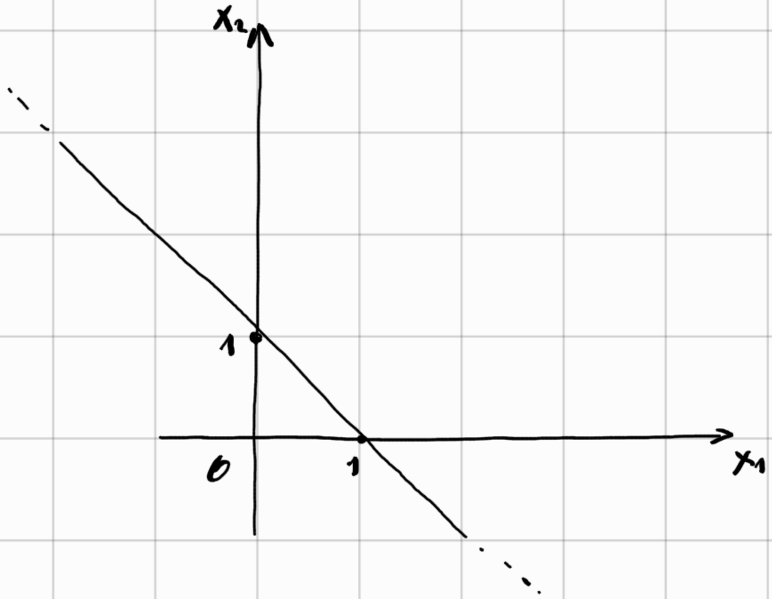
PIANI SEPARATORI nel PIANO 2D

a) i. $\mathbf{w}^T \mathbf{x} = w_1 x_1 + w_2 x_2 + w_0 = 0$

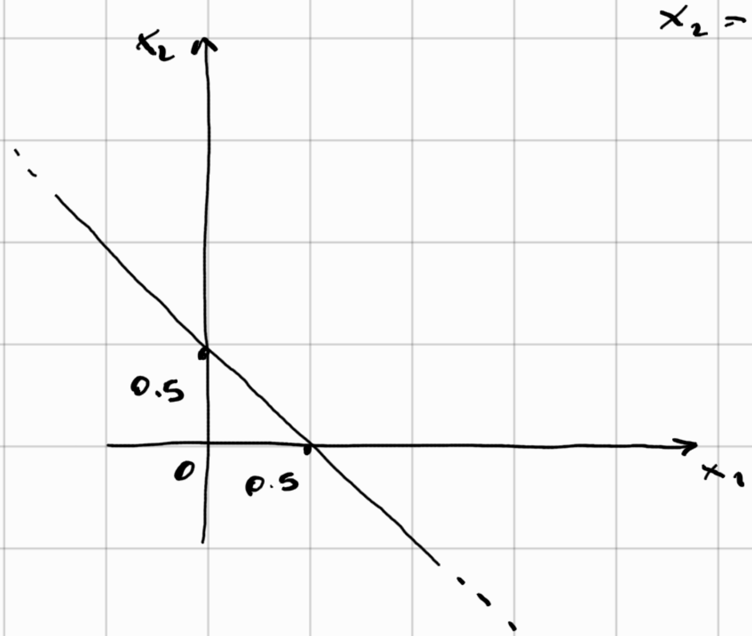
$$x_2 = \frac{-w_1 x_1 + w_0}{w_2}$$

- $x_1 + x_2 - 1 = 0 \Rightarrow w_1 = 1, \quad w_2 = 1, \quad w_0 = -1$



- $x_1 + x_2 - 0.5 = 0 \Rightarrow w_1 = 1, \quad w_2 = 1, \quad w_0 = -0.5$

$x_2 = 0.5 - x_1$



ii. Il decision boundary gode di free scaling: non cambia se moltiplicato per una costante

b) $x_1 \wedge x_2 \wedge x_4 \Leftrightarrow y$ con modello lineare

$1x_1 + 1x_2 + 1x_4 \geq 2.5$

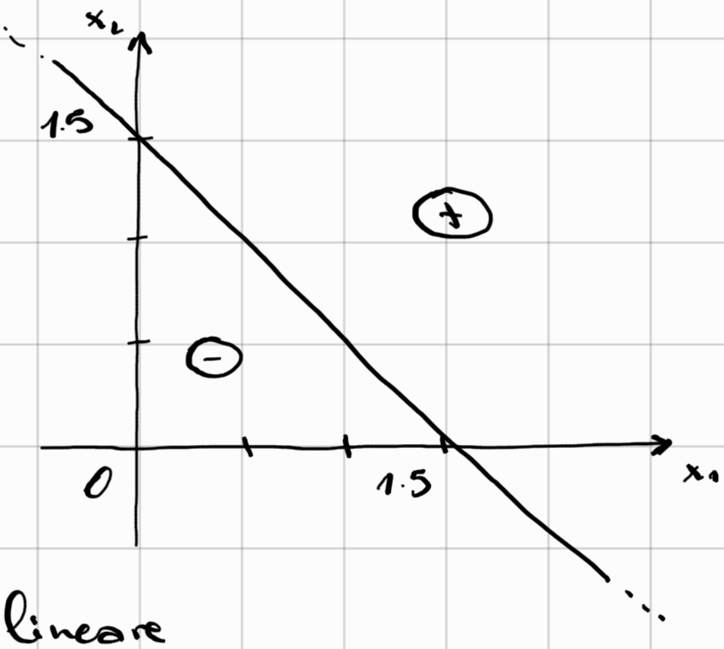
$$h(x) = \begin{cases} 1 & \text{se } wx \geq 0 \\ 0 & \text{alt} \end{cases}$$

c) $x_1 \wedge x_2 \Leftrightarrow y$ con modello lineare

$$h(x) = \begin{cases} 1 & \text{se } wx \geq 0 \\ 0 & \text{alt} \end{cases}$$

$1x_1 + 1x_2 \geq 1.5$

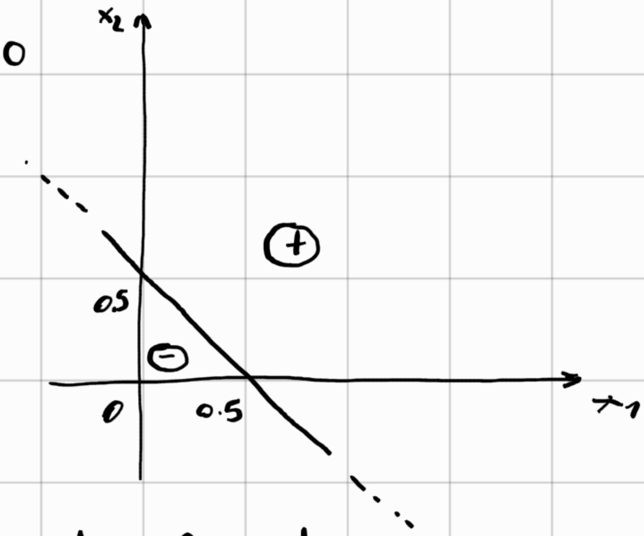
/// : ok



d) $x_1 \vee x_2$ con modello lineare

$$h(x) = \begin{cases} 1 & \text{se } wx \geq 0 \\ 0 & \text{alt} \end{cases}$$

$1x_1 + 1x_2 \geq 0.5$



e) Il decision boundary resta lo stesso, il segno dei due semi-iperpiani individuati viene invertito

MODELLO LINEARE ESTREMAMENTE RIDOTTO

a) $h(x) = w_0$

b) Trovare w_0 con un parametro di training:

y_i = valore di target

$$\frac{\delta}{\delta w_0} E(w) = \frac{\delta}{\delta w_0} (y - h_w(x))^2 =$$

$$= \frac{\delta}{\delta w_0} y - h_w(x) \cdot 2(y - h_w(x)) =$$

$$= -2(y - w_0) = -2y + w_0$$

Pongo $\frac{\delta}{\delta w_0} E(w) = 0$: $-2y + w_0 = 0 \iff w_0 = y$.

c) Trovare w_0 con ℓ parametri di training.

$$\frac{\delta}{\delta w_0} E(w) = \sum_{i=1}^{\ell} \frac{\delta}{\delta w_0} (y_i - w_0)^2 = \sum_{i=1}^{\ell} \frac{\delta}{\delta w_0} (y_i^2 - 2y_i w_0 + w_0^2) =$$

$$= \sum_{i=1}^{\ell} 2(w_0 - y_i) = \sum_{i=1}^{\ell} -2y_i + 2\ell w_0$$

Pongo $\frac{\delta}{\delta w_0} E(w) = 0$: $2\ell w_0 = \sum_{i=1}^{\ell} +2y_i$

$$w_0 = \bar{y}.$$

d) Il risultato e' la media aritmetica dei valori di training.

e) $h(w) = \text{sign}(w_0) = \text{sign}(\bar{y})$

DEFINIRE UN TASK

a) $u, t \in \mathbb{R}$, continue

b) output $= \begin{cases} 1 & \text{se piacevole,} \\ 0 & \text{alt.} \end{cases}$

$y :=$ target, $\quad y = \mathbb{R} \times \mathbb{R} \to$ Boolean

c) $(\mathbf{x}_p, y_p)$ coppie con $\mathbf{x}_p := (u_p, t_p)$ con $p \in [1; l] \cap \mathbb{N}$

d) supervisionato, Concept Learning

e) $h(x) = \text{sign}(\mathbf{w}^T \mathbf{x}) = \text{sign}(w_0 + w_1 u + w_2 t)$

f) apprendimento: trovare $h(x)$ che minimizzi l'errore su tutto l'insieme dei dati

training: minimizzare la funzione d'errore scegliendo opportunamente: parametri w_1, w_2, w_0.

FIND-S e CANDIDATE ELIMINATION

x_1	x_2	x_3	x_4	y
0	0	1	0	0
0	1	0	0	0
0	0	1	1	1
1	0	0	1	1
0	1	1	0	0
1	1	0	0	0
0	1	0	1	0

Find-S: $h_0 = \langle \emptyset, \emptyset, \emptyset, \emptyset \rangle$

$h_1 = h_0$

$h_2 = h_0$

$h_3 = h_0$

$h_4 = \langle 0, 0, 1, 1 \rangle$

$h_5 = \langle ?, 0, ?, 1 \rangle$

$h_6 = h_5$

$h_7 = h_5$

$h(x) = \neg x_2 \wedge x_4.$

Candidate-elimination:

$S_0 = \langle \emptyset, \emptyset, \emptyset, \emptyset \rangle$, $G_0 = \langle ?, ?, ?, ? \rangle$

$S_1 = S_0$, $G_1 = \text{not} \langle 0, 0, 1, 0 \rangle$

$S_2 = S_0$, $G_2 = \{ \langle 1, ?, ?, ? \rangle, \langle ?, ?, ?, 1 \rangle \}$

$S_3 = S_0$, $G_3 = \langle ?, ?, ?, 1 \rangle$

$S_4 = h_4$ $G_4 = G_3$

$S_5 = h_5$, $G_5 = G_3$

$S_6 = S_5$, $G_6 = G_3$

$S_7 = G_7$

$h(x) = \neg x_2 \wedge x_4.$