Comp 352 Winter 2019
Tutorial Week 2

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Outline

What is Algorithms?

- What is Algorithms?
- What do we mean by Complexity?
- How to measure Complexity?

What is Algorithms?

- Algorithm is a sequence of computational steps that transform the input into the output.
- To analyze an algorithm is to determine the amount of resources (such as time and storage) necessary to execute it.
- The complexity of an algorithm is the cost, measured in running time, or storage, or whatever units are relevant, of using the algorithm to solve one of those problems.

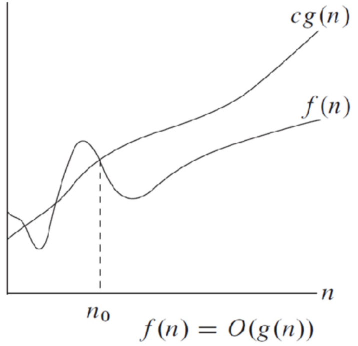
Big O

$$O\big(g(n)\big) = \{f(n) :$$

there exist positive constants c and n_0

such that

$0 \le f(n) \le cg(n)$ for all $n \ge n_0\}$



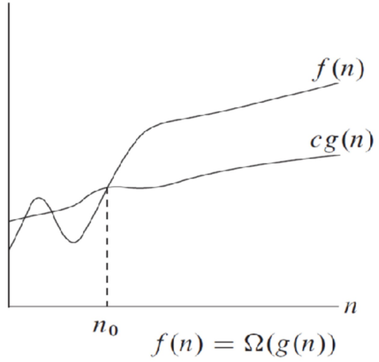
Big Ω

$$\Omega(g(n)) = \{f(n) :$$

there exist positive constants c and n_0

such that

$0 \le cg(n) \le f(n)$ for all $n \ge n_0\}$



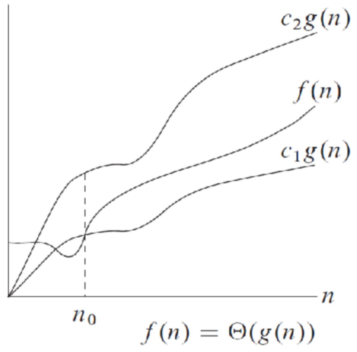
Big Θ

$$\Theta(g(n)) = \{f(n) :$$

there exist positive constants c_1, c_2 and n_0

such that

$0 \le c_1 g(n) \le f(n) \le c_2 g(n)$ for all $n \ge n_0\}$



Infinite series

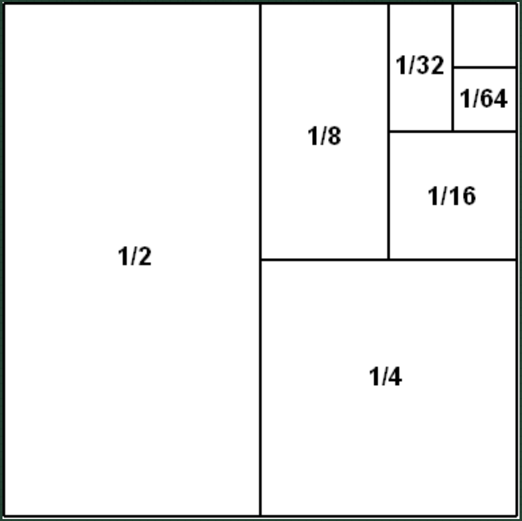
$$1 + 2 + 3 + \cdots + (n-1) + n = \frac{n(n+1)}{2}$$

Geometric series

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots = 1$$

$$\Rightarrow \frac{1}{2}n + \frac{1}{4}n + \frac{1}{8}n + \frac{1}{16}n + \cdots = 1 \cdot n = n$$

Geometric series



Logarithms & Exponentials

$$\log_b(x \cdot y) = \log_b x + \log_b y$$
$$\log_b(x/y) = \log_b x - \log_b y$$
$$\log_b(x^a) = a \log_b x$$
$$\log_b a = \log_x a / \log_x b$$

$$a^{b+c} = a^b \cdot a^c$$
$$a^{b \cdot c} = (a^b)^c$$
$$a^b / a^c = a^{b-c}$$
$$b = a^{\log_a b}$$
$$b^c = a^{c \cdot \log_a b}$$

R4.8

The number of operations executed by algorithms A and B is $40n^2$ and $2n^3$, respectively. Determine n_0 such that A is better than B for $n \geq n_0$.

R4.11

What is the sum of all the even numbers from 0 to $2n$, for any positive integer n?

R4.14

Show that if $d(n)$ is $O\big(f(n)\big)$, then $ad(n)$ is $O\big(f(n)\big)$ for any constant $a > 0$.

R4.15

Show that if $d(n)$ is $O\big(f(n)\big)$ and $e(n)$ is $O\big(g(n)\big)$, then the product $d(n)e(n)$ is $O\big(f(n)g(n)\big)$.

R4.30

Show that 2^{n+1} is $O(2^n)$.

R4.38

Given an n-element array X, algorithm D calls algorithm E on each elements $X[i]$. Algorithm E runs in $O(i)$ time when it is called on element $X[i]$. What is the worst-case running time of algorithm D?

C4.11

Show that $\log_b f(n)$ is $\Theta\big(\log f(n)\big)$ if $b > 1$ is a constant.

True or False

1. $10^8 n^2 + 5n^4 + 7000n^3 + n \in \Theta(n^6)$
2. $n^n \in O(n!)$
3. $1000n^3 + 0.000000001n^7 \in O(n^3)$
4. $2n^5 \log n \in O(n^7 \log n)$
5. $n^2 + 0.00000001n^5 \in \Omega(n^3)$
6. $n! \in \Omega(2^n)$

C4.13

Bob built a web site and gave the URL only to his n friends, which he numbered from 1 to n. He told friend number i that he/she can visit the web site at most i times. Now Bob has a counter C keeping track of the total number of visits to the site (but not the identities of who visits). What is the minimum value for C such that Bob should know that one of his friends has visited his/her maximum allowed number of times?

C4.12

1. Describe a method for finding both the minimum and maximum of n numbers using fewer than $3n/2$ comparisons. (Hint: First construct a group of candidate minimums and a group of candidate maximums).
2. Tournament Tree.
3. How about the second largest number?