XYZ Corporation water usage sourcing optimization analysis

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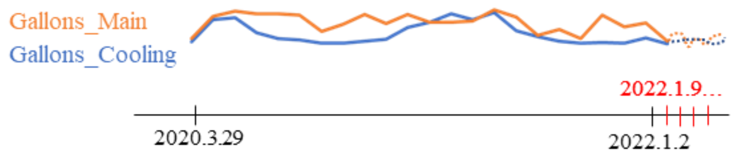
I. Project introduction

The project goal is to assist XYZ Corporation in optimizing the overall costs of water usage in Building T based on our forecast for the next four-week water usage. Our team provided consulting suggestions on two decision-making problems for the management, including (1) Which contract from The Water Co. should XYZ Corporation purchase, and (2) The optimal water allocation from The Water Co. and the company's own water storage tank.

II. Methodology

Our project consists mainly of two parts. First, we predicted the water usage of building T for the next four weeks, based on the historical data. From the historical data, we have usage amounts in gallon-unit with two uses (Cooling and Main) and need to predict the next four weeks (dash-lines), that is from week Jan.9, 2022, to Jan.30, 2022, respectively.
We drew the water usage throughout the time and found out there is no clear sign that "Main" and "Cooling" are interrelated. Therefore, we assumed that the water usage for the two purposes is independent and decided to separately forecast the future water usage using different models.



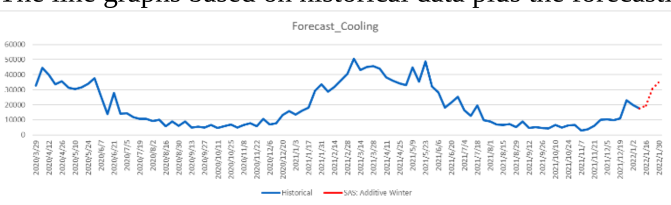
We utilized four different tools to do the forecast, and the numbers in red are what we chose for the optimization model input in objective2. Firstly, we observed the obvious increasing trend in cooling use in January from historical data, and we also wanted to pursue a robust model with smaller error and other metric measures.

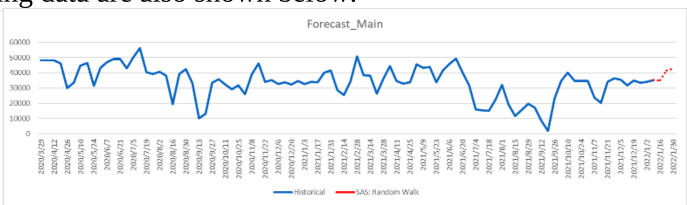
Due to the model measurement tools and the ease of use, we decided to choose the models from SAS. The model we tried included Random Walk models, Moving Average models, Exponential Smoothing models, and ARIMA models, considering different argument options. To compare the performance with the same standards, we aimed to find the models with the smallest AIC. After repeated testing, Winter's additive exponential smoothing model fit the cooling usage data the best, with 1,446.95 AIC, while the random walk model considering seasonality fit the main data the best, with 852.5327 AIC. The forecast usages of each model are shown in the following, and the one we chose is highlighted in red.

	Forecast_Main				
Method	2022/1/3-9	2022/1/10-16	2022/1/17-23	2022/1/24-30	SubTotal
SAS: Additive Season	34468.9	34402.8	40543.8	41924.8	151340.3
SAS: Random Work	35339	35273	41414	42795	154821
Minitab	34069.4	34069.4	34069.4	34069.4	136277.6
Excel	28419.7	28251.4	28083.1	27914.8	112669
Python: AR1	34390.1	34390.1	34390.1	34390.1	137560.4
Python: AR2	33316.5	33805.3	33518.5	33612.9	134253.2
		...			

	Forecast_Cooling				
Method	2022/1/3-9	2022/1/10-16	2022/1/17-23	2022/1/24-30	SubTotal
SAS: Additive Season	16798	18799.6	29869.5	34209	99676.1
SAS: Additive Winter	17658.3	19658.6	30727.1	35065.3	103109.3
Minitab	22033.4	24236.4	26439.5	28642.5	101351.8
Excel	19787	19670.5	19553.9	19437.4	78448.8
Python: AR1	11701.7	12802.5	14018.2	15325.1	53847.5
Python: AR2	19463.4	19684.6	19879.2	20044.8	79072
		...			

The line graphs based on historical data plus the forecasting data are also shown below.





The second part of our project is to construct an optimization model based on the contracts with The Water Co., the regulation of EECSI, and XYZ's internal policy. According to the these conditions, the company should comply to these water usage regulations:

(1) The company would have a minimum purchase amount from The Water Co., either 25,000 or 35,000 gallons, depending on which contract it would sign for the next four weeks.

(2) To maintain internal water supply, the storage amount of the water tank should never be below 30,000 gallons during any week over the end of the next four weeks. The storage of the water tank consists of the initial storage amount, 62,500 gallons, adding the expected precipitation, and deducting the usage in the specific week and all over the next four weeks. XYZ expects to add 12,000 gallons to the Water Storage Tank in the first week, 18,000, 20,000, and 22,000 gallons in the following weeks.

(3) As an outstanding member of EECSI, at least 25% of total water usage in Building T should come from the water storage tank.

(4) With a more conservative water usage attitude, the management also requires the sum of water usage from both purchase and storage be greater than our forecasted water demand for each week.

These regulations are the constraints of our optimization model, which serves as the dependence of optimized solutions. The corresponding mathematical formula and programming for each regulation are listed as follows: (**Note:** Here we only listed the mathematical formula of the ***first week*** to demonstrate the rationale of our model, since the following weeks would apply the same calculation logic, and more calculation would be done on SAS.)

Given the water usage in the first week would be sourced from The Water Co. x gallons and from the storage tank y gallons.

(1) x >=25,000 or x >=35,000 depending on the optimal contract XYZ would sign.

```
proc optmodel;
set WCCONTRACT=/WC1 WC2/;   /***Set index for data WCCONTRACT***/
set DETAIL=/1 2 3 4/;   /***Set index for data DETAIL***/

put WCCONTRACT=;
put DETAIL=;

num costwc{WCCONTRACT} = [0.15 0.12];   /***Set costwc values for WCCONTRACT***/
num mindemand{WCCONTRACT} = [25000 35000];   /***Set mindemand values for WCCONTRACT***/
num costst{DETAIL} = [0.18 0.18 0.1 0.1];   /***Set costst values for data DETAIL***/
num weeklydemand{DETAIL} = [52997.3 54931.6 72141.1 77860.3];   /***Set weeklydemand values for DETAIL***/
num precipitation{DETAIL} = [12000 18000 20000 22000];   /***Set precipitation values for DETAIL***/
/***Declare a binary variable x for WCCONTRACT and declare a constraint statement to decide on whether go for contract one or contract two***/
var x{WCCONTRACT} binary;
con selectcon: sum{i in WCCONTRACT} x[i] = 1;

/***Declare an implicit variable selectweekly to calculate the weekly minimum demand based on the contract chosen***/
impvar selectweekly=sum{i in WCCONTRACT} mindemand[i] * x[i];
/***Declare a constraint statement on weekly demand of Water Co. equal or greater than the weekly minimum demand based on the contract chosen***/
con weeklymindemand{i in DETAIL}: demandwcweekly[i]>=selectweekly;
```

We first set the index of WCCONTRACT and DETAIL, and some num variables. Then, set a binary variable x in hope of deciding which contract to choose, the minimum weekly demand it would be according to the contract, and the constraint statement that the weekly demand from Water Co. should be greater than this according value.

```
/***Alternative for the binary output.***/
impvar y{i in WCCONTRACT} = 1-x[i];
/***Declare an implicit variable altweekly to calculate the weekly minimum demand based on the alternative contract***/
impvar altweekly=sum{i in WCCONTRACT} mindemand[i] * y[i];
/***Declare a constraint statement on weekly demand of Water Co. equal or greater than the weekly minimum demand given the alternative contract***/
con altweeklymindemand{i in DETAIL}: altdemandwcweekly[i]>=altweekly;
```

We also set a binary variable y to understand the same situation if XYZ signs the alternative contract instead of the recommended one.

(2) y >= (x+y)*0.25

```
/***Declare demandstweekly variable for DETAIL and set lower bound***/
var demandstweekly{DETAIL};
for {i in DETAIL}
do;
  demandstweekly[i].lb=0;
end;
/***Declare a variable demandwcweekly and set lower bound***/
var demandwcweekly{DETAIL};
for {i in DETAIL}
do;
  demandwcweekly[i].lb=0;
end;

/***Declare a constraint statement on weekly storage tank demand equals or larger than 25% of weekly total demand***/
con leastdemandstweekly{i in DETAIL}: demandstweekly[i]>= weeklydemand[i]*0.25;
/***Declare demandstweekly variable for DETAIL and set lower bound given the alternative contract***/
var altdemandstweekly{DETAIL};
for {i in DETAIL}
do;
  altdemandstweekly[i].lb=0;
end;
/***Declare a variable altdemandwcweekly and set lower and upper bound given the alternative contract***/
var altdemandwcweekly{DETAIL};
for {i in DETAIL: not missing(weeklydemand[i])}
do;
  demandwcweekly[i].lb= 0;
  demandwcweekly[i].ub= weeklydemand[i]*0.75;
end;
/***Declare a constraint statement on weekly storage tank demand equals or larger than 25% of weekly total demand given the alternative contract***/
con altleastdemandstweekly{i in DETAIL}: altdemandstweekly[i]>= altotalstwcdemand[i]*0.25;
```

We set the weekly demand of both the lower bounds of Storage Tank and Water Co. as zero. Then, we declared the constraint statement that weekly demand for Storage Tank should be equal or greater than 25% of the total weekly demand. Declare similar statements for the alternate contract as well.

(3) 62,500 + 12,000 – y >= 30,000

The storage amount of the water tank for each week is cumulative. Therefore, 62,500 + 12,000 – y would become the beginning amount of the next period.

```
/***Set current water level of storage tank as 62500 gallons***/
num tanklevel = 62500;

/***Set implicit variables currentlevel and do the cumulative demand over the four weeks***/
impvar currentlevel{i in DETAIL} =
if i=1 then tanklevel + precipitation[i] - demandstweekly[i]
else currentlevel[i-1] + precipitation[i] - demandstweekly[i];
/***Declare a constraint statement on the weekly level of Storage Tank equal or greater than 30000 gallons***/
con weeklylevel{i in DETAIL}:currentlevel[i] >= 30000;
/***Set implicit variables altcurrentlevel and do the cumulative demand over the four weeks given the alternative contract***/
impvar altcurrentlevel{i in DETAIL} =
if i=1 then tanklevel + precipitation[i] - altdemandstweekly[i]
else altcurrentlevel[i-1] + precipitation[i] - altdemandstweekly[i];

/***Declare a constraint statement on the weekly level of Storage Tank equal or greater than 30000 gallons***/
con altweeklylevel{i in DETAIL}:altcurrentlevel[i] >= 30000;
```

We set the current level in the water tank as 62,500 gallons and declared an implicit variable to add the first week precipitation and subtract the weekly demand. Loop through this process for four weeks and set another implicit variable for the alternative contract.

(4) x+y >= 52,997.3

```
/***Declare a variable totalstwcdemand***/
impvar totalstwcdemand {i in DETAIL}=
demandwcweekly[i] + demandstweekly[i];
/***Declare a constraint statement on weekly total demand equals or less than the sum of weekly Water Co. demand and Storage Tank demand***/
con weeklydemandcon{i in DETAIL}: weeklydemand[i] <= totalstwcdemand[i];
/***Declare a variable altotalstwcdemand given the alternative contract***/
impvar altotalstwcdemand {i in DETAIL}=
altdemandwcweekly[i] + altdemandstweekly[i];
/***Declare a constraint statement on weekly total demand equals or less than the sum of weekly Water Co. demand and Storage Tank demand given the alternative contrac
con altweeklydemandcon{i in DETAIL}: weeklydemand[i] <= altotalstwcdemand[i];
```

The number 52,997.3 is our forecast water usage for week 1. In this part, we declared an implicit variable as the sum of the weekly demand of Water Co. and Storage Tank and set a constraint statement that this sum should be equal or greater than the expected weekly demand from objective 1. Set the same situation for the alternative contract.

(5) Objective function for the minimum total cost

```
/***Set an implicit variable selectweeklycost to calculate the weekly cost of Water Co. given the chosen contract***/
impvar selectweeklycost=sum{i in WCCONTRACT} costwc[i] * x[i];

/***Set an implicit variable altweeklycost to calculate the weekly cost of Water Co. given the alternative contract***/
impvar altweeklycost=sum{i in WCCONTRACT} costwc[i] * y[i];
/***Set implicit variables totakcostwc and totalcostst to calculate the total cost of Water Co. and Storage Tank respectively***/
impvar totalcostwc= sum{i in DETAIL} demandwcweekly[i]*selectweeklycost;
impvar totalcostst= sum{i in DETAIL} demandstweekly[i]*costst[i];

/***Set implicit variables alttotakcostwc to calculate the total cost of Water Co. given the alternative contract***/
impvar alttotalcostwc= sum{i in DETAIL} altdemandwcweekly[i]*altweeklycost;
impvar altotalcostst= sum{i in DETAIL} altdemandstweekly[i]*costst[i];
/***Set the optimization target***/
min TotalCost = totalcostwc + totalcostst;
/***Set the optimization target given the alternative contract***/
min ALT_TotalCost = alttotalcostwc + altotalcostst;
expand;
solve with lso / maxtime=600 nthreads=4 primalin;   /***Solve the problem using the LSO Solver***/
```

Lastly, we set the total cost as an objective function to minimize, and run the solutions by SAS with the LSO solver.

III. Conclusion

1) Based on our model, our consulting team would suggest the company sign **contract 2, with 12 cents ($0.12) per gallon and a minimum of 35,000 gallons purchased per week**, to minimize the cost of purchasing from The Water CO. over the next four weeks.

[1]	x
WC1	0
WC2	1

Based on our optimization model, we recommended XYZ corporation to **sign contract 2**. The calculation of the other related output variables from are as follows:

[1]	costst	weeklydemand	demandstweekly	demandwcweekly	precipitation
1	0.18	52997	13249	39748	12000
2	0.18	54932	13733	41199	18000
3	0.10	72141	36794	35347	20000
4	0.10	77860	40724	37137	22000

2) According to the output, the total demand from Water Co. for the next four weeks would be 39,748+ 41,199+ 35,347+ 37,137 = 153,431 gallons, and the overall payment to The Water Co. would be 153,431 * 0.12 = **$18,411.72**

3) The usage from the water storage tank in the next four week would be 13,249, 13,733, 36,794, and 40.724 gallons. And the cost from the usage of water storage tank would be
13,249* $0.18 + 13,733 * $0.18 + 36,794 * $0.10 + 40,724 * $0.10 = **$12,608.56**

4) From 2) and 3), the total water cost at the end of the next four weeks would be
$18,411.72 + $12,608.56 = **$31,020.28**

TotalCost.SOL
31020

5) Based on the weekly demand from both the purchase and the water storage tank, the projected water storage tank inventory could be calculated as follows:

Week	**Calculations** (Note: Ending inventory= Beginning inventory + precipitation - weekly water tank usage)	**Ending inventory**	**Equal or Greater than 30,000?**
1	62,500 + 12,000 – 13,249 =	61,251	Y
2	61,251 + 18,000 – 13,733 =	65,518	Y
3	65,518 + 20,000 – 36,794 =	48,724	Y
4	48,724 + 22,000 – 40.724 =	**30,000**	Y

6) By choosing the alternative contract, XYZ would have the following calculation for the output variables and total water cost. XYZ would thus save $35,630 – $31,020 = **$4,610** by choosing the recommended contract over the alternative contract.

[1]	costst	weeklydemand	altdemandstweekly	altdemandwcweekly	precipitation
1	0.18	52997	13252	39756	12000
2	0.18	54932	13741	41222	18000
3	0.10	72141	33262	38879	20000
4	0.10	77860	44246	33615	22000

ALT_TotalCost.SOL
35630

7) In the situation of the recommended contract, the water level at the end of the four week period would be 30,000 gallons. In the situation of the alternative contract, the water level would be 29,999 gallons. (**Note:** The ending inventory of the alternative contract should be 30,000, according to the constraint we set. The calculated difference resulted from the default setting of SAS software. If the numbers are significantly large, SAS will only show the integer in the output solutions, such as in column "altdemandstweekly".)
Due to this default setting, we calculated the level at the end of the four-week period as 29,999 gallons, which would be 30,000 gallons by adding in the hidden decimals. Therefore, there would not be a difference in the water level comparing the recommended contract to the alternative one.

Week	**Calculations** (Note: Ending inventory= Beginning inventory + precipitation - weekly water tank usage)	**Ending inventory**	**Equal or Greater than 30,000?**
1	62,500 + 12,000 – 13,252 =	61,248	Y
2	61,248 + 18,000 – 13,741 =	65,507	Y
3	65,507 + 20,000 – 33,262 =	52,245	Y
4	52,245 + 22,000 – 44,246 =	**29,999**	Y