Calculus III ⇒ Math 265

Multivariate Analysis Worksheet

#1) Find the domain for each:

A) $f(x,y) = \sqrt{16 - 4x^2 - y^2}$

B) $f(x,y) = \ln(x^2 + y^2)$

#2) Find the partial derivative of each:

A) f_x for $f(x,y) = e^{xy}(\cos x \sin y)$

B) f_y for $f(x,y,z) = \sqrt{x^2 + y^2 + z^2}$ @ $(3, \sqrt{11}, -4)$

C) f_{xyz} for $f(x,y,z) = x^3yz - 2x^3y^2z^2 + 3xz - 4y^3z$

D) $\frac{\partial^2 z}{\partial x \partial y}$ for $z = x^2 - 3xy + 8xy^2 + 2y^4$

E) $\frac{\partial^2 z}{\partial x \partial x}$ for $z = x^3 - 10xy + \frac{x}{y^2} + 6y^3$

#3) Find the actual change:

A) $f(x,y) = x^2 - xy$ from $(2,4)$ to $(2.16, 3.98)$

B) $f(x,y) = x^3 - 2xy^2$ from $(2,3)$ to $(2.06, 3.10)$

#4) Use the differential to approximate the change in:

A) $z = \ln(2x^2 + y^2)$ as (x,y) moves from $(0,1)$ to $(0.02, 1.1)$

B) $z = \cos(4x - y)$ as (x,y) moves from $(0.25, 0.5)$ to $(0.3, 0.75)$

#5) The volume of a right circular cone is given by $V = \frac{1}{3}\pi r^2 h$. The radius of the base measures 10 inches and the height measures 16 inches. The possible error in each measurement is 0.02 inches. Use differentials to approximate the maximum possible percentage error in the volume.

#6) Find the following partial derivatives of w and evaluate at t:

A) $w = xyz^2 + y^2z$, $x = t$, $y = t^2$, $z = t^3$
Find dw/dt when $t = 1$

B) $w = \sin(2x + y)$, $x = s^2$, $y = t^2$
Find dw/dt when $s = 2$ and $t = 1$

C) $w = \sqrt{x + 3y}$, $x = 2t$, $y = t$
Find dw/dt when $t = 2$

D) $w = \ln(x - y)$, $x = 5t$, $y = 3t$
Find dw/ds when $s = 4$ and $t = 1$

#7) Find ∇f, the gradient of f:

A) $f(x,y,z) = 2x^2 + 2y^2 + z^3e^z$ @ $(0,4,-1)$

B) $f(x,y) = 2x \ln(x^2y)$ @ $(-2,6)$

#8) Calculate the directional derivative of:

A) $f(x,y) = 2 - x^2 - y^2/2$ @ $(1,2)$ in the direction $u = \cos\pi/4\,\hat{i} + \sin\pi/4\,\hat{j}$

B) $f(x,y,z) = z^3e^{xy}$ @ $(-1,0,3)$ in the direction of $v = 3\hat{i} - \hat{j} - 5\hat{k}$

#9) Given $f(x,y) = x^2 \ln 2y$, find the maximum value of the directional derivative at the point $(1/2, 2)$.

#10) Let $f(x,y,z) = x^2 + y^2 + z^2 - 3xy + 2xz - yz$

A) Find the gradient of f @ $(1,2,-1)$.

B) Find the directional derivative of f at the point $(1,2,-1)$ in the direction of $v = 3\hat{i} - 2\hat{j} + 5\hat{k}$.

c) Find the maximum value of this directional derivative (in part B).