# GENERATIVE ADVERSARIAL NETWORK (GAN) MODEL IN ASSET PRICING

by

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## ABSTRACT

An important topic in asset pricing is explaining the variation in expected returns of financial assets. No-arbitrage pricing theory suggests the idea of a pricing kernel that governs asset prices. However, it remains a challenge to estimate the asset-pricing kernel. The difficulties include (1) choosing the right factors, (2) estimating the pricing kernel's functional form, and (3) selecting the right portfolio to estimate the kernel. Recently, Chen et al. (2021) proposed a Generative Adversarial Network (GAN) model that attempts to solve all three challenges in a single setup and claim to achieve the best performance compared to all existing models. This paper seeks to empirically validate Chen et al. (2021)'s research based on the United States stock data with the United Kingdom (UK) London Stock Exchange 1998 - 2017 data. This paper found that the GAN model outperformed the benchmark four-factor model in terms of Sharpe ratio.



**KEYWORDS: Generative Adversarial Network, Asset Pricing, Stochastic Discount Factor, Machine Learning, Deep Learning, Stock Returns**

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# 1 INTRODUCTION

An important goal in asset pricing is to explain the variation in expected returns of financial assets. No-arbitrage pricing theory suggests the idea of a pricing kernel, also known as the stochastic discount factor (SDF), that governs asset prices. The literature on determining the appropriate form of the SDF has a long history and perhaps one of the most extensive analyses in finance. However, it remains a challenge to estimate the pricing kernel. There are three challenges in estimating the asset-pricing kernel: (1) choosing the right factors (data features) to estimate the pricing kernel, (2) estimating the functional form of the pricing kernel and, (3) selecting the right portfolio (combination of individual assets) to estimate the pricing kernel.

Factor models such as Fama & French (1993)'s three-factor model and Carhart (1997)'s four-factor model constructed different factors based on empirical findings and estimated the pricing kernel through a linear combination of these constructed factors. These factor models have been performing well empirically, and are considered the benchmark for assets pricing models. Besides exploring various new possible factors, recent literature also attempts to estimate the pricing kernel using non-linear and non-parametric methods. For example, Gu et al. (2020) found that neural network models outperform linear models in estimating the pricing kernel. Furthermore, Gu et al. (2021) also realised that imposing economic theory onto machine learning models can improve model performance further.

With reference to existing literature, Chen et al. (2021) proposed a Generative Adversarial Network (GAN) model that imposes the no-arbitrage condition in the neural network model. GAN models are a class of neural network frameworks which consist of two competing network models in a zero-sum game. A discriminative neural network attempts to price the asset prices using no-arbitrage condition, while a generative neural network attempts to increase the pricing error through different combinations of assets and factors. Through the alternating training, the discriminative neural network estimates the functional form of the pricing kernel while the generative network constructs portfolios that no-arbitrage condition is least able to explain, thus solving the three problems in empirical asset pricing theory in a single setup. As compared to previous research that focused on firm characteristics data, Chen et al. (2021) included macroeconomic data using long short-term memory (LSTM), a class of neural network architecture commonly used in predictions with a time series structure. Their findings have shown that the inclusion of macroeconomic data did enhance the model performance. Therefore, Chen et al. (2021) analyses have outperformed all previous models in Sharpe ratio, explained variation and pricing errors.

However, similar to most literature, Chen et al. (2021)'s research focus solely on the United States (U.S.) market. In fact, Karolyi (2012) pointed out that only few papers in top finance journals have explored the model performance in non-U.S. markets. The lack of comparison of model performance on non-U.S. markets thus motivates this paper to empirically validate the external validity of Chen et al. (2021)'s research with the United Kingdom London Stock Exchange data from 1998 to 2017. This paper is organised as follows: Section 2 provides the literature review, Section 3 explains the methodology, Section 4 and 5 goes in-depth on the model training and evaluation metrics, Section 6 and 7 explains the data and empirical findings. This paper will end with discussion in Section 8 and finally, conclusion in Section 9. Overall, this paper concludes that the GAN model outperformed benchmark four-factor model when evaluated against the Sharpe ratio. Replication files are available on the author's GitHub account github.com/lingjie00/asset_pricing.

# 2 LITERATURE REVIEW

The literature on determining the appropriate form of the SDF has a long history and perhaps is one of the largest analysis in finance. The first asset-pricing model proposed by Sharpe (1964), Lintner (1965) and Mossin (1966) in the 1960s was the Capital Asset Pricing Model (CAPM). CAPM explains the variation in asset returns with the assets' exposure to market risk. The exposure is measured by a linear pricing kernel against a wealth portfolio which includes all possible asset classes. However, in reality, broad stock market index is commonly used as a proxy for the wealth portfolio. The resultant regression coefficient in this single-factor model is coined as "beta", and investors continued to use “beta” as a measure of the systematic risk of assets.

Ross (1976) proposed the arbitrage pricing theory (APT) and shown that if returns are generated by a linear factor model, there is a SDF linear in factors that prices the returns. However, the exact factors remained unknown. Fama & French (1993) proposed their version of candidate factors based on the APT and derived a three-factor model that expands on the CAPM model by including size and value factors. Fama & French (1993) found that small market capitalisation stocks and low-value stocks outperformed the market, and including the size and value factors improves the performance of the CAPM model. The size factor is the difference in returns between the smallest and largest stocks measured by market capitalisation. On the other hand, the value factor is the difference in returns between the cheapest and most expensive stocks measured by price to book ratio. Carhart (1997) built on the three-factor model to include a momentum factor: the difference in returns between the best and worst-performing stocks. Fama & French (2015) later expanded on the three-factor model to include profitability and investment. The profitability factor is the difference in returns between stocks with high and low operating profitability, and the investment factor is the difference in returns between high and low capital investment stocks. The three, four, and five factor models have been a benchmark for asset pricing models and remain relevant today. In recent years, different research papers have attempted to find new factors to improve the factor model performance. The term “factor zoo” was used by Feng et al. (2020) to describe the hundreds of factors proposed over the years in the literature. Feng et al. (2020) and Freyberger et al. (2017) investigated a wide range of such proposed factors and found that only a handful are statistically significant in explaining the asset returns.

The factor models mentioned earlier have explored different factors while assuming a linear functional form for the SDF. Recent literature attempt to estimate the SDF with non-linear, non-parametric models such as decision tree-based methods and neural network models. Non-parametric models do not require a priori knowledge of the SDF’s functional form and allow estimation of flexible non-linear functions. Moreover, Freyberger et al. (2017) show that non-linear relationships are essential in SDF estimation.

Gu et al. (2020) compared different machine learning models and show that tree and neural network models marginally outperform linear models, with the neural network model emerging as the best performing model, measured by the out-of-sample $R^2$.

Traditional machine learning models do not consider economic theories during estimation. Gu et al. (2021) found that imposing constraints based on economic theory can further improve the neural network models. Furthermore, Chen et al. (2021) imposed a no-arbitrage condition on neural networks through a generative adversarial network (GAN) model.

Goodfellow et al. (2014) first introduced GAN models for image recognition tasks, and GAN models continue to be heavily used in image and video tasks, as suggested by S & Durgadevi (2021)'s recent survey. GAN models differ from classical neural network models by including two competing neural networks in a zero-sum game. In the context of Chen et al. (2021) and this paper, one of the neural networks will price assets through no-arbitrage condition while the other neural network will attempt to find factors and mispriced assets to increase pricing error. This paper will explain the GAN model in detail in the methodology section.

Consumption-based asset pricing is an alternative approach that derives SDF directly from a utility function. However, Campbell & Cochrane (2000) show that the consumption-based model cannot account for the time-varying nature of SDF, resulting in poor performance compared to the factor models. Therefore, this paper focuses on factor models.

Current literature estimates SDF solely based on the firm characteristics data. However, Pelger & Xiong (2020) found that including macroeconomic data can further improve the performance of machine learning models, which is also evident in Chen et al. (2021)'s results. Chen et al. (2021) included a recurrent neural network with long short-term memory (LSTM) architecture in the GAN model to capture the time series dynamics of the macroeconomic data. Hochreiter & Schmidhuber (1997) first proposed the LSTM architecture, and a recent review on the applications of LSTM by Houdt et al. (2020) found that LSTM is still commonly used in sequence data to capture dynamics in data, such as language-related tasks. This paper will also explain the different LSTM components in the methodology section.

The main contribution of this paper is in examining the external validity of Chen et al. (2021)'s model using the United Kingdom (U.K.) London Stock Exchange (LSE) data. Karolyi (2012) found that most empirical papers consider the data-rich United States, with only around 20% of the papers in top finance journals investigating countries outside the United States (U.S.). Likewise, Chen et al. (2021) model was also trained based on the U.S. data. Therefore, this paper explores the model performance in a different context, such as the U.K. market. There are several works of literature done on LSE data, but none answers the question this paper seeks to address. Bhatnagar & Ramlogan (2012) and Korajczyk & Viallet (1989) focus only on linear factor models, while Tobek & Hronec (2018) forecast asset prices without considering economic theories. Therefore, one of the critical contributions of this paper is to provide insights into the potential benefits advanced models can bring to the empirical asset pricing of UK securities.

# 3 METHODOLOGY

This chapter focuses on explaining the empirical approach used in this paper. First, we present the no-arbitrage asset-pricing model that is the foundation of our base model: the Fama-French factor model and the GAN model. Next, we expand the no-arbitrage condition to construct a pricing loss function used in the GAN model. Then, we describe a simple feed-forward neural network before exploring a recurrent neural network model that considers data dynamics. Following that, we explain the GAN model, which consists of two neural network models competing based on the pricing loss function and end the chapter with an explanation for the Fama-French factor model.

Training neural network models is an empirical challenge. Regularization techniques are commonly used in machine learning to prevent over-fitting and improve model training results. Chen et al. (2021) adopted a dropout strategy proposed by Srivastava et al. (2014). This paper further includes early stopping rules introduced by Yao et al. (2007). The discussion on neural networks presented in this section is referenced from the Hands-

On Machine Learning with Scikit-Learn and TensorFlow by Géron (2017).

## 3.1 No-arbitrage asset-pricing model

The no-arbitrage asset pricing model assumes the existence of an asset independent, time-dependent pricing kernel, also known as a stochastic discount factor (SDF) $M_t$, such that there is no excess return in expectation. Let $R_{t+1,i}$ denote the asset $i$'s return in time $t+1$ and $R^f_{t+1}$ as the return of a risk-free asset. The excess return is then defined as $R^e_{t+1,i} := R_{t+1,i} - R^f_{t+1}$. Summarising the no-arbitrage condition, we have the equation

$$E_t[M_{t+1}R^e_{t+1,i}] = 0.$$

SDF can be expressed as an affine transformation of a mean-variance efficient tangency portfolio. The mean-variance efficient frontier refers to the maximum possible balance between the mean excess returns and risk factor, measured by the variance of the excess returns. A portfolio that maximises mean-variance efficiency is considered as a tangency portfolio. Let $\omega_t$ denote the SDF weights used to construct the tangency portfolio based on all the assets in the market, and $F_{t+1} := \omega_t^T R^e_{t+1}$ denote the tangency portfolio. Using the covariance formula $E(XY) = E(X)E(Y) + Cov(X,Y)$ and correlation formula $E(XY) = E(X)E(Y) + \rho(X,Y)\sigma(X)\sigma(Y)$, we have $E(M_{t+1})E(R^e_{t+1,i}) = -\rho(M_{t+1}, R^e_{t+1,i})\sigma(M_{t+1})\sigma(R^e_{t+1})$. Since $\rho(\cdot) \in (0,1)$, we can express the bounds of the excess return as a function of the risk of the asset, forming the mean-variance frontier, illustrated in figure 1

$$|E_t(R^e_{t+1,i})| \leq \left(\frac{\sigma_t(M_{t+1})}{E_t(M_{t+1})}\right)\sigma_t(R^e_{t+1}).$$

Since the tangency portfolio occurs at the mean-variance frontier where $\rho(M_{t+1}, F_{t+1}) = -1$, it is perfectly correlated to the SDF which is given by an affine transformation of the SDF ($M_{t+1} = a - bF_{t+1}$). Since any mean-variance efficient return carries all the pricing information, without loss of generality, we consider one of the possible $(a,b) = (1,1)$

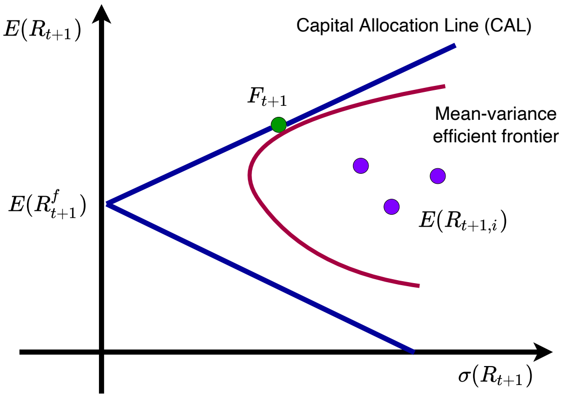



Figure 1: MEAN-VARIANCE FRONTIER

$$M_{t+1} = 1 - F_{t+1} = 1 - \omega_t^T R^e_{t+1}.$$

Therefore, the no-arbitrage condition reduces the asset-pricing problem into estimating an SDF weight function $\omega$ such that the following equation holds. We will use this relationship to construct our loss function that trains the GAN model

$$E\left[\left(1 - \omega_t^T R^e_{t+1}\right) R^e_{t+1,i}\right] = 0.$$

## 3.2 Pricing loss function

The no-arbitrage condition reduces the asset pricing problem into estimating the SDF weight function $\omega$. This paper assumes $\omega$ is a function of macroeconomic data $I_t$ and firm characteristic data $I_{t,i}$. However, suppose the no-arbitrage condition alone is insufficient to explain the differences in asset prices. In that case, we can define a function $g(I_t, I_{t,i})$ that selects firm characteristic and assets and output some factor unexplained by the no-arbitrage to price the assets. The relationship between $\omega(I_t, I_{t,i})$ and $g(I_t, I_{t,i})$ can be described by the following equation. We define this as our pricing loss function. The pricing loss function can be viewed as a competition between the no-arbitrage condition and an alternative theory that explains the variation in asset excess returns, which is the key in building the GAN model proposed by Chen et al. (2021)

$$E\left[\left(1-\omega(I_t,I_{t,i})^T R^e_{t+1}\right) R^e_{t+1,i} g(I_t,I_{t,i})\right]=0.$$

## 3.3 Neural network model

A neural network model is a flexible non-parametric model that is able to recover effectively the non-linear relationships, including the interaction effects, between input and output data. In this paper, we build two neural network models to (1) model the stochastic discount factor (SDF) weight and (2) model factors unexplained by the no-arbitrage condition. The GAN model section provides an in-depth explanation of these two neural networks. Each of the two neural network models combines feed-forward and recurrent neural networks. We first explain the standard feed-forward neural network before expanding to the recurrent neural network in long short-term memory (LSTM) architecture under subsection 3.4. Regularization techniques are employed to minimise model overfitting and are explained under subsection 4.1.

### 3.3.1 Feed-forward neural network

A standard feed-forward neural network (FFN) consists of one input layer, one or multiple hidden layer(s) and one output layer. In essence, FFN performs a linear combination of covariates before passing the intermediate output to a non-linear function. The output from the non-linear function is then linearly combined again, and the procedure repeats until the output layer. This paper first illustrates this relationship using Figure 2 before introducing the individual components.

The first layer in Figure 2 is the *input layer*. The number of input units in the input layer corresponds to the number of covariates available. Therefore, an input data $\mathbf{X}=(X_1,X_2,\cdots,X_p)$ with $p$ covariates will have $p$ input units in the input layer. The input data in this paper includes both macroeconomic factors and firm characteristics data.

The second layer in Figure 2 is the *hidden layer*. The input units are linked to the hidden layer as a directed acyclic graph (DAG). Each hidden unit will first linearly combine the input units before passing the intermediate output to a non-linear function $h(\alpha_{0m} + \alpha_m^T\mathbf{X})$, where $h(\cdot)$ is the non-linear function, $m = 1, \cdots, M$ is the $m$th hidden unit, and $\alpha$ is the weights in the hidden unit. The output in the hidden units is a non-linear transformation of the input units. Although the figure only shows a single hidden layer, there might be multiple hidden layers in practice, forming a deep neural network. The output of the hidden units will be passed as the input to the next hidden layer, and the procedure repeats.

The third and final layer in Figure 2 is the *output layer*. The number of output units in the output layer depends on the nature of the predicted variable. For example, in the case of a continuous $Y$, we have one output unit and in the case of categorical $Y$ the number of output units will follow the number of categories. In the continuous $Y$ case, the outputs from the hidden units are linearly combined to produce a single value as the final output of the FFN.

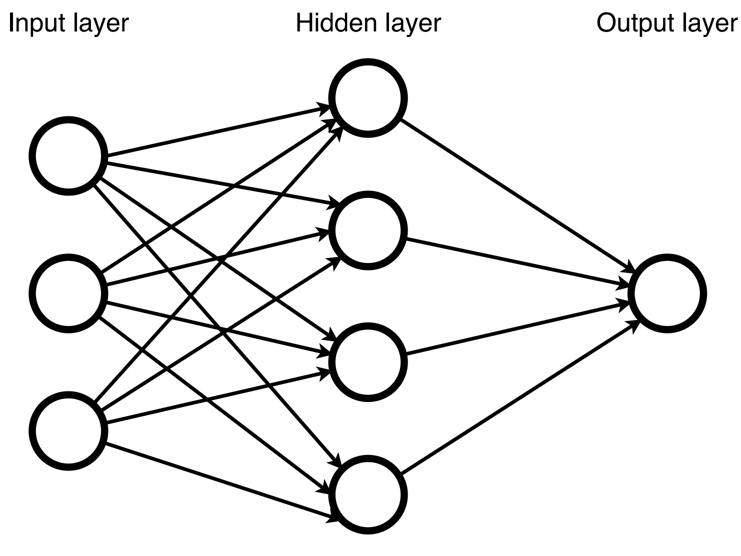



Figure 2: FEED-FORWARD NEURAL NETWORK

### 3.3.2 Network components

Let $\mathbf{X}$ be the vector of inputs, $y$ be the numeric output being predicted when $Y$ is continuous, $M$ be the total number of hidden units, $\alpha$ be the constants in the hidden units and $\beta$ be the weights in the output unit. The neural network structure can be summarized with the following equations

$$\begin{aligned} Z_m &= h(\alpha_{0m} + \alpha_m^T \mathbf{X}),\ m = 1, \cdots, M, \\ \mathbf{Z} &= (Z_1, Z_2, \cdots, Z_m), \\ y &= \beta_0 + \beta^T \mathbf{Z}. \end{aligned}$$

### 3.3.3 Activation function

Multiple activation functions are used in the literature. This paper uses one of the most commonly used activation functions known as rectified linear unit (ReLU), shown in Figure 3. $ReLU(x) := \max(x, 0)$ effectively removes the negative values. Nair & Hinton (2010) shown in their paper that ReLU improves the convergence rate of stochastic gradient descent compared to other activation functions such as the logistic function.

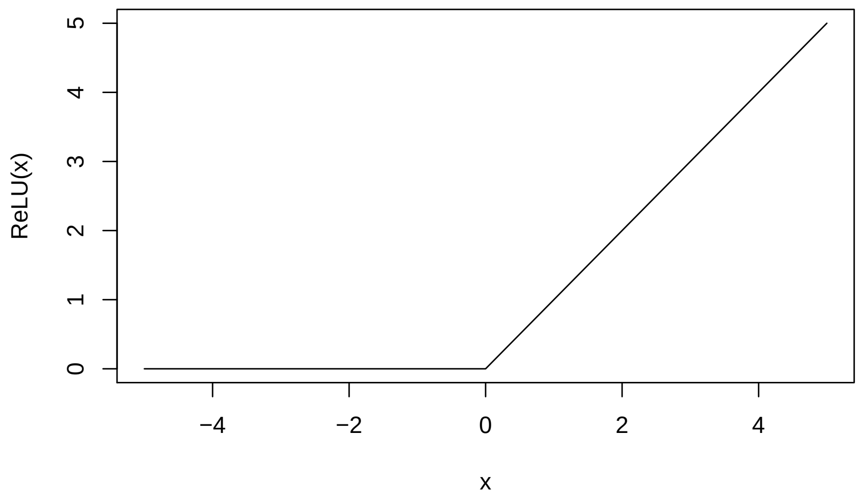



Figure 3: RECTIFIED LINEAR UNIT (RELU) ACTIVATION FUNCTION

### 3.3.4 Loss function

After the data is passed from the input layer to the output layer, network predictions are evaluated using an appropriate loss function. One commonly used loss function is the squared loss $L(y, \hat{y}) = (y - \hat{y})^2$. This paper implements a custom pricing loss function described in the pricing loss function under subsection 3.2.

### 3.3.5 Back-propagation

After the loss is calculated, a back-propagation training algorithm is then used to train the network. Back-propagation algorithm minimises a given loss function by updating the weights and constant terms in the network model through gradient descent methods. This paper adopted adaptive moment estimation (Adam) as the optimizer as described in subsection 4.1.

## 3.4 Long short-term memory network architecture

A recurrent neural network (RNN) is a class of neural networks that considers the past data in future prediction and hence is particularly suited for the prediction of time series data. Long Short-Term Memory (LSTM) is a sub-class of RNN that considers more extended dynamics through a gate controlling system. We focus our discussion here on LSTM. In contrast to a feed-forward network that receives only input data $\mathbf{x}_{(t)}$ at time $t$, an LSTM also gets a short term state $\mathbf{h}_{(t)}$ and a long term state $\mathbf{c}_{(t)}$.

As shown in figure 4, the long term state $\mathbf{c}_{(t-1)}$ from the previous iteration travels through a forget gate and an input gate before being passed through to the next iteration. The long term state $\mathbf{c}_{(t)}$ is also being passed as a short term state $\mathbf{h}_{(t)}$ through an output gate (which is also the output of the cell $\mathbf{y}_{(t)}$). The gates are being controlled by the functions $\mathbf{f}_{(t)}, \mathbf{i}_{(t)}, \mathbf{o}_{(t)}$, with logistic activation functions that produce 1 to keep the gate open and 0 to close the gate. The decision on the gates is based on the input data $\mathbf{x}_{(t)}$ and the short term state $\mathbf{h}_{(t)}$. The function $\mathbf{q}_{(t)}$ takes in the data and short term state to be combined with the long term state. Summarising the LSTM computation, we have the following equations. $\mathbf{W}_{x\cdot}$ are the weights for $\mathbf{x}_{(t)}$, and $\mathbf{W}_{h\cdot}$ are the weights for $\mathbf{h}_{(t-1)}$ and $\mathbf{b}_i$ are the constants for $i$ layers, $\otimes$ are element-wise multiplication. Therefore, using an LSTM neural network layer allows the GAN model to extract long term dynamics from macroeconomics data without specifying the number of lags as a hyper-parameter.

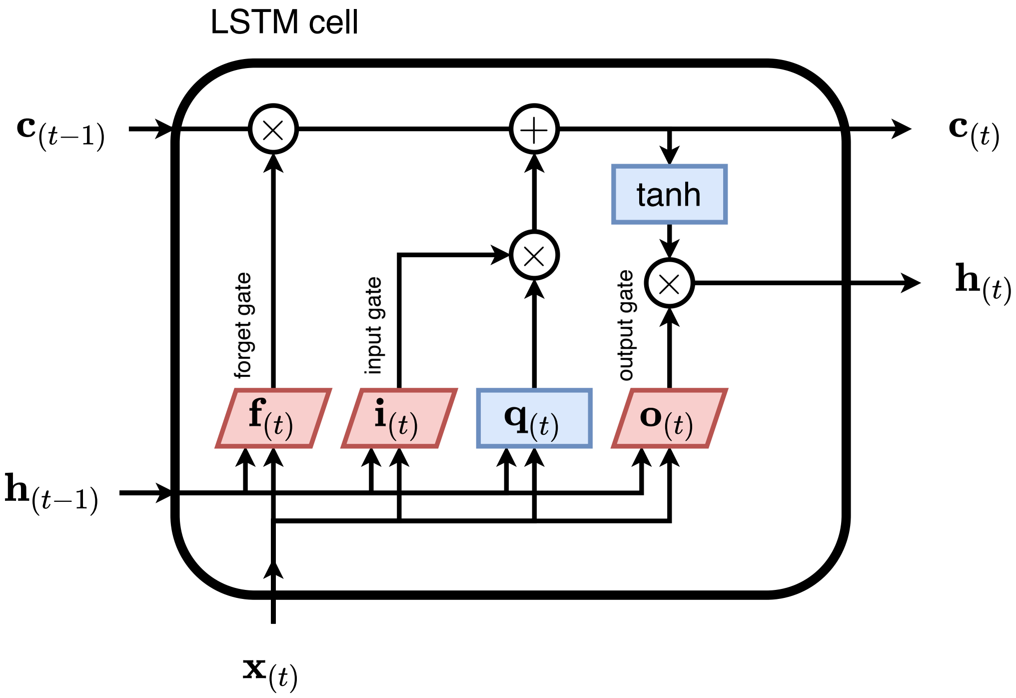



Figure 4: LONG SHORT-TERM MEMORY CELL

$$\mathbf{i}_{(t)} = \sigma(\mathbf{W}_{xi}^{T}\mathbf{x}_{(t)} + \mathbf{W}_{hi}^{T}\mathbf{h}_{(t-1)} + \mathbf{b}_i)$$

$$\mathbf{f}_{(t)} = \sigma(\mathbf{W}_{xf}^{T}\mathbf{x}_{(t)} + \mathbf{W}_{hf}^{T}\mathbf{h}_{(t-1)} + \mathbf{b}_f)$$

$$\mathbf{o}_{(t)} = \sigma(\mathbf{W}_{xo}^{T}\mathbf{x}_{(t)} + \mathbf{W}_{ho}^{T}\mathbf{h}_{(t-1)} + \mathbf{b}_o)$$

$$\mathbf{q}_{(t)} = tanh(\mathbf{W}_{xg}^{T}\mathbf{x}_{(t)} + \mathbf{W}_{hg}^{T}\mathbf{h}_{(t-1)} + \mathbf{b}_g)$$

$$\mathbf{c}_{(t)} = \mathbf{f}_{(t)} \otimes \mathbf{c}_{(t-1)} + \mathbf{i}_{(t)} \otimes \mathbf{q}_{(t)}$$

$$\mathbf{y}_{(t)} = \mathbf{h}_{(t)} = \mathbf{o}_{(t)} \otimes tanh(\mathbf{c}_{(t)})$$

## 3.5 GAN model

The GAN model consists of two competing neural networks called discriminator and generator. The discriminator in this paper will estimate the SDF weight function $\omega(I_t, I_{t,i})$, and the generator will estimate the factor function $g(I_t, I_{t,i})$. Both models use an LSTM network to extract signal from macroeconomic data and a feed-forward network to extract signal from firm characteristic data. The GAN architecture under section subsection 3.5

explains the model structure in-depth

$$\text{Discriminator: } \omega(I_t, I_{t,i}), \text{ Generator:} g(I_t, I_{t,i}).$$

Therefore, the GAN training procedure can be viewed as a zero-sum game where the discriminator minimises the pricing loss. In contrast, the generator tries to maximise the pricing loss. Note that the functions estimated by discriminator and generator are both time and asset independent. Given a data size $N$, we summarise the GAN training as the following min-max optimisation problem with the pricing loss function motivated by the no-arbitrage condition

$$\min_{\omega} \max_{g} L(\omega, g | I_t, I_{t,i}) = \frac{1}{N} \sum_{i=1}^{N} \left\{ E\left[ \left( 1 - \sum_{j=1}^{N} \omega(I_t, I_{t,j}) R^e_{t+1,j} \right) R^e_{t+1,j} g(I_t, I_{t,i}) \right] \right\}^2 .$$

## 3.6 Factor models

The Carhart (1997) four-factor model builds on Fama & French (1993)'s three-factor model. This paper uses the four-factor model as the benchmark against the GAN model.

The three-factor model aims to explain the assets' excess returns through (1) market risk ($R^e_{mt}$), (2) outperformance of small market capitalisation companies relative to large market capitalisation companies (small minus big, SMB) and (3) the outperformance of high book-to-market value companies versus low book-to-market value companies (high minus low, HML). On the other hand, the four-factor model further included a momentum (UMD) factor to account for the speed of price change. If the factor model correctly explains the variation in asset prices, we expect a no-intercept regression with $E(\alpha_i) = 0$. Formally the model can be described by the following equation, where $R^e_{t+1,i}$ is asset i's excess return,

$$R^e_{t+1,i} = \alpha_i + \beta_i R^e_{mt} + s_i SMB_t + h_i HML_t + \omega_i UMD_t + \epsilon_{t,i}.$$

Therefore, in contrast to the non-linear, non-parametric GAN model, the factor model can be considered as a parametric model where assets' excess return is a linear combination of the constructed factors.

# 4 MODEL TRAINING

This chapter explains the methodology used to train the GAN and the factor model. We first describe a general neural network training before focusing on the GAN model.

## 4.1 Training a neutral network

Training a neural network is an empirical experiment. We used Chen et al. (2019)'s best-performing hyper-parameters choices, including the number of hidden units and number of hidden layers, while following the best practice of neural network training, including dynamic learning rate and regularization.

### 4.1.1 Adam Optimizer

As explained in the subsection 3.3, back-propagation is one critical component in the training procedure. We used adaptive moment estimation (Adam) proposed by Kingma & Ba (2015), which combines momentum optimization and RMSProp, another optimizer popular before Adam. We define $\theta$ as the multi-variable weights, $\nabla_\theta L(\theta)$ as the multi-variable gradient with respect to a loss function $L(\cdot)$, $\eta$ as the learning rate, $\mathbf{m}$ as the momentum vector, and $\mathbf{s}$ as the squared of momentum vector, $\beta_1$ as the momentum rate, $\beta_2$ as the decay rate, $\epsilon$ as the smoothing parameter to avoid zero division, $\otimes$ as the element wise multiplication, and $\oslash$ as the element wise division. In contrast to a gradient descent algorithm where the updating rule is independent of the previous gradients: $\theta \leftarrow \theta - \eta\nabla_\theta L(\theta)$, a general momentum algorithm includes an additional parameter $\mathbf{m}$ that captures the value of previous gradients, allowing for faster convergence. Adaptive gradient methods scale down the gradient by the past gradient value $\sqrt{\mathbf{s}}$, decaying the steeper gradients more than the smoother gradients, allowing the parameter to convergence even faster. Adam combined the momentum and adaptive gradients techniques.

A momentum algorithm is described as:

1. $\mathbf{m} \leftarrow \beta_1 \mathbf{m} - \eta \nabla_\theta L(\theta)$
2. $\theta \leftarrow \theta + \mathbf{m}$

An adaptive gradient algorithm is described as:

1. $\mathbf{s} \leftarrow \beta_2 \mathbf{s} + (1 - \beta_2) \nabla_\theta L(\theta) \otimes \nabla_\theta L(\theta)$
2. $\theta \leftarrow \theta - \eta \nabla_\theta L(\theta) \oslash \sqrt{\mathbf{s} + \epsilon}$

The Adam algorithm is described as:

1. $\mathbf{m} \leftarrow \beta_1 \mathbf{m} - (1 - \beta_1) \nabla_\theta L(\theta)$
2. $\mathbf{s} \leftarrow \beta_2 \mathbf{s} + (1 - \beta_2) \nabla_\theta L(\theta) \otimes \nabla_\theta L(\theta)$
3. $\hat{\mathbf{m}} \leftarrow \frac{\mathbf{m}}{1 - \beta_1^t}$
4. $\hat{\mathbf{s}} \leftarrow \frac{\mathbf{s}}{1 - \beta_2^t}$
5. $\theta \leftarrow \theta + \eta \hat{\mathbf{m}} \oslash \sqrt{\hat{\mathbf{s}} + \epsilon}$

#### 4.1.2 Dynamic learning rate with learning schedule

The learning rate $\eta$ affects the extent to which gradients are updated by changing the step size in the gradient descent method. A too high learning rate prevents gradient descent from converging while a too low learning rate significantly increases the training time. Instead of using a fixed learning rate, this paper adopts exponential scheduling where the learning rate is updated as the training epochs $t$ increase. We denote $\eta_0$ as the initial learning rate, $s$ as a hyper-parameter step that decreases the impact of the initial $s$ training epochs. As a result, the learning rate decreases faster as the training epochs increase

$$\eta(t) = \eta_0 0.1^{t/s}.$$

#### 4.1.3 Regularization

Regularization refers to techniques used to prevent over-fitting. Over-fitting occurs when the model is tuned solely on the training data and does not generalise well in unseen test data. For example, LASSO and Ridge modify least square regression by including $\ell 1$ and $\ell 2$ penalties. Deep learning provides additional regularization techniques, including the dropout and early stopping adopted in this paper.

**Regularization with dropout** Dropout is a simple yet powerful algorithm, users set a hyper-parameter dropout rate $p$, and during training, $p\%$ of the hidden units will not be included in the gradient calculation. Intuitively, dropout forces the neural network to train a new but dependent model during each training epoch, avoiding relying on the same information each time. Dropout is not used during prediction.

**Regularization with early stopping** Early stopping is a technique that stops model training before the model over-fits the training data. Stopping criteria is pre-set by the users, and for a standard neural network, the decrease in loss function is often used as the criteria. For example, training stops if the decrease in loss between $t$ and $t+1$ is less than $\epsilon$. However, it is harder to decide on stopping criteria for GAN models, and this paper uses the Sharpe ratio as the stopping criteria.

## 4.2 GAN architecture

As explained in subsection 3.5, a GAN model alternates between training a discriminator and generator. To further increase the convergence speed, we first train the individual network separately in a standard neural network training procedure before training them in a GAN framework.

### 4.2.1 Discriminator network structure

As illustrated in Figure 5, the discriminator network has two input layers, one for macroeconomic data and another for firm characteristics data. The network concatenates the latent variables produced by the LSTM layer and firm characteristics to produce a single output corresponding to the stochastic discount factor (SDF) weight $\omega_{\mathbf{t}}$. Therefore, the training for discriminator can be expressed as the following equation

$$\min_{\omega} L(\omega|\hat{g}, I_t, I_{t,i}).$$

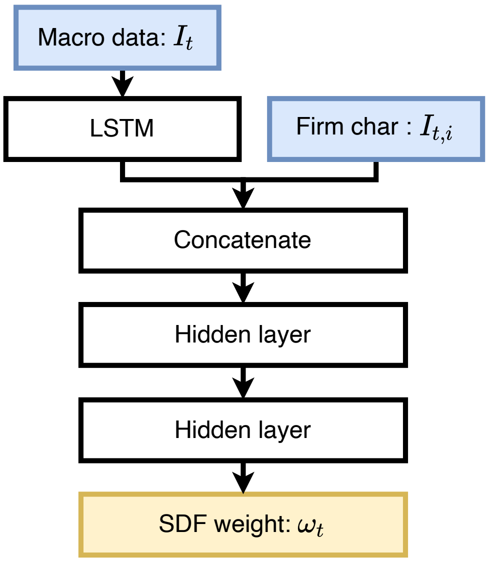



Figure 5: DISCRIMINATOR STRUCTURE

#### 4.2.2 Generator network structure

As illustrated in Figure 6, the generator network shares a similar structure as the discriminator network. The only difference is in the output layer, where the generator network will select factors representing a combination of assets and firm characteristics unexplained by the no-arbitrage condition. Therefore, the generator training can be expressed as the following equation

$$\max_{g} L(g|\hat{\omega}, I_t, I_{t,i}).$$

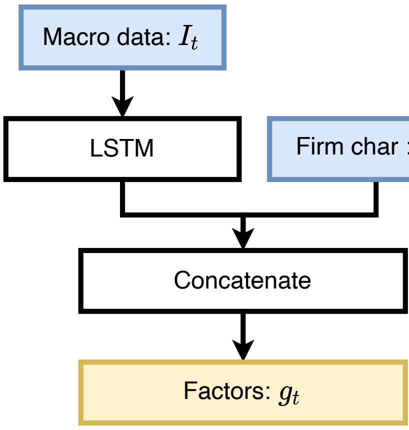



Figure 6: GENERATOR STRUCTURE

### 4.2.3 GAN network structure

As shown in Figure 7, the discriminator and generator are linked by a single pricing loss function. Discriminator aims to decrease the pricing loss while generator seeks to increase the pricing loss. Note that we require the SDF weights multiplied by the excess returns before the pricing loss calculation in constructing the SDF.

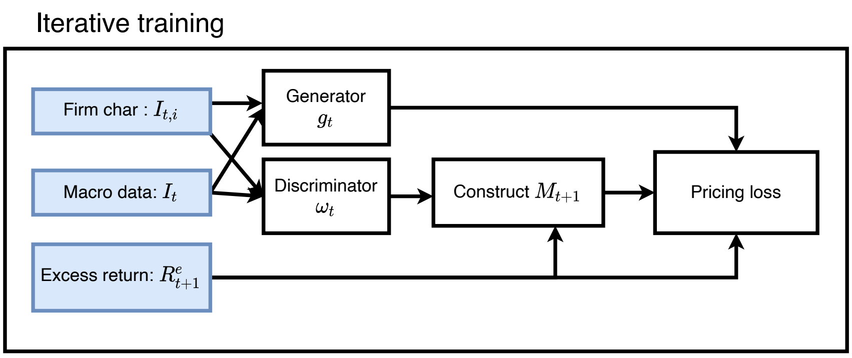



Figure 7: GAN MODEL STRUCTURE

### 4.2.4 Empirical no arbitrage asset pricing loss

As the firms exist for different duration, to incorporate unbalanced data, we weighted the pricing loss to the number of non-missing data $T_i$

$$\min_{\omega} \max_{g} L(\omega, g|I_t, I_{t,i}) = \frac{1}{N}\sum_{i=1}^{N}\frac{T_i}{T}\left[\frac{1}{T_i}\sum_{t\in T_i}\left(1-\sum_{j=1}^{N}\omega(I_t, I_{t,j})R^e_{t+1,j}\right)R^e_{t+1,i}g(I_t, I_{t,i})\right]^2.$$

## 4.3 Training the factor model

The factor model can be trained using least squares estimation. This paper uses ordinary least squares method for estimation.

# 5 EVALUATION METRICS

This paper evaluates the performance of the GAN model against the four-factor model based on the Sharpe ratio.

## 5.1 Sharpe ratio

Sharpe (1966) proposed the Sharpe ratio to evaluate the performance of risk-adjusted assets relative to a risk-free asset. A Sharpe ratio ($S$) for an asset $i$ is defined as the asset's historical excess return $E_t(R_i^e)$ weighted by the risk of the asset, where the standard deviation of the excess return $\sigma_i$ approximates the risk of an asset. The excess return is the difference between the asset's return and the risk-free return $E_t(R_i^e) = E_t(R_i - R^f)$. Sharpe ratio measures the reward investors get when investing in the asset while considering the risk of the assets. Therefore, a higher Sharpe ratio implies the asset is better, and the tangency portfolio $F_{t+1}$ that lies on the mean-variance frontier will have the highest Sharpe ratio possible. We compare the Sharpe ratio of the tangency portfolio by both GAN and four-factor models to decide on the model with better performance

$$S(R_i) = \frac{E_t[R_i - R^f]}{\sigma_i}.$$

### 5.1.1 GAN Sharpe ratio computation

The discriminant network in the GAN model estimates the SDF weight $\omega_t$. Therefore, by construction, the tangency portfolio $F_{t+1}$ will be defined as the dot product between the SDF weight and assets' excess returns $R_{t+1}^e$ as described in the equation

$$F_{t+1} = \omega_t^T R_{t+1}^e.$$

### 5.1.2 Factor model Sharpe ratio computation

Let $\omega$ be the optimal portfolio weight for factor models, $f$ be the factors, $\Sigma$ be the covariance matrix between factors and $R^f$ be the risk free rate. Then we can solve Sharpe ratio explicitly through the following maximisation problem

$$\max_{\omega} \frac{E(\omega^T f - R^f)}{\sqrt{\omega^T \Sigma \omega}}.$$

The optimal weight $\omega^*$ is the solution to the maximisation problem, which is

$$w^* = \Sigma^{-1} E(f).$$

The optimal Sharpe ratio $S^*$ is then the value function

$$S^* = \sqrt{E(f)^T \Sigma^{-1} E(f)}.$$

# 6 DATA

This paper extracts January 1998 to December 2017 London Stock Exchange (LSE) monthly stock prices from Yahoo! Finance and splits the complete data into twelve years of training data (1998 - 2009), four years of validation data (2010 - 2013) and four years of out-of-sample testing data (2014 - 2017). Gregory et al. (2011) provide the risk-free rate used to calculate excess returns, and the monthly factor data containing small minus big (SMB), high minus low (HML), momentum (UMD) factors and value-weighted market portfolio returns. Gregory et al. (2011) provide a detailed explanation for the factor construction in their paper. In addition to the price data, this paper also extracts sixteen firm income statement data from *Finage LTD* (2022), a financial data provider which retrieves income statements through company reports. The income statements follow standard accounting naming convention. This paper interpolated quarterly income statement data into monthly data and transformed level data using log differences.

The analysis in this paper follows Chen et al. (2021) to remove stocks without complete characteristics data in a particular month. However, the Finage database only contains limited income statement data for a subset of stocks. Therefore, we perform two different data splits to maximise the data available for training. The first dataset contains factor data and past returns data for 942 stocks. The second dataset used for training includes factor data, past returns and income statement data which accounts for 242 stocks. Furthermore, we refer to the dataset based on only factor and past returns as *factor data* while the dataset based on the factor, past returns and income statement data as *fundamental data.* Table 1 summarises the training data.

Additionally, we retrieve a large macroeconomic dataset from Coulombe et al. (2021). The dataset contains 112 monthly macroeconomic indicators comprising of nine categories from domestic productions to price index, international trade and interest rates. Coulombe et al. (2021) have transformed the macroeconomic data into stationary time series, so this paper does not require any further data processing. Their paper and website contain detailed information on data construction and handling.

Table 1: FIRM-SPECIFIC TRAINING DATA

| Characteristics | Description | Factor data | Fundamental data | Data Source |
|---|---|---|---|---|
| excess returns | Return of an asset minus risk free rate | Y | Y | Yahoo! Finance, Gregory et al. (2011) |
| rmrf | Excess return for market portfolio, market risk premium factor | Y | Y | Gregory et al. (2011) |
| hml | High minus Low, value factor | Y | Y | Gregory et al. (2011) |
| smb | Small minus Big, size factor | Y | Y | Gregory et al. (2011) |
| umd | Momentum | Y | Y | Gregory et al. (2011) |
| r2_1 | Short-term momentum, computed as lagged one-month return | Y | Y | Yahoo! Finance |
| r12_7 | Intermediate momentum, cumulative return from 12 to 7 months before the return prediction | Y | Y | Yahoo! Finance |
| cost and expenses | Cost and expense | N | Y | Finage |
| depreciation and amortization | Depreciation and Amortization | N | Y | Finage |
| ebitda | Earnings Before Interest, Taxes, Depreciation, and Amortization | N | Y | Finage |
| ebitdaratio | EBITDA to sales ratio | N | Y | Finage |
| eps | Earnings per share | N | Y | Finage |

| Characteristics | Description | Factor data | Fundamental data | Data Source |
|---|---|---|---|---|
| epsdiluted | Diluted Earnings per Share | N | Y | Finage |
| income before tax | Earnings before tax | N | Y | Finage |
| income before taxRatio | Pretax profit margin | N | Y | Finage |
| netincome | Net income, the amount an individual or business makes after deducting costs, allowances and taxes | N | Y | Finage |
| netincomeratio | Net profit margin | N | Y | Finage |
| operatingincome | Amount of profit generated from a business operation, after deducing operating expenses | N | Y | Finage |
| revenue | Revenue, sales generated from business operations | N | Y | Finage |
| weighted averageshsout | Weighted average of outstanding shares | N | Y | Finage |
| weighted averageshsout dil | Diluted weighted average of outstanding shares | N | Y | Finage |

# 7 EMPIRICAL FINDINGS

The Data section describes two types of datasets: *factor* and *fundamental* data. The *factor data* training contains more assets but lesser characteristic data, while *fundamental data* training contains more characteristic data with lesser assets. Therefore, readers should consider differences across the datasets used when interpreting the results. Generally, neural network models require a large dataset for training. However, as *Google LLC* (2019) recommended in their developer website, there is no common consensus among literature and the required size of dataset may be context-dependent.

## 7.1 Sharpe ratio

GAN models generally achieve a higher out-of-sample Sharpe ratio than the four-factor model. The GAN model trained on factor and macroeconomic data has achieved the highest Sharpe ratio of 1.99 in the test period compared to 0.42 in the four-factor model. However, the GAN model trained solely on factor data has achieved a Sharpe ratio of 0.33, which was lower than the four-factor model. Meanwhile, the GAN models trained on fundamental data achieve a higher out-of-sample Sharpe ratio at 0.49 (without macroeconomic data) and 0.69 (with macroeconomic data) than the four-factor models. Therefore, the overall trend in the results shows that the GAN models outperform the four-factor model, which is consistent with Chen et al. (2021)'s finding.

Additionally, this paper finds that adding macroeconomic data can improve the GAN models in both datasets. The Sharpe ratio of GAN models trained on macroeconomic factors outperformed the GAN models trained on the same dataset, without macroeconomic data. The improvement in model performance suggests that macroeconomic data has a role in estimating the SDF, which aligns with Chen et al. (2021)'s finding.

However, the Sharpe ratio is lower than the GAN model trained without the additional fundamental data. Although fundamental data was relevant in Chen et al. (2021)'s paper, the GAN models trained with income statement data do not outperform those trained without income statement data in this paper. The two possible reasons for the lower performance could be either the irrelevance of income statement data or the narrower range of fundamental data available for the UK. In our results, the GAN model trained solely on fundamental data outperformed the GAN model trained solely on factor data. Therefore, this paper suggests that the lower performance could be due to the smaller dataset. As mentioned earlier, fundamental sample contains 242 stocks, while the factor sample contains 942 stocks. Since all GAN models have the same parameter setting, there could be insufficient data for GAN models trained with fundamental data, and income statements could still be relevant in estimating the SDF.

In contrast to Chen et al. (2021)'s result, the Sharpe ratios in this paper do not decrease sharply from the in-sample period to the out-of-sample period. Chen et al. (2021) achieved a Sharpe ratio of 2.68 in the training period, 1.43 in the validation period and 0.75 in the testing period. However, the best performing GAN model in this paper achieved a Sharpe ratio of 1.88 in the training period, 1.76 in the validation period and even higher Sharpe ratio of 1.99 in the testing period. Even the factor model achieve a higher Sharpe ratio of 0.42 in the testing period than the Sharpe ratio of 0.29 in the training period. The stable Sharpe ratio suggests that although we expect the SDF functional form to be time-varying, the UK SDF functional form might be relatively consistent across the 5 to 10 year period. Table 2 summarises the Sharpe ratio for different models.

Table 2: SHARPE RATIO RESULTS

| Model | Data used | Included Macro data | Train | Valid | Test |
|---|---|---|---|---|---|
| Factor | factor | N | 0.29 | 0.62 | 0.42 |
| GAN | factor | N | 0.65 | 0.49 | 0.33 |
| GAN | factor | Y | 1.88 | 1.76 | 1.99 |
| GAN | fundamental | N | 0.70 | 0.35 | 0.49 |
| GAN | fundamental | Y | 1.18 | 0.39 | 0.69 |

## 7.2 Variable importance

To better understand how each variable affects the SDF weight, we have computed sensitivity score following Chen et al. (2021)'s paper. The sensitivity score measures the change in SDF weight $\omega$ as a variable changes. Mathematically, this is the average absolute gradient expressed in the following equation. We define $C$ normalising constant scaling sensitivity scores between 0 and 1. A higher sensitivity means the variable has a larger effect on the SDF weight $\omega$ with 0 being not relevant and 1 being the most important variable.

$$\text{Sensitivity}(x_j) = \frac{1}{C} \sum_{i=1}^{N} \sum_{t=1}^{T} \left| \frac{\partial \omega(I_t, I_{t,i})}{\partial x_j} \right| .$$

In this paper, we focus our analysis on the top performing model: the GAN trained on factor and macroeconomic data. With reference to Figure 8, the top four most important variables are the four factors in the factor model (RMRF, HML, UMD, SMB). This thus suggests that the factors are indeed very crucial in estimating the SDF weight. Following the factors, we have the two firm specific past return data and this implies that the firm specific characteristics are important as well. For macroeconomic data, we notice that the top ten macroeconomic data contributed the most to the SDF weight. The top ten macroeconomic data revolve around indexes affecting consumers, business and the general stock markets. The top ten macroeconomic data include producer price indices (PPI) of different UK domestic markets, Composite leading indicator (CLI), Consumer and Business confidence index (CCI, BCI) iShares MSCI United Kindom ETF (UK_focused_equity) which tracks the mid and large size companies in UK market, and Standard & Poor 500 (SP500). Interestingly, the UK SDF weight is also affected by the US market, suggesting the possible benefits of including more US market data in future studies.

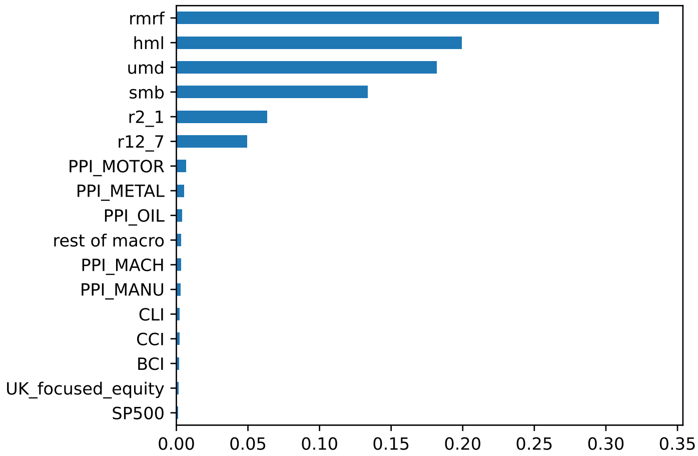
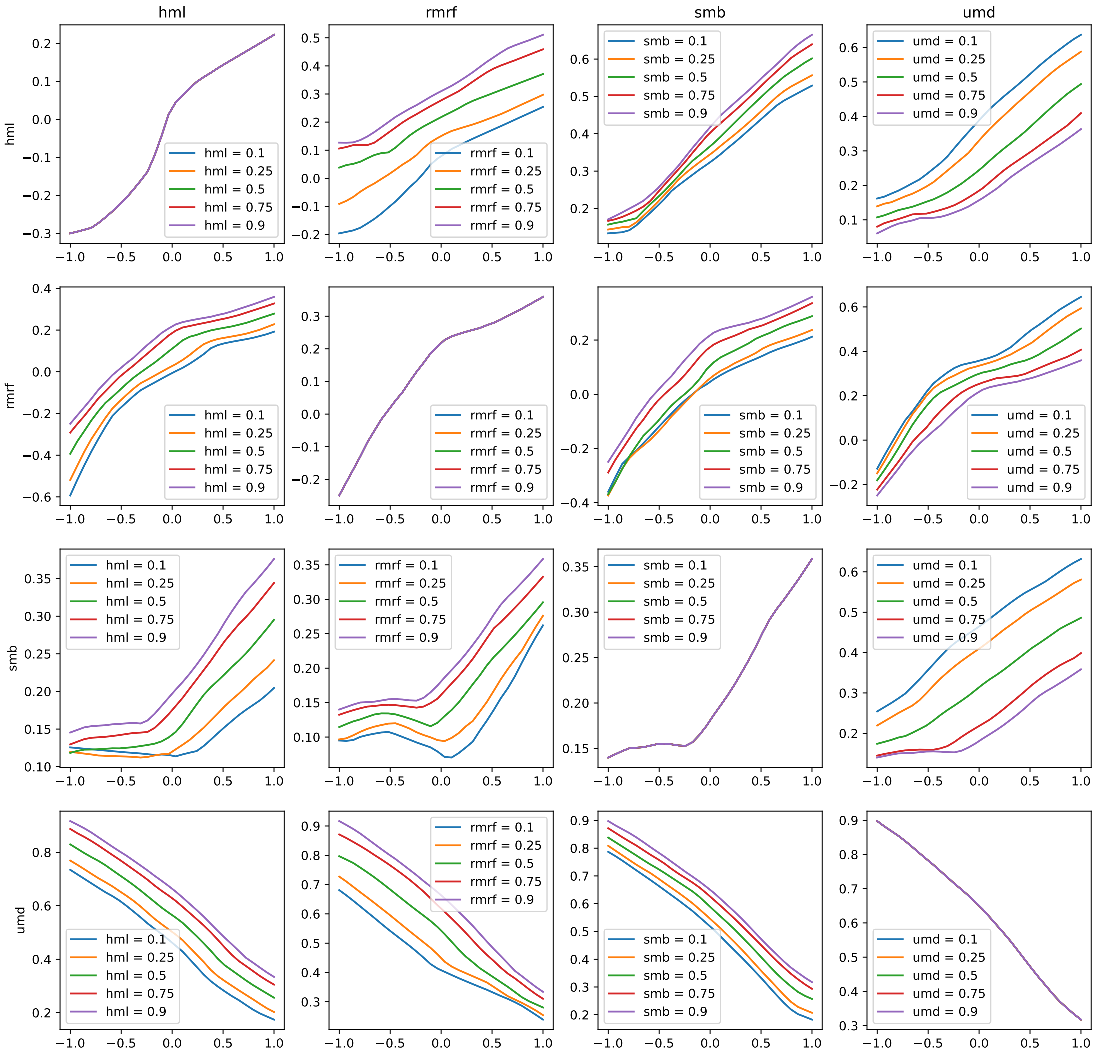



Figure 8: VARIABLE IMPORTANCE

### 7.3 SDF structure

We also studied the SDF weight structure as a function of the factors and discovered the same two observations in Chen et al. (2021)'s paper. Firstly, factors in the four factor model have a linear relationship with the SDF weight. Secondly, there is an interaction between these factors. Figure 9 plots the different combinations of factors in a matrix form. The order of row and columns represents value factor (HML), market excess return (RMRF), size factor (SMB), momentum (UMD). The figure is achieved by keeping all other factors at the mean level while changing one of the factors. The diagonal entries show the general relationship between the factors and the SDF weight. Moreover, the off-diagonal entries illustrate the interaction effects between factors. The different coloured lines correspond to different values of the respective factors. Upon analysis, all factors at the diagonal entries have a near linear relationship with the SDF weight. This explains why linear factor models generally work well. On the other hand, we observe a non-trivial interaction effects between the factors at the off-diagonal entries. Therefore, non-linear models such as the GAN model which takes into account in the interactions between the factors are able to perform better than linear models.




The different coloured lines correspond to different values of the respective factors

Figure 9: GAN SDF WEIGHT AS A FUNCTION OF FACTORS

# 8 DISCUSSION

This paper empirically validates Chen et al. (2021)'s GAN model using UK data. However, the smaller dataset used in this paper poses a challenge to the empirical approach. Furthermore, this paper lacks discussion on the affordability and environmental impact of neural network training.

Firstly, despite the effort of this paper to maximise the data available, there is a considerable gap between the dataset from Chen et al. (2021)'s paper and ours. Chen et al. (2021) data include 50 years of historical data with over 10,000 stocks. Although the number of macroeconomic data available is comparable (178 in Chen et al. (2021)'s paper and 112 in this paper), the number of firm characteristics data available and the number of stocks available are limited. As Karras et al. (2020) and Zhao et al. (2020) pointed out, training GAN models with little data generally leads to the discriminator network over-fitting the training data. However, proposed solutions in the current literature are only applicable to image data and is not applicable to this paper. The data-poor nature of the U.K. market could be one of the crucial factors resulting in most of the literature focusing on the U.S data, as mentioned in our literature review. Even the paid subscription provider was not able to furnish complete and rich data comparable to the US. Therefore, the results in this paper are still relevant in assessing the performance of the GAN model when subject to data limitations.

Secondly, this paper did not mention the affordability and environmental impact of using the GAN model. Neural network models such as the GAN model require Graphics Processing Unit (GPU). GPU is a specialised electronic circuit initially built for the gaming industry and later adopted to neural network training due to the multiple simultaneous computations available. Entry-level GPU used in this paper costs a few hundred USD while the GPU used in Chen et al. (2021)'s costs over three thousand USD, according to the price listed in the GPU supplier *Nvidia Corporation* (2022). Moreover, Chen et al. (2021) used eight such GPUs for the model training. Training neural networks is also an energy-intensive task, and Strubell et al. (2019) show that the literature seldom discuss the high carbon emissions produced by training complicated models.

# 9 CONCLUSION

In conclusion, this paper delivers four key findings. Firstly, this paper found that Chen et al. (2021)'s GAN model is applicable to the U.K. market, where the model outperformed the benchmark of four-factor model in terms of Sharpe ratio. The Sharpe ratio of the GAN models trained on all datasets is higher than the four-factor model, except the GAN model trained only on past return data. Secondly, similar to Chen et al. (2021)'s finding, macroeconomic data is essential in estimating the SDF functional form. The GAN models estimated with macroeconomic data have a higher Sharpe ratio than the GAN models estimated without macroeconomic data, and the best performing GAN model requires macroeconomic data. Lastly, the Sharpe ratio in the training period is comparable to the out-of-sample test period, suggesting that the U.K. SDF functional form seems to be consistent over time. Having said that, the consistency applies to the period explored in this paper. It is uncertain if the SDF will remain consistent after Brexit or COVID-19. This paper empirically showed that the factors relevant in estimating the SDF. Furthermore, factors are linear in the SDF, providing justifications for linear factor models. However, there is an interaction effect between the factors and models like the GAN which considers such interaction effect would likely outperforms linear models. With reference to the U.K. LSE 1998 - 2017 data, this paper concludes that considering the GAN model does value added to the empirical asset pricing.

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