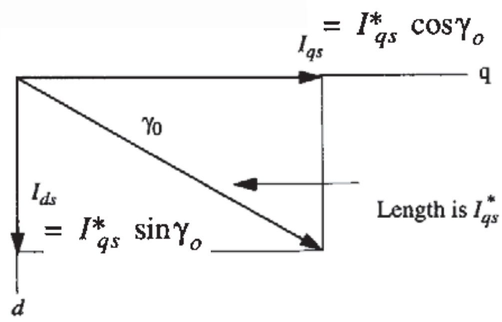
$$E = \frac{T_e}{|I_{qs}|} = \frac{1.0}{1.22} = 0.820\ \text{pu}$$

$$pf = \frac{|E + r_s I_{qs}|}{|V|} = 0.869$$



- b) Assume the encoder is incorrectly aligned such that $\gamma_0 = 20^\circ$. Find and plot T_e vs. I^*_{qs} from zero to rated value, also find the terminal voltage at rated $I = I^*_{qs}$.

$(\gamma \neq 0)$



$$I_f = \left|\frac{E}{\omega L_{md}}\right| = \frac{0.820}{(1.0)\,(1.1)} = 0.751 \quad \text{pu}$$

$$T_e = L_{md} I_f I_{qs} + (L_{ds} - L_{qs})\, I_{ds} I_{qs}$$

$$T_e = L_{md} I_f I^*_{qs}\cos\gamma_o + (L_{ds} - L_{qs})\,(I^*_{qs})^2 \sin\gamma_o \cos\gamma_o$$

$$L_{md} = 1.1 \quad I_f = 0.751 \quad L_{ds} = 1.2 = L_{md} + L_{lds}$$

$$L_{qs} = 0.4 = L_{mq} + L_{lqs}$$

$$V = r_s\left(I_{qs} - jI_{ds}\right) + X_{ds} I_{ds} + jX_{qs} I_{qs} + E$$

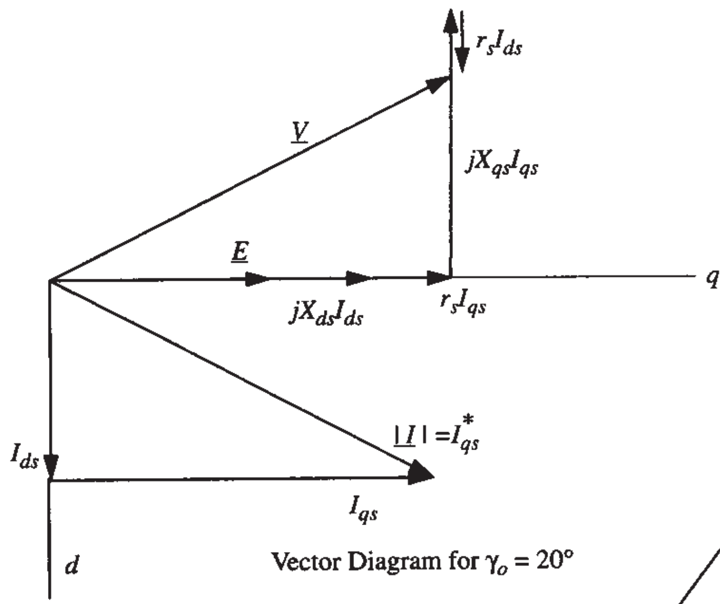
$$= 0.82 + j\,(1.2)\,(-jI^*_{qs}\sin\gamma_o) + j\,(0.4)\,(I^*_{qs}\cos\gamma_o)$$

$$+ (0.04)\,(-jI^*_{qs}\sin\gamma_o) + (0.04)\,(I^*_{qs}\cos\gamma_o)$$

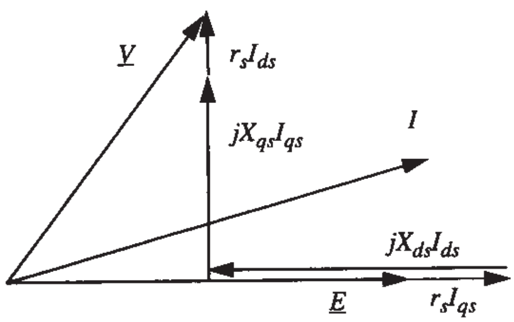
For $\gamma_o = 20^\circ$, $I^*_{qs} = 1.22$, $V = 1.44\ \angle 17.9^\circ$

- (c) Same as (b), but $\gamma_0 = -20^\circ$

$\underline{V} = 0.599\ \angle 52.5^\circ$



Vector Diagram for $\gamma_o = 20^\circ$



Vector Diagram for $\gamma_0 = -20^\circ$

- d) Find the torque $T_e(t)$ following a step input of i_{qs}^* (rated value) for cases (a), (b), and (c). Express time in seconds.

$$v_{qr} = r_{qr}i_{qr} + p\lambda_{qr} = 0 \qquad \lambda_{qr} = L_{lqr}i_{qr} + L_{mq}(i_{qr} + i_{qs})$$

$$\Rightarrow \quad v_{qr} = 0 = r_{qr}i_{qr} + p\left(L_{lqr}i_{qr} + L_{mq}(i_{qr} + i_{qs})\right)$$

$$\Rightarrow \quad i_{qr} = \frac{-L_{mq}p}{r_{qr} + L_{qr}p}i_{qs} \qquad L_{qr} = L_{lqr} + L_{mq}$$

$$v_{dr} = r_{dr}i_{dr} + p\lambda_{dr} = 0 \qquad \lambda_{dr} = L_{ldr}i_{dr} + L_{md}(i_{dr} + i_{ds} + i_f)$$

$$\Rightarrow \quad v_{dr} = 0 = r_{dr}i_{dr} + p\left(L_{ldr}i_{dr} + L_{md}(i_{dr} + i_{ds} + i_f)\right)$$

$$\Rightarrow \quad i_{dr} = \frac{-L_{md}pi_{ds}}{r_{dr} + L_{dr}p} + \frac{-L_{md}pi_f}{r_{dr} + L_{dr}p}$$

$$T_e = \lambda_{ds}i_{qs} - \lambda_{qs}i_{ds}$$

$$T_e = [L_{ls}i_{ds} + L_{md}(i_{ds} + i_{dr} + i_f)]\,i_{qs} - [L_{ls}i_{qs} + L_{mq}(i_{qs} + i_{qr})]\,i_{ds}$$

$$T_e = L_{md}(i_f + i_{dr})\,i_{qs} - L_{mq}(i_{qr}i_{ds}) + (L_{md} - L_{mg})\,i_{ds}i_{qs}$$

Let $i_{qs} = i_{qs}^* \cos\gamma_o u(t)$ and $i_{ds} = i_{qs}^* \sin\gamma_o u(t)$ and note that $pi_f = 0$.
In Laplace Transform notation:

$$I_{qr}(s) = \frac{i_{qs}^* \cos\gamma_o}{s} \qquad I_{dr}(s) = \frac{i_{qs}^* \sin\gamma_o}{s}$$

$$I_{dr}(s) = \frac{-L_{md}s}{r_{dr} + L_{dr}s}\;\frac{i_{qs}^* \sin\gamma_o}{s} \qquad I_{qr}(s) = \frac{-L_{mq}s}{r_{qr} + L_{qr}s}\;\frac{i_{qs}^* \cos\gamma_o}{s}$$

$$I_{dr}(s) = \frac{-L_{md}i_{qs}^* \sin\gamma_o}{r_{dr} + L_{dr}s} \qquad I_{qr}(s) = \frac{-L_{mq}i_{qs}^* \cos\gamma_o}{r_{qr} + L_{qr}s}$$

$$\Rightarrow \quad i_{dr}(t) = -\frac{L_{md}}{L_{dr}}i_{qs}^* \sin\gamma_o e^{-\frac{R_{dr}}{L_{dr}}t} \qquad \Rightarrow \quad i_{qr}(t) = -\frac{L_{mq}}{L_{qr}}i_{qs}^* \cos\gamma_o e^{-\frac{R_{qr}}{L_{qr}}t}$$

$$T_e = L_{md} i_f i_{qs} + L_{md} i_{dr} i_{qs} - L_{mq} i_{qr} i_{ds} + (L_{md} - L_{mq}) i_{ds} i_{qs}$$

$$T_e = L_{md} i_f i_{qs}^{*} \cos\gamma_o \qquad \text{Field Torque}$$

$$+ \left(\frac{-L_{md}^2}{L_{dr}} i_{qs}^{*2} \quad \sin\gamma_o \cos\gamma_o \right) e^{-\frac{R_{dr}}{L_{dr}} t} \qquad \text{D–Damper torque}$$

$$+ \left(\frac{L_{mq}^2}{L_{qr}} i_{qs}^{*2} \quad \sin\gamma_o \cos\gamma_o \right) e^{-\frac{R_{dr}}{L_{dr}} t} \qquad \text{Q–Damper Torque}$$

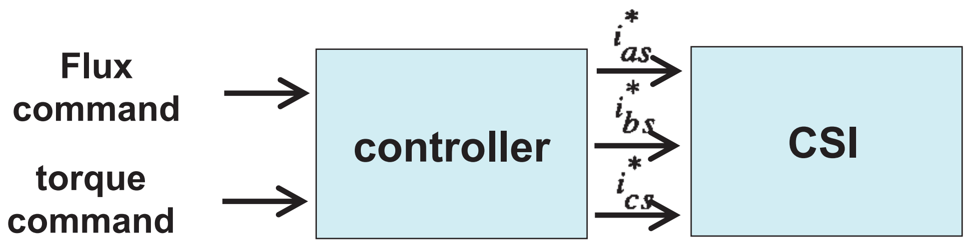
$$+ (L_{md} - L_{mq}) i_{qs}^{*2} \quad \sin\gamma_o \cos\gamma_o \qquad \text{Reluctance Torque}$$

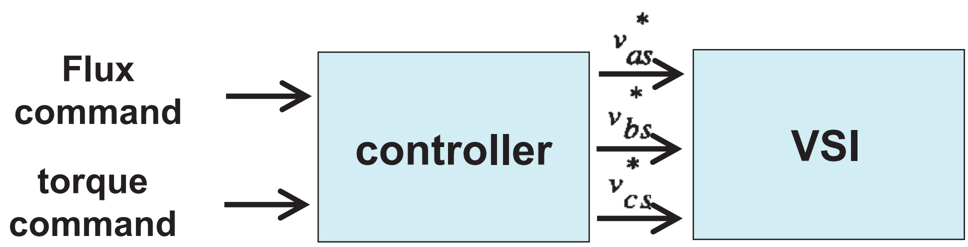
The time constants are:

$$\frac{L_{dr}\,(\mathrm{pu})}{r_{dr}\,(\mathrm{pu})} \frac{z_{base}/\omega_{base}}{z_{base}} = \frac{L_{dr}\,(\mathrm{pu})}{\omega_{base} r_{dr}\,(\mathrm{pu})} = \frac{(1.1 + 0.1)}{(377 \ \mathrm{sec}^{-1}) \ (0.04x)} = 79.6 \ \ \mathrm{ms}$$

$$\frac{L_{qr}\,(\mathrm{pu})}{r_{qr}\,(\mathrm{pu})} \frac{z_{base}/\omega_{base}}{z_{base}} = \frac{L_{qr}\,(\mathrm{pu})}{\omega_{base} r_{qr}\,(\mathrm{pu})} = \frac{(0.3 + 0.15)}{(0.08) \ (377 \ \mathrm{sec}^{-1})} = 14.9 \ \ \mathrm{ms}$$

6.7 Field Orientation Using Voltage as the Controlled Variable





6.7.1 Stator voltage equations in terms of rotor flux-IM

Rotor flux oriented:

$$v_{qs}^e = f_{qs}\left(i_{qs}^e, \lambda_{qs}^e, \lambda_{ds}^e\right) \qquad\Rightarrow\qquad v_{qs}^e = f_{qs}\left(i_{qs}^e, \lambda_{qr}^e, \lambda_{dr}^e\right)$$

$$v_{ds}^e = f_{ds}\left(i_{ds}^e, \lambda_{qs}^e, \lambda_{ds}^e\right) \qquad\Rightarrow\qquad v_{ds}^e = f_{ds}\left(i_{ds}^e, \lambda_{qr}^e, \lambda_{dr}^e\right)$$

6.7.1 Stator voltage equations in terms of rotor flux-IM

$$\lambda_{qs}^e = \left(L_s - \frac{L_m^2}{L_r}\right) i_{qs}^e + \frac{L_m}{L_r}\lambda_{qr}^e = L_s' i_{qs}^e + \frac{L_m}{L_r}\lambda_{qr}^e \quad (6.7\text{–}1)$$

$$\lambda_{ds}^e = \left(L_s - \frac{L_m^2}{L_r}\right) i_{ds}^e + \frac{L_m}{L_r}\lambda_{dr}^e = L_s' i_{ds}^e + \frac{L_m}{L_r}\lambda_{dr}^e \quad (6.7\text{–}2)$$

$$v_{qs}^e = r_s i_{qs}^e + p\lambda_{qs}^e + \omega_e \boxed{\lambda_{ds}^e}$$

$$v_{ds}^e = r_s i_{ds}^e + p\lambda_{ds}^e - \omega_e \boxed{\lambda_{qs}^e}$$

$$\begin{cases} \lambda_{ds}^e = L_{ls} i_{ds}^e + L_m\left(i_{ds}^e + i_{dr}^e\right) \\ \lambda_{qs}^e = L_{ls} i_{qs}^e + L_m\left(i_{qs}^e + i_{qr}^e\right) \end{cases} \qquad \begin{cases} \lambda_{dr}^e = L_{lr} i_{dr}^e + L_m\left(i_{ds}^e + i_{dr}^e\right) \\ \lambda_{qr}^e = L_{lr} i_{qr}^e + L_m\left(i_{qs}^e + i_{qr}^e\right) \end{cases}$$

$$v_{qs}^e = (r_s + L_s' p)\, i_{qs}^e + \frac{L_m}{L_r} p\lambda_{qr}^e + \omega_e\left(L_s' i_{ds}^e + \frac{L_m}{L_r}\lambda_{dr}^e\right) \quad (6.7\text{–}4)$$

$$v_{ds}^e = (r_s + L_s' p)\, i_{ds}^e + \frac{L_m}{L_r} p\lambda_{dr}^e - \omega_e\left(L_s' i_{qs}^e + \frac{L_m}{L_r}\lambda_{qr}^e\right) \quad (6.7\text{–}5)$$

6.7.2 Decoupling equations for field orientation

- **Rotor flux oriented:**

  forcing $\lambda_{qr}^{e} = 0$ (6.7–6)

$$
\begin{cases}
v_{qs}^{e} = (r_s + L_s' p)\, i_{qs}^{e} + \omega_e \left(L_s' i_{ds}^{e} + \dfrac{L_m}{L_r} \lambda_{dr}^{e} \right) \\
v_{ds}^{e} = (r_s + L_s' p)\, i_{ds}^{e} - \omega_e L_s' i_{qs}^{e} + \dfrac{L_m}{L_r} p \lambda_{dr}^{e}
\end{cases}
$$

Voltage decoupler

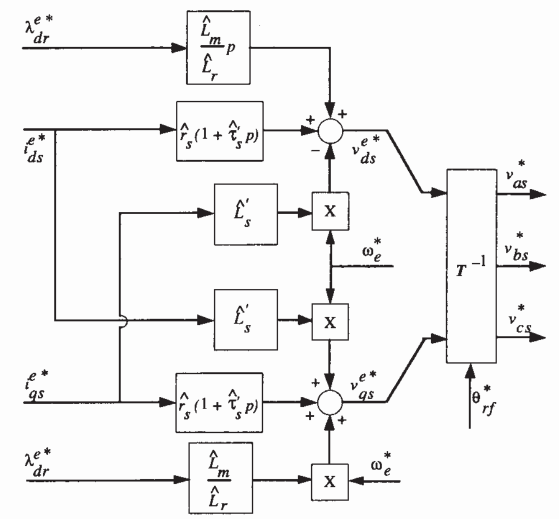


Figure 6.30 Computation of voltage commands for voltage controlled field orientation in induction machines (voltage decoupler)

- **constant rotor flux at steady state:**

  (w/o field weakening)

$$\lambda_{dr}^{e} = L_{lr} i_{dr}^{e} + L_m (i_{ds}^{e} + i_{dr}^{e})$$

$$\Rightarrow \lambda_{dr}^{e} = L_m I_{ds}^{e} \quad (6.7\text{–}9)$$

$$
\begin{cases}
v_{qs}^{e} = (r_s + L_s' p)\, i_{qs}^{e} + \omega_e \left(L_s' + \dfrac{L_m^2}{L_r} \right) I_{ds}^{e} & (6.7\text{–}10) \\
\quad = (r_s + L_s' p)\, i_{qs}^{e} + \omega_e L_s I_{ds}^{e} & (6.7\text{–}11) \\
v_{ds}^{e} = r_s I_{ds}^{e} - \omega_e L_s' i_{qs}^{e} & (6.7\text{–}12)
\end{cases}
$$

Constant flux voltage decoupler

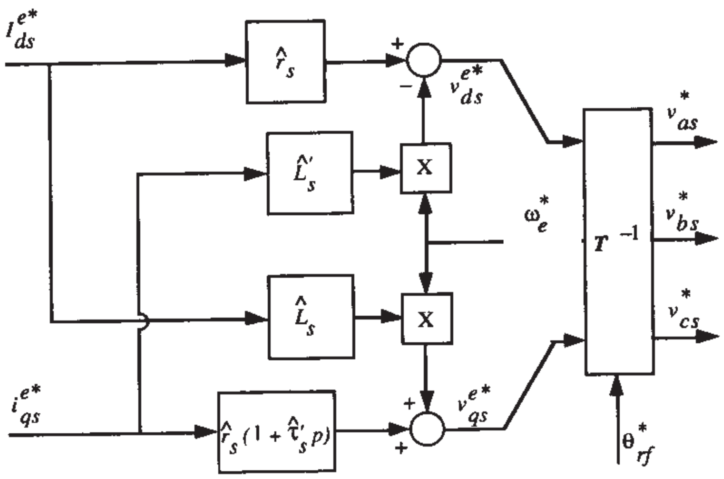


Figure 6.31 Computation of voltage commands for voltage controlled field orientation in induction machines with constant flux (constant flux voltage decoupler)

6.7.2 Example of field orientation using voltage controlled inverters

• indirect field orientation controller

• direct field orientation controller

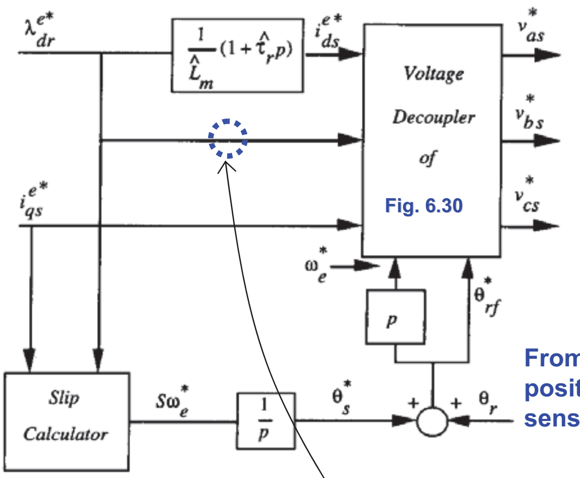


Figure 6.32 Indirect field orientation controller using voltage controlled inverter

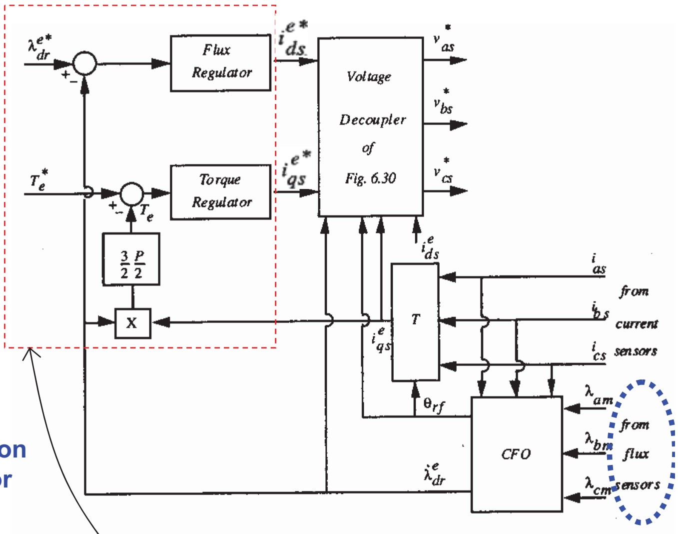


Figure 6.33 Induction machine direct orientation controller using flux sensors and a voltage controlled inverter

• flux compensation can be omitted when constant flux operation is desired

• because of external regulators, stator resistance-transient time constant can be eliminated

6.8 Stator Flux Based Field Orientation

- Eliminations of position sensor in IFO system and flux sensor in DFO system
  - Estimate rotor flux from terminal quantities based on the rotor flux based field orientation

	i_{qds}	i'_{qdr}	λ_{qds}	λ'_{qdr}	λ_{qdm}	Torque Expression
1	‡	‡				$T_e = \frac{3}{2}\frac{P}{2}L_m Im\{i'^{\dagger}_{qdr} i_{qds}\}$
2	‡				‡	$T_e = \frac{3}{2}\frac{P}{2} Im\{\lambda^{\dagger}_{qdm} i_{qds}\}$
3	‡		‡			$T_e = \frac{3}{2}\frac{P}{2} Im\{\lambda^{\dagger}_{qds} i_{qds}\}$
4	‡			‡		$T_e = \frac{3}{2}\frac{P}{2}\frac{L_m}{L_r} Im\{i_{qds}\lambda'^{\dagger}_{qdr}\}$
5		‡			‡	$T_e = \frac{3}{2}\frac{P}{2} Im\{i'^{\dagger}_{qdr}\lambda_{qdm}\}$
6		‡		‡		$T_e = \frac{3}{2}\frac{P}{2} Im\{i'^{\dagger}_{qdr}\lambda'_{qdr}\}$
7		‡	‡			$T_e = \frac{3}{2}\frac{P}{2}\frac{L_m}{L_s} Im\{i'^{\dagger}_{qdr}\lambda_{qds}\}$
8			‡	‡		$T_e = \frac{3}{2}\frac{P}{2}\frac{L_m}{\sigma L_s L_r} Im\{\lambda'^{\dagger}_{qdr}\lambda_{qds}\}$

rotor flux can be estimate by:

$$\hat{\underline{\lambda}}_s = \int(\underline{v}_s - r_s\underline{i}_s)\,dt$$

$$\hat{\underline{\lambda}}_r = \frac{L_r}{L_m}(\hat{\underline{\lambda}}_s - \sigma L_s \underline{i}_s)$$

$$\sigma = 1 - \frac{L_m^2}{L_s L_r} \qquad (6.8\text{–}3)$$

Stator Flux Based Field Orientation

- Problem for estimating rotor flux from terminal quantities based on the rotor flux based field orientation
  - Accuracy of the estimated stator resistance
    - Slow variation with temperature : stator flux can be estimated accuracy ($\hat{\underline{\lambda}}_s = \int (\underline{v}_s - r_s \underline{i}_s)\, dt$)
  - Leakage inductances vary with operating conditions
    - Rotor flux accuracy can be improved by fbk path of a closed loop system relies on the **accuracy of the fbk signal**

$$\boxed{\hat{\lambda}_r = \frac{L_r}{L_m}(\hat{\lambda}_s - \sigma L_s \underline{i}_s)}$$

    - The most accuracy signal should be chosen as the fbk signal, which leads to the implementation of the **stator flux oriented system**

6.8.1 Mathematical model of a stator flux oriented IM

	$\underline{i}_{qds}$	$\underline{i}'_{qdr}$	$\underline{\lambda}_{qds}$	$\underline{\lambda}'_{qdr}$	$\underline{\lambda}_{qdm}$	Torque Expression	
1	‡	‡				$T_e = \frac{3}{2}\frac{P}{2}L_m Im\{\underline{i}'^{\dagger}_{qdr}\underline{i}_{qds}\}$	
2	‡				‡	$T_e = \frac{3}{2}\frac{P}{2} Im\{\underline{\lambda}^{\dagger}_{qdm}\underline{i}_{qds}\}$	
3	‡		‡			$T_e = \frac{3}{2}\frac{P}{2} Im\{\underline{\lambda}^{\dagger}_{qds}\underline{i}_{qds}\}$	stator flux oriented IM
4	‡			‡		$T_e = \frac{3}{2}\frac{P}{2}\frac{L_m}{L'_r} Im\{\underline{i}_{qds}\underline{\lambda}'^{\dagger}_{qdr}\}$	rotor flux oriented IM
5		‡			‡	$T_e = \frac{3}{2}\frac{P}{2} Im\{\underline{i}'^{\dagger}_{qdr}\underline{\lambda}_{qdm}\}$	
6		‡		‡		$T_e = \frac{3}{2}\frac{P}{2} Im\{\underline{i}'^{\dagger}_{qdr}\underline{\lambda}'_{qdr}\}$	
7		‡	‡			$T_e = \frac{3}{2}\frac{P}{2}\frac{L_m}{L_s} Im\{\underline{i}'^{\dagger}_{qdr}\underline{\lambda}_{qds}\}$	
8			‡	‡		$T_e = \frac{3}{2}\frac{P}{2}\frac{L_m}{\sigma L_s L_r} Im\{\underline{\lambda}'^{\dagger}_{qdr}\underline{\lambda}_{qds}\}$	

$$T_e = \frac{3}{2}\frac{P}{2}\frac{L_m}{L_r}(\lambda^e_{dr} i^e_{qs} - \underbrace{\lambda^e_{qr}}_{=0\text{ : field orientation}} i^e_{ds}) \Rightarrow T_e = \frac{3}{2}\frac{P}{2}(\lambda_{ds} i_{qs} - \underbrace{\lambda_{qs}}_{=0\text{ : field orientation}} i_{ds})$$

Force the q-axis stator flux to be zero

$$(1+\sigma\tau_r p)\,L_s i_{qs}^{es} - S\omega_e\tau_r(\lambda_{ds}^{es} - \sigma L_s i_{ds}^{es}) = 0 \qquad (6.8\text{–}4)$$

$$(1+\tau_r p)\,\lambda_{ds}^{es} = (1+\sigma\tau_r p)\,L_s i_{ds}^{es} - S\omega_e\tau_r\sigma L_s i_{qs}^{es} \qquad (6.8\text{–}5)$$

$$T_e = \frac{3P}{4}\lambda_{ds}^{es} i_{qs}^{es} \qquad (6.8\text{–}6)$$

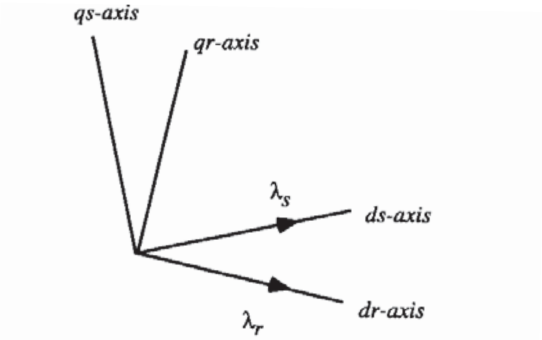


Figure 6.34 Rotor and stator flux linkage reference frames

6.8.2 Design of a decoupler for direct stator flux oriented system

Compare rotor flux oriented system (6.2-12) to stator flux oriented system (6.8-5)

$$0 = r_r i_{dr}^e + p\lambda_{dr}^e \qquad (6.2\text{–}12)$$

- d-axis rotor flux can be controlled directly by the d-axis rotor current

$$(1+\tau_r p)\,\lambda_{ds}^{es} = (1+\sigma\tau_r p)\,L_s i_{ds}^{es} - S\omega_e\tau_r\sigma L_s i_{qs}^{es} \qquad (6.8\text{–}5)$$

- d-axis rotor flux depends on the d-axis rotor current, q-axis stator current and slip frequency
- Any change in torque command (q-axis stator current) will cause a transient in the stator flux

Decouple the coupling between the q-axis stator current and stator flux

$$(1+\tau_r p)\,\lambda_{ds}^{es} = (1+\sigma\tau_r p)\,L_s i_{ds}^{es} - S\omega_e\tau_r\sigma L_s i_{qs}^{es} \qquad (6.8\text{–}5)$$

$$\Rightarrow \quad K_0\lambda_{ds}^{es} = \boxed{K_1 i_{ds}^{es}} + K_2 i_{qs}^{es}$$

let $K_1 i_{ds}^{es} = K_1 i_{ds(1)}^{es} + K_1 i_{ds(2)}^{es}$

$$\Rightarrow \quad K_0\lambda_{ds}^{es} = K_1 i_{ds(1)}^{es} + \boxed{K_1 i_{ds(2)}^{es} + K_2 i_{qs}^{es}}$$

- Concept of decoupler

let $K_1 i_{ds(2)}^{es} + K_2 i_{qs}^{es} = 0$

$$\Rightarrow \quad K_0\lambda_{ds}^{es} = K_1 i_{ds(1)}^{es}$$

which is analogous to the rotor flux oriented system:

$$0 = r_r i_{dr}^{e} + p\lambda_{dr}^{e} \qquad (6.2\text{–}12)$$

Design of a decoupler

define $i_{dq}^{es} = i_{ds(2)}^{es}$

$$(1+\sigma\tau_r p)\,L_s i_{dq}^{es} - S\omega_e\tau_r\sigma L_s i_{qs}^{es} = 0 \qquad (6.8\text{–}9)$$

$$i_{dq}^{es} = \frac{S\omega_e\tau_r\sigma L_s i_{qs}^{es}}{(1+\sigma\tau_r p)\,L_s} \qquad (6.8\text{–}10)$$

$$S\omega_e = \frac{(1+\sigma\tau_r p)\,L_s i_{qs}^{es}}{\tau_r(\lambda_{ds}^{es} - \sigma L_s i_{ds}^{es})} \qquad (6.8\text{–}11)$$

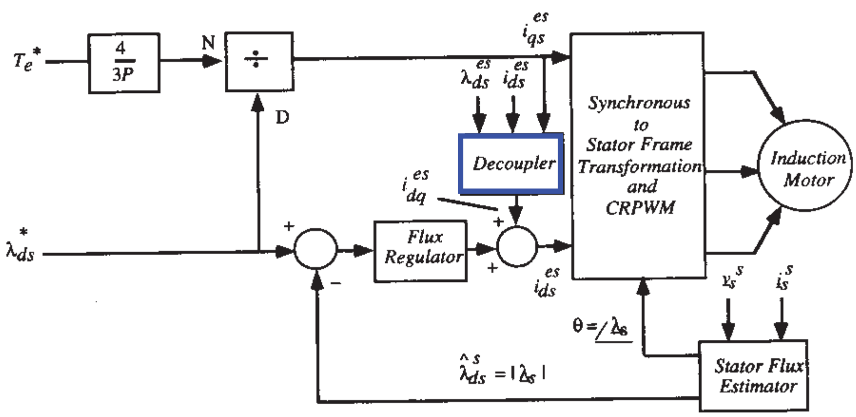


Figure 6.35 Block diagram of stator flux based field orientation system

Design of a decoupler

Flux regulator

$$\underbrace{i_{ds}^{es} = \boxed{\left(K_p + \frac{K_i}{p}\right)(\lambda_{ds}^* - \hat{\lambda}_{ds})} + i_{dq}^{es}} \qquad (6.8\text{–}7)$$

$$(1+\tau_r p)\,\lambda_{ds}^{es} = (1+\sigma\tau_r p)\,L_s \boxed{i_{ds}^{es}} - S\omega_e\tau_r\sigma L_s i_{qs}^{es} \qquad (6.8\text{–}5)$$

$$\Rightarrow \quad (1+\tau_r p)\,\lambda_{ds}^{es} = (1+\sigma\tau_r p)\,L_s\left(K_p + \frac{K_i}{p}\right)(\lambda_{ds}^* - \hat{\lambda}_{ds}) + (1+\sigma\tau_r p)\,L_s i_{dq}^{es} - S\omega_e\tau_r\sigma L_s i_{qs}^{es} \qquad (6.8\text{–}8)$$

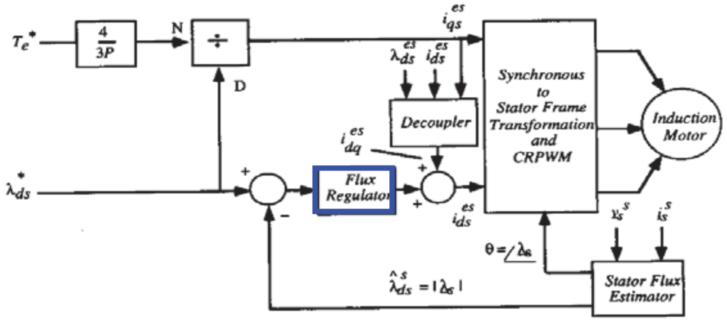


Figure 6.35 Block diagram of stator flux based field orientation system