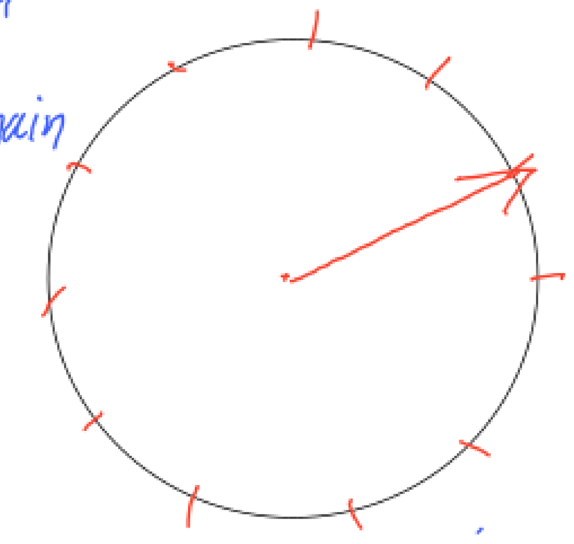
$$d_\ell = \begin{bmatrix} k_1^\ell \\ k_2^\ell \end{bmatrix} = k \begin{bmatrix} \cos\left(\frac{2\pi\ell}{L}\right) \\ \sin\left(\frac{2\pi\ell}{L}\right) \end{bmatrix}, \quad \ell = 0, \dots, L-1$$

$$x \longmapsto e^{ik d_\ell \cdot x} \cdot v_\ell(x)$$

In Fourier domain $\delta_{\hat{d}_\ell}(\omega)$

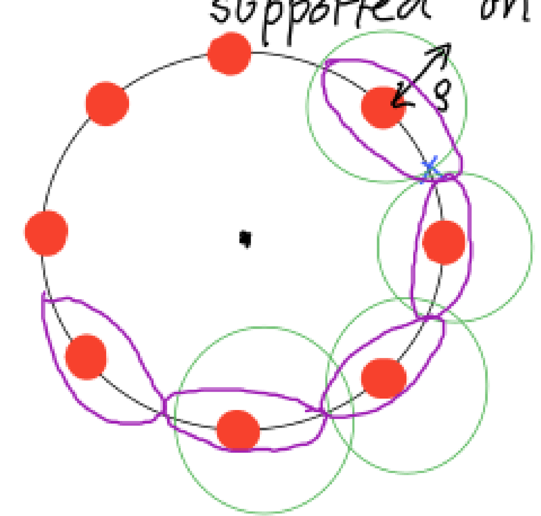


$$\Delta u + k^2 u = 0 \text{ in } \mathbb{R}^2$$ → Fourier domain

$\hat{u} \hat{=}$ Fourier trf.

$$\Rightarrow \hat{u}(\omega)(\omega^2 - k^2) = 0$$

↑ supported on characteristic circle



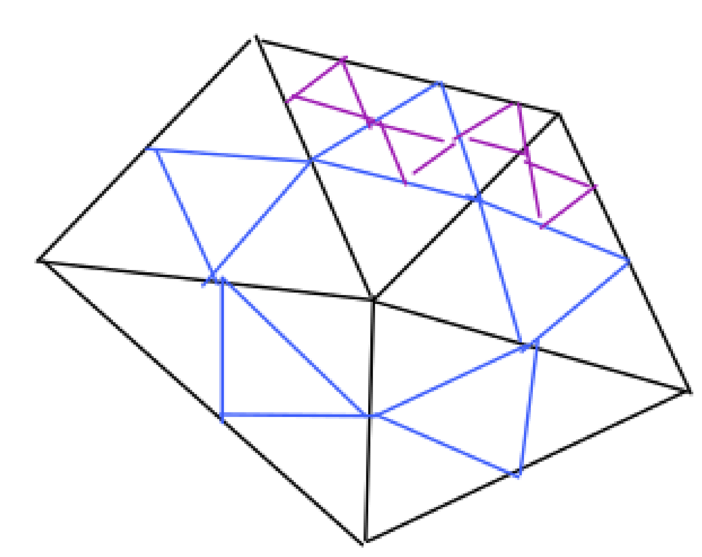
$$\hat{u}(\omega) = \sum_{\ell=1}^{L} \hat{v}_\ell(\omega) * \delta_{\hat{d}_\ell} = \sum_{\ell=1}^{L} \hat{v}_\ell(\omega - \hat{d}_\ell)$$

A sum of $\hat{d}_\ell$-translates of functions supported in a small neighborhood of 0 can cover the support of $\hat{u}$.

▷ Can choose $\hat{v}_\ell \in \mathcal{U} := \{\hat{f}$ supported in $|\omega| \leq \varrho\}$

↑ "Low-frequency functions"

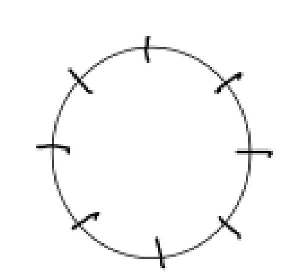
↳ can be represented on a coarse grid



On finest grid: Std FE space

On next coarser grid:

On coarsest grid:



MG in FE context:

a linear form on V_h

$$u_h \in V_h : a(u_h, v_h) = f(v_h) \quad \forall v_h \in V_h$$

residual of $w_h \in V_h$:

$$v_h \longmapsto f(v_h) - a(w_h, v_h) =: r(v_h)$$

$\hat{=}$ a linear form on V_h

Coarse space V_H + prolongation $P: V_H \to V_h$ [injective]

"Restriction of residual to V_H" yields

$$v_H \longmapsto r(P v_H)$$

$\hat{=}$ linear form on V_H