Epidemiology

Agenda and Checklist

The main goals of this worksheet are:

- Gain a deeper understanding of compartment models and stock and flow diagrams.
- Implement optimizations in the SIR model.
- Understand how sweeping multiple parameters can be used to identify patterns of behavior in a model.
- Practice mathematical analysis and understand its use as a tool for answering modeling questions.

☑ Write your name here: Lilo Heinrich

☑ Write the name(s) of your studio partner(s) here: Luke Nonas-Hunter

☐ By midnight on Friday, October 11: Scan this worksheet and submit it on Canvas.

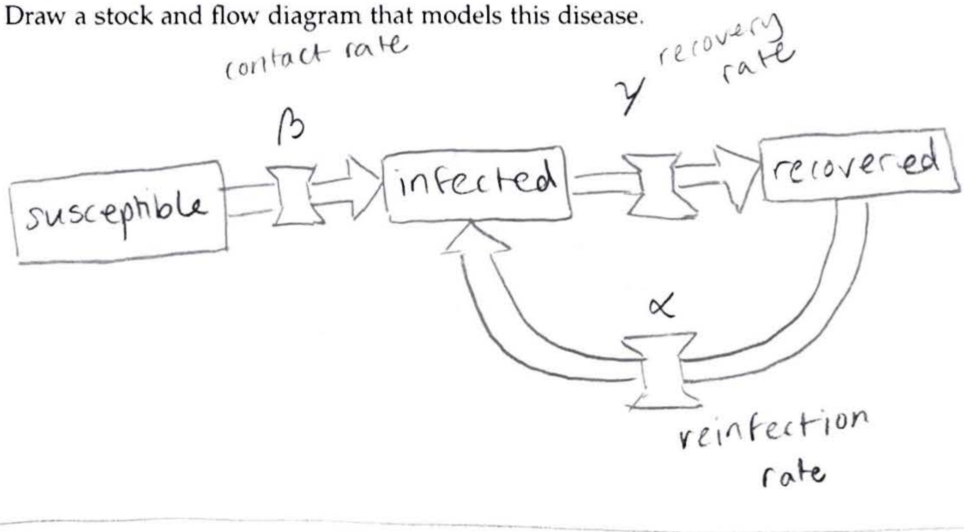
The SIR Model

Reading

Q1: The SIR model considers three populations, susceptible, infectious, and recovered, represented by the variables s, i, and r, respectively. The beginning of Section 11.3 has three differential equations that describe the changes of these populations over time. Why does r never appear in these equations?

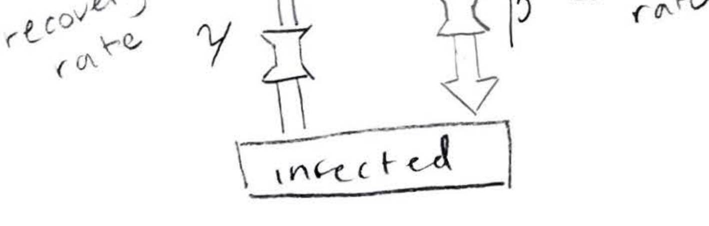
r never appears in the equation because the model assumes that once someone has recovered, they are effectively immunized and will not be re-infected.

Q2: Suppose that we are considering a disease in which recovered people *can* get reinfected. We will assume that recovered people are just as susceptible to infection as people who have never had the disease. Draw a stock and flow diagram that models this disease.



V1

V2

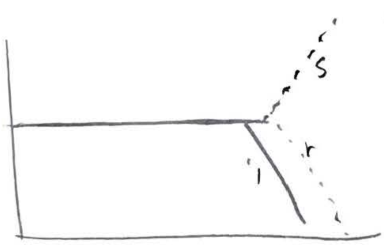


Notebook

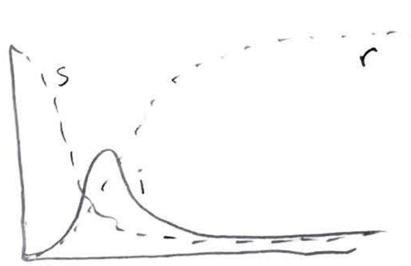
Q3: Execute the cells in the Chapter 11 notebook until the exercise. (You should not have to write any code for this to work.) Keep track of the results you get. Now find the first occurrence of `update_func` in the Chapter 11 notebook. Replace the parameter `t` with an underscore (`_`) and execute the cells again. At what point, if any, does the code fail to run? If the code successfully runs, what does this tell you about the parameter `t`?

It never fails or gets errors
this is because the update function does not actually use t, or the time, since all of the rates are constants.

Q4: The last exercise of this notebook asks you to change the time between contacts `tc` and the recovery time `tr` and plot the results of the simulation. Write a function `plot_simulation(tc, tr)` that takes values of `tc` and `tr`, sets up and runs the simulation, and plots the results. Then test your function with various choices of these parameters, sketch the results of two plots you found particularly interesting, and argue why these plots are interesting or useful.



(0.25, 1)
since tc is <1, this makes the contact rate greater than 100% of the population, which is a bug in this simulation because then the amount of recovered people becomes negative.



(1,4)
with a contact rate that is short by comparison to a longer recovery rate, we see that almost everyone becomes infected and recovers pretty quickly, due to the high probability of infection per individual.

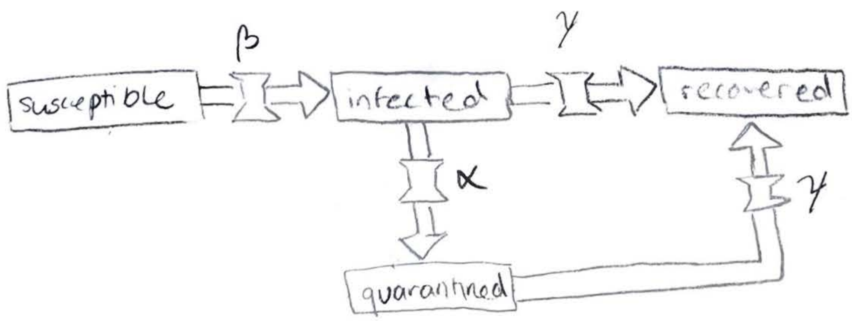
Optimization

These questions refer to the last two exercises of the Chapter 12 notebook.

Q5: If the price per dose of the vaccine is low enough, it is actually optimal to spend *all* of the budget on vaccines, and none on a hand-washing campaign. What is the lowest price per dose (in integer dollar amounts) where it is optimal to still spend some of the budget on a hand-washing campaign? (If you want to challenge yourself, try implementing this as a parameter sweep.)

$53 is the lowest price per dose where spending at least some money on hand washing is still the most efficient solution.

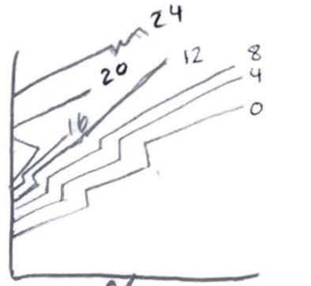
Q6: The last exercise discusses the possibility of quarantining infected students until they are no longer infectious. Assume that for each infected student, we quarantine α per day. (The recovery rate is unchanged.) Draw a stock and flow diagram for this model.



Sweeping Multiple Parameters and Analysis

Q7: In Chapter 13 (both the book and the notebook), you can see how to sweep multiple parameters to determine the fraction of infected individuals for different values of beta and gamma. Sweep various values of beta and gamma to determine their relationship to *either* the peak fraction of individuals infected at a single time, *or* the day on which the epidemic peaks (the fraction of infected individuals is highest). In the space below, sketch a plot showing this relationship (either multiple sweeps on a single graph or a contour plot) and comment on the results.

$= \frac{max(i(t))}{N}$ β



γ

when $\gamma \downarrow$ t in $max(i(t)) \uparrow$

when $\beta \downarrow$ t in $max(i(t)) \uparrow$

(γ)
when recovery rate is lower, the epidemic tends to peak later, because the number of infected individuals doesn't decrease as quickly.

the contact rate must always be greater than the recovery rate in order for the infection to spread.

Lastly, as the contact rate (β) increases, the epidemic tends to peak sooner.

Q8: In this question, you will do the analysis from Section 14.3 with a more general set of assumptions. Suppose that $i(0) = i_0$, $s(0) = s_0$, fractions $i(\infty) = i_\infty$, and $s(\infty) = s_\infty$. Also assume that at the start of the epidemic, nobody has recovered from the disease yet, so $r(0) = 0$. What is the equation that relates the contact number c to i_0, s_0, i_∞, and s_∞?

$$C = \frac{\gamma}{\beta}$$

$i_0 = 0$	$s_0 = 1$
$i_\infty = 0$	$s_\infty = ?$

① $\frac{di}{dt} = \beta si - \gamma i$

② $\frac{ds}{dt} = -\beta si$

③ $\frac{di}{ds} = \frac{\beta si - \gamma i}{-\beta si} = -1 + \frac{\gamma i}{\beta si} = -1 + \frac{\gamma}{\beta s} = -1 + \frac{1}{cs}$

④ $i = -s + \frac{1}{c}\log s + q$

⑤ $q = i + s - \frac{1}{c}\log s$

⑥ $q_0 = i_0 + s_0 - \frac{1}{c}\log(s_0) = 0 + 1 - \frac{1}{c}\log 1 = 1$

⑦ $q_\infty = i_\infty + s_\infty - \frac{1}{c}\log(s_\infty) = 0 + s_\infty - \frac{1}{c}\log(s_\infty)$

⑧ $1 = s_\infty - \frac{1}{c}\log(s_\infty)$

⑨ $c = \frac{\log(s_\infty)}{s_\infty - 1}$

Q9: In this question, you will use your equation from Q8 to estimate the value of c based on some knowledge of the epidemic. Specifically, we will assume that the total population is 90, and that the epidemic starts with a "patient zero," that is, a single infected individual. Also assume that at the end of the epidemic, there are no more infected individuals, so $i_\infty = 0$. Use this information to plot an estimate of c for various values of s_∞ over the existing data. (Your graph should look like Figure 14.2 in the book; feel free to use code from the Chapter 14 notebook as a starter.) Sketch your estimate on the graph below.

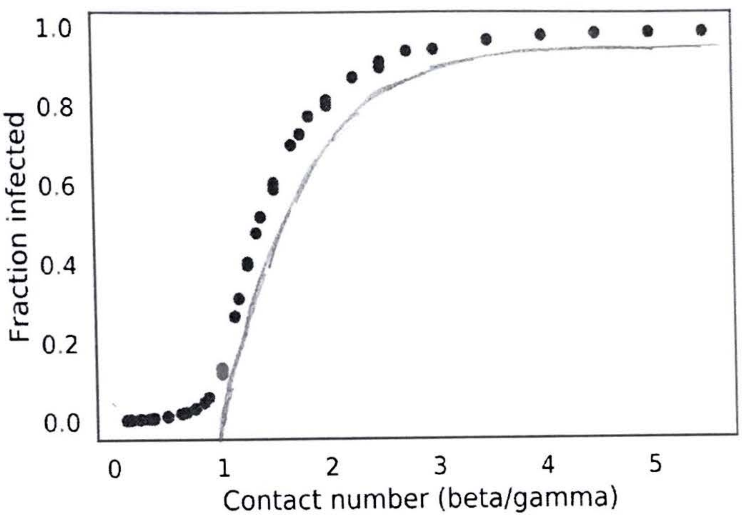
$N = 90$

$i_0 = \frac{1}{N} = \frac{1}{90}$

$i_\infty = 0$

$s_0 = \frac{N-1}{N} = \frac{89}{90}$

$s_\infty = ?$ ←



$q = i + s - \frac{1}{c}\log s$

$q_0 = \frac{1}{90} + \frac{89}{90} - \frac{1}{c}\log\left(\frac{89}{90}\right) = 1 - \frac{1}{c}\cdot -0.0117 = 1 + \frac{0.0117}{c}$

$q_\infty = 0 + s_\infty - \frac{1}{c}\log(s_\infty)$

$1 + \frac{0.0117}{c} = s_\infty - \frac{1}{c}\log(s_\infty)$

$\frac{1}{c}\log(s_\infty) + \frac{0.0117}{c} = s_\infty - 1$

$\frac{\log(s_\infty) + 0.0117}{s_\infty - 1} = c$ (contact number)

$y = \frac{89}{90} - \frac{s_\infty}{90}$ (fraction infected)

graphing fraction infected by contact number

Worksheet Reflection Questions

repairs
new bikes
docks

1. In Chapter 12, you saw an example of how we could use optimization to determine which solutions to use in response to a problem. Choose a problem domain that you have seen previously in this course, and write a modeling question that brings optimization into that example. (The beginnings of Sections 12.3 and 12.4 have some ideas on how you might phrase your question.)

   Bikeshare:
   with a fixed budget to make improvements to a bikeshare system, we have to decide how much to spend on repairs, new bikes, and docks.

2. In Chapters 13 and 14, you saw how sweeping multiple parameters and mathematical analysis could help answer more complex modeling questions. Choose a problem domain that you have seen previously in this course (different from the one you selected above), and describe how you could use a multi-parameter sweep or analysis to answer a question in this domain.

   Population Growth:
   sweep growth rate and death rate and make a contour graph

3. In Project 1 and Chapters 11–14, you used many new techniques for working with data and implementing models in Python. For example, you likely encountered error messages and how to interpret them, as well as system objects and a variety of plot types. Which have you found most helpful or useful, and why?

   - using matplotlib to plot data and control what is on both the x & y axes has been really helpful
   - also sweepseries are always pretty useful even if they only store 1 column of data