3MARTLE – 162

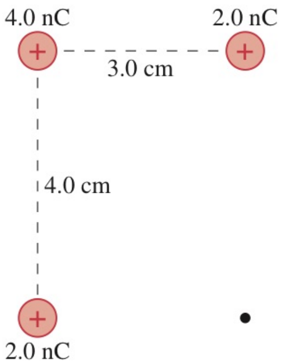


FIGURE EX28.24

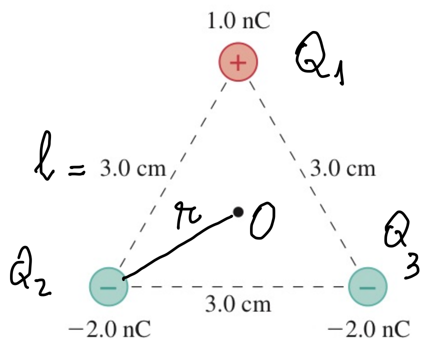


FIGURE EX28.25

25. | What is the electric potential at the point indicated with the dot in FIGURE EX28.25?

pag. 834

$$r = \frac{l\sqrt{3}}{2} \cdot \frac{2}{3} = \frac{l\sqrt{3}}{3}$$

$$V_0 = \frac{kQ_1}{r} + \frac{kQ_2}{r} + \frac{kQ_3}{r} = \frac{k}{r}(Q_1+Q_2+Q_3)$$

$$= \frac{k}{\frac{l\sqrt{3}}{3}}(Q_1+Q_2+Q_3) = \frac{k\,3}{l\sqrt{3}}(Q_1+Q_2+Q_3) =$$

$$= \frac{k\sqrt{3}}{l}(Q_1+Q_2+Q_3) = \frac{9\cdot10^{9}\cdot\sqrt{3}}{3\cdot10^{-2}}(-3\cdot10^{-9})\,V$$

$$= -9\sqrt{3}\cdot10^{2}\,V \simeq -9\cdot1{,}73\cdot10^{2}\,V$$

$$= -15{,}57\cdot10^{2}\,V = -1557\,V.$$

27. ‖ A −2.0 nC charge and a +2.0 nC charge are located on the x-axis at $x = -1.0$ cm and $x = +1.0$ cm, respectively.

a. Other than at infinity, is there a position or positions on the x-axis where the electric field is zero? If so, where?
b. Other than at infinity, at what position or positions on the x-axis is the electric potential zero?
c. Sketch graphs of the electric field strength and the electric potential along the x-axis.

pag. 834.

$Q = 2nC$

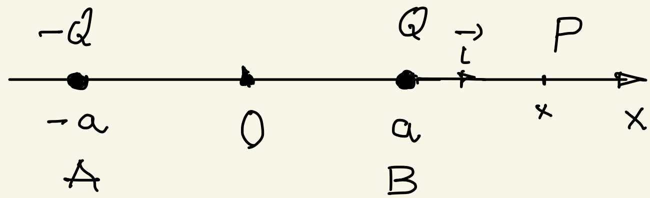
$a = 1cm$

a) $E = 0 \quad x = ?$

b) $V = 0 \quad x = ?$

c) Grafic $E(x), V(x)$

a)



$$\vec{E}_P = \vec{E}_{-QP} + \vec{E}_{QP} =$$

$$= \frac{k(-Q)}{|\vec{AP}|^3} \cdot \vec{AP} + \frac{kQ}{|\vec{BP}|^3} \cdot \vec{BP}$$

$$\vec{AP}\,(x+a, 0, 0) \Rightarrow \vec{AP} = (x+a)\,\vec{i}$$

$$|\vec{AP}| = \sqrt{\text{aoleu}} = \sqrt{(x+a)^2} =$$

$$= |x+a|$$

$$\overrightarrow{BP}\,(x-a,0,0) \Rightarrow \overrightarrow{BP} = (x-a)\,\vec{i}$$

$$|\overrightarrow{BP}| = |x-a|$$

$$\vec{E}_P = \frac{k(-Q)}{|x+a|^3}(x+a)\,\vec{i} + \frac{kQ}{|x-a|^3}\cdot(x-a)\,\vec{i}$$

$$= kQ\left[-\frac{x+a}{|x+a|^3} + \frac{x-a}{|x-a|^3}\right]\vec{i}$$

$$E_{P,x} = kQ\left[-\frac{x+a}{|x+a|^3} + \frac{x-a}{|x-a|^3}\right] = 0$$

$$\Rightarrow -\frac{x+a}{|x+a|^3} + \frac{x-a}{|x-a|^3} = 0$$

$$-\infty \qquad -a \qquad a \qquad \infty$$

1) $\underline{x \in (-\infty, -a)}$

$$-\frac{x+a}{\left[-(x+a)\right]^3} + \frac{x-a}{(a-x)^3} = 0$$

$$\frac{1}{(x+a)^2} - \frac{1}{(a-x)^2} = 0 \Rightarrow \frac{1}{(x+a)^2} = \frac{1}{(a-x)^2}$$

$$(x+a)^2 = (a-x)^2$$

$$\cancel{x^2} + 2ax + \cancel{a^2} = \cancel{x^2} + \cancel{a^2} - 2ax$$

$$2ax = -2ax \Rightarrow x = 0 \notin (-\infty, -a) \Rightarrow$$

$\Rightarrow E_{P,x}$ nu se anulează pe $(-\infty, -a)$

2) $\underline{x \in (-a, a)}$

$$-\frac{x+a}{|x+a|^3} + \frac{x-a}{|x-a|^3}$$

$$-\frac{x+a}{(x+a)^3} + \frac{x-a}{(a-x)^3} = 0$$

$$-\frac{1}{(x+a)^2} - \frac{1}{(a-x)^2} = 0$$ $\Rightarrow$ nu are soluție $\Rightarrow$

$\Rightarrow E_{P,x}$ nu se anulează

3) $\underline{x \in (a, \infty)}$

$$-\frac{x+a}{(x+a)^3} + \frac{x-a}{(x-a)^3} = 0$$

$$-\frac{1}{(x+a)^2} + \frac{1}{(x-a)^2} = 0$$

$$\frac{1}{(x+a)^2} = \frac{1}{(x-a)^2} \Rightarrow (x+a)^2 = (x-a)^2$$

$$x^2 + a^2 + 2ax = x^2 + a^2 - 2ax$$

$$2ax = -2ax \Rightarrow x = 0 \notin (-a, \infty)$$

$\Rightarrow E_{P,x}$ nu se anulează.

b) $$V_P = \frac{k(-Q)}{|x+a|} + \frac{kQ}{|x-a|}$$

$$V_P = \frac{kQ}{r}$$

$$V_P(x) = kQ\left[-\frac{1}{|x+a|} + \frac{1}{|x-a|}\right] = 0$$

$$\Rightarrow -\frac{1}{|x+a|} + \frac{1}{|x-a|} = 0$$

$$\frac{1}{|x+a|} = \frac{1}{|x-a|}$$

$$|x+a| = |x-a|$$

I) $\underline{x \in (-\infty, -a)}$

$$-x - a = -x + a \Rightarrow -a = a \ (F) \Rightarrow$$

$\Rightarrow V_P(x)$ nu se anulează

2) $x \in (-a, a)$

$$x + a = a - x \Rightarrow 2x = 0 \Rightarrow x = 0 \in (-a, a)$$

Am găsit o soluție : $x = 0 \longrightarrow V_P(0) = 0.$

3) $x \in (a, \infty)$

$x + a = x - a \Rightarrow 2a = 0$ (F) $\Rightarrow V_P(x)$ nu se anulează.

c) Grafice

Graficul lui $E_{P,x}(x)$

$$E_{P,x} = kQ\left[-\frac{x+a}{|x+a|^3} + \frac{x-a}{|x-a|^3} \right]$$

$$x \in (-\infty, -a) \Rightarrow E_{P,x} = kQ\left[-\frac{x+a}{[-(a+x)]^3} + \frac{x-a}{(a-x)^3} \right]$$

$$= kQ\left[\frac{1}{(x+a)^2} - \frac{1}{(a-x)^2} \right] =$$

$$= kQ \frac{(a-x)^2 - (x+a)^2}{(x+a)^2(a-x)^2} =$$

$$= kQ \frac{-4ax}{(x+a)^2(a-x)^2} = -4kaQ \frac{x}{(x+a)^2(a-x)^2}$$

$\underline{x \in (-a, a)} \Rightarrow E_{P,x} = kQ \cdot \left[-\frac{x+a}{(x+a)^3} + \frac{x-a}{(a-x)^3} \right]$

$$= kQ \left[-\frac{1}{(x+a)^2} - \frac{1}{(a-x)^2} \right] = -kQ \frac{2(x^2+a^2)}{(x+a)^2(a-x)^2}$$

$\underline{x \in (a, \infty)} \Rightarrow E_{P,x} = kQ \left[-\frac{x+a}{(x+a)^3} + \frac{x-a}{(x-a)^3} \right]$

$$= kQ \left[-\frac{1}{(x+a)^2} + \frac{1}{(x-a)^2} \right] =$$

$$= kQ \left[\frac{1}{(x-a)^2} - \frac{1}{(x+a)^2} \right] =$$

$$= kQ \frac{4ax}{(x-a)^2(x+a)^2} = 4kaQ \frac{x}{(x-a)^2(x+a)^2}$$

$$E_{P,x}(x) = \begin{cases} -4kaQ \dfrac{x}{(x+a)^2(a-x)^2}, & x \in (-\infty, -a) \\[2ex] -kQ \dfrac{2(x^2+a^2)}{(x+a)^2(a-x)^2}, & x \in (-a, a) \\[2ex] 4kaQ \dfrac{x}{(x-a)^2(x+a)^2}, & x \in (a, \infty) \end{cases}$$

$$\lim_{x\to-\infty} E_{P,x}(x) = 0 = \lim_{x\to+\infty} E_{P,x}(x)$$

$$\lim_{x\nearrow -a} E_{P,x}(x) = \lim_{x\nearrow -a}\left(-4kaQ\frac{x}{(x+a)^2(a-x)^2}\right) =$$

$$= -4kaQ \lim_{x\nearrow -a}\frac{x}{(x+a)^2(a-x)^2}$$

$$= -4kaQ\frac{(-a)}{0_+\cdot 4a^2} = +\infty$$

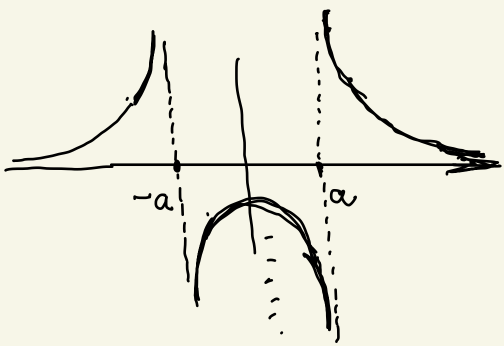
$$\lim_{x\searrow -a} E_{P,x}(x) = \lim_{x\searrow -a}\left(-kQ\frac{2(x^2+a^2)}{(x+a)^2(a-x)^2}\right) =$$

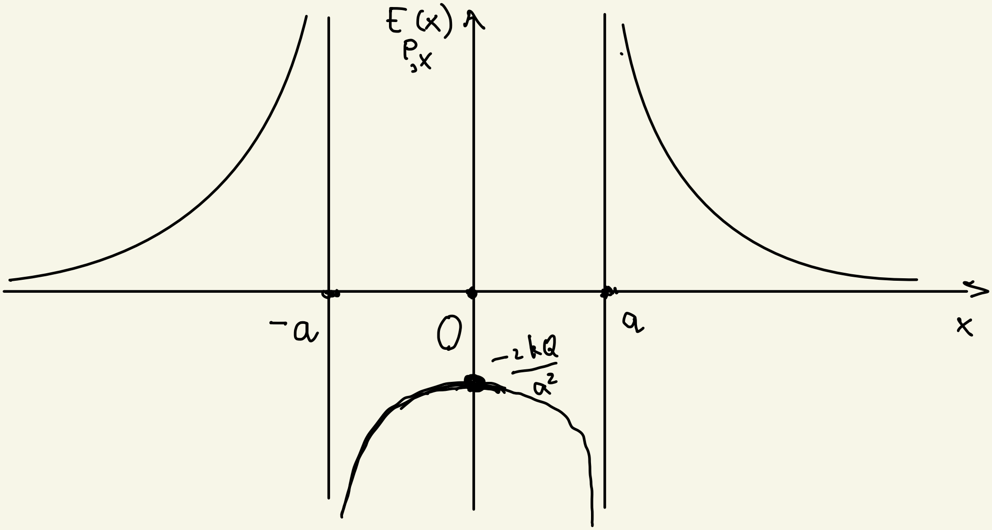
$$= -2kQ\lim_{x\searrow -a}\frac{x^2+a^2}{(x+a)^2(a-x)^2} =$$

$$= -2kQ\frac{2a^2}{0_+\cdot 4a^2} = -\infty$$

$$\lim_{x\nearrow a} E_{P,x}(x) = -\infty$$

$$\lim_{x\searrow a} E_{P,x}(x) = \infty$$





$$E_{p,x}(0) = -kQ \left.\frac{2(x^2+a^2)}{(x+a)^2(a-x)^2}\right|_{x=0} = -kQ\frac{2a^2}{a^2\cdot a^2}$$

$$E_{p,x}(0) = -\frac{2kQ}{a^2} < 0 \qquad -\frac{2\cdot 9\cdot 10^{9}\cdot 2\cdot 10^{-9}}{10^{-4}} = -36\cdot 10^{+4}$$

$$= -360000$$

$$= -3{,}6\cdot 10^{5}$$

<u>Graficul potențialului</u>

$$V_p(x) = \frac{k(-Q)}{|x+a|} + \frac{kQ}{|x-a|} = kQ\left(-\frac{1}{|x+a|} + \frac{1}{|x-a|}\right)$$

$$\boxed{V_p(x) = kQ\left(-\frac{1}{|x+a|} + \frac{1}{|x-a|}\right)}$$

$$x \in (-\infty, -a) \Rightarrow V_p(x) = kQ\left[-\frac{1}{-(x+a)} + \frac{1}{a-x}\right] =$$

$$= kQ\left(\frac{1}{x+a} - \frac{1}{x-a}\right) = kQ\,\frac{-2a}{(x+a)(x-a)}$$

$$= -2kQa\,\frac{1}{(x+a)(x-a)}$$

$$x \in (-a, a) \Rightarrow V_p(x) = kQ\left[-\frac{1}{x+a} + \frac{1}{a-x}\right] =$$

$$= kQ\left(-\frac{1}{x+a} - \frac{1}{x-a}\right) =$$

$$= -kQ\left(\frac{1}{x+a} + \frac{1}{x-a}\right) = -kQ\,\frac{2x}{(x+a)(x-a)} =$$

$$= -2kQ\,\frac{x}{(x+a)(x-a)}$$

$$x \in (a, \infty) \Rightarrow V_p(x) = kQ\left[-\frac{1}{x+a} + \frac{1}{x-a}\right] =$$

$$= kQ\left(\frac{1}{x-a} - \frac{1}{x+a}\right) = kQ\,\frac{2a}{(x-a)(x+a)}$$

$$= 2kQa\,\frac{1}{(x-a)(x+a)}$$

$$V_p(x) = \begin{cases} -2kQa \dfrac{1}{(x+a)(x-a)}, & x \in (-\infty, -a) \\ -2kQ \dfrac{x}{(x+a)(x-a)}, & x \in (-a, a) \\ 2kQa \dfrac{1}{(x-a)(x+a)}, & x \in (a, \infty) \end{cases}$$

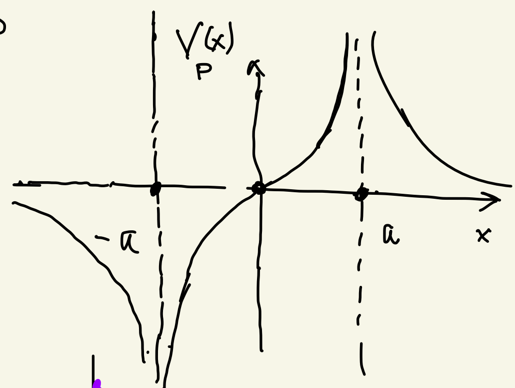
$$\lim_{x \to -\infty} V_p(x) = 0 = \lim_{x \to \infty} V_p(x)$$

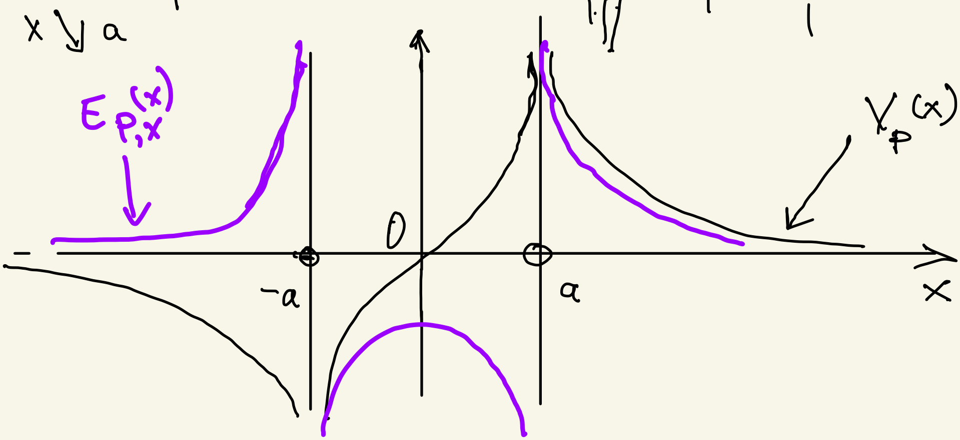
$$\lim_{x \nearrow -a} V_p(x) = \lim_{x \nearrow -a} \left(-2kQa \frac{1}{(x+a)(x-a)} \right) = -\infty$$

$$\lim_{x \searrow -a} V_p(x) = -\infty$$

$$\lim_{x \nearrow a} V_p(x) = +\infty$$

$$\lim_{x \searrow a} V_p(x) = +\infty$$





$$\vec{E}_p = -\operatorname{grad} V_p$$

$$Ox: \quad E_{p,x} = -\frac{\partial V_p}{\partial x} = -\frac{dV_p}{dx}.$$

$$\boxed{E_{p,x}(x) = -\frac{dV_p}{dx}}$$

$$kQ\left(-\frac{(x+a)}{|x+a|^3}+\frac{(x-a)}{|x-a|^3}\right)$$

$a = 7.1$

0.1 500

$k = 9\cdot 10^9$

$k = 9\times 10^9$

$Q = 2\cdot 10^{-9}$

$Q = 2\times 10^{-9}$

$$f(x) = kQ\left(-\frac{1}{|x+a|}+\frac{1}{|x-a|}\right)$$

$$-\frac{d}{dx}f(x)$$