ESERCIZIO 1

$F = (10, 3, 7, 6)$, troncamento

a) $x = 2021$, calcolare $\tilde{x} := fl(x)$.

$$x = 2021 = +10^4(0.2021)$$
$$\Rightarrow \quad \tilde{x} = 10^4(0.202)$$

b) Si determinino tutti i numeri $k \in \mathbb{R} \mid fl(k) = \tilde{x}$.

$$\tilde{x} = 10^4(0.202)$$
$$\Rightarrow \{\text{def. di troncamento}\}$$
$$k \in [2020, 2030[$$

c) u? Determinare che numeri $z \in F \mid \frac{|z - x|}{|x|} < u$.

$$u = \beta^{1-t} = 10^{1-3} = 10^{-2}$$
$$\frac{|z - x|}{|x|} < 10^{-2}$$

$$\Rightarrow |z - x| < 10^2(0.2021)$$
$$|z - x| < 20.21$$

e.g. $z = 10^4(0.201) \vee z = 10^4(0.203)$

ESERCIZIO 2

$F = (10, 3, m, M)$, troncamento

a) Hp. $x_1, x_2 \in \mathbb{R}$, $x_1 \in \,]0; x_2[$

Th. $fl(x_1) \leq fl(x_2)$

{Definizione di fl}

$fl(x_1) = \tilde{x}_1 = 10^{p_1}\left(\sum_{i=1}^{t} d_{i_1} \beta^{-i}\right)$,

la definizione di x_2 e' analoga;

{Hp: $x_1 \in \,]0; x_2[$ }

$10^{p_1}\left(\sum_{i=1}^{t} d_{i_1} \beta^{-i}\right) \in \left]0; 10^{p_2}\left(\sum_{i=1}^{t} d_{i_2} \beta^{-i}\right)\right[$

$\Rightarrow$ Th.

Q.E.D.

b) Dimostrare con un esempio che

$fl(x_1) \leq fl(x_2) \not\Rightarrow x_1 \leq x_2$

Si considera $x_1 = 2028$, $x_2 = 2021$

$fl(x_1) = fl(x_2) = 2020 \wedge$

$x_1 > x_2$

$\Rightarrow$ Th.

Q.E.D.

ESERCIZIO 3

$F = (10, 3, 3, 3)$, troncamento

a) Trovare $a, b \in F \mid (a \oplus b) \oslash 2 = a$.

$a = \tilde{a} = 10(0.100)$,

$b = \tilde{b} = a$

$\Rightarrow (a \oplus b) = 10(0.100) \oplus 10(0.100)$

$= fl(1 + 1) = fl(2) = 10(0.200)$

$\Rightarrow (a \oplus b) \oslash 2 = 10(0.200) \oslash 2 =$

$fl(10(0.200)/2) = 10(0.100)$.

b) Trovare $a, b \in F \mid (a \oplus b) \oslash < \min\{a, b\}$

$a = \omega = 10^{-3-1} = 10^{-4}$,

$b = 0$

$\Rightarrow (a \oplus b) = 10^{-4} \oplus 0 = 10^{-4}$

$\Rightarrow (a \oplus b) \oslash 2 = 10^{-4} \oslash 2 =$

$fl(10^{-4}/2) = 0$.

ESERCIZIO 4

$F = \mathcal{F}(\beta, t, m, M)$

a) ω ? Ω ?

$$\omega = \beta^{-m-1} \quad , \quad \Omega = \beta^{M}(1-\beta^{-t})$$

b) $\frac{1}{\omega} \in F$? $\frac{1}{\Omega} \in F$?

$$\frac{1}{\omega} = \frac{1}{\beta^{-m-1}} = \beta^{m+1}$$

$\Rightarrow \frac{1}{\omega} \in F$ sse

$$\beta^{m+1} < \beta^{M}(1-\beta^{-t})$$

$$\frac{1}{\Omega} = \frac{1}{\beta^{M}(1-\beta^{-t})} = \beta^{-M}(1-\beta^{-t})$$

$\Rightarrow \frac{1}{\Omega} \in F$ sse

$$\beta^{-M}(1-\beta^{-t}) > \beta^{-m-1}$$

c) Esaminare il caso $\beta = 2,\ t = 8,\ m = M = 6$

$$\Rightarrow \omega = 2^{-7}, \quad \Omega = 2^{6}(1-2^{-8})$$

$$\Rightarrow \frac{1}{\omega} \notin F, \quad \frac{1}{\Omega} \in F.$$

ESERCIZIO 5

Determinare il massimo $N \in \mathbb{Z} \mid (\forall x \in \mathbb{Z},\ x \in [-N;N].$
$x = fl(x) \wedge x \in F(2,t,m,M))$.

Quanto vale N in aritmetica IEEE in doppia precisione?

ESERCIZIO 6

$$f = (x_1, \dots, x_n) = \sum_{i=1}^{n} x_i^2$$

a) Studiare il condizionamento di $f(x)$.

$$\varepsilon_{in} = \frac{f(\tilde{x}) - f(x)}{f(x)} \quad \text{con } x = (x_1, \dots, x_n)$$

$$f(\tilde{x}) = fl\left(\sum_{i=1}^{n}\left((\tilde{x}_i) \otimes (\tilde{x}_i)\right)\right) =$$

$$= fl\left(\sum_{i=1}^{n}\left((x_i(1+\varepsilon_{x_i})) \otimes (x_i(1+\varepsilon_{x_i}))\right)\right) =$$

$$= fl\left(\sum_{i=1}^{n}\left((x_i + x_i\varepsilon_{x_i}) \otimes (x_i + x_i\varepsilon_{x_i})\right)\right) =$$

$$\doteq fl\left(\sum_{i=1}^{n}\left(x_i^2 + 2x_i^2\varepsilon_{x_i}\right)\right) =$$

DA FINIRE !

b) Si fornisca un algoritmo per il calcolo di $f(x)$ e se ne studi la stabilità.

DA FARE!

ESERCIZIO 7

$$\frac{1-x^4}{1-x} = 1+x+x^2+x^3$$

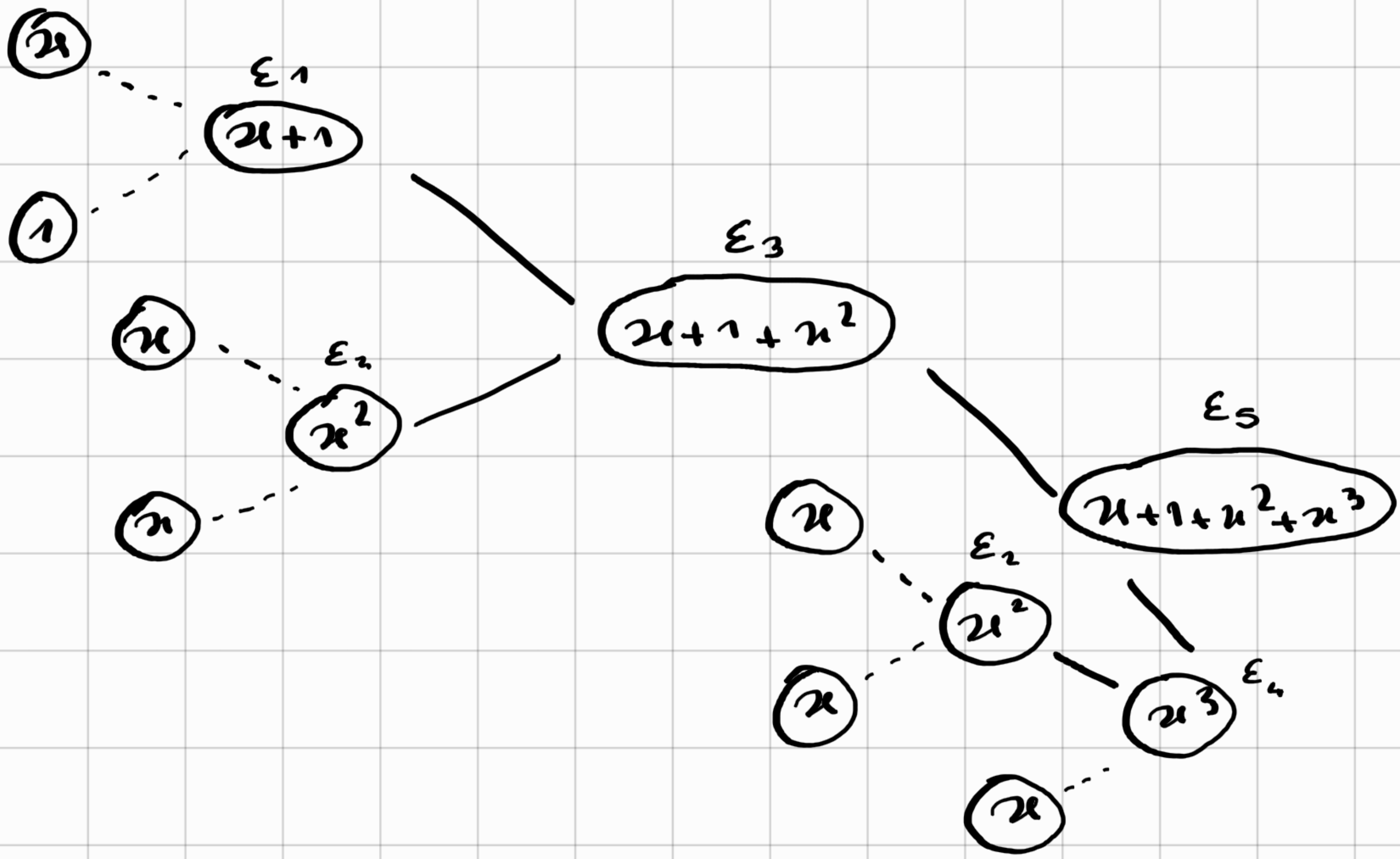
Quale dei due algoritmi è più stabile?

$$g_1(x) = 1 \oplus x + (x \otimes x) + (x \otimes x \otimes x)$$

$$z^1 = 1 \oplus x$$

$$z^2 = z^1 + x \otimes x$$

$$z^3 = z^2 + x \otimes x \otimes x$$



DA FINIRE!