

$$f) \|\vec{w}\| = \sqrt{3^2 + 1^2 + -1^2} = \sqrt{9+1+1} = \boxed{\sqrt{11}}$$

$$g) \text{Proj}_{\vec{w}_2} \vec{u} = \frac{\vec{u} \cdot \vec{v}(\vec{v})}{\|\vec{v}\|^2} \vec{v} = \frac{(\langle 2, 1, -4 \rangle \cdot \langle 1, 0, 2 \rangle) \langle 1, 0, 2 \rangle}{\sqrt{1+4}}$$

$$\frac{2 \cdot 1 + 1 \cdot 0 + -4 \cdot 2}{2 + -8} =$$

$$\frac{-6 \langle 1, 0, 2 \rangle}{\sqrt{5}} = \frac{\langle -6, 0, -12 \rangle}{\sqrt{5}}$$

$$\boxed{\langle \frac{-6}{\sqrt{5}}, 0, \frac{-12}{\sqrt{5}} \rangle}$$

$$h) \vec{w}_2 \text{ for part G } \langle 2, 1, -4 \rangle + \langle \frac{6}{\sqrt{5}}, 0, \frac{12}{\sqrt{5}} \rangle$$

$$\vec{w}_2 = \vec{u} - \vec{w}_1 \quad 2 + \frac{6}{\sqrt{5}}, 1 - 0, -4 + \frac{12}{\sqrt{5}}$$

$$4.683281573, 1, 1.366563146$$

$$\vec{w}_2 = \boxed{\langle 4.69, 1, 1.37 \rangle}$$

$$i) \text{direction cosines for } \vec{w} \quad \langle 3, 1, -1 \rangle \quad \sqrt{3^2 + 1^2 + -1^2} = \sqrt{11}$$

$$\cos \alpha = \frac{x}{\|\vec{w}\|} = \frac{3}{\sqrt{11}}$$

$$\cos \beta = \frac{y}{\|\vec{w}\|} = \frac{1}{\sqrt{11}}$$

$$\cos \gamma = \frac{z}{\|\vec{w}\|} = \frac{-1}{\sqrt{11}}$$

$$j) \text{direction angles for } \vec{w}$$

$$25.24^\circ$$

$$72.45^\circ$$

$$107.55^\circ$$

$$k) \text{angle between } \vec{u} \text{ and } \vec{v}$$

$$\cos \theta = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \|\vec{v}\|}$$

$$\frac{\langle 2, 1, -4 \rangle \cdot \langle 1, 0, 2 \rangle}{\sqrt{2^2 + 1^2 + 4^2} \sqrt{1^2 + 0^2 + 2^2}} = \frac{2+0+-8}{\sqrt{21} \sqrt{5}}$$

$$\frac{2+0+-8}{\sqrt{105}} = \frac{-6}{\sqrt{105}}$$

$$\cos \theta = \frac{-6}{\sqrt{105}}$$

$$\theta = \cos^{-1} \frac{-6}{\sqrt{105}} = \boxed{125.84^\circ}$$