8

Diffusion

Diffusion is an important social process. Administrators are interested in the diffusion of information and opinions manufacturers seek the adoption of new techniques and products, and all of us have a vivid interest in not acquiring contagious diseases. Diffusion processes are being studied in the communication sciences, social psychology and sociology, public administration, marketing, and epidemiology.

In this chapter, we present diffusion processes from a network point of view. Diffusion is a special case of brokerage, namely brokerage with a time dimension. Something – a disease, product, opinion, or attitude – is handed over from one person to another in the course of time. We assume that social relations are instrumental to the diffusion process: they are channels of social contagion and persuasion.

If personal contacts are important, then the structure of personal ties is relevant to the diffusion process and not just the personal characteristics that make one person more open to innovations than another. We investigate the relation between structural positions of actors and the moment at which they adopt an innovation.

8.1 Example

Educational innovations have received a lot of attention in the tradition of diffusion research. Our example is a well-known study into the diffusion of a new mathematics method in the 1950s. This innovation was instigated by top mathematicians and sponsored by the National Science Foundation of the United States as well as the U.S. Department of Education. The diffusion process was successful because the new method was adopted in a relatively short period by most schools.

The example traces the diffusion of the modern math method among school systems that combine elementary and secondary programs in Allegheny County (Pennsylvania, U.S.A.). All those school superintendents who were in office at least two years were interviewed. They are the gatekeepers to educational innovation because they are in the position to make the final decision. The researchers obtained data from sixty-one of sixty-eight superintendents, fifty-one of whom had adopted by 1963 (84%).

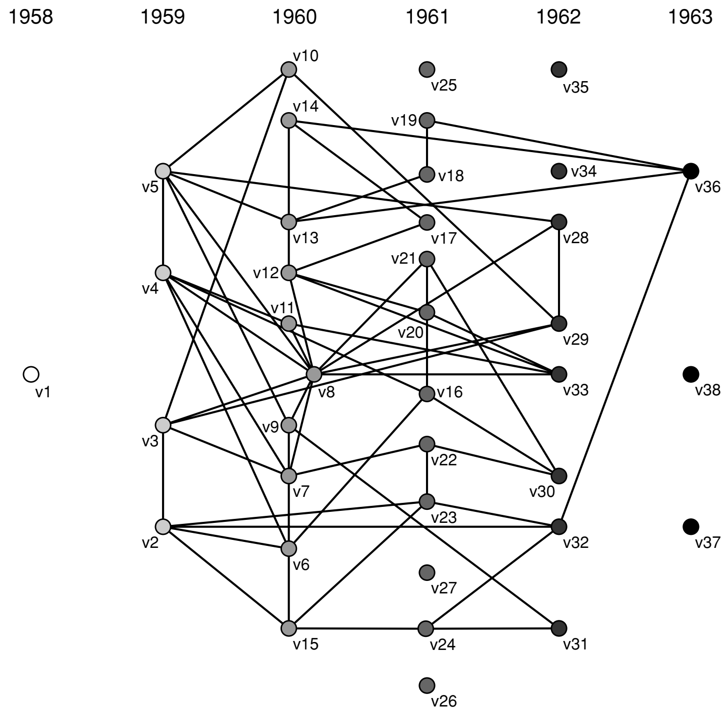

Figure 75. Friendship ties among superintendents and year of adoption.

Among other things, the superintendents were asked to indicate their friendship ties with other superintendents in the county with the following question: Among the chief school administrators in Allegheny County, who are your three best friends? The researcher analyzed the friendship choices among the thirty-eight interviewed superintendents who adopted the method and were in position at least one year before the first adoption, so they could have adopted earlier. Unfortunately, the researcher did not include the friendship choices by superintendents who received no choices themselves; they are treated as isolates. In the original network, some friendship choices are reciprocated and others are not (`ModMath_directed.net`) but we use the symmetrized network (`ModMath.net`), which is depicted in Figure 75. A line in this network indicates that at least one superintendent chooses the other as his friend.

As you may infer from Figure 75, adoption started in 1958 and all the schools researched had adopted by 1963. The year of adoption by a superintendent's school is coded in the partition `ModMath_adoption.clu`: 1958 is class (time) 1, 1959 is class (time) 2, and so on. The first adopter (v1) is a superintendent with many contacts outside Allegheny County but few friends within the county. He is a "cosmopolite" and cosmopolites usually are early adopters but they are often too innovative to be influential in a local network.

Anwendungsbeispiele in Pajek ignorieren

Application

Draw>Draw-Partition

Layers>in y direction

For a first visual impression of a diffusion process, open the Pajek project file `ModMath.paj` and draw the sociogram in the order of adoption time (see Figure 75; we manually added the years to the top of this figure). To do this, the adoption time of vertices must be specified in a partition (e.g., `ModMath_adoption.clu`). Draw the sociogram with vertex colors defined by the partition (*Draw>Draw-Partition* or *Ctrl-p*) and select the command *Layers>in y direction* to arrange the vertices by adoption time. Note that this procedure is available only when a network with partition is drawn.

Layers>Optimize layers in x direction

In most cases, the vertices are not optimally placed within each level. To improve their positions, use the *Optimize layers in x direction* command. You can let this command adjust all levels (i.e., classes) or you can restrict the optimization to a range of levels. Play around with the options (*Forward*, *Backward*, *Complete*) until you obtain a layout without lines that cross vertices with which they are not incident. In Figure 75, this was not possible because superintendent v8 is connected to too many vertices in his adoption class, so we decided to move him away from the line of his class. Even for our small diffusion network, the sociogram needs a lot of fine-tuning, so you should not expect a clear picture if you are working with large networks.

Move>Fix>y

Sometimes it helps to rearrange the vertices within a layer by hand. If you do this but you want to be sure that the vertices within a class remain aligned, activate the option *y* in the *Fix* menu of the Draw screen. Now, you can move vertices horizontally only.

[Draw screen] Options> Transform> Rotate 2D

The layers are drawn in the *y* direction: from the top down. In Figure 75, however, time flows from left to right on the *x* axis, which is the standard way to represent time. We obtained this figure by rotating the standard layout of layers by 90 degrees. Select the command *Rotate 2D* from the *Options>Transform* submenu in the Draw screen. Type 90 in the dialog box captioned *Angle in degrees* and press the OK button.

8.2 Contagion

Information is important to the diffusion of new opinions, products, and the like. In most societies, the mass media are central to the spreading of information, so we ought to pay attention to mass communication. Several models have been proposed for the process of mass communication, one of which is consistent with a network approach: the two-step flow model. According to this model, mass communication consists of two phases. In the first phase, mass media inform and influence opinion leaders. In the second phase, opinion leaders influence potential adopters within their communities or social systems.

Network models of diffusion focus on the second phase, assuming that opinion leaders use social relations to influence their contacts. Social ties are thought to be important because innovations are new, hence risky. Personal contacts are needed to inform and persuade people of the benefits associated with the innovation. Note that salient social relations for

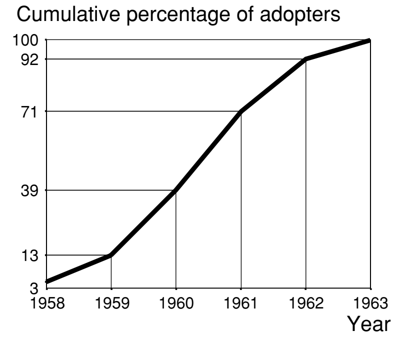

Figure 76. Adoption of the modern math method: diffusion curve.

spreading information may be different from relations used for persuasion. The relations most commonly investigated are advice and friendship relations.

Basically, network models see diffusion as a process of contamination, just like the spread of an infectious disease. Therefore, passing on an innovation via social ties is called *social contagion*. This perspective is backed by the empirical fact that many innovations diffuse in a pattern that is similar to the spread of infectious diseases. First, an innovation is adopted by few people but their number increases relatively fast. Then, large numbers adopt but the growth rate decreases. Finally, the number of new adopters decreases rapidly and the diffusion process slowly stops. This diffusion pattern is characteristic for a chain reaction in which people contaminate their contacts, who contaminate their contacts in the next step, and so on.

The adoption of the modern math method is represented by a *diffusion curve* (Figure 76). The *x* axis shows the moment of adoption and the *y* axis represents the *prevalence* of the innovation, which is the percentage of all interviewed superintendents who have adopted the modern math method by that year. Note that prevalence is represented by cumulative percentages, that is, the sum of all percentages of previous adopters: in 1958, 3 percent of the superintendents adopt and in 1959 another 10 percent adopt, so the cumulative percentage of adopters is 13 percent in 1959.

The diffusion curve has the logistic S-shape, which is characteristic of a chain reaction. We find a similar curve when we take a random network and choose a vertex as a source of contamination (the white vertex in Figure 77). When we assume that a vertex contaminates its neighbors at time 1, who contaminate their neighbors at time 2, and so on, we obtain the typical diffusion curve of Figure 78 (bold line). Note that the number of new adopters increases faster and faster in the first three steps (vertices with numbers 1, 2, and 3) and that the absolute number of new adopters decreases sharply after the fourth step. This example illustrates that contagion through network ties may explain the logistic spread of an

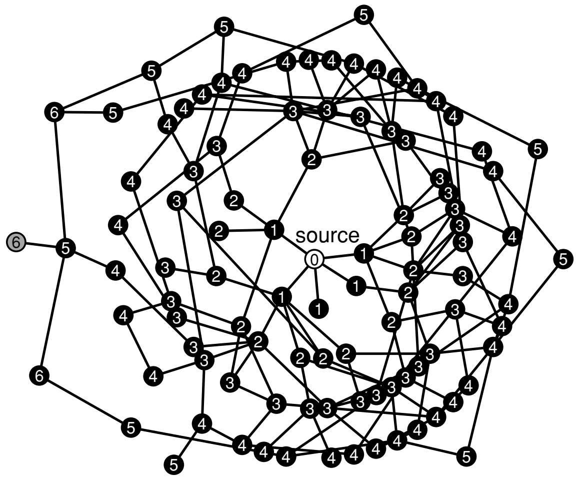

Figure 77. Diffusion by contacts in a random network ($N = 100$, vertex numbers indicate the distance from the source vertex).

innovation or a disease. If we find a diffusion curve that does not have the typical S shape, it is quite unlikely that network ties are important to the diffusion process and diffusion is probably propelled predominantly by other forces such as mass media campaigns.

When contagion drives the diffusion process, the structure of an information or contact network conditions the diffusion of information, innovations, diseases, and so on. Using the measures introduced in previous chapters, some broad hypotheses are easily derived as follows:

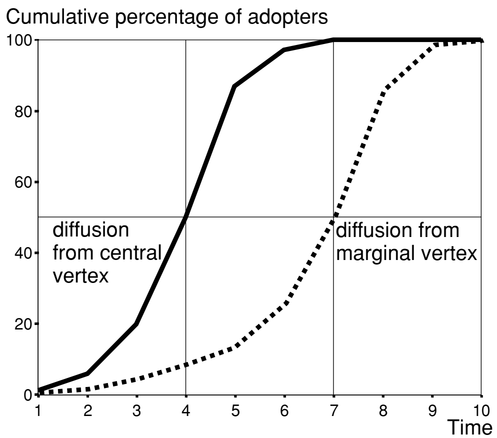

Figure 78. Diffusion from a central and a marginal vertex.

- In a dense network an innovation spreads more easily and faster than in a sparse network,
- In an unconnected network diffusion will be slower and less comprehensive than in a connected network,
- In a bi-component diffusion will be faster than in components with cut-points or bridges,
- The larger the neighborhood of a person within the network, the earlier s/he will adopt an innovation,
- A central position is likely to lead to early adoption,
- Diffusion from a central vertex is faster than from a vertex in the margins of the network.

The *adoption rate* is the number or percentage of new adopters at a particular moment.

The speed of the diffusion process is measured by the *adoption rate*, which is the number or percentage of new adopters at a particular moment. It is easy to see that the adoption rate is higher when an innovation spreads from a central vertex than when it starts at a marginal vertex. Figure 78 shows the diffusion curves for the diffusion from the central white source vertex in Figure 77 (bold line) and from the peripheral gray vertex (dotted line). Both curves have the typical S shape but it takes considerably more time for a diffusion to reach half or all of the population when it is triggered by a vertex in the periphery.

The hypotheses presented above highlight the impact of network structure on the diffusion process. We should note, however, that personal characteristics and the type of innovation also influence the rate of adoption. The perceived risk of an innovation, its perceived advantage over alternatives, and the extent to which the innovation complies with social norms that govern the target group determine whether it is adopted quickly, reluctantly, or not at all. A risky innovation, for instance, will diffuse slower regardless of the network's density and connectivity.

Application

Info>Partition

The diffusion curve is constructed from a simple frequency tabulation of adoption time, which is displayed by the *Info>Partition* command (Table 10). The table shows the cumulative relative frequencies that are plotted on the y axis in the chart of Figure 76. The class numbers represent the moments that are displayed on the x axis. Note that the table and chart are basic statistical techniques, which may be produced in any statistical software package or spreadsheet.

Exercise I

Net>Random Network> Vertices Output Degree

Create a simple random network with fifty vertices that have an outdegree of 1 or 2 (use the *Net>Random Network>Vertices Output Degree* command with a minimum outdegree of 1 and a maximum outdegree of 2 and no multiple lines). Pick a vertex as the source of a diffusion process

Table 10. *Adoption in the Modern Math Network*

Class	Freq	Freq%	CumFreq	CumFreq%	Representative
1	1	2.63	1	2.63	v1
2	4	10.53	5	13.16	v2
3	10	26.32	15	39.47	v6
4	12	31.58	27	71.05	v16
5	8	21.05	35	92.11	v28
6	3	7.89	38	100.00	v36
SUM	38	100.00			

and determine the adoption time of all vertices and the adoption rate at each point in time, assuming that a vertex will adopt at the first time point after it has established direct contact with an adopter. Note that the adoption time of a vertex is equal to its distance (see Chapter 7) from the source vertex under this assumption. Ignore the direction of the lines in the network.

8.3 Exposure and Thresholds

In the previous section, we assumed that every person is equally susceptible to contagion. One infected neighbor is enough to get infected; friendship with one adopter is enough to persuade someone to adopt. This is not very realistic because some people are more receptive to innovations than other people. There are two different ways to conceptualize the innovativeness of people, namely relative to the system and relative to their personal networks: adoption categories and threshold categories.

Adoption categories classify people according to their adoption time relative to all other adopters. These typologies are very popular in product marketing. A standard classification distinguishes between the early adopters (the first 16 percent who adopt), the early majority (the next 34 percent), the late majority (the next 34 percent), and late adopters or laggards (the last 16 percent to adopt). To classify people, we have to know only their adoption time. Then, we can simply mark the first 16 percent of all adopters as early adopters, and so on. This classification is useful for marketing purposes because it enables the marketing manager to identify the social and demographic characteristics of early adopters.

In the modern math example, early adopters are characterized by higher professionalism ratings and more accurate knowledge about the spread of educational innovations in their district. In addition, the superintendents who adopted early were not recruited from the school staff but they came from outside.

We concentrate on the second approach to innovativeness, *threshold categories*, which considers the personal network of actors. The network model of diffusion is based on contagion: an adopter spreads the innovation to his or her contacts. It is quite natural to assume that the chance that a person will adopt increases when he or she is linked to more people who

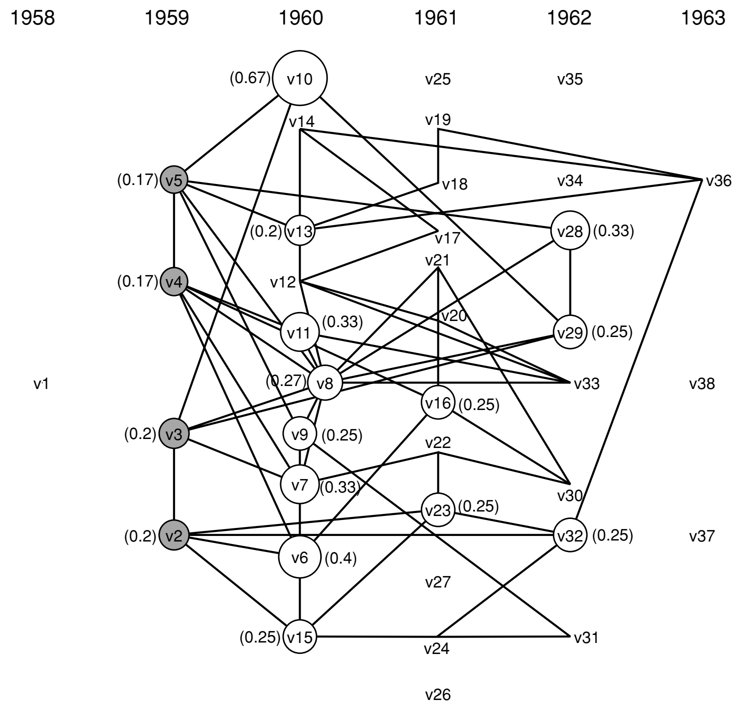

Figure 79. Adoption (vertex color) and exposure (in brackets) at the end of 1959.

already have adopted, that is, when he or she is exposed to more adopters. Hearing about the benefits of an innovation from different sources will persuade a person to adopt. The amount of exposure varies over time and among individuals, which explains that some people adopt early although they are not close to the sources in a diffusion process. The exposure of a person is expressed as a proportion so it may be the thought of as a chance to adopt.

> The *exposure* of a vertex in a network at a particular moment is the proportion of its neighbors who have adopted before that time.

Figure 79 shows the modern math network with the exposure of vertices in 1959 indicated by vertex size and by the numbers in brackets. Note that invisible vertices have zero exposure: none of their neighbors adopted in or before 1959. Eight of the ten superintendents who adopted in 1960 had friends among the 1959 adopters, so they were exposed. Clearly, superintendent v10 was most exposed: two of his three friends adopted in 1959, so his exposure was 0.67 at the end of 1959. However, not all exposed superintendents adopted in 1960: superintendents v16 and v23 adopted in 1961 and v28, v29, and superintendent v32 adopted in 1962. They were not less exposed than several superintendents who adopted immediately in 1960, for instance, v8, v9, v13, and v15, so we would expect them also to adopt in 1960. They contradict the simple contagion model which presupposes that all actors need the same amount of exposure to adopt.

In fact, statistical analyses of diffusion data do not always find a systematic relation between exposure and adoption. This means that either exposure and contagion are irrelevant to adoption or people need different levels of exposure before they adopt. If we pursue the latter option, we assume that some people are easily persuaded (e.g., they need only one contact with an adopter), whereas others are talked into adopting an innovation with difficulty. Some people are more susceptible than others, which is an established fact for media exposure as well as social exposure.

In the network model of diffusion, the *innovativeness* of a person is perceived as his or her threshold to exposure. An individual's threshold is the degree of exposure that he or she needs to adopt an innovation. Now, differences between individual thresholds may account for the fact that only part of the people adopt who are equally exposed.

> The *threshold* of an actor is his or her exposure at the time of adoption.

In our example, four superintendents (gray vertices in Figure 79) adopted the new math method in 1959. They exposed thirteen superintendents (white vertices) to their experience with this method and eight of them adopted the method in the next year. However, five superintendents adopted two or three years later. Why? Each of the exposed superintendents who adopted after 1960 has one or two friends among the colleagues who adopted in 1960 or 1961. By the time they adopted, these friends had also adopted, so their exposure was higher than at the end of 1959. According to the threshold hypothesis, their exposure had not reached the required threshold in 1959 but it did in 1960 or 1961. This explains why they adopted later.

At the end of 1959, for example, one of the four friends of superintendent v23 had adopted the modern math method, so his exposure was 0.25. In 1960, one more friend (v15) adopted and his exposure increased to 0.5. Then, superintendent v23 adopted, so we assume that his threshold was 0.5 or somewhere between 0.25 and 0.5.

We should note that individual thresholds are computed from the diffusion network after the fact: they are predictions with hindsight and they are not very informative by themselves. It is important to make sense of them or to validate them, which means that they should be associated with other indicators of innovativeness, for instance, adoption time or personal characteristics.

Thresholds indicate personal innovativeness, a lower threshold means more innovative, and we expect innovative people to adopt an innovation earlier than noninnovative people. Therefore, individual thresholds must be related to adoption time: innovative people have low thresholds and adopt early. If we find such a relation, we obtain some support for the assumption that individual thresholds indicate innovativeness.

At least to some extent, however, a positive relation between adoption time and individual thresholds is an artifact of the contagion model that we use. The first adopters cannot be exposed to previous adopters, so their thresholds are zero by definition. Within the network of adopters, the last adopters are very likely to be connected to previous adopters, so their exposure and thresholds are high at the time of adoption. When measurement of adoption time is restricted to a small number of moments, this will automatically produce a relation between individual thresholds and adoption time.

Therefore, it is also important to compare individual thresholds to external characteristics of the actors that usually indicate innovativeness. In general, innovativeness and low thresholds are supposed to be related to broad media use, many cosmopolitan contacts (contacts outside your local community), a high level of education, and high socioeconomic status.

Anwendungsbeispiel in Pajek-Software

Application

Net>Transform>Arcs→Edges>All

Let us compute exposure levels in the modern math network at one moment, for instance, at time 2, 1959 (see Figure 79). The procedure consists of several steps, which illuminate the calculation and exact meaning of the exposure concept. We assume that the network is undirected. If not, symmetrize it (*Net>Transform>Arcs→Edges>All* and remove any multiple lines, e.g., take the sum or minimum line value).

Partition>Binarize

Partition>Make Vector

First, we identify the adopters in the network at the selected time, which is 1959 or time 2 in our example. Make a binary partition from the adoption time partition where adoption times 1 and 2 are assigned a score of 1 (adopted) and others are assigned a score of zero (notadopted yet) with the *Partition>Binarize* command, selecting classes 1 through 2 in the dialog boxes. In Figure 79, the adopters are gray and the nonadopters are white. Then turn this partition into a vector to use it for computation (*Partition>Make Vector* or simply press *Ctrl-v*).

Operations>Vector>Summing up Neigbours

Second, compute the number of adopters in each actor's neighborhood with the command *Operations>Vector>Summing up Neigbours>Input, Output*, or *All*. A dialog box appears that asks whether a vertex should be included in its own neighborhood; answer no. Pajek does not count the number of neighbors but it sums the class numbers of the neighbors of a vertex. Because we use a binary partition in which an adopter has class number 1 and a nonadopter has class number zero, this sum is equal to the number of adopters in the neighborhood. It is a little trick, but it works.

Vectors>First Vector

Vectors>Second Vector

Vectors>Divide First by Second

Third, the number of adopters in the neighborhood of a vertex must be divided by its total number of neighbors because we defined exposure as the percentage of neighbors who have adopted. The division can be done in the *Vectors* menu. The vector we just made must be selected as the first vector in this menu (*Vectors>First Vector*). Next, we must make a vector with the total number of neighbors of a vertex. Recall that the degree of a vertex in a simple undirected network specifies the number of neighbors of a vertex, so we can make a degree partition in the usual way (*Net>Partitions>Degree*) and turn it into a vector (*Partition>Make Vector* – do not use the Normalized Indegree vector!). This vector must be used as the second vector in the *Vector* menu (*Vector>Second Vector*). Finally, we divide the number of adopters in a vertex's neighborhood by the total number of neighbors with the *Vectors>Divide First by Second* command. Now, we obtain a vector with the exposure of vertices at the end of 1959 (time 2).

[Main] Options>Read/ Write>0/0

Note that the computation of exposure is not straightforward if the network contains isolated vertices. An isolated vertex has no neighbors, so its degree is zero. The division described in the previous paragraph would ask Pajek to divide by zero, which is mathematically incorrect. In this case, Pajek assigns the value zero to the exposure of the vertex, that is, if the default setting of *0/0* to zero was not changed in the *Options>Read/Write* submenu of the Main screen.

Macro>Play

The calculation of exposure consists of a considerable number of steps. If you want to compute exposure at several points in time, you have to repeat these steps over and over again. This is not very efficient, so Pajek contains the possibility of executing a number of steps in one command, which is called a *macro*. A macro is a file that consists of a list of commands that are executed when you play the macro in Pajek. We prepared the macro `exposure.mcr`, which you can execute by clicking on the *Play* command in the *Macro* menu and selecting the file `exposure.mcr`, which is located in the directory with the data accompanying this chapter. Make sure that the original undirected network and the adoption time partition are selected before you execute the macro. When you open this file, Pajek starts to execute the commands. It displays the dialog boxes that allow you to select the first time of adoption (*Select clusters from*), which is 1 in our example, and the time for which you want to compute exposure (*Select clusters from 1 to*), for instance, time 3. Upon completion of the macro, several new partitions and vectors have been created and the last vector contains the exposure at the requested time.

Macro>Record

Macro>Add message

Creating a macro yourself is fairly simple. In essence, Pajek records all commands that you execute between the first and second time you click the *Record* command in the *Macro* menu. It prompts for a filename with the extension `.mcr` in which to store the recorded commands. While recording, you can add messages to the macro (*Macro>Add message*) that will be displayed in the Report screen when the macro is played afterwards. Make sure that you have the relevant network, partition, and vector selected in the drop-down menus before you record the macro and check the results when you play it for the first time.

Now that we have computed the exposure at one time, let us turn our attention to the calculation of thresholds. The *threshold* of a vertex is the proportion of its neighbors who have adopted before ego does, so we have to divide the number of prior adopters among the neighbors of a vertex by the size of its neighborhood. The computation of thresholds is fairly simple once you realize that the number of neighbors who have adopted prior to ego is equal to the indegree of ego in a directed network in which each line points from an earlier adopter to a later adopter. Figure 80,

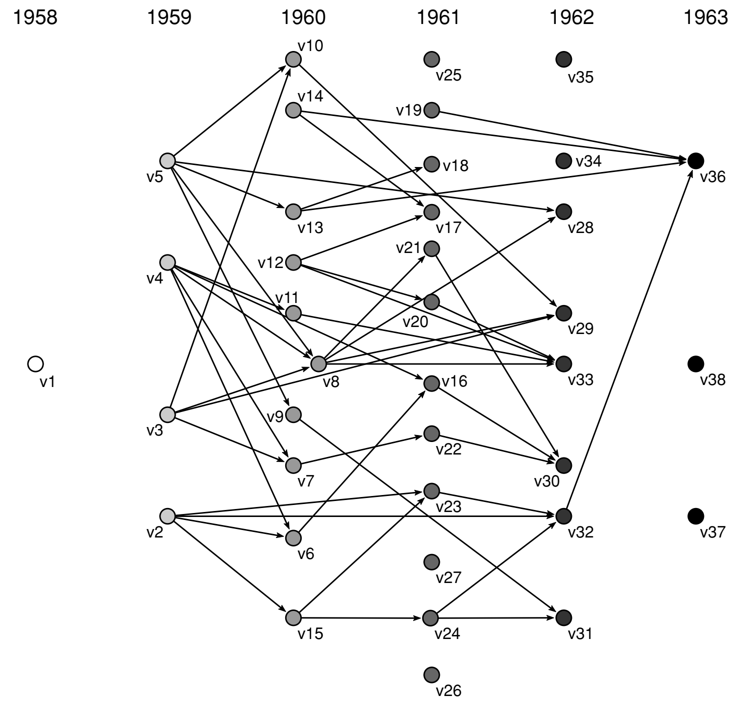

Figure 80. Modern math network with arcs pointing toward later adopters.

for instance, shows the modern math network if edges are replaced by arcs that point to later adopters. If social ties are used to spread the innovation, the arcs represent the direction of the spread. Note that ties within an adoption class are omitted because they are not supposed to spread the innovation.

Operations> Transform> Direction

In Pajek, we can change an undirected network (e.g., the modern math network) into a directed network with all arcs pointing from an earlier adopter to a later adopter with the commands in the *Operations> Transform>Direction* submenu. The commands are located in the *Operations* menu, so you need a network and something else, namely a partition that specifies the (adoption) classes to which the vertices belong. There are two commands: *Lower→Higher* and *Higher→Lower*. The first command replaces an edge in an undirected network by an arc that points toward the vertex with the higher class number. In our case, the partition contains adoption time classes so the *Lower→Higher* command produces arcs toward later adopters. This command issues a dialog box asking whether lines within classes must be deleted. In a directed diffusion network, we do not want to have lines within adoption classes normally, so answer yes. Now, we obtain the network shown in Figure 80. Applied to a directed network, the *Direction* procedure selects the arcs that conform to the selected option (lower to higher or higher to lower).

Now, we can simply compute the thresholds of all vertices in the diffusion network by dividing the indegree of vertices in the transformed directed network by their degree in the original undirected network provided that both networks contain neither multiple lines nor loops. Select the vector with normalized indegree in the directed network as the first vector in the *Vectors* menu, select the corresponding vector for the undirected network as the second vector, and divide the first by the second to obtain a vector with the individual thresholds. Make sure that a division of zero by zero (no neighbors) yields zero in the *Options>Read/Write* menu.

Net>Partitions>Degree>Input

Vectors>Divide First by Second

Exercise II

Compute the thresholds of the vertices in the modern math diffusion network as explained in this section. Is the threshold higher for vertices that adopt later as one would expect when thresholds really matter?

8.4 Critical Mass

Some diffusion processes are successful because almost everybody in the target group adopts the innovation. For instance, the modern math method was adopted by fifty-one of sixty-one superintendents in Allegheny County within a period of six years. Diffusion, however, may also fail because too few people adopt and spread the innovation. Once again, a biological metaphor is illuminating: a bacteria may either succeed to overcome the resistance of the human body and develop into a disease or does not gain the upper hand and is oppressed and finally eliminated by antibodies. The spread of a disease has a critical limit: once it is exceeded, the bacteria multiplies quickly.

> The *critical mass* of a diffusion process is the minimum number of adopters needed to sustain a diffusion process.

In the diffusion of innovations theory, a similar limit is hypothesized to exist. It is called the critical mass of a diffusion process and it is defined as the minimum number of adopters needed to sustain a diffusion process. In the first stage of a diffusion process, outside help is needed (e.g., an advertisement campaign) but once a sufficient number of opinion leaders have adopted, social contagion fuels the process and causes a chain reaction that ensures wide and rapid diffusion. Then, no more outside input to the diffusion process is required.

The critical mass of a particular diffusion process is difficult to pinpoint, so it is hard to prove that it exists and when it occurs. Recall that the two-step flow model combines contagion with external events. We need detailed information about the effects of external events, such as media

Table 11. *Adoption Rate and Acceleration in the Modern Math Diffusion Curve*

Time	Cum% of Adopters	Cum # of Adopters	Adoption Rate	Acceleration
↓			1	
1958 (1)	2.63	1		3
↓			4	
1959 (2)	13.16	5		6
↓			10	
1960 (3)	39.47	15		2
↓			12	
1961 (4)	71.05	27		−4
↓			8	
1962 (5)	92.11	35		−5
↓			3	
1963 (6)	100.00	38		−3
↓			0	

campaigns, versus the effect of social contagion on the diffusion process to know when critical mass is reached. Only afterwards, we may evaluate whether a diffusion process was successful. We present some approaches that try to overcome this problem.

There is an empirical rule of thumb that tells us something about the number of people who will eventually adopt an innovation. In many diffusion processes, a particular phenomenon occurs when the innovation has been adopted by 16 (or 10 to 20) percent of all people who will adopt eventually: the acceleration of the adoption rate decreases although the adoption rate still increases in absolute numbers. This is known as the first *second-order inflection point* of the S-curve.

In the modern math network, for instance, the number of new adopters (adoption rate) rises from 1 to 4 from 1958 to 1959 (see the fourth column in Table 11), which is three more than the number of adopters in 1958, so there is an acceleration of 3 (see the fifth column in Table 11). Note that adoption rates are placed between the moments because they reflect the change between two measurements. In the next year, ten superintendents adopt, which is an even larger acceleration, but in 1961 the acceleration drops to 2 because the number of new adopters grows only from 10 to 12; the number of new adopters still rises but it rises less sharply. In 1959, we may conclude, the acceleration of the adoption rate is highest and we can see that 13 percent of all adopters have adopted (see the column "Cum% of adopters" in Table 11) as predicted by the rule of thumb.

Because of this empirical relation between the first second-order inflection point of the diffusion curve and the final spread of an innovation, diffusion analysts say that critical mass is attained when the diffusion curve reaches this inflection point. In this approach, any diffusion process in which the adoption rate first accelerates and then declines is thought to be driven by the chain reaction characteristic for contagion models. Social contagion is assumed to take over the diffusion process at this point, so we may conclude that the process has reached its critical mass.

A similar argument has been made for the first-order inflection point of the logistic diffusion curve, which is the period with the highest adoption rate, that is, the largest absolute increase in new adopters. Usually, the first-order inflection point occurs when approximately 50 percent of all eventual adopters have adopted. In the modern math network, the highest adoption rate is 12 and it was realized between 1960 and 1961. In this period, the percentage of adopters rose from 39 to 71 percent.

We ought to realize that this approach completely presupposes the relation between contagion and critical mass; it does not prove that critical mass occurs, it merely assumes so. Nevertheless, it is useful for practical purposes. We may monitor the diffusion process and watch out for the moment in which the first decline in growth acceleration occurs (but we should ignore incidental declines). When it occurs, we may estimate the final number of adopters at about five to ten times the number of adopters at the time of the largest increase because about 10 to 20 percent has adopted then. If this estimated number of adopters is not enough according to our target, we can try to boost the diffusion process with additional media campaigns and the like. If this leads to acceleration of the diffusion, the critical mass becomes larger and the diffusion process will probably reach more people in the end. However, we have no guarantee that this will actually happen. After all, we are working with a simple rule of thumb.

In another perspective, a diffusion process is assumed to attain its critical mass when the most central people have adopted. Once they have adopted, so many actors in the network are exposed to adopters that many individual thresholds have been reached and an avalanche of adoptions occurs. Betweenness-centrality seems to be associated with critical mass in particular. Targeting the actors with highest betweenness-centrality is a good strategy for launching an innovation. In general, the position of the first adopters in the network is relevant to the diffusion process. If the first adopters are central and directly linked, their neighbors have higher exposure rates, so they are more likely to adopt.

Why does critical mass boost the diffusion of an innovation? On the one hand, the reason may be purely quantitative: once a sufficient number of well-connected people have adopted, enough people are exposed to the innovation to adopt, after which even more people are exposed. This is the mechanism we described for the case that the central actors adopt. On the other hand, reaching the critical mass has been thought of as a qualitative change to the system, namely a sudden lowering of individual thresholds. During the diffusion process, individual thresholds may be lowered as a consequence of the rate of adoption in the entire social system. People are supposed to monitor their social system. If they perceive wide acceptance of an innovation, they feel confident or even obliged to adopt it. Lower thresholds lead to easier adoption, so the diffusion process strengthens itself and it will most probably not wither away.

The lowering of thresholds is expected to occur particularly when actors are interdependent with regard to an innovation. New communication technology products (e.g., buzzers or SMS) are a case in point. When more people have one, their benefits and value increase. The first adopters

can reach few people with the new communication products but the late majority can contact many more users. This kind of innovation is called an *interactive innovation*. Even in the case of noninteractive innovations, such as the modern math method, the qualitative mechanism may be operative. Superintendents may be persuaded to adopt the new method because they know that most of their peers have adopted, regardless of the number of adopters in their circle of friends.

> A *threshold lag* is a period in which an actor does not adopt although he or she is exposed at the level at which he or she will adopt later.

The lowering of thresholds when critical mass is attained in the diffusion process may explain the occurrence of a threshold lag, that is, a period in which the exposure has reached the individual threshold but the individual does not adopt. In this case, adoption occurs after the critical mass is reached, and the individual's threshold is lowered. In the modern math network, superintendents v28 and v29 reached the level of exposure at which they would eventually adopt in 1960 because all of their friends had adopted by that year. However, they did not adopt immediately in 1961. There is a delay of one year, which is their threshold lag. Perhaps, the diffusion process reached its critical mass in 1961, which lowered their thresholds and induced them to adopt in 1962.

We should note that this approach to thresholds and threshold lags does not prove that individuals have certain thresholds and threshold lags; it merely defines them in a particular way. In an empirical diffusion network, we can always compute an actor's exposure at the moment of adoption (threshold) and how long this actor had been exposed at this level before he or she adopted (threshold lag). But this does not rule out the possibility that the individual threshold was actually lower and his or her threshold lag was longer. We should also consider the possibility that the individual's original threshold was even higher than the exposure at the time of adoption, so there was no threshold lag at all, namely when the diffusion process reached a critical mass lowering individual thresholds or when outside events (e.g., a media campaign) convinced individuals to adopt before they reached their thresholds. The launching of Sputnik I in October, 1957, for example, is known to have spurred a wave of innovations in science and education in the United States.

Therefore, we need empirical data supplementing the diffusion network data to validate the actors' thresholds, notably, psychological information, relevant social characteristics, or a record of past adoptions. Then we can estimate the most likely adoption time and compare it to the actual adoption time to determine threshold lags and critical mass effects. If threshold lags coincide with external events, it is likely that these events have an impact on the diffusion process. In contrast, if a media campaign does not coincide with the end of relatively many lags, it is probably not very influential.

Table 12. *Fragment of Table 11*

Time	Cum% of Adopters	Cum # of Adopters	Adoption Rate	Acceleration
↓			1	
1958 (1)	2.63	1		3
↓			4	
1959 (2)	13.16	5		6
↓			10	
1960 (3)	39.47	15		2

Anwendungsbeispiel in Pajek-Software

Application

The absolute adoption rates and their acceleration can be calculated from the frequency tabulation of adoption times, discussed in Section 8.2. The absolute growth or adoption rate is just the number of new adopters between two moments (e.g., ten superintendents adopt the modern math method between the end of 1959 and the end of 1960). In Table 12, adoption rates are again placed between the moments because they reflect the change between two measurements. The acceleration of the adoption rate at a particular moment is the difference between the adoption rate directly before and after this moment: subtract two successive adoption rates. In 1959, the acceleration is 6, because ten schools adopted in the year after the end of 1959 and four adopted in the year before. It is easy to spot the moment in which the acceleration starts to decrease while the absolute growth (adoption rate) is still increasing.

Info>Partition

If the critical mass is the first moment at which the most central vertices have adopted, we may simply calculate the betweenness centrality of the vertices and check at which time all or most of the central actors have adopted (*Net>Vector>Centrality>Betweenness*). We advise computing betweenness-centrality in the undirected network, so symmetrize a directed network first (*Net>Transform>Arcs→Edges>All*, avoid multiple lines). List the most central vertices with the *Info>Vector* command by entering a positive number in the dialog box captioned *Highest/lowest or interval of values*. You can check their adoption time in the adoption time partition or in the layered sociogram (see Section 8.1). In our example, the most central superintendents (v8, v13, and v12) are found among the adopters in 1960 or 1959 (v5), so critical mass was reached in 1960. Note, however, that not all central actors have adopted then: the fifth (v36) and sixth (v32) most central superintendents adopted as late as 1963 and 1962.

Net>Vector>Centrality>Betweenness

Net>Transform>Arcs→Edges>All

Info>Vector

For the calculation of threshold lags, it is important to note that a vertex reaches its threshold when all prior adopters in its neighborhood have adopted. In the modern math network, for instance, superintendent v28 reached his threshold of 0.67 in 1960, when superintendent v8 adopted. At the end of 1960, all prior adopters in his neighborhood had adopted (superintendents v5 and v8). The third contact in his friendship network, superintendent v29, is irrelevant to the threshold of v28 because he or

she adopted at the same time as v28. The threshold lag is calculated as the difference between the time of adoption of an actor (his or her class number in the adoption partition) and the maximum adoption time of the neighbors that adopted before him or her. We have to subtract 1 from the threshold lag if we consider exposure at one moment to cause adoption at the next moment. The threshold lag of v28 is equal to 1962–1960–1, which is one year.

threshold lag.mcr

In Pajek, the last contact of a vertex to adopt prior to this vertex is easily found in the directed network that we introduced to calculate thresholds. When all lines point from earlier to later adopters, the neighbor with the highest adoption time on the input side of a vertex is its closest predecessor. However, the computation of thresholds includes some tricks we do not want to discuss here, so we prepared a macro (`Threshold_lag.mcr`) you can use to obtain threshold lags from the original undirected network and adoption time partition. When you play this macro (see Section 8.3), some new networks, partitions, and vectors are created. The last vector contains the threshold lags.

In the modern math network, we find threshold lags only for superintendents v28 and v29 (one-year lag). This small number of lags does not suggest that external events or critical mass influenced the diffusion process. Most vertices have adopted right after one or more of their friends had adopted, which is in line with the simple exposure and threshold model.

8.5 Summary

Innovations and infectious diseases diffuse in a particular manner that is represented by the typical S shape of the diffusion curve. At first, few actors adopt the innovation but the adoption rate accelerates. When 10 to 20 percent of the actors have adopted, the acceleration levels off while the absolute number of new adopters is still increasing, causing a sharp rise of the total number of adopters. Finally, the number of new adopters decreases and the diffusion process slowly reaches its end.

This growth pattern is typical for a chain reaction caused by contagion. Therefore, network models approach diffusion as a contagion process in which personal contacts with adopters expose people to an innovation. They learn about the innovation and their contacts persuade them to adopt. Once exposure reaches their threshold for adopting the innovation, which depends on their personal characteristics and on characteristics of the innovation, they will adopt the innovation and start infecting others. As a consequence, the network structure and the positions of the first adopters in the network, who are usually opinion leaders, influence the rate at which an innovation diffuses. This is a very likely mechanism but it is difficult to prove that diffusion actually works this way.

At a particular moment in time, a successful diffusion process is hypothesized to reach a critical mass, which means that the diffusion process can sustain or even accelerate itself without help from outside (e.g., media campaigns). Even with hindsight it is not easy to pinpoint the moment when critical mass is reached, but according to an empirical rule of thumb this happens when the innovation has spread to 10 to 20 percent of the actors who adopt eventually. This is the first second-order inflection point of the S-shaped diffusion curve: the moment when the adoption rate no longer accelerates although it is still increasing. Alternatively, the critical mass may be placed at the moment when the most central actors have adopted or when relatively many actors adopt although their exposure is not increasing. In the latter case, the critical mass or external events are thought to lower individual thresholds.

We are not sure whether critical mass occurs and whether it has the hypothesized impact on the diffusion process. Ongoing research in the diffusion of innovations tradition must clarify this matter. Nevertheless, the concept offers some practical tools for monitoring and guiding a diffusion process.

In theory, knowledge about other people's adoption without personal contact may count as exposure too, especially in the case of status similarities. Knowing that people with similar network positions have adopted although you are not directly linked to them may persuade you to also adopt. In this chapter, we have presented contagion by contact only and not by status imitation. Structural approaches to status and roles will be introduced in Part V of this book.