CTI 162

LICU MIHAI-GEORGE

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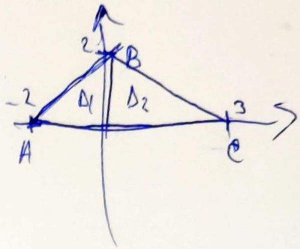
Lucrare scrisă
Matematici Speciale

1) $\int_{\gamma} (x+1)\,dx + (x^3+2)\,dy$

$A(-2,0)$
$B(0,2)$
$C(3,0)$



$= \iint_{D} (3x^2 - 0)\,dx\,dy =$

$\omega : \begin{cases} AB : y = x+2 \\ BC : y = -\frac{2}{3}x + 2 \\ AC : y = 0 \end{cases}$

$= \int_{-2}^{0} \int_{0}^{x+2} 3x^2\,dy\,dx + \int_{0}^{3} \int_{0}^{-\frac{2}{3}x+2} 3x^2\,dy\,dx =$

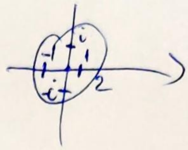
$= \int_{-2}^{0} 3x^2 y \Big|_{0}^{x+2} dx + \int_{0}^{3} 3x^2 y \Big|_{0}^{-\frac{2}{3}x+2} dx =$

$= \int_{-2}^{0} 3x^3 + 6x^2\,dx + \int_{0}^{3} -2x^3 + 6x^2\,dx = \left(\frac{3x^4}{4} + 2x^3\right)\Big|_{-2}^{0} + \left(-\frac{x^4}{2} + 2x^3\right)\Big|_{0}^{3} =$

$= 0 + 0 - \left(\frac{3 \cdot (16)}{4} - 16\right) + \left(-\frac{81}{2} + 54\right) - (0+0) = -(12-16) + \left(-\frac{81}{2} + \frac{108}{2}\right) =$

$= 4 + \frac{27}{2} = \frac{35}{2}$

2) $$\int\limits_{|z|=2} \frac{z+\cos z}{z^4-1}\,dz =$$



$$z^4-1=0 \Rightarrow z_1=1$$
$$z_2=-1$$
$$z_3=i$$
$$z_4=-i$$

$$\int\limits_{|z|=2} \frac{z+\cos z}{z^4-1}\,dz = \int\limits_{|z|=2}\frac{z}{z^4-1}\,dz + \int\limits_{|z|=2}\frac{\cos z}{z^4-1}\,dz$$

$$\int\limits_{|z|=2}\frac{z}{z^4-1}\,dz = 2\pi i\left(\sum_{i=1}^{4}\mathrm{Rez}(f,z_i)\right) = 2\pi i\left(\frac{1}{0!}\cdot\lim_{z\to 1}\left((z-1)\cdot\frac{z}{z^4-1}\right) + \lim_{z\to -1}\left((z+1)\cdot\frac{z}{z^4-1}\right) +\right.$$

$$\left. + \lim_{z\to i}\left((z-i)\frac{z}{z^4-1}\right) + \lim_{z\to -i}\left((z+i)\frac{z}{z^4-1}\right)\right) = 2\pi i\left(\frac{1}{(1+1)(1+i)(1-i)} + \frac{-1}{(-1-1)(-1+i)(-1-i)} + \frac{i}{(i+1)(i-1)(i+i)}\right.$$

$$\left. + \frac{-i}{(-i+1)(-i-1)(-i-i)}\right) =$$

$$= 2\pi i\left(\frac{1}{2\cdot 2} + \frac{-1}{-4} + \frac{i}{-4i} + \frac{-i}{4i}\right) = 2\pi i\left(\frac14+\frac14-\frac14-\frac14\right) = 0$$

$$\int\limits_{|z|=2}\frac{\cos z}{z^4-1}\,dz = 2\pi i\left(\frac{\cos 1}{2\cdot 2} + \frac{\cos(-1)}{-4} + \frac{\cos i}{-4i} + \frac{\cos(-i)}{4i}\right) =$$

$$= 2\pi i\left(\frac{\cos(1)-\cos(1)}{4} + \frac{\cos(i)-\cos(i)}{4i}\right) = 0$$

$$\Rightarrow \int\limits_{|z|=2}\frac{z+\cos z}{z^4-1}\,dz = 0+0=0$$

3) $y'' - 5y' + 4y = e^{-t}$ $\quad y(0)=0 \quad y'(0)=1$

$$\mathcal{L}[y''](p) - 5\,\mathcal{L}[y'](p) + 4\,\mathcal{L}[y](p) = \mathcal{L}[e^{-t}](p)$$

$$Y(p) = \mathcal{L}[y](p)$$

$$\mathcal{L}[y'](p) = p\,Y(p) - 0 = p \cdot Y(p)$$

$$\mathcal{L}[y''](p) = p \cdot \mathcal{L}[y'](p) + 1 = p^2 Y(p) + 1$$

$$p^2 Y(p) + 1 - 5\left(p\,Y(p)\right) + 4Y(p) = \frac{1}{p+1}$$