



Autonomous Racing

1/10 the scale. 10 times the fun!

## Advanced Path Planning and Obstacle Avoidance

Madhur Behl  
(University of Virginia)

# Assignment 4 Demo

- Wednesday, April 26 @ 2:00pm
- Rice 024
- Complete wall following laps
  - Extra Credits:
    - Launch file
    - Velocity PID
    - Collision Avoidance – stop when there is an obstacle in the front of the car.

# Final Race

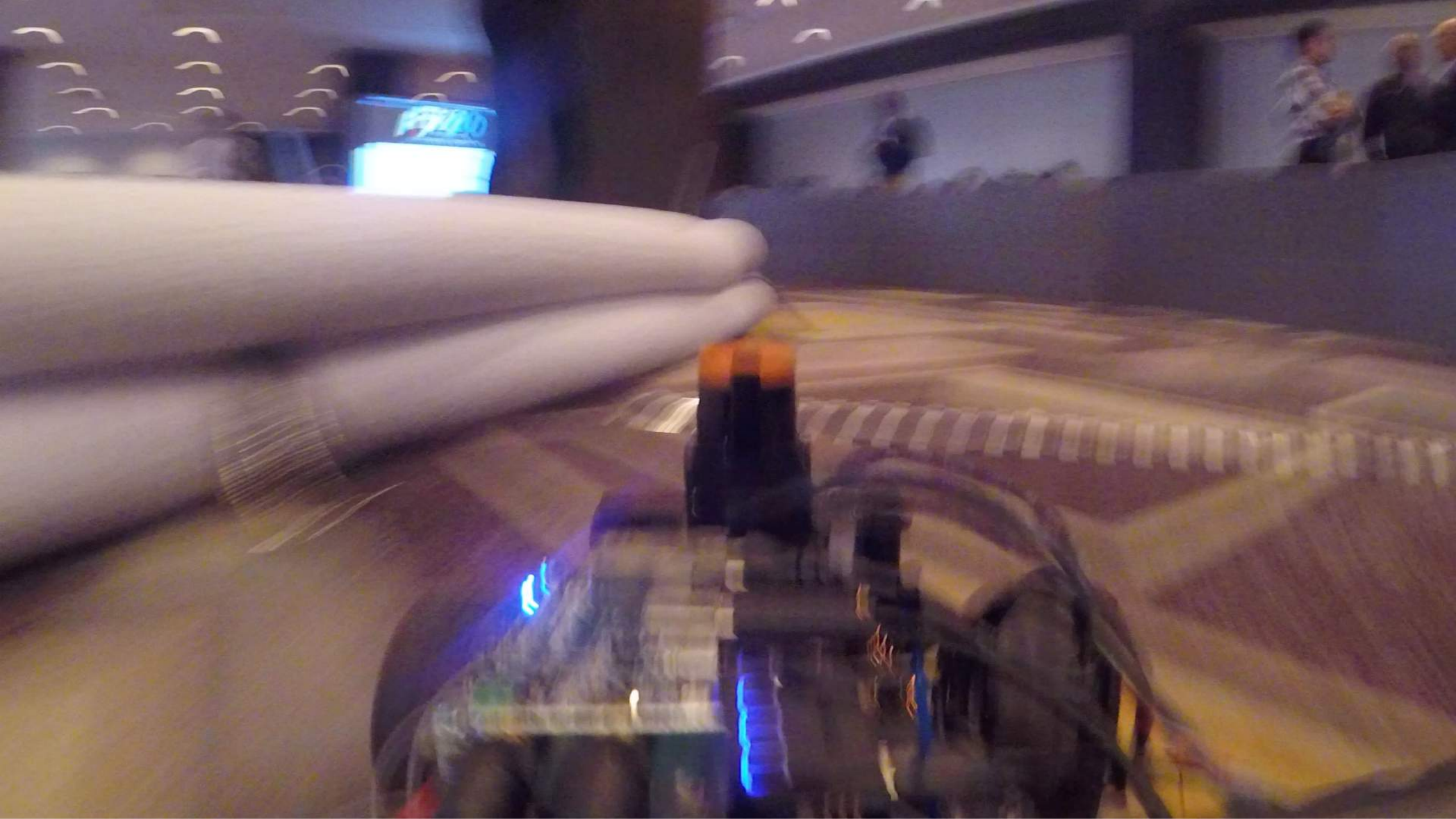
- Friday, May 10, from noon-2:00pm
- Link Lab Arena
  - New racetrack will be setup (slightly bigger and challenging)
  - Time trials – round robin.



# 4<sup>th</sup> F1/10 International Autonomous Racing Competition











# Head-to-head Autonomous Racing

## 4th F1/10 International Autonomous Racing Competition









# A simple path planning algorithm: Gap Finding

Conditions:

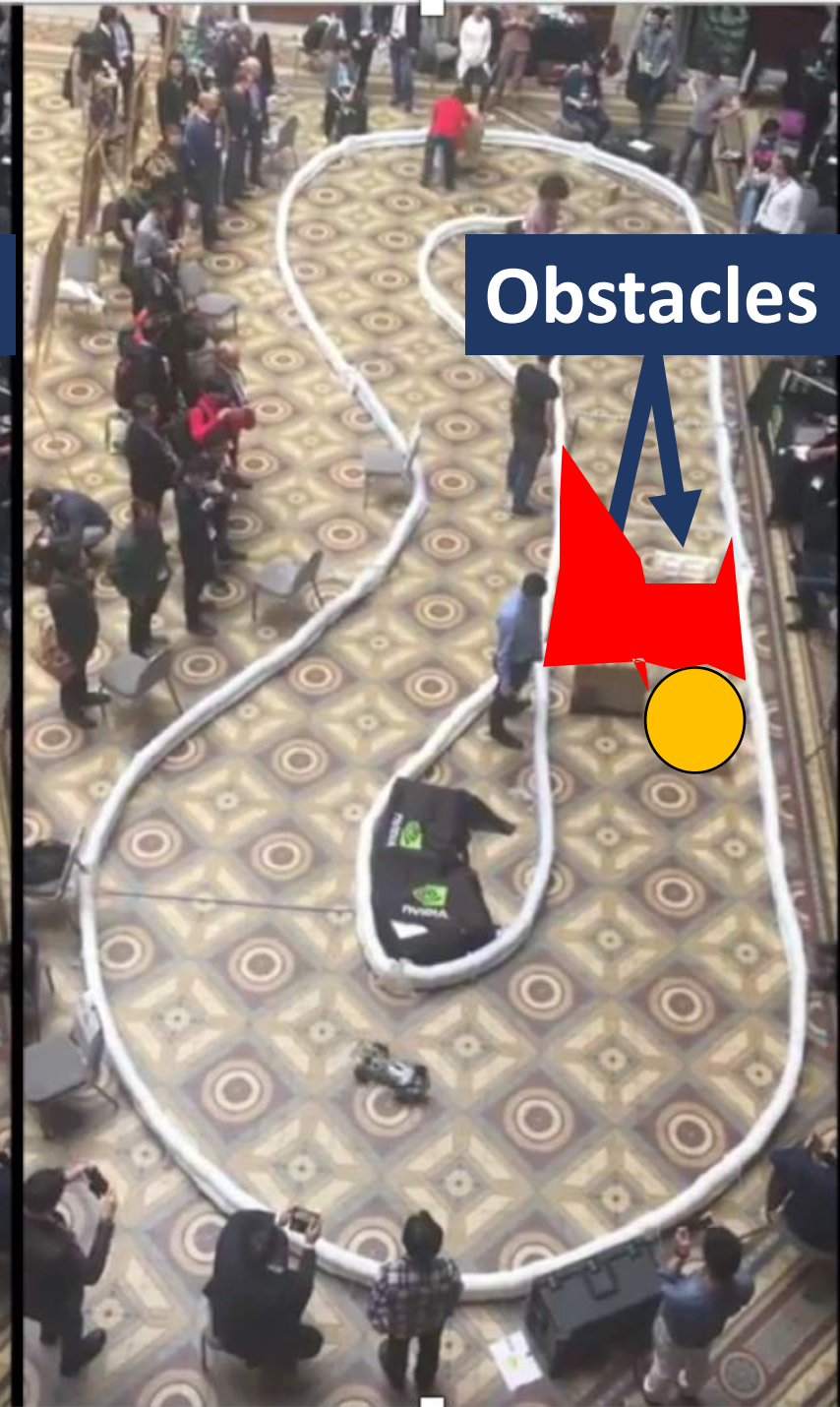
- Closed track
- Always move forward – no choices

Idea: find the largest gap in front of the car and go through it



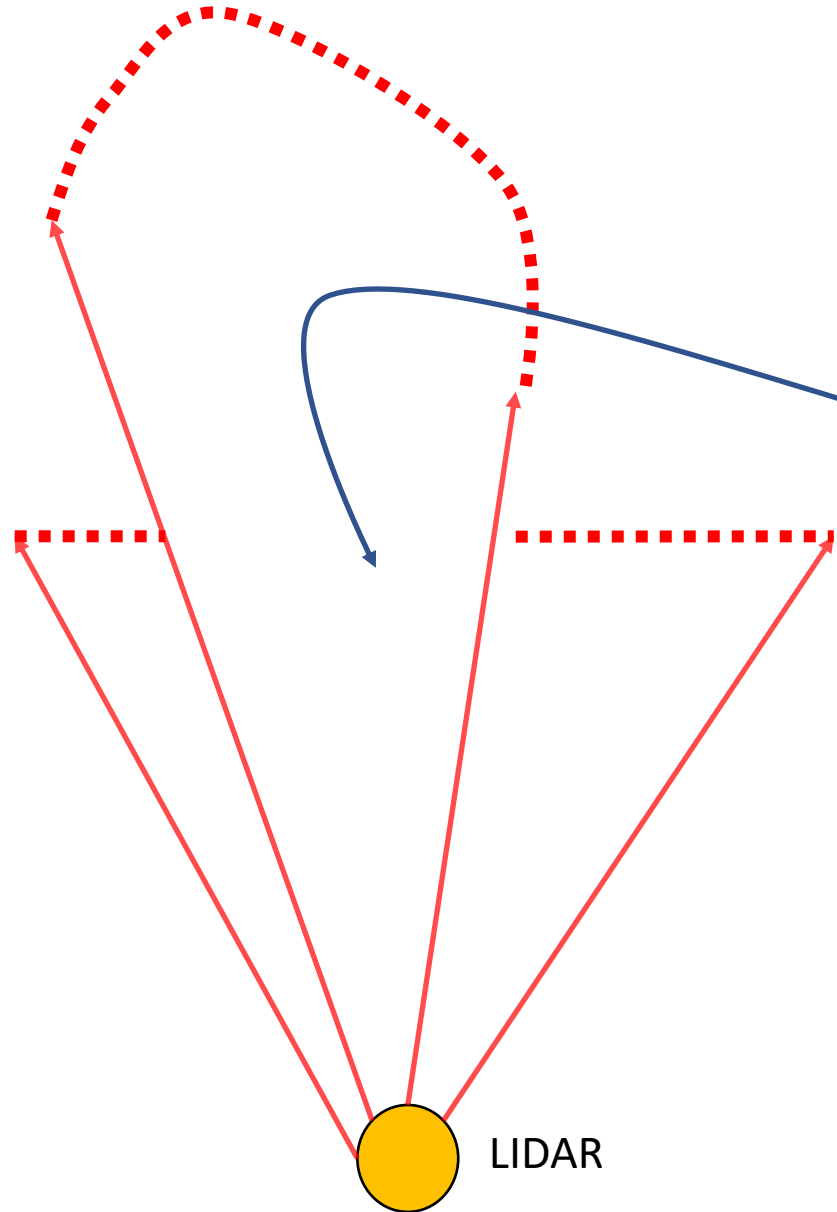
Competition track in Porto, April 2018

Find the gap



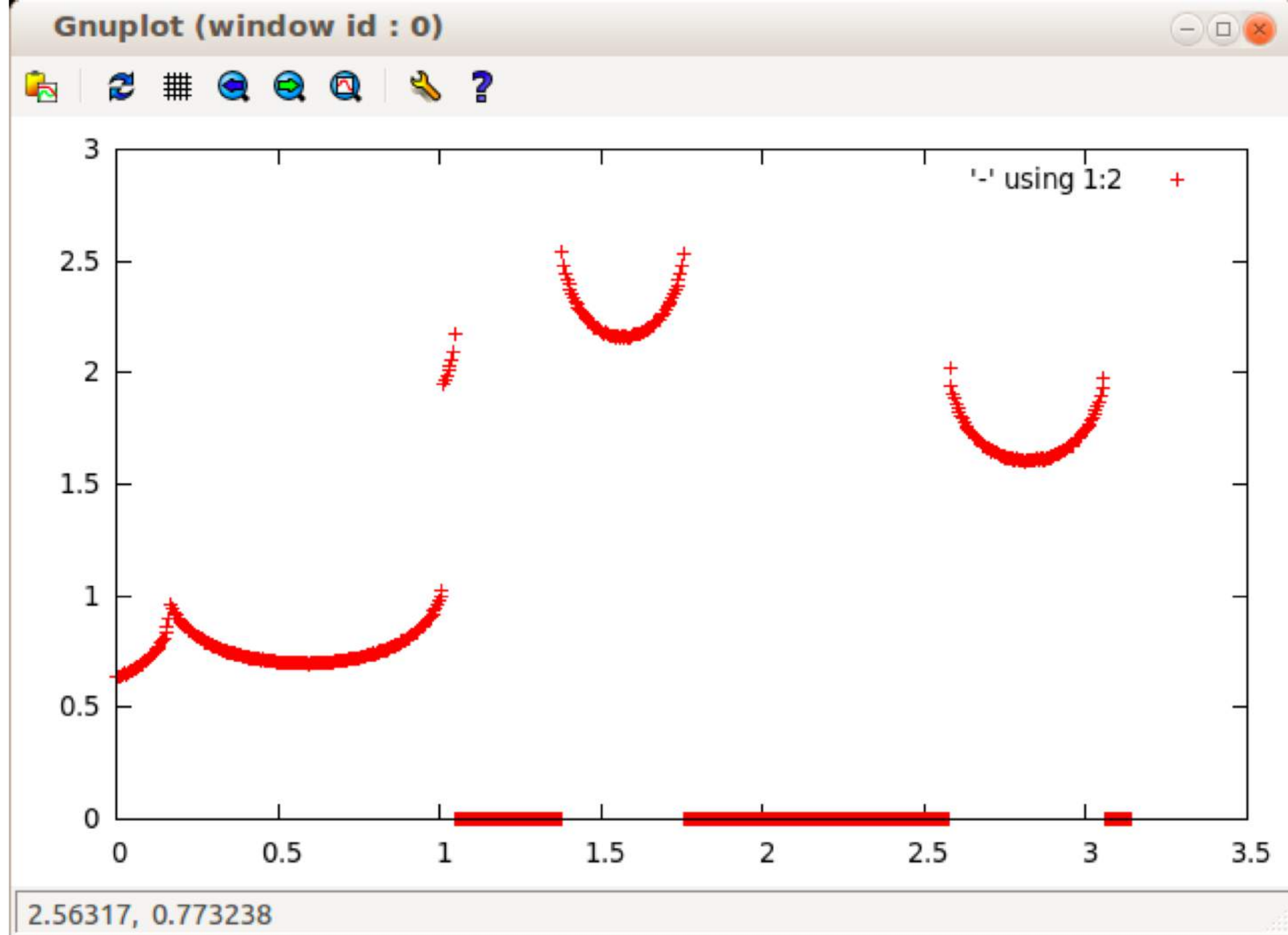


# Find the gap



Find this gap and  
calculate its width  
(this *has* to be the  
way forward)

Where should  
the car go ?





# Gap finding

- **Find the gaps**
- Calculate the width of each gap
- Determine the widest gap
- Optional: Determine the “deepest” gap

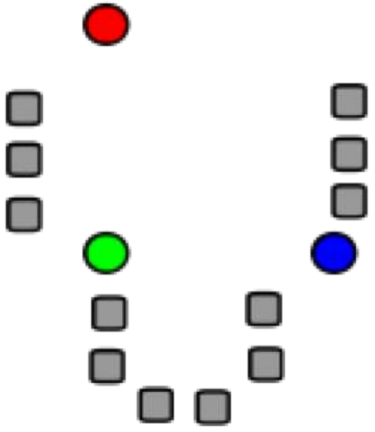
# Find the gap

- What determines what a gap is?
- Intuitively, it's a sequence of adjacent ranges that have a similar value.
- We want to divide the 1080 ranges into *clusters* of values, such that values within one cluster are “close”. Each cluster is a gap candidate.
  - How many clusters?



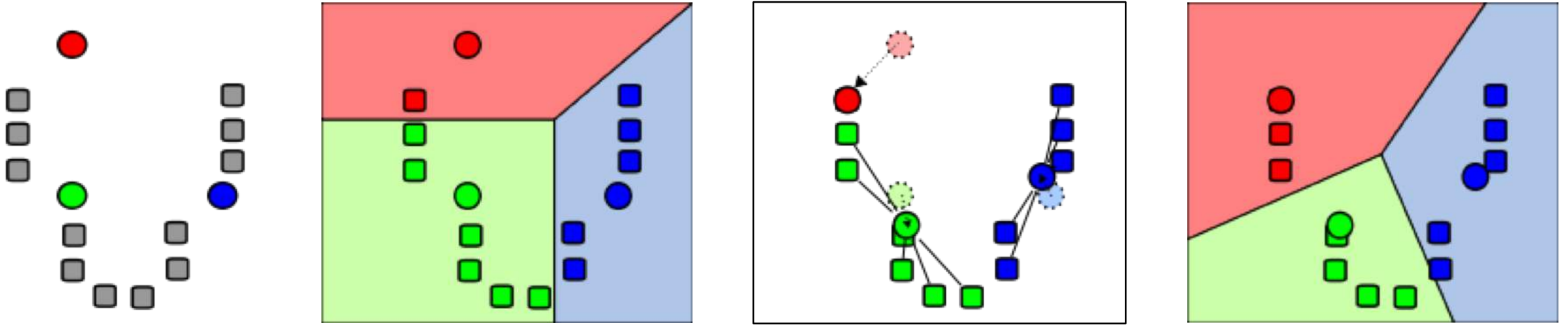


# K-means clustering



2D example: each data item has 2 features

# K-means clustering



## Initialization:

Select a target number of clusters,  $k$  (here,  $k=3$ ) (How?)

Select  $k$  cluster centers (How?)

## Assignment step:

Assign each data item to its nearest cluster center

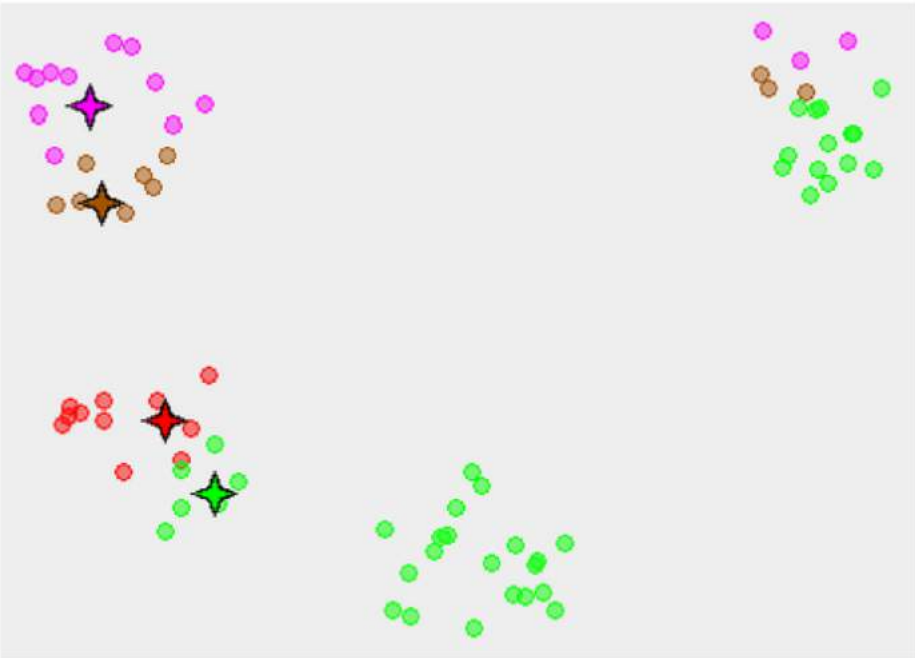
## Update step:

Set each cluster to be the average of the items assigned to its cluster





# K-means clustering



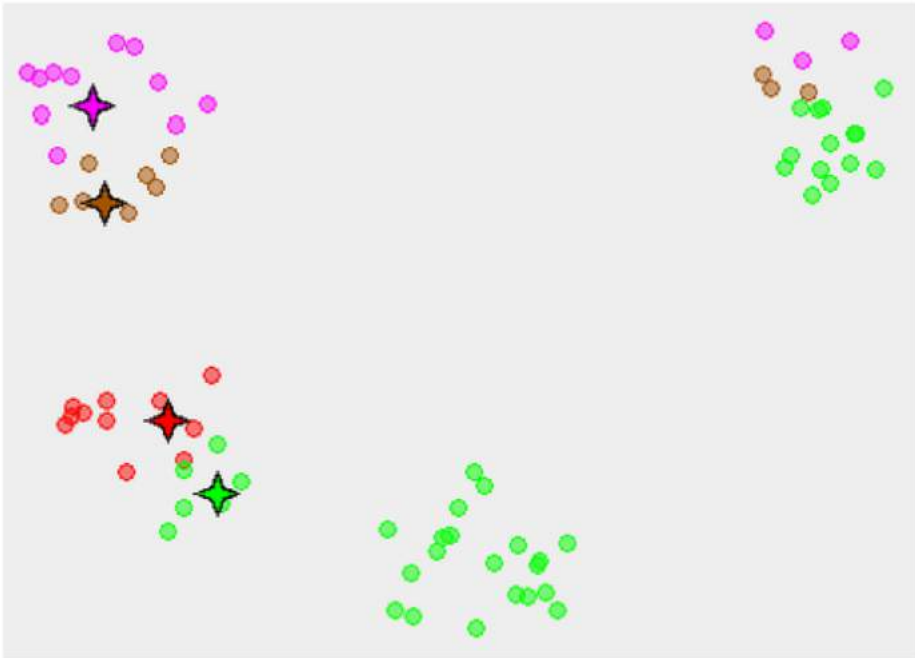
**Initialization:** Select a target number of clusters,  $k$ , and select  $k$  cluster centers

**Assignment step:** Assign each data item to its nearest cluster center

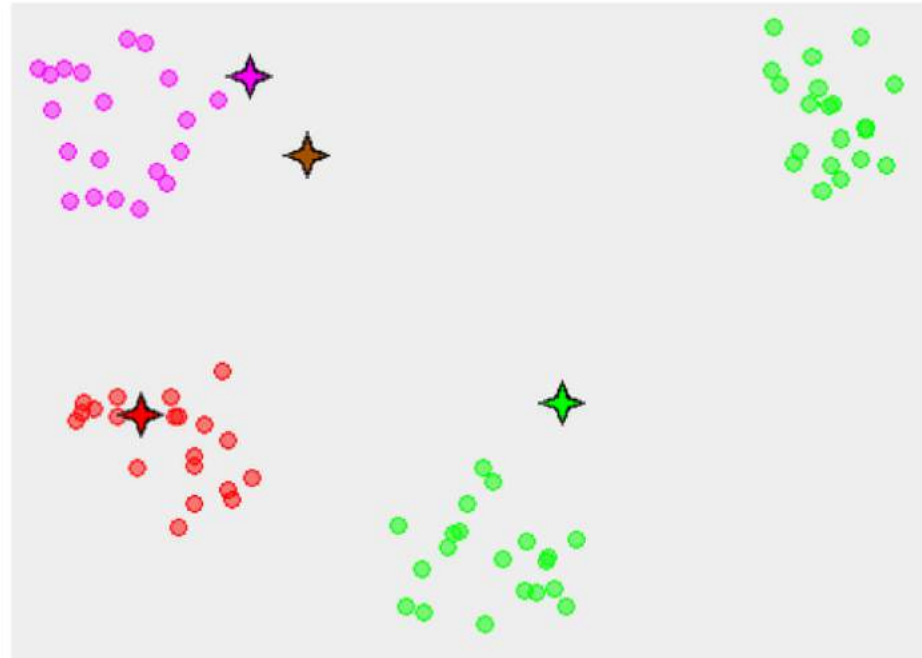
**Update step:** Set each cluster to be the average of the items assigned to its cluster

# K-means clustering

Initialize and Assign



Update centers and re-assign



**Initialization:** Select a target number of clusters,  $k$ , and select  $k$  cluster centers

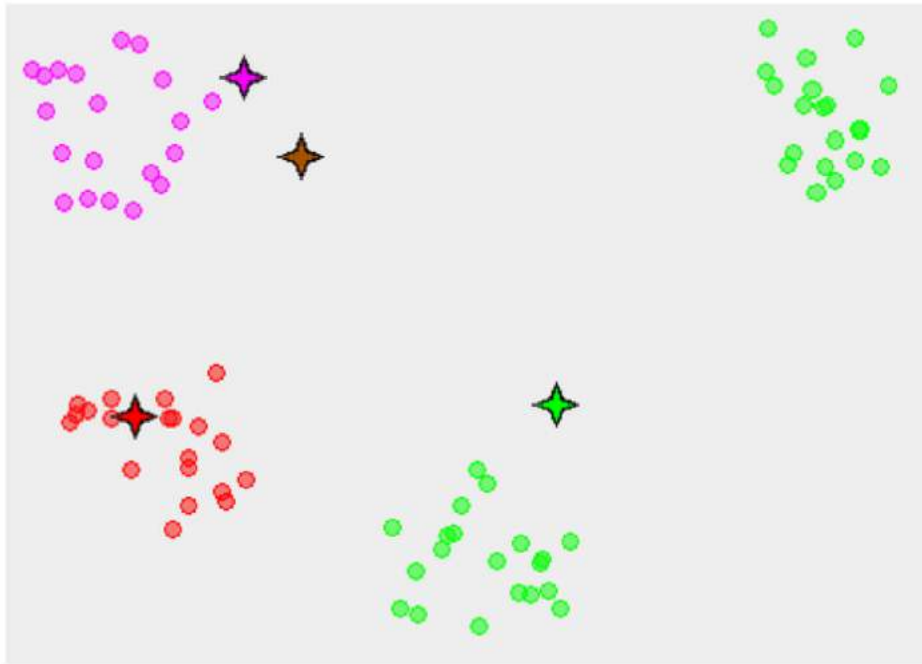
**Assignment step:** Assign each data item to its nearest cluster center

**Update step:** Set each cluster to be the average of the items assigned to its cluster

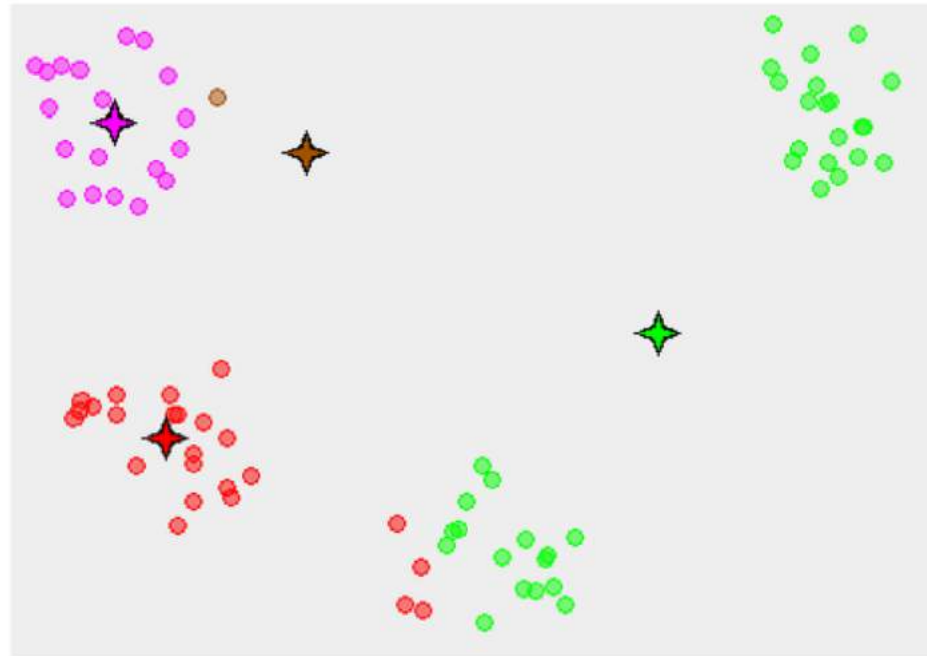


# K-means clustering

(previous state)



Update centers and re-assign



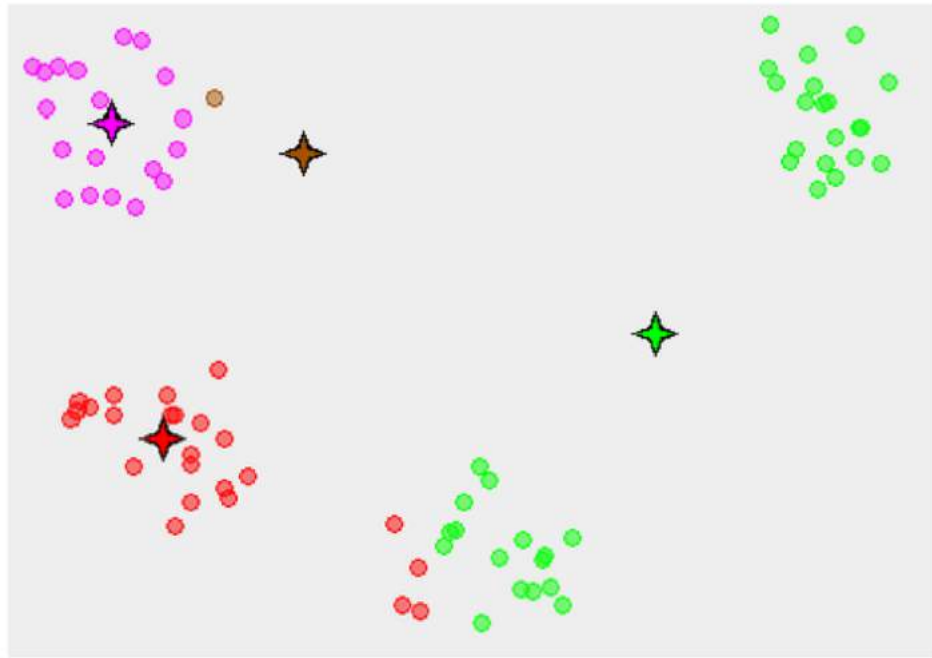
**Initialization:** Select a target number of clusters,  $k$ , and select  $k$  cluster centers

**Assignment step:** Assign each data item to its nearest cluster center

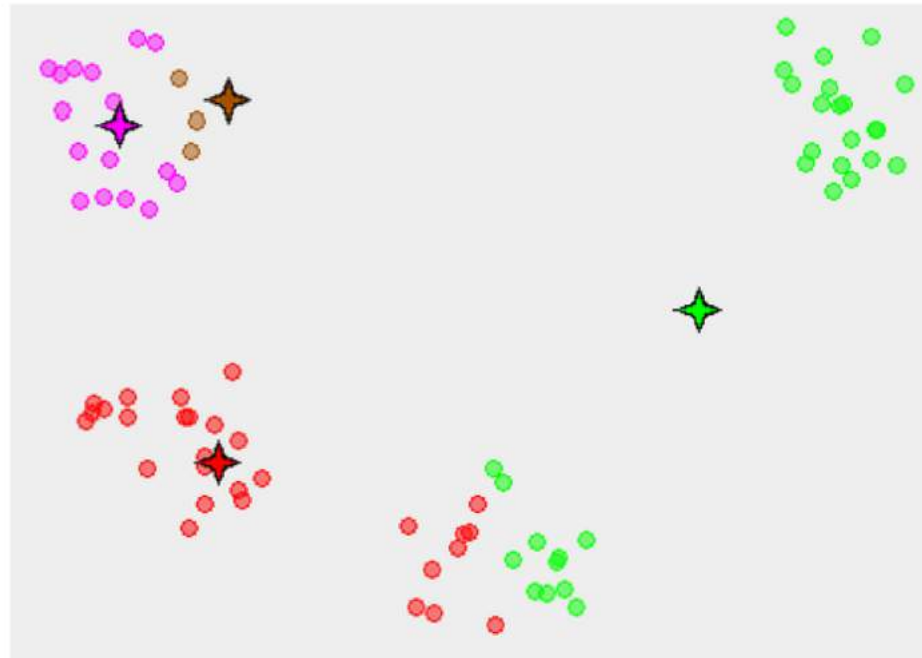
**Update step:** Set each cluster to be the average of the items assigned to its cluster

# K-means clustering

(previous state)



Update centers and re-assign



**Initialization:** Select a target number of clusters,  $k$ , and select  $k$  cluster centers

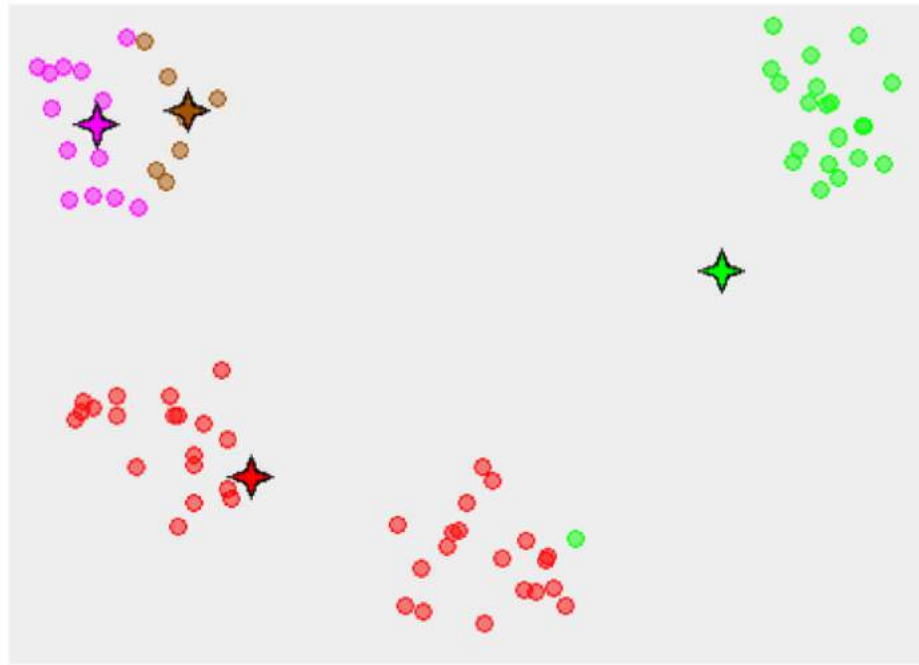
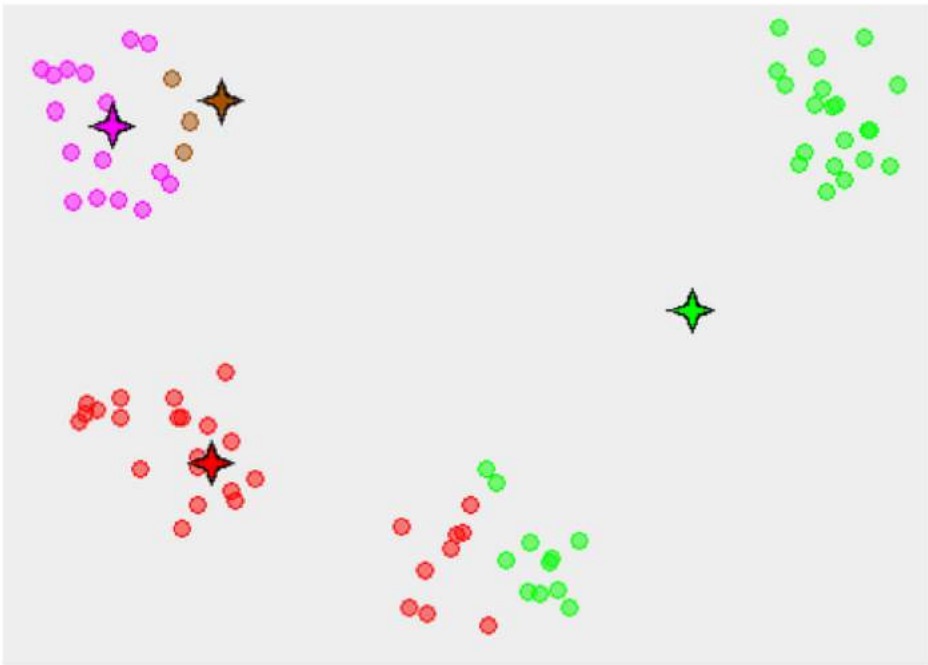
**Assignment step:** Assign each data item to its nearest cluster center

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# K-means clustering

(previous state)

Update centers and re-assign



**Initialization:** Select a target number of clusters,  $k$ , and select  $k$  cluster centers

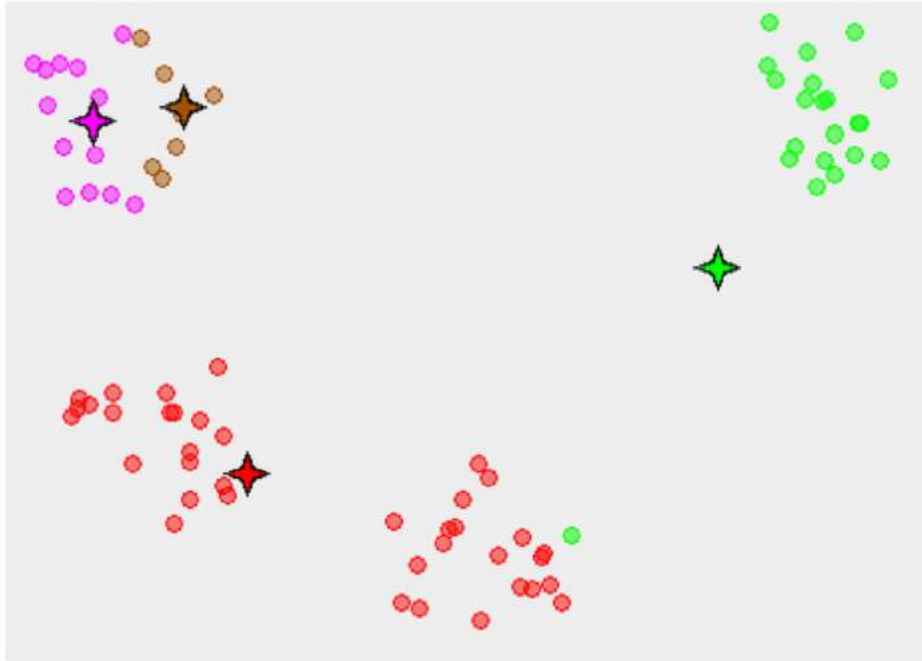
**Assignment step:** Assign each data item to its nearest cluster center

**Update step:** Set each cluster to be the average of the items assigned to its cluster

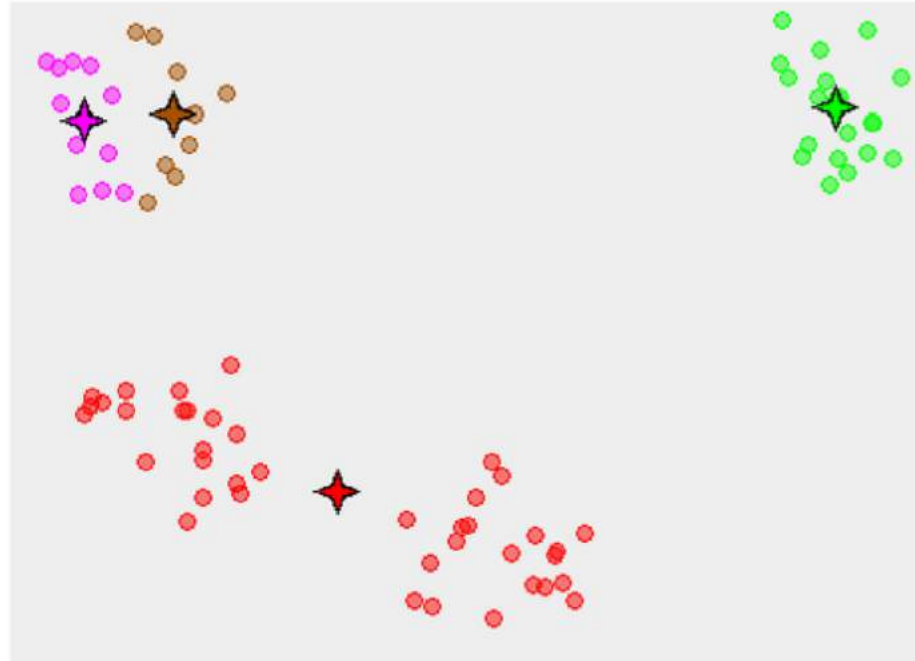


# K-means clustering

(previous state)



Update centers and re-assign...not much movement → stop



**Initialization:** Select a target number of clusters,  $k$ , and select  $k$  cluster centers

**Assignment step:** Assign each data item to its nearest cluster center

**Update step:** Set each cluster to be the average of the items assigned to its cluster

# K-means clustering

- K-means is minimizing the intra-cluster distances to generate compact clusters

$$\arg \min_{\mathbf{S}} \sum_{i=1}^k \sum_{\mathbf{x} \in S_i} \|\mathbf{x} - \boldsymbol{\mu}_i\|^2 = \arg \min_{\mathbf{S}} \sum_{i=1}^k |S_i| \text{Var } S_i$$

Choice of clusters

$i^{\text{th}}$  cluster

Data item

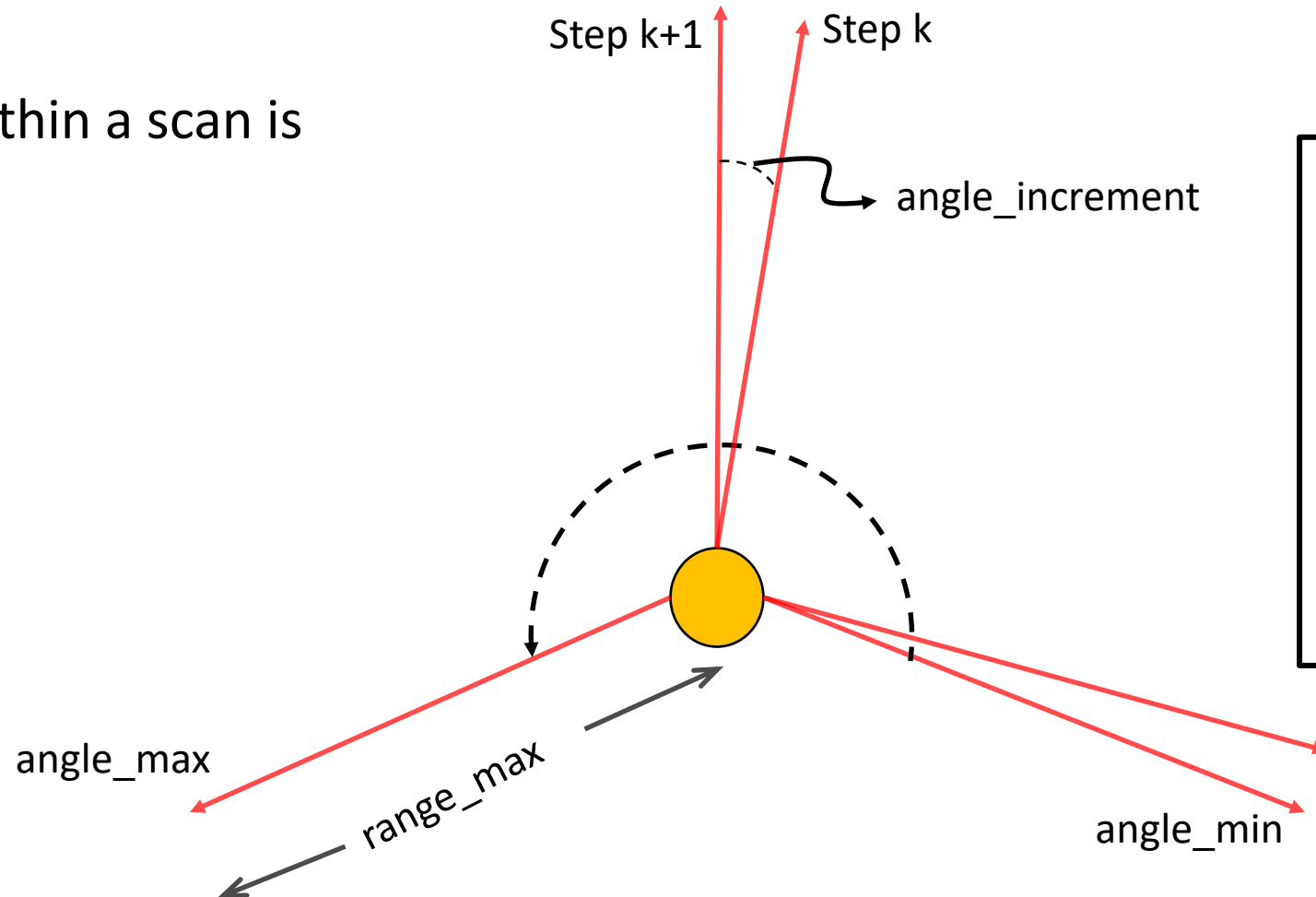
Centroid

Appropriate distance function

# What features to use?

Each data item / measurement within a scan is characterized by

- Range
- Angle



```
std_msgs/Header header
float32 angle_min
float32 angle_max
float32 angle_increment
float32 time_increment
float32 scan_time
float32 range_min
float32 range_max
float32[] ranges
float32[] intensities
```

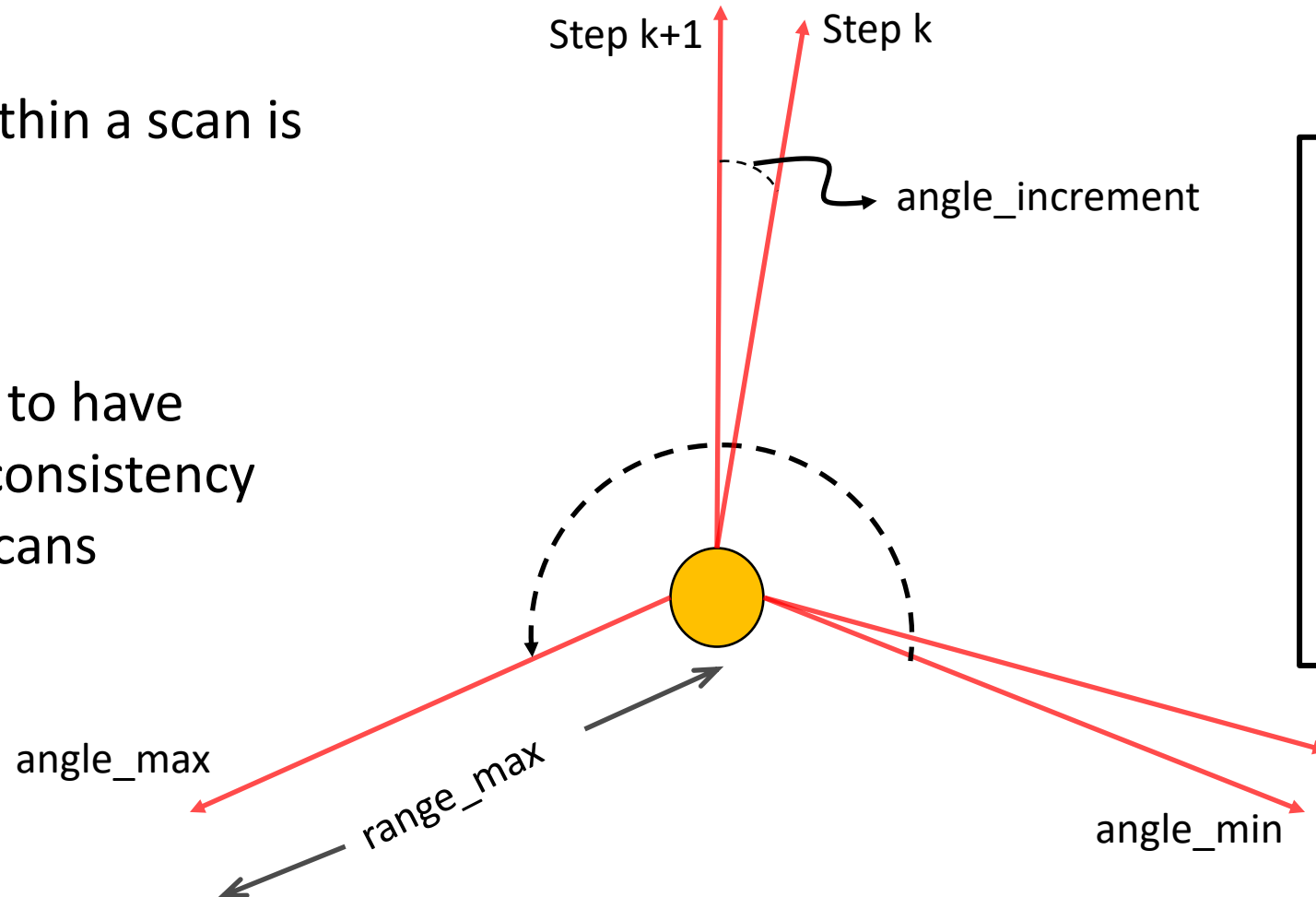


# What features to use?

Each data item / measurement within a scan is characterized by

- Range
- Angle

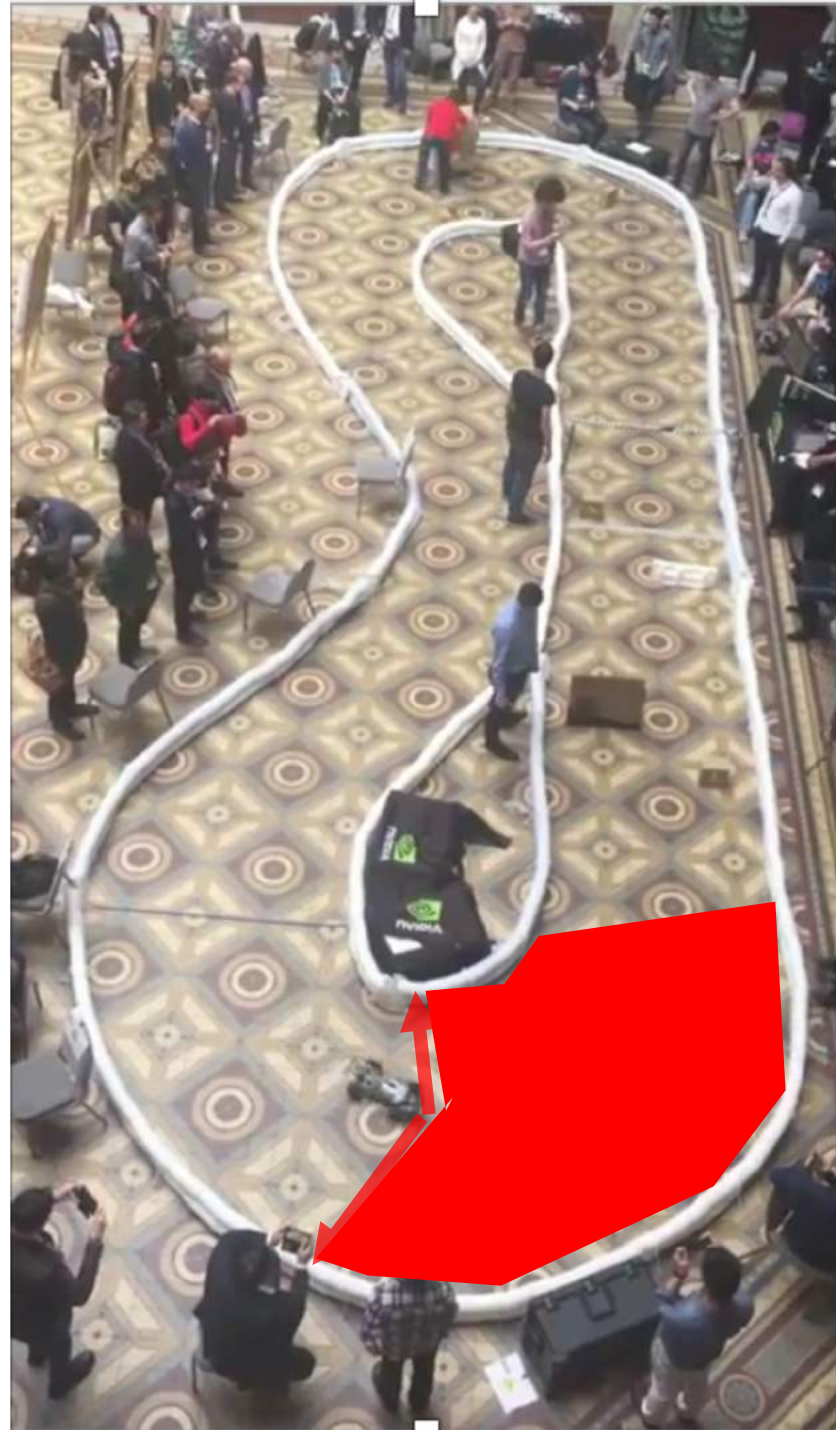
It probably helps to have some filtering = consistency check between scans



```
std_msgs/Header header
float32 angle_min
float32 angle_max
float32 angle_increment
float32 time_increment
float32 scan_time
float32 range_min
float32 range_max
float32[] ranges
float32[] intensities
```

# Difficulties

Smooth curves and progressive  
openings  
– where do you draw the line?



# Dynamic Path Planning

Aim is of avoiding unexpected obstacles along the robot's trajectory to reach the goal.

## Methods

- Bug Algorithms
- Artificial Potential Field (APF) Algorithm
- Harmonic Potential Field (HPF) Algorithm
- Virtual Force Field (VFF) method
- Virtual Field Histogram (VFH) method
- Follow the Gap Method (FGM)

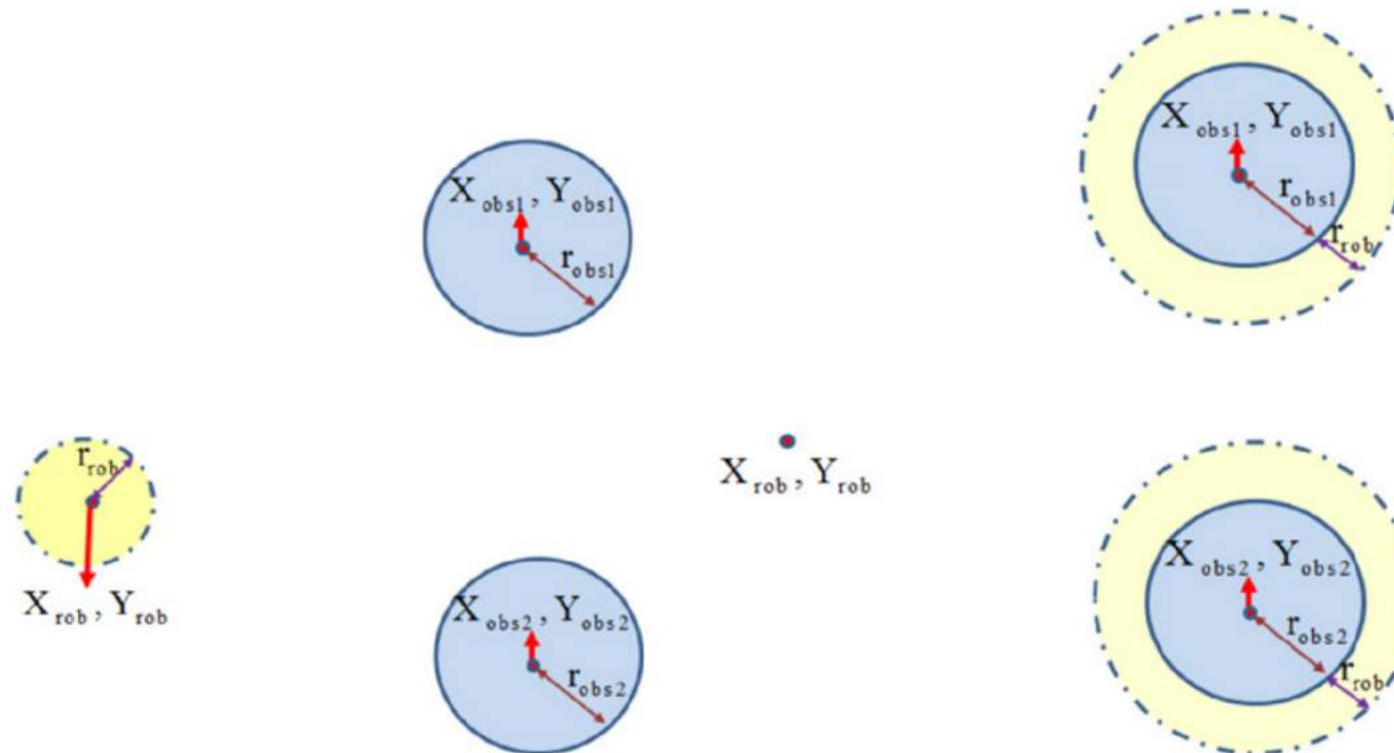


# Some terms of concern

- Point Robot Approach
- Field of view of Robot
- Non-holonomic constraints

# Point Robot Approach

- Robot and Obstacles are assumed circular.
- Radius of robot is added to radius of obstacles
- The Robot is reduced to a point, while Obstacles are equally enlarged.

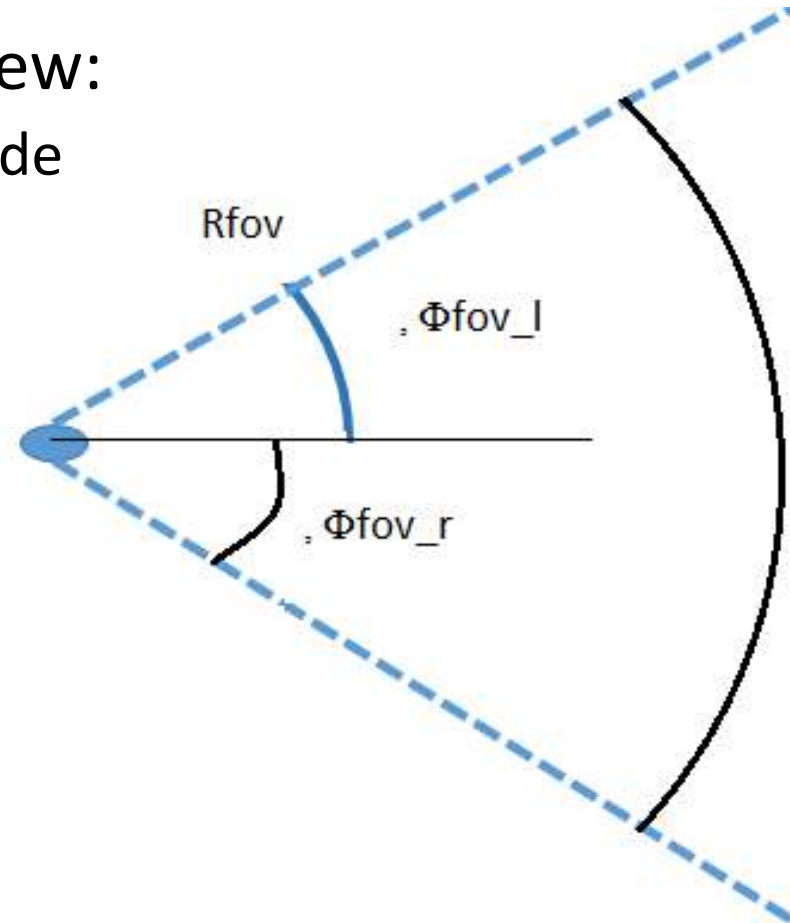


(a) Circular robot with circular obstacles.

(b) Point robot with enlarged obstacles.

# Field of view

- The sector region within the range of robot's sensors to get information of environment.
- Two quantitative measures of field of view:
  - End angles of the sector on right and left side
  - Radius of the sector.

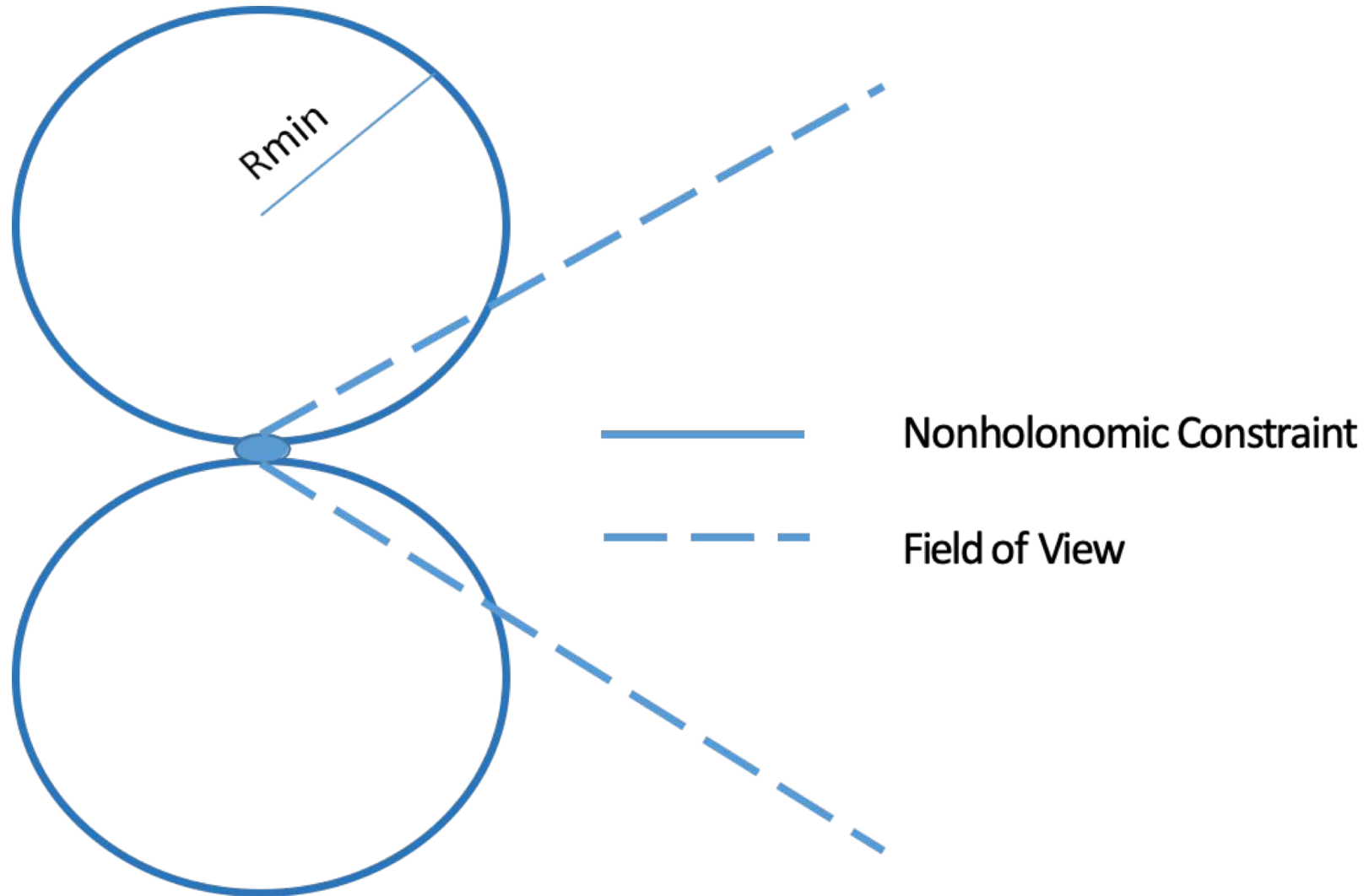




# Nonholonomic Constraints

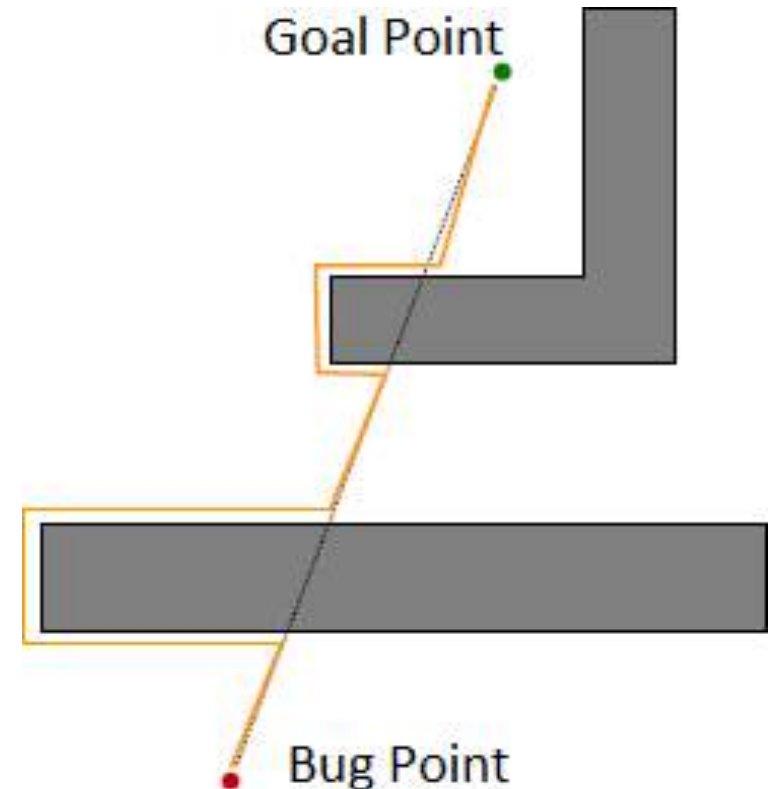
- If the vector space of the possible motion directions of a mechanical system is restricted
- And the restriction can not be converted into an algebraic relation between configuration variables.
- Can be visualized as, inability of a car like vehicle to move sideways, it is bound to follow an arc to reach a lateral co-ordinate.

# Nonholonomic Constraints and Field of View of Robot

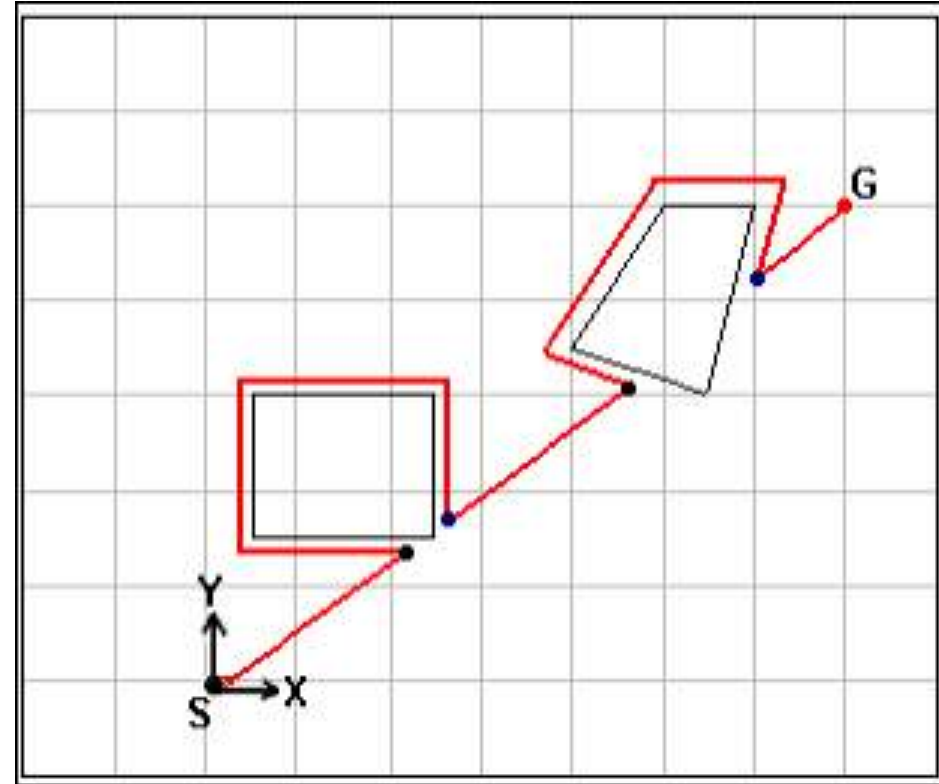
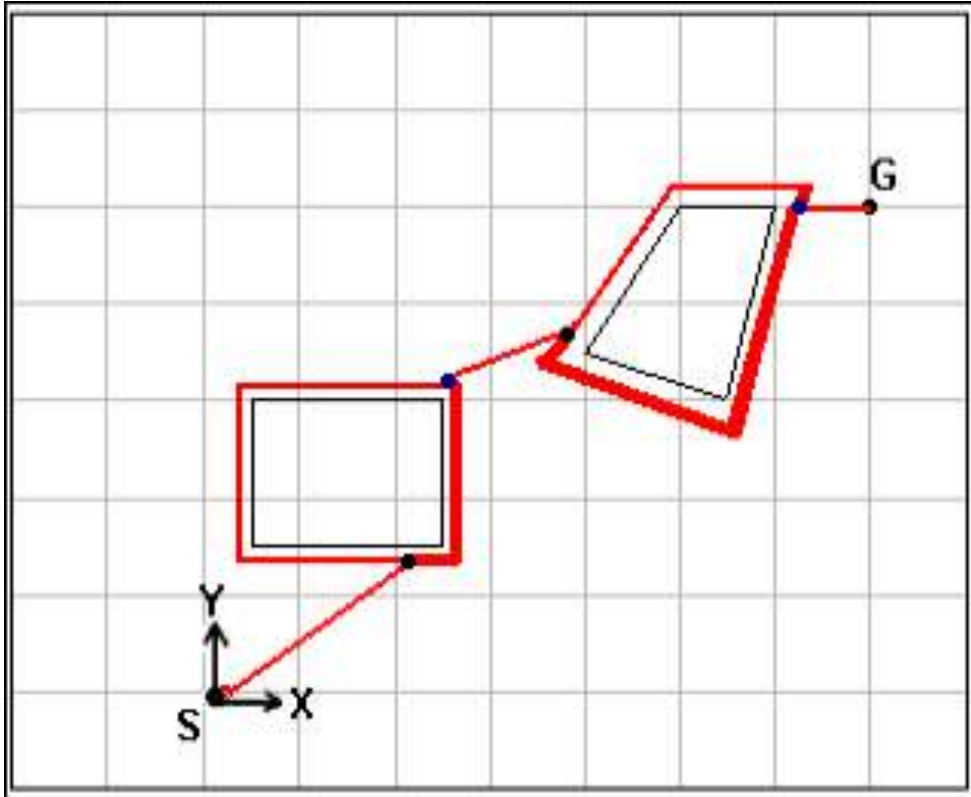


# Bug Algorithms

- Common sense approach of moving directly to goal.
- Contour the obstacle when found, until moving straight to goal is possible again.
- Path chosen – often too long
- Robot prone to move close to obstacles

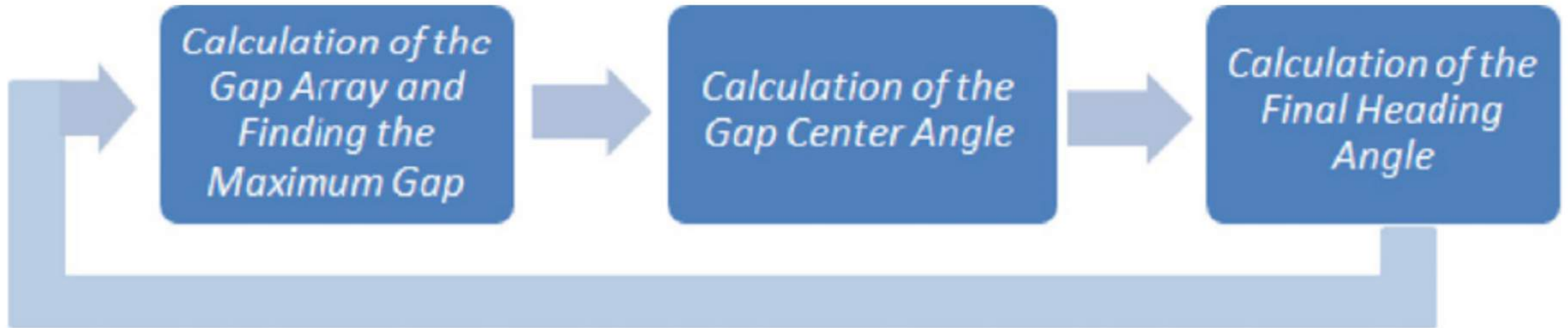


# Possible paths with Bug Algorithm





# Follow the Gap Method (FGM)



**Fig. 3.** Steps of the Follow the Gap method.

# Follow the Gap Method (FGM)

- Point Robot Approach
- Obstacle representation
- Construction a gap array among obstacles.
- Determination of maximum gap, considering the Goal point location.
- Calculation of angle to Center of Maximum gap
- Robot proceeds to center of maximum gap.

# Problem Definition

- The Algorithm
  - Should find a purely reactive heading to achieve goal co-ordinates
  - Should avoiding obstacles with as large distance as possible
  - Should consider measurement and nonholonomic constraints
  - for obstacle avoidance must collaborate with global planner
- Goal point – obtained from the global planner
- Obstacle co-ordinates - change with time

# Point Robot Approach

$X_{rob}$  = Abscissa of robot point

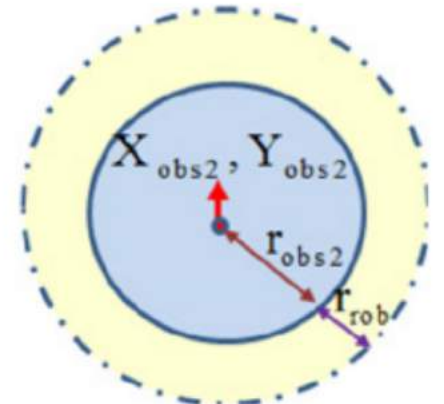
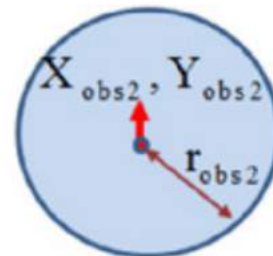
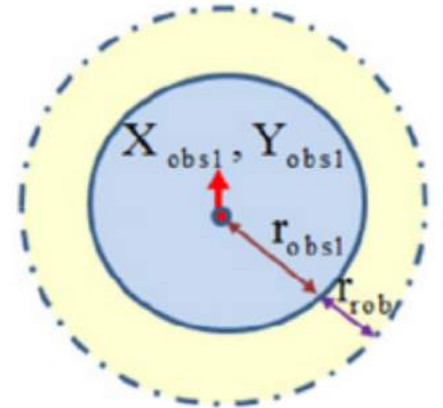
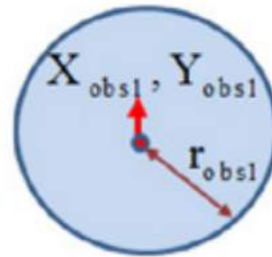
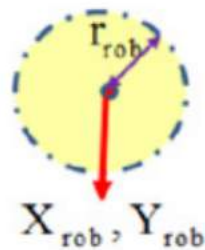
$Y_{rob}$  = Ordinate of robot point

$R_{rob}$  = Robot circle's radius

$X_{obsn}$  = Abscissa of nth obstacle

$Y_{obsn}$  = Ordinate of nth obstacle

$R_{obsn}$  = nth obstacle's circle's radius



(a) Circular robot with circular obstacles.

(b) Point robot with enlarged obstacles.

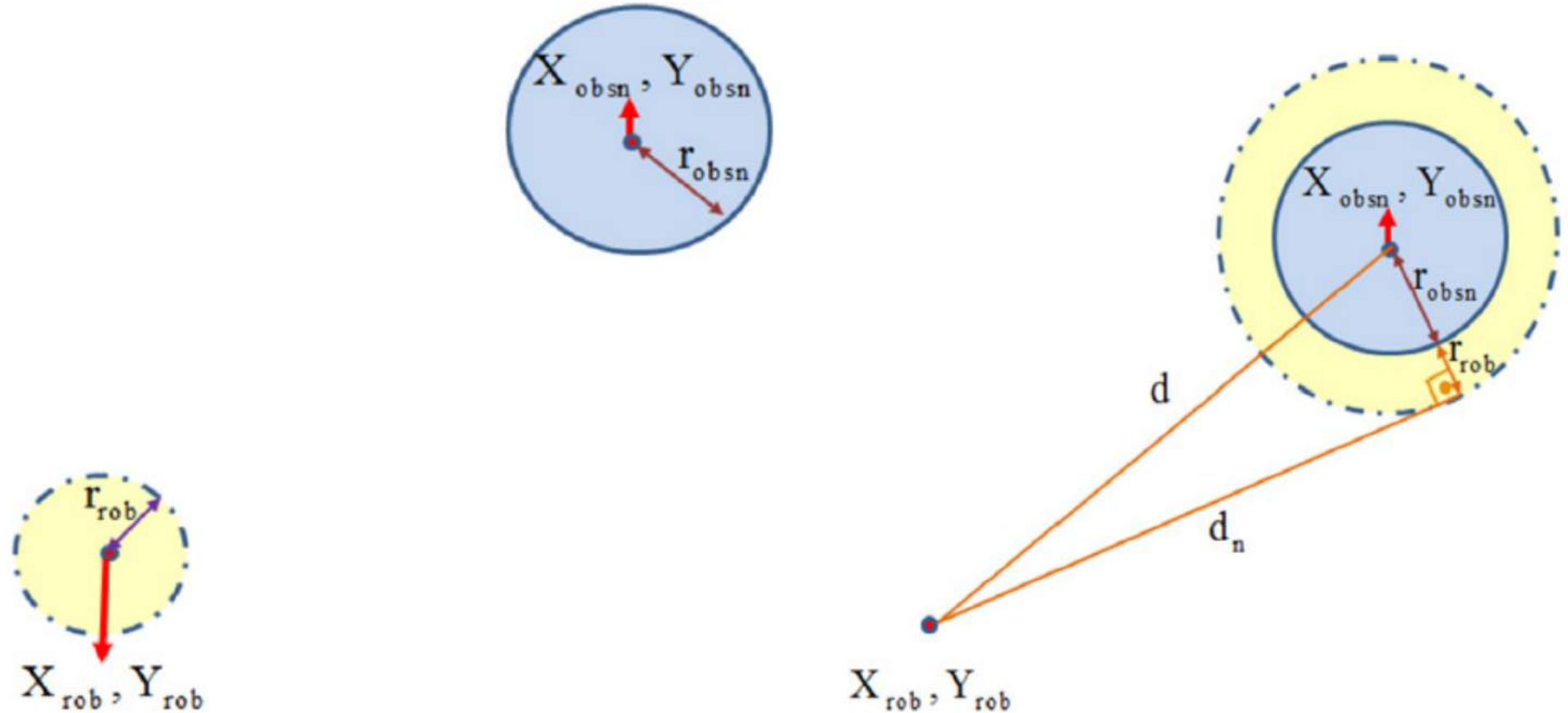


# Distance to Obstacle

$$d = \sqrt{(X_{obsn} - X_{rob})^2 + (Y_{obsn} - Y_{rob})^2}$$

$$d_n^2 + (r_{obsn} + r_{rob})^2 = d^2$$

$$\Rightarrow d_n = \sqrt{(X_{obsn} - X_{rob})^2 + (Y_{obsn} - Y_{rob})^2 - (r_{obsn} + r_{rob})^2}.$$

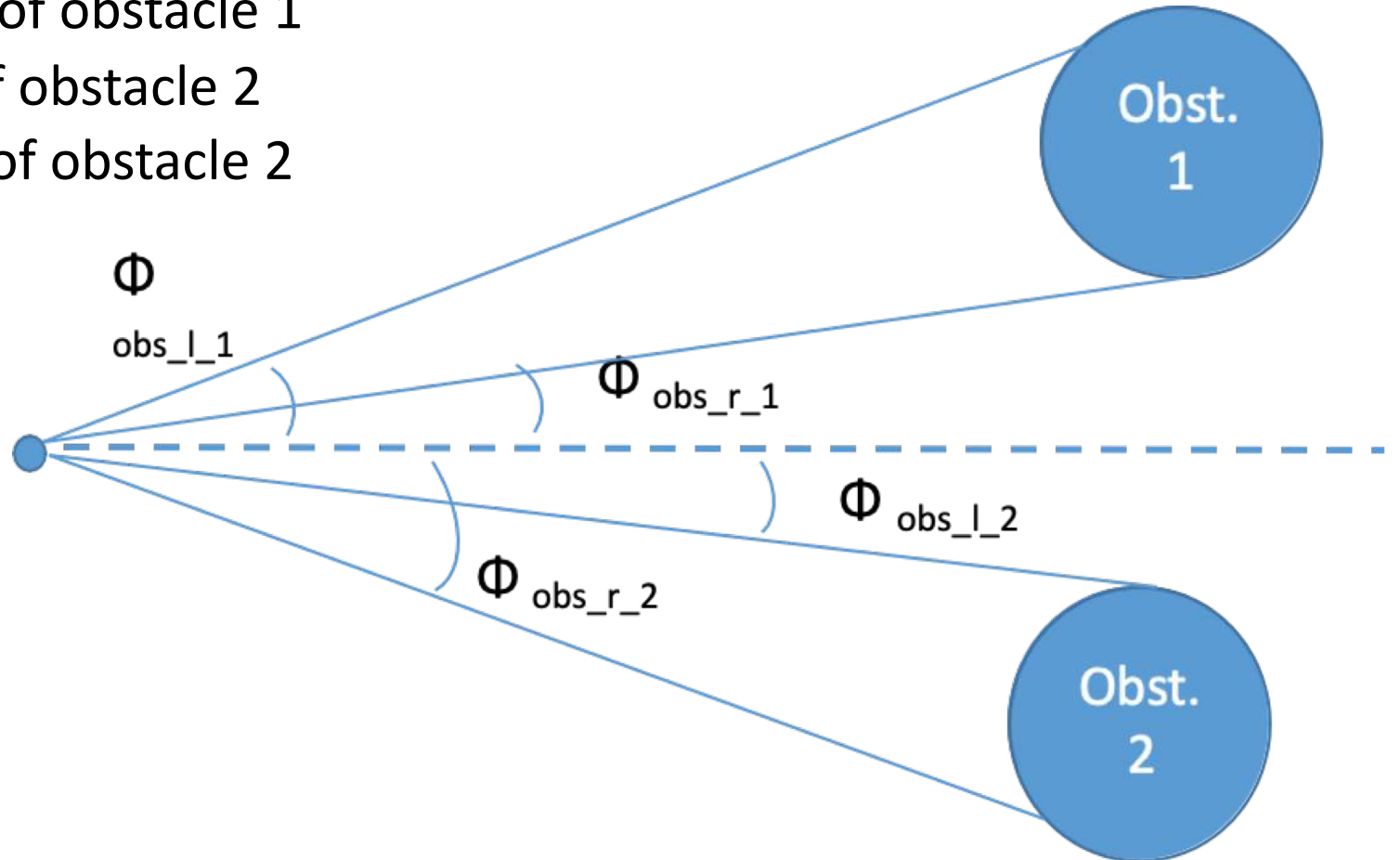


(a) Circular robot and circular obstacle parameters.

(b) Distance to obstacle geometry.

# Obstacle Representation

- Two parameter representation
  - $\Phi_{\text{obs}_l_1}$  – Border left angle of obstacle 1
  - $\Phi_{\text{obs}_r_1}$  – Border right angle of obstacle 1
  - $\Phi_{\text{obs}_l_2}$  – Border left angle of obstacle 2
  - $\Phi_{\text{obs}_r_2}$  – Border right angle of obstacle 2



# Gap Border Evaluation

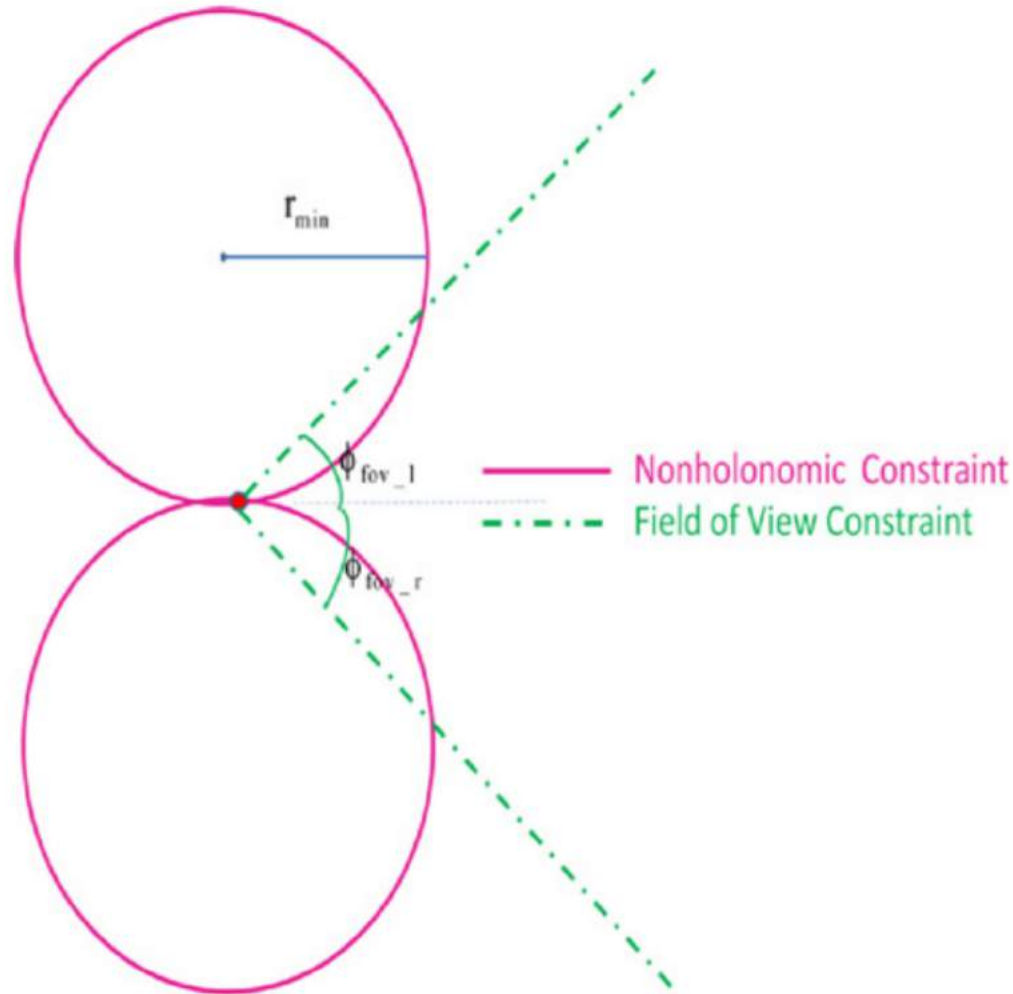
In order to understand which boundary is active for a boundary obstacle, *decision rule* are illustrated as follows:

$$d_{nhol} < d_{fov} \Rightarrow \phi_{lim} = \phi_{nhol}$$

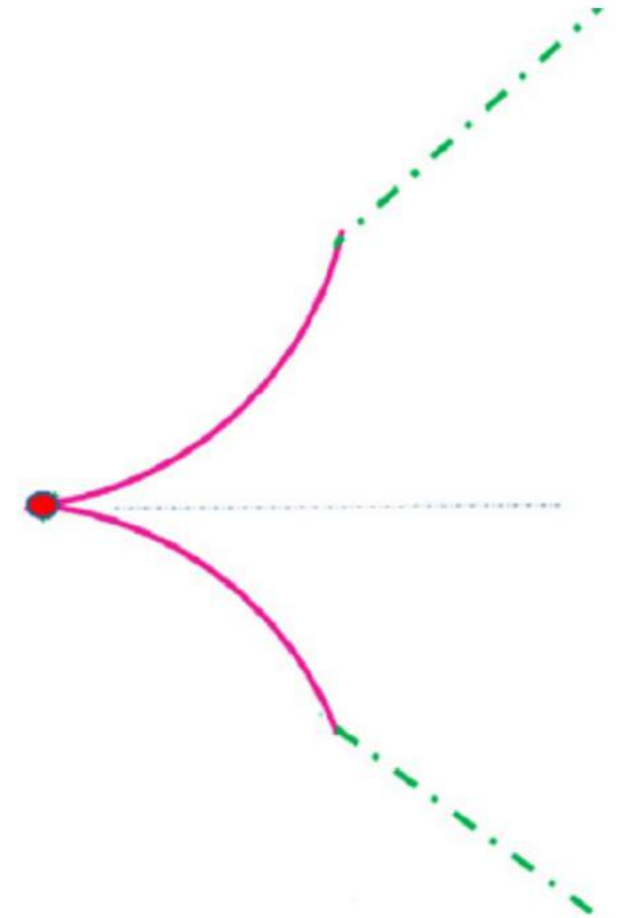
$$d_{nhol} \geq d_{fov} \Rightarrow \phi_{lim} = \phi_{fov}$$

where

$\phi_{lim}$ : Gap border angle. ( $\phi_{lim\_l}$



(a) Field of view and nonholonomic movement constraint separately.



(b) Gap border.

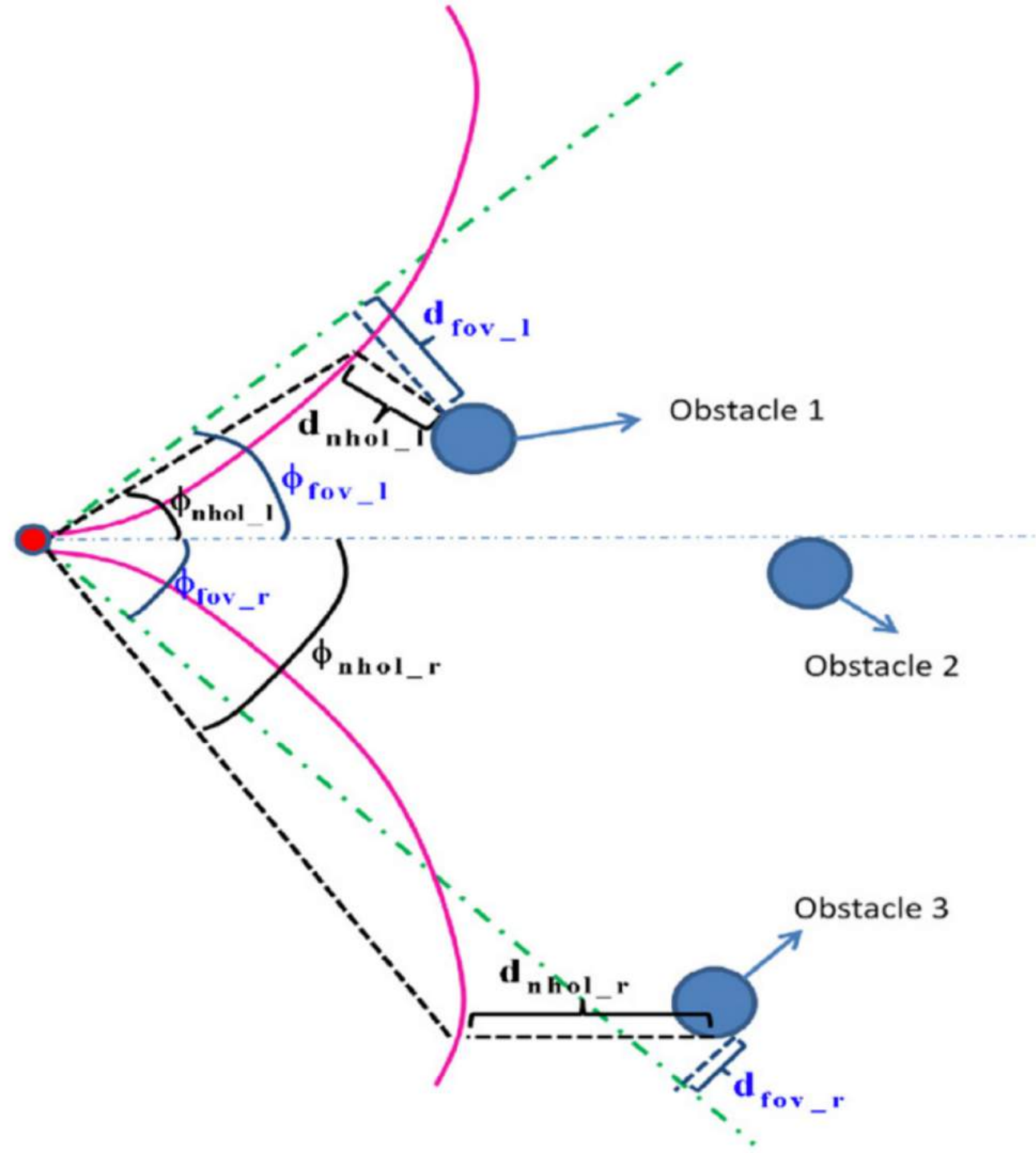
# Gap border parameters

1.  $\Phi_{lim}$ : Gap border angle
2.  $\Phi_{nhol}$ : Border angle coming from nonholonomic constraint
3.  $\Phi_{fov}$ : Border angle coming from field of view
4.  $d_{nhol}$ : Nearest distance between nonholonomic constraint arc and obstacle border
5.  $d_{fov}$ : Nearest distance between field of view line and obstacle border



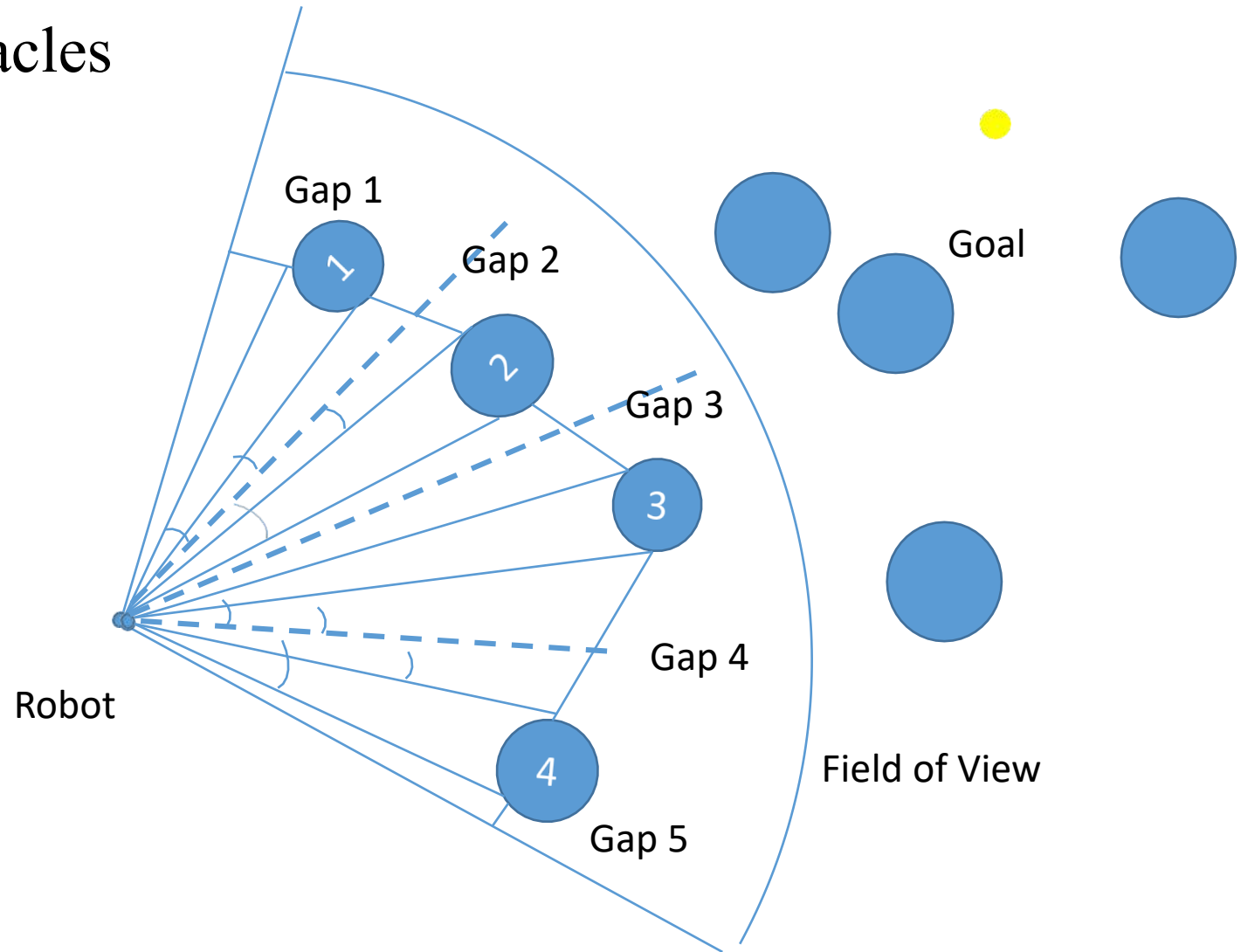
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# Construction of gap array

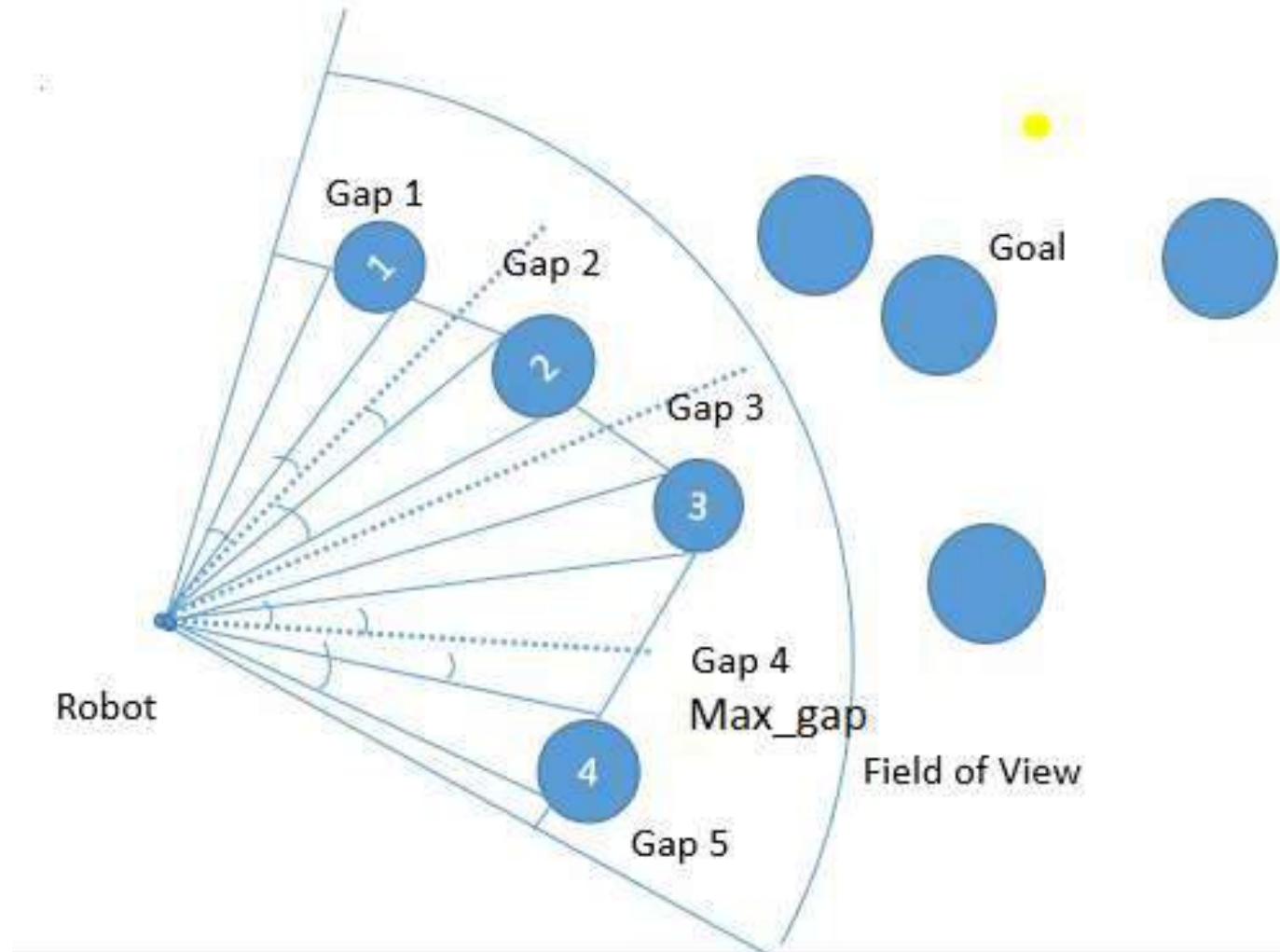
□  $N + 1$  gaps for  $N$  obstacles



# Gap array and Maximum Gap

- $\text{Gap}[N+1] = [(\Phi_{\text{lim}_1} - \Phi_{\text{obs1}_1})(\Phi_{\text{obs1}_r} - \Phi_{\text{obs2}_1}) \dots (\Phi_{\text{obs}(n-1)_r} - \Phi_{\text{obs}(n-1)_1})(\Phi_{\text{obsn}_r} - \Phi_{\text{lim}_r})]$
- Maximum gap is determined with a sorting algorithm in program.

# Gap array and Maximum Gap

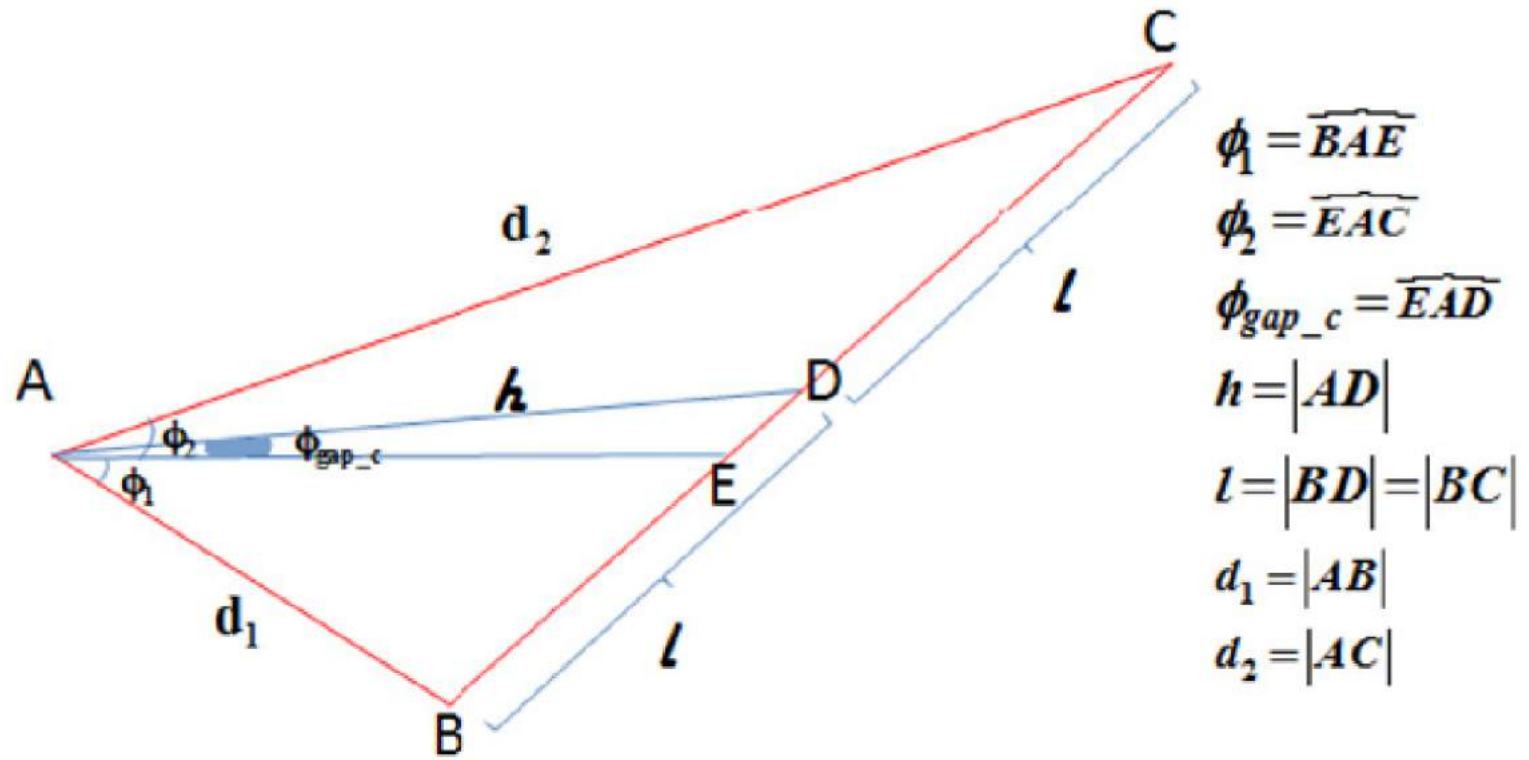




# Follow the Gap Method (FGM)

- Point Robot Approach
- Obstacle representation
- Construction a gap array among obstacles.
- Determination of maximum gap, considering the Goal point location.
- Calculation of angle to Center of Maximum gap
- Robot proceeds to center of maximum gap.

# Gap Center angle Calculation



Firstly, the Cosine Rule is applied to the ABC triangle:

$$(2l)^2 = d_1^2 + d_2^2 - 2d_1d_2 \cos(\phi_1 + \phi_2)$$

$$l^2 = \frac{d_1^2 + d_2^2 - 2d_1d_2 \cos(\phi_1 + \phi_2)}{4}.$$

After that, the Apollonius theorem is applied to the ABC triangle.

$$d_1^2 + d_2^2 = 2l^2 + 2h^2$$

**Fig. 8.** Gap center angle parameterization.

# Gap center angle

- The gap center angle ( $\phi_{gap\_c}$ ) is found in terms of the measurable  $d_1$ ,  $d_2$ ,  $\phi_1$ ,  $\phi_2$  parameters

$$\phi_{gap\_c} = \arccos \left( \frac{d_1 + d_2 \cos(\phi_1 + \phi_2)}{\sqrt{d_1^2 + d_2^2 + 2d_1d_2 \cos(\phi_1 + \phi_2)}} \right) - \phi_1$$

# Follow the Gap Method (FGM)

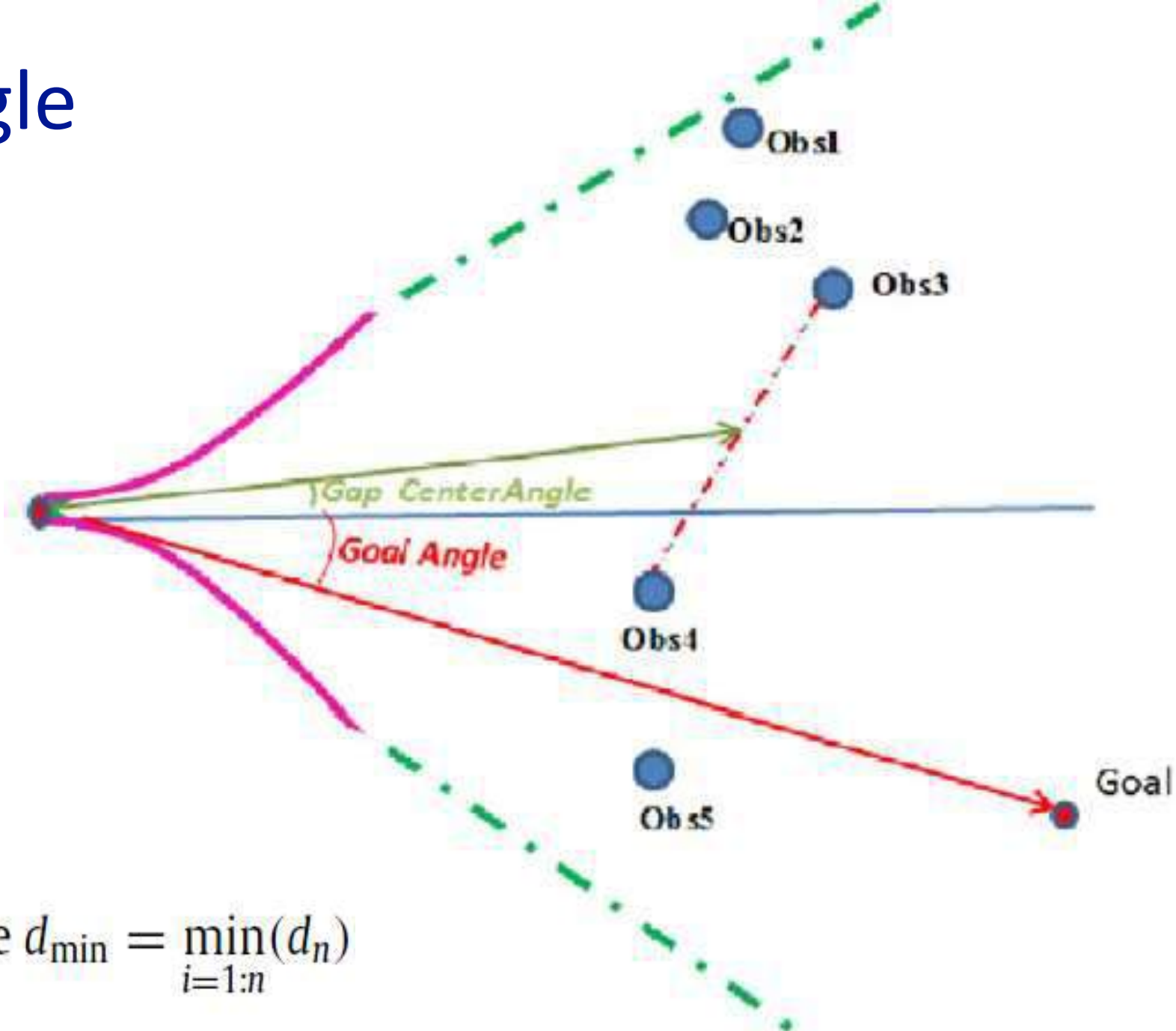
- Point Robot Approach
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# Calculation of final heading angle

- Final angle is Combination of angle of center of maximum gap and Goal point angle.
- Determined by fusing weighted average function of gap center angle and goal angle.
- $\alpha$  is the weight to obstacle gap.
- $\alpha$  acts as tuning parameter for FGM.
- $\beta$  weight to goal point (assumed 1 for simplicity)
- $d_{min}$  is minimum distance to the approaching obstacle.



# Final Heading Angle



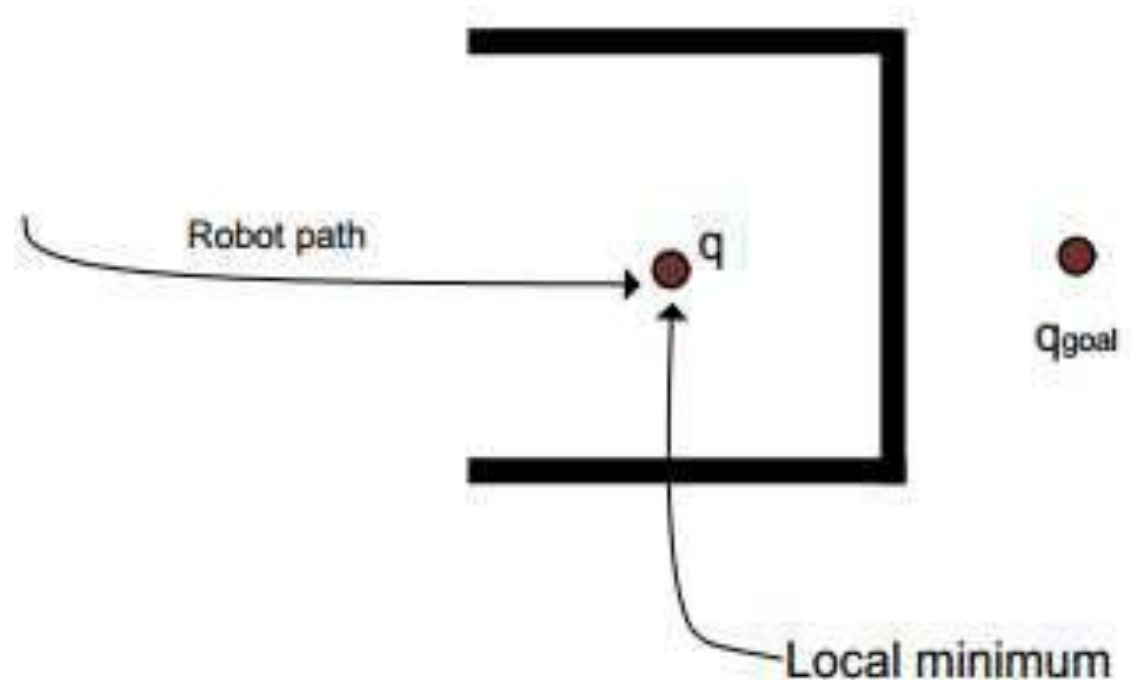
$$\phi_{final} = \frac{\frac{\alpha}{d_{min}} \phi_{gap\_c} + \phi_{goal}}{\frac{\alpha}{d_{min}} + 1} \quad \text{where } d_{min} = \min_{i=1:n}(d_n)$$

# Role of $\alpha$ value

- Weightage to gap angle is  $\alpha/d_{\min}$
- $\alpha$  makes the path goal oriented or gap oriented.
- For  $\alpha = 0$ ,  $\phi_{\text{final}}$  is equal to  $\phi_{\text{goal}}$
- Increasing values of alpha brings  $\phi_{\text{final}}$  closer to  $\phi_{\text{gap\_c}}$  and vice versa

# Dead end Scenario

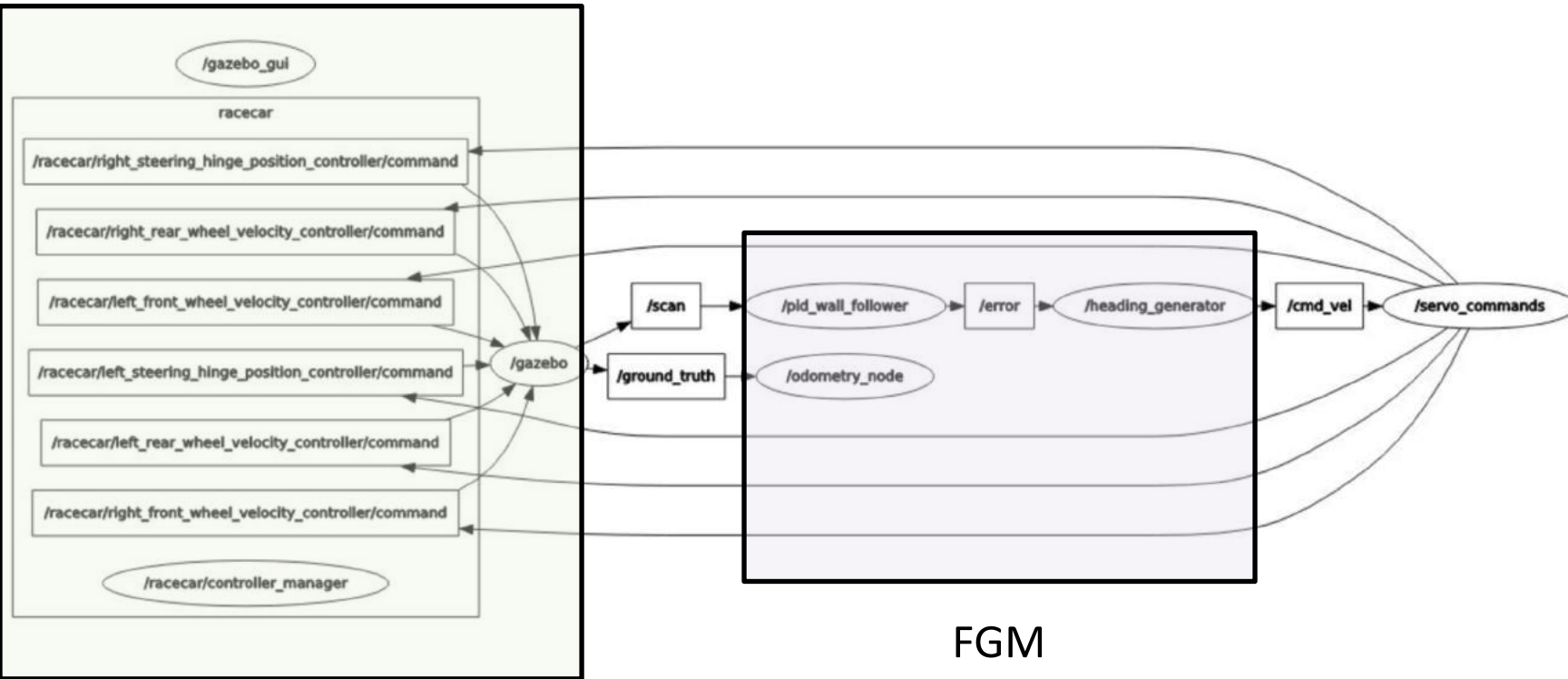
- A dead-end scenario of U-shaped obstacles is a problem for FGM as it is for APF as both are more sort of local planners.
- It needs upper level of intelligence.
- Can be solved by approaches like Virtual Obstacle Method, Multiple Goal Point method etc.



# Advantages of FGM

- Single tuning parameter ( $\alpha$ ) in weightage to gap center angle ( $\alpha/d_{\min}$ )
- Considers nonholonomic constraints for the robot.
- Only feasible trajectories are generated, lesser ambiguity to decision, lesser computation time.
- Field of view of robot is taken into account.
- Robot does not move in unmeasured directions.
- Passage through maximum gap center – Safest path.

# Follow the Gap navigation and planning on the F1/10 Car



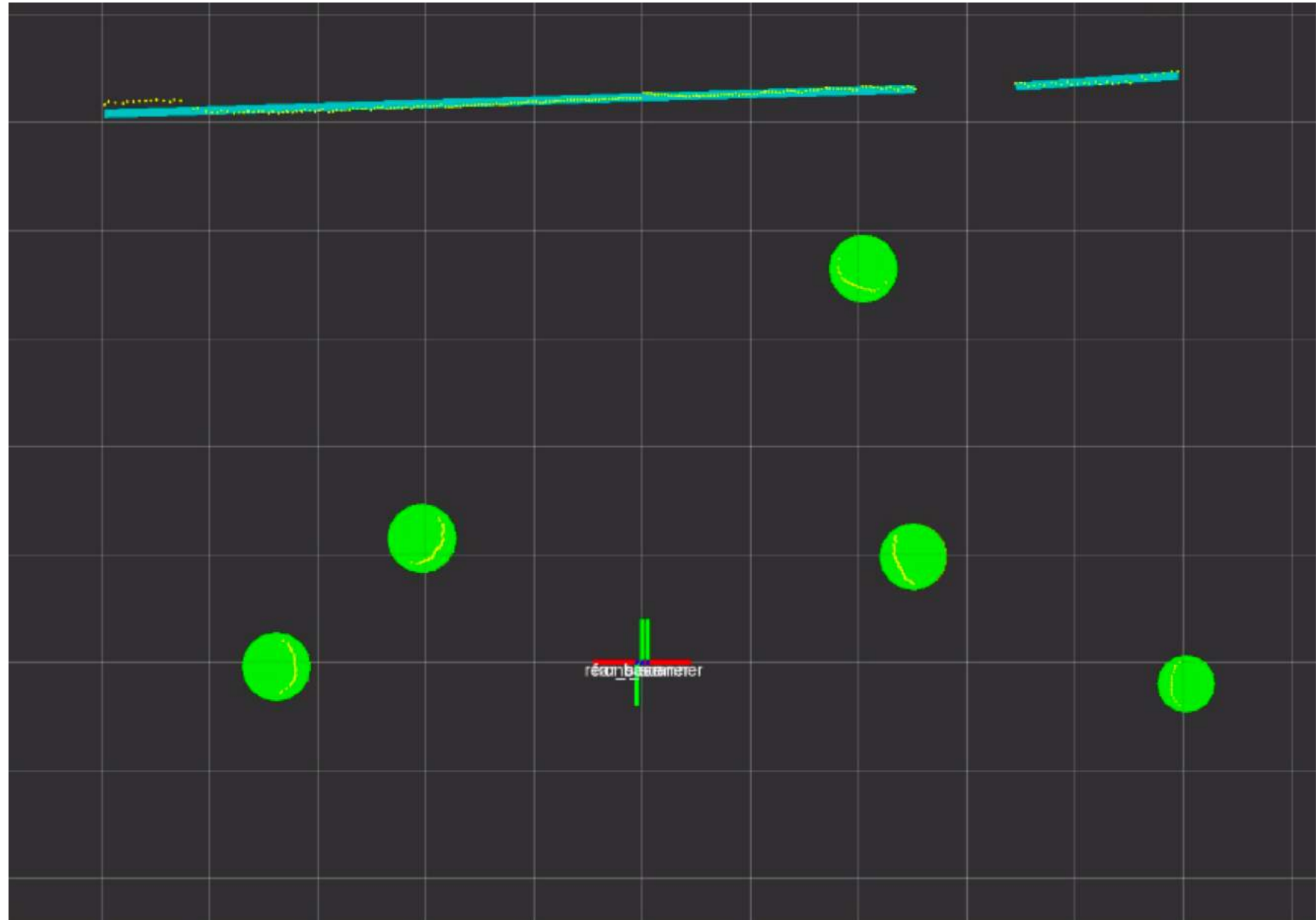
Simulator





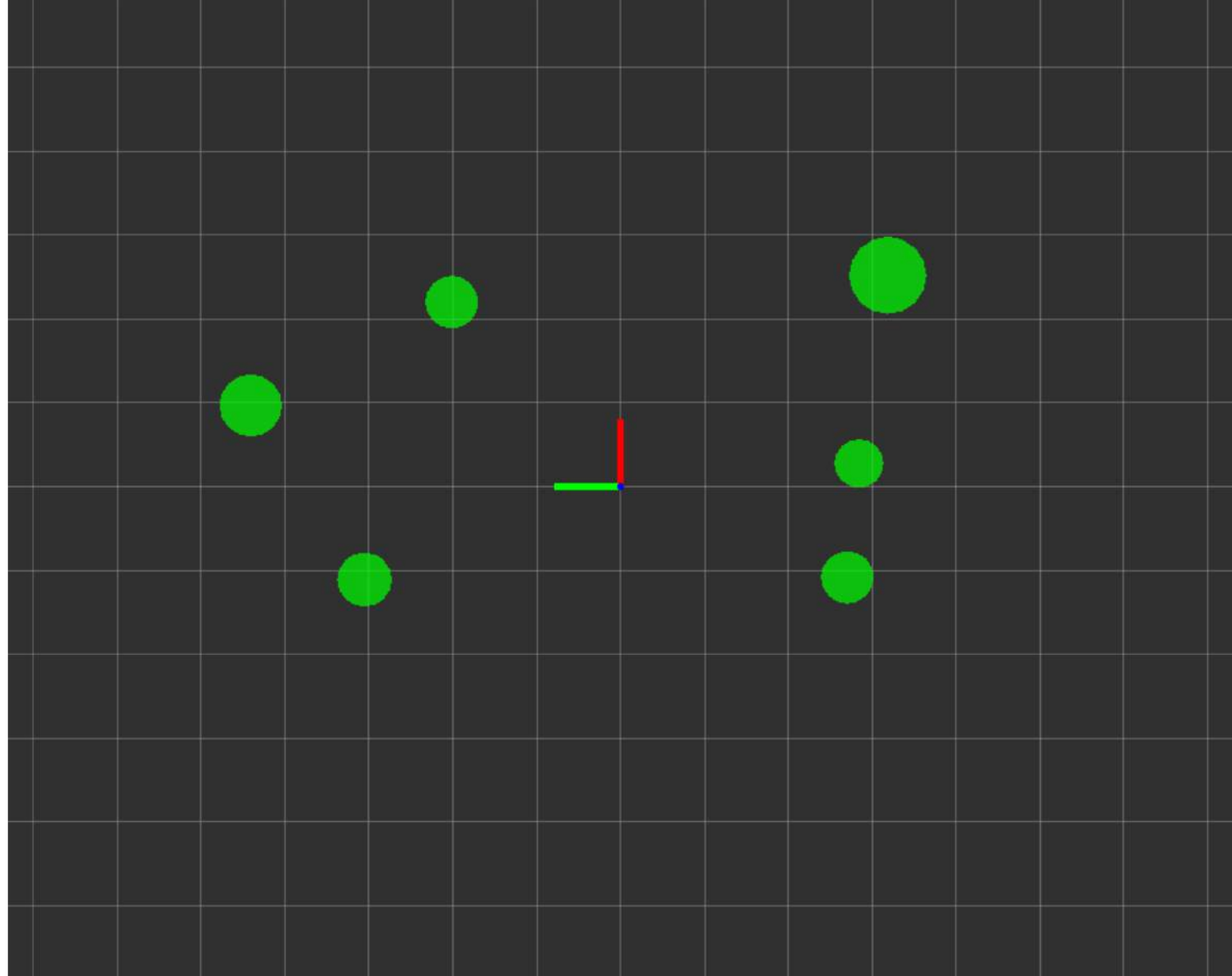
[https://github.com/tysik/obstacle\\_detector](https://github.com/tysik/obstacle_detector)

Fig. 1. Visual  
example of  
obstacle detector  
output.



[https://github.com/tysik/obstacle\\_detector](https://github.com/tysik/obstacle_detector)

Visual example of  
'obstacle\_tracker'  
output.



# Head-to-head Autonomous Racing

## 4th F1/10 International Autonomous Racing Competition

**F1**  
**TENTH**  
[ftenth.org](http://ftenth.org)

