Newton's Laws of Motion

Agenda and Checklist

The main goals of this worksheet are:

- Understand and apply Newton's second law of motion.
- Incorporate air resistance into a projectile model.
- Practice using second order differential equations, event functions, and trajectories.
- Recreate a model directly from code.

☑ Write your name here: Lilo

☑ Write the name(s) of your studio partner(s) here: Jen

☐ By midnight on Friday, November 8: Scan this worksheet and submit it on Canvas.

Applying Newton's Second Law

Falling Objects

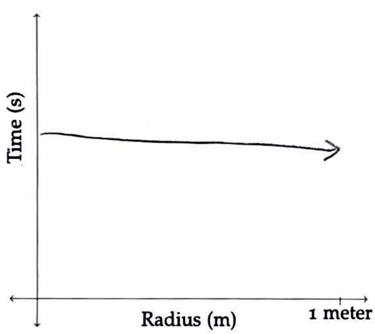
$h = 381\ m$

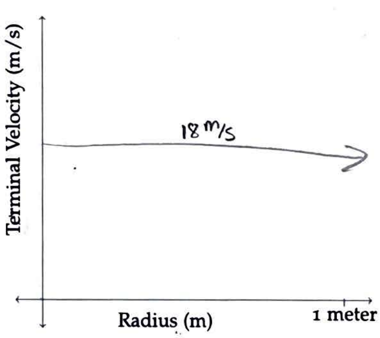
Q1. In Chapter 21, we modeled a penny falling off the Empire State Building, in which the two main forces were gravity and air resistance. In this exercise, we will consider larger pennies. Use the Chapter 21 notebook as a starting point to graph time to hit the sidewalk and terminal velocity as functions of coin radius. Draw the plot below. You should assume that the height of the penny stays constant at 1.52 *mm* and the density of copper is 8960 *kg*/*m*³.

$F = ma = mg$

$m = 1.52 \cdot 10^{-3} (\pi r^2) \cdot 8960 =$

$v = \sqrt{\frac{2mg}{\rho A C}}$ bc ratio of SA : mass





Q2. Explain why you observe the results that you do. Hint: Think about the equation for each force acting on the penny.

$v = \sqrt{\frac{2mg}{\rho A C}}$ where $m \propto A$

Q3. How do our model's assumptions change as the radius increases? Are there any assumptions violated or extra assumptions we should add?

Realistically the coin should flip as it is falling. Since the ratio of thickness to face area is changeing, this could affect the result slightly.

Since the frontal area of the penny is changing, it is changing the shape of the projectile causing the drag coefficient to become less accurate.

Q4. Now let's try some different objects! Choose 2-3 objects that you'd like to drop off the Empire State Building and run your model on these objects. Record the following information in the table below. Estimates of drag coefficients for some common shapes can be found here: `https://en.wikipedia.org/wiki/Drag_coefficient`.

$$v = \sqrt{\frac{2mg}{\rho A C}}$$

What did you need to update in your code?

Object	Mass (kg)	Frontal Area (m^2)	Drag Coefficient	Terminal Velocity (m/s)	Time to Hit the Sidewalk (s)
model rocket	30 g	250 mm^2	0.75	50 m/s	11 s
formula car	600 kg	1.8 m^2	1.4	62 m/s	10 s
tesla	2000 kg	3 m^2	0.24	210 m/s	9 s

$2 \times 1.5 = 3$

$1.8 \times 1 = 1.8$

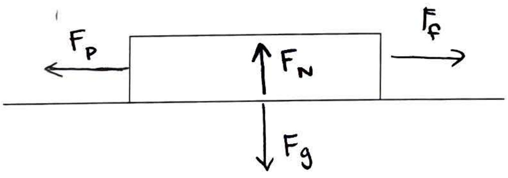
πr^2

Free Body Diagrams

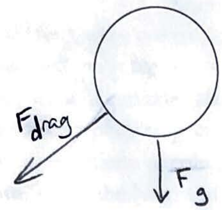
Q5. Recall from lecture that a free body diagrams are used to visualize the forces on a given body. Like stock and flow diagrams, free body diagrams will help us create models in this module. That is, once we have the free body diagram for an object in question, we can translate this into a set of differential equations and a model implementation.

For each of the following, draw all forces acting on the given object.

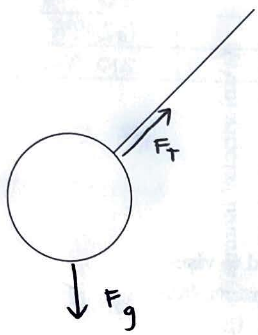
A. Book being pushed to the left across a table.



B. Baseball traveling at a 45 degree angle through the air.



C. Ball on a string.



Mystery Motion

Alice secretly wrote a model for some object in motion, but she won't tell us what it is. Luckily, Steve discovered the update function from her code below. In this exercise, we'll reconstruct the model from this function.

```
def update_func(state, angle, system):
    """
    Input: ??
    Output: ??
    """
    tension, x, y = state
    angular_velocity = system.angular_velocity
    gravity, radius, mass = system.gravity, system.radius, system.mass
    dangle = system.dangle

    dtension_dangle = -mass*gravity*math.cos(angle)
    dx_dangle = -radius*math.sin(angle)
    dy_dangle = radius*math.cos(angle)

    tension += dtension_dangle*dangle
    x += dx_dangle*dangle
    y += dy_dangle*dangle

    return State(tension = tension, x = x, y = y)
```

Q6. What are the parameters of the model? What are the state variables?

State var: tension, x, y

param: angular_velocity, gravity, radius, mass, dangle

Q7. Write the differential equation for each state variable below. Notice that these derivatives are in terms of `angle` rather than `t`.

$$\frac{dT}{d\theta} = -mg\cos\theta$$

$$\frac{dx}{d\theta} = -r\sin\theta$$

$$\frac{dy}{d\theta} = r\cos\theta$$

Q8. Integrate the equations above to find the functions for each state variable (up to a constant).

$$T(\theta) = -mg(-\sin(\theta)) = mg\sin\theta + C$$

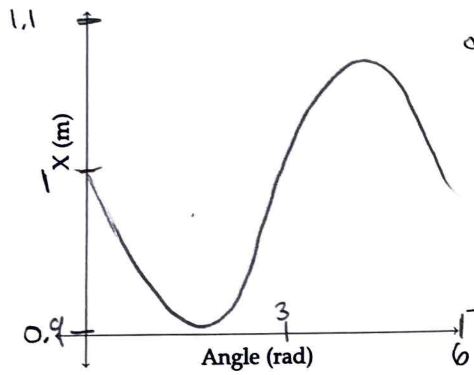
$$x(\theta) = -r\cos\theta + C$$

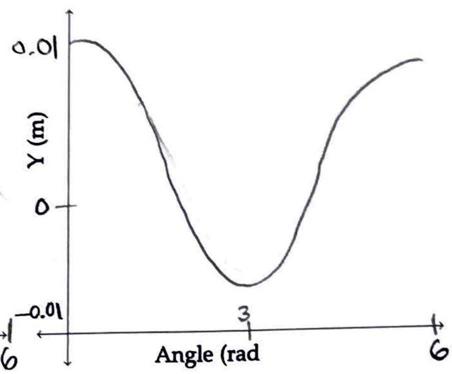
$$y(\theta) = -r\sin\theta + C$$

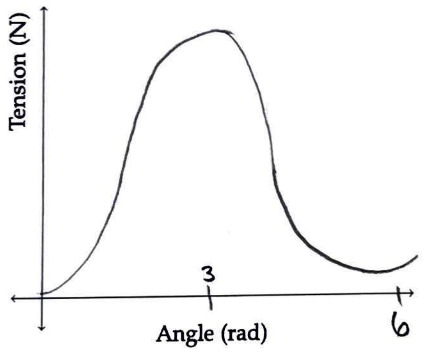
Q9. Write code to run a simulation of the above code. Use the line of code from Alice's simulation below to set up your system. Record the output using a TimeFrame. Then plot the three state variables over time below. What do you notice about each plot? What do you think Alice is modeling?

```
system = make_system(init=State(tension=2.45,x=1,y=0), angular_velocity=3.14,
                     radius=1,mass=0.25,gravity=9.8,angle0=0,angle_end=7, dangle=0.01)
```

thetas = linspace (0,7, 0.01)







~~pendulum~~ swinging

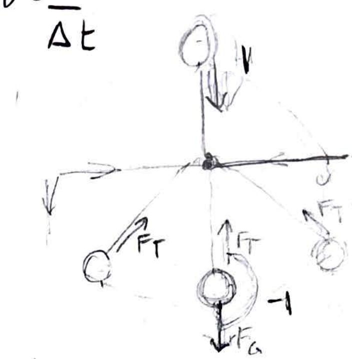
something around in a circle

Q10. When objects are traveling in a circular path, the angular velocity v is the change in angle over time ($\Delta \text{rad}/\Delta s$). A centripetal force is a force orthogonal to the direction of motion (i.e., a force that points towards or away from the center of motion). By Newton's laws, the sum of centripetal forces is equal to

$$F_c = \frac{mv^2}{r}.$$

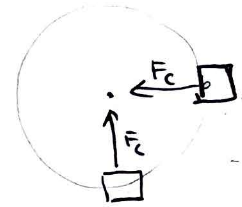
Consider a mass on a string. Using this formula and the free body diagram you drew in question 5C, solve for the tension in the string given the angle of the mass. What happens if the velocity is very small? How does this relate to the equations you solved for in question 8?

$$v = \frac{\Delta \theta}{\Delta t}$$



$$F_c = F_T + mg\sin\theta$$
$$F_T = F_c - mg\sin\theta$$
$$= \frac{mv^2}{r} - mg\sin\theta$$

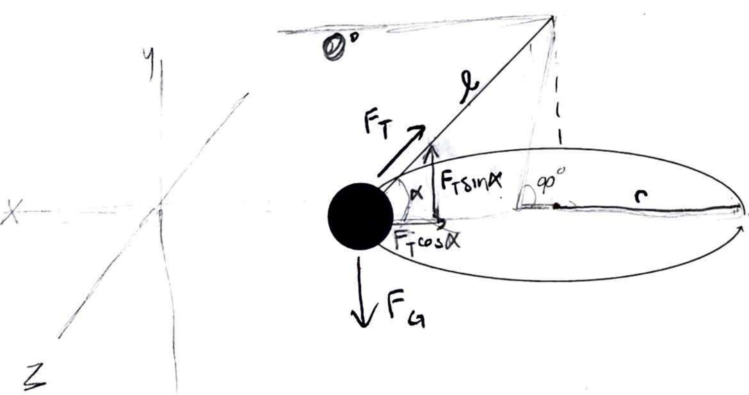
if $F_T <= 0$ because v is too small to counteract gravity, the ball won't swing all the way around the circle



Challenge! Other Spinning Objects

Implement a model for a mass spinning horizontally on a string. See the picture below. You should start with a free body diagram.[1]

[1] If you finish the rest of the worksheet in class, then move on to this challenge!



$\frac{1}{\sin}$ $\frac{1}{\tan}$

$$F_T \sin\theta = F_G = mg$$
$$F_T = \frac{mg}{\sin\alpha} = mg\csc\alpha$$
$$\frac{dF_T}{d\alpha} = mg(-\csc\alpha\cot\alpha)$$
$$r = l\cos\alpha$$
$$x =$$

$$F_T\cos\alpha = F_c = \frac{mv^2}{r} = \frac{mv^2}{l\cos\alpha}$$
$$\frac{\cancel{m}g\cos\alpha}{\sin\alpha} = \frac{\cancel{m}v^2}{l\cos\alpha}$$
$$v = \sqrt{\frac{lg\cos^2\alpha}{\sin\alpha}}$$

$$F_T = \sqrt{F_c^2 + F_G^2}$$
$$=$$

Worksheet Reflection Questions

1. **We've now practiced translating from a model to an implementation (many times!) and translating from an implementation to a model. What do you think the benefits of practicing the latter are?**

Translating a model into an impementation is a useful skill that we have practiced, but translating an implementation into a model allows us to check our understanding as well as learn to understand others' work better.

2. **Consider the current state of your "modeling toolkit" after working through Chapters 20–23 of the ModSimPy book. What kinds of physical systems are you now equipped to model, and what kinds of useful questions might you be able to answer about them?**

we have learned how to model thermal systems, HIV-infected cells, and kinematics using differential equations. Now we can model any other system where we can calculate or quantify the rate of change of something.

3. **Give a few examples of physical systems are you *not* equipped to model using the tools you currently have. Why not? How do you think people model them?**

Any system where all of the factors affecting the rate of change can't be quantified easily, such as thermal/ ocean systems, present a very difficult modelling challenge.