Bogriu
Alexandra-Adriana
grupa 234
i=84

Examen

1) $(0,84] \cap \mathbb{N}$

a) : 3

$P = \frac{28}{84} = \frac{14}{42} = \frac{7}{21} = \frac{1}{3}$

$3 \cdot 1, 3 \cdot 2, 3 \cdot 3, \dots 3 \cdot 28$
$\Rightarrow$ 28 nr. : 3

b) pp

$P = \frac{9}{84} = \frac{3}{28}$

$1^2, 2^2, 3^2, 4^2, 5^2, 6^2, 7^2, 8^2, 9^2$
$\Rightarrow$ 9 nr pp.

c) prime

$P = \frac{23}{84}$

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83
23 nr prime

2) $X \sim \begin{pmatrix} -1 & 0 & 1 \\ \frac{84}{1000} & \frac{1}{100} & c \end{pmatrix}$

a) $\frac{84}{1000} + \frac{1}{100} + c = 1$

$\frac{84}{1000} + \frac{10}{1000} + \frac{1000c}{1000} = 1$

$1000c + 94 = 1000$
$1000c = 1000 - 94 = 906$
$c = \frac{906}{1000} = 0.906$

b) $Var(x) = E(x^2) - [E(x)]^2$

$E(x) = (-1) \cdot \frac{84}{1000} + 0 \cdot \frac{1}{100} + 1 \cdot 0.906 = -\frac{84}{1000} + 0.906 = -0.084 + 0.906 = 0.822$

$[E(x)]^2 = (0.822)^2 = 0.675684$

$E(x^2) = (-1)^2 \cdot \frac{84}{1000} + 1^2 \cdot 0.906 = 0.084 + 0.906 = 0.99$

$Var(x) = 0.99 - 0.675684 = 0.314316$

c) $Var(2x) = E(4x^2) - [E(2x)]^2 = 4E(x^2) - 4(E(x))^2 = 4(E(x^2) -$

$-(E(X))^2] = 4 \cdot Var_2(X) = 4 \cdot 0.314316 = 1.257264$

4) 200 monede

84 tipul A, prob. avers 0.5 $\quad P(D_H|A) = 0.5$

200-84 = 116 tipul B, prob. avers 0.8 $\quad P(D_H|B) = 0.8$

$P(A) = \frac{84}{200} = 0.42$

$P(B) = \frac{116}{200} = 0.58$

Prob. predictivă a priori:

$P(D_H) = P(D_H|A) \cdot P(A) + P(D_H|B) \cdot P(B) = 0.5 \cdot 0.42 + 0.8 \cdot 0.58$

$= 0.21 + 0.464 = 0.674$

Prob. predictivă a posteriori:

$P(D_H|D) = P(D_H|A) \cdot P(A|D) \to P(D_H|B) \cdot P(B|D)$

Tabelul Bayes

H	P(H)	P(D|H)	P(D|H) · P(H)	P(H|D)
A	0.42	0.5	0.21	$\frac{0.21}{0.674} \simeq 0.311$
B	0.58	0.8	0.464	$\frac{0.464}{0.674} \simeq 0.688$
Total	1		0.674	1

$P(D_H|D) = 0.5 \cdot 0.311 + 0.8 \cdot 0.688 = 0.1555 + 0.5504 = 0.7059$

3) $[0, 84]$ pdf comună $f(x) = cxy$

a) $f(x,y) \geq 0$

$$\int_0^{84} \int_0^{84} f(x,y)\, dy\, dx = 1$$

$$\int_0^{84} \int_0^{84} cxy\, dy\, dx = 1 \iff c \int_0^{84} \int_0^{84} xy\, dy\, dx = 1 \iff$$

$\Leftrightarrow c\int_0^{84} x\cdot \frac{y^2}{2}\Big|_0^{84} dx = c\cdot\int_0^{84} x\cdot 3528\,dx = 3528\,c\int_0^{84} x\,dx =$

$= 3528\,c\cdot \frac{x^2}{2}\Big|_0^{84} = 3528^2\cdot c = 1$

$c = \frac{1}{3528^2}$

b) $P(X\le 3) = F_X(3)$

Pt. $x\in[0,84]$ și $y>84$

$F(x,y) = \int_0^x\int_0^{84} \frac{1}{3528^2}\,x\,y\,dx\,dy = \frac{1}{3528^2}\int_0^x\int_0^{84} xy\,dx\,dy =$

$= \frac{1}{3528^2}\int_0^{84}\left(\int_0^x x\,dx\right)y\,dy = \frac{1}{3528^2}\int_0^{84}\frac{x^2}{2}y\,dy = \frac{x^2}{2\cdot 3528^2}\cdot\frac{84^2}{2}$

$= \frac{x^2 84^2}{4\cdot 3528^2}$

$P(X\le 3) = F_X(3) = \frac{3^2\cdot 84^2}{4\cdot 3528^2}$

c) Pt $x>84$ și $y\in[0,84]$

$F(x,y) = \int_0^{84}\int_0^y \frac{1}{3528^2}\,xy\,dx\,dy = \frac{1}{3528^2}\int_0^{84}\int_0^y xy\,dx\,dy =$

$= \frac{1}{3528^2}\int_0^y\left(\int_0^{84} x\,dx\right)y\,dy = \frac{1}{3528^2}\int_0^y \frac{84^2}{2}y\,dy = \frac{84^2}{2\cdot 3528^2}\cdot$

$\cdot\int_0^y y\,dy = \frac{84^2 y^2}{4\cdot 3528^2}$

Pt. $(x,y)\in(0,84]\times(0,84)$

$F(x,y) = \int_0^{84}\int_0^{84} \frac{1}{3528^2}\,xy\,dy = \frac{1}{3528^2}\int_0^y\left(\int_0^x x\,dx\right)y\,dy$

$= \frac{1}{3528^2}\int_0^y \frac{x^2}{2}y\,dy = \frac{x^2}{2\cdot 3528^2}\cdot\frac{y^2}{2} = \frac{x^2y^2}{4\cdot 3528^2}$

Verif. dacă $F(x,y) = F_X(x) \cdot F_Y(y)$

$$\frac{x^2y^2}{4 \cdot 3528^2} = \frac{x^2 \cdot 84^2}{4 \cdot 3528^2} \cdot \frac{y^2 \cdot 84^2}{4 \cdot 3528^2}$$

$x^2y^2 = x^2y^2 \cdot 84^4$ fals $\Rightarrow$ variabilele sunt dependente

3) 110 ori $\rightarrow$ 84 aversuri
$\searrow$ 26 reversuri

Tabel de actualizare Bayesiană

hypothesis	prior	likelihood	Bayes numerator	posterior
θ	$1 \cdot d\theta$	$C_{110}^{84} \theta^{84}(1-\theta)^{26}$	$C_{110}^{84} \cdot \theta^{84}(1-\theta)^{26} d\theta$	$f(\theta \mid x_1) \cdot d\theta$
			$T = C_{110}^{84} \cdot \int_0^1 \theta^{84}(1-\theta)^{26} d\theta$	1
total	1			

pdf: $f(\theta / x_1) = c \cdot \theta^{84}(1-\theta)^{26} = \frac{(a+b-1)!}{(a-1)!(b-1)!} \cdot \theta^{84}(1-\theta)^{26}$,

unde $a, b \in$ repartiției beta(a, b), $a, b \in \mathbb{N}^*$

6) Interval simetric de $\frac{84}{200} = 0.42$ - probabilitate; normală standard

$1 - 0.42 = 0.58$ cantitate de probabilitate rămasă

$\frac{0.58}{2} = 0.29$ în fiecare parte