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Calcul Numeric

Examen - proba scrisă

CTI Anul I

1) $\begin{cases} x-y-2z=7 \\ x+y-3z=-3 \\ x+2y-2z=12 \end{cases}$ GPT

$$\bar{A}=[A|b]=\left(\begin{array}{ccc|c} 1 & -1 & -2 & 7 \\ 1 & 1 & -3 & -3 \\ 1 & 2 & -1 & 12 \end{array}\right) \xRightarrow{L_1 \leftrightarrow L_2} \begin{pmatrix} 1 & 1 & -3 & -3 \\ 1 & -1 & -2 & 7 \\ 1 & 2 & -1 & 12 \end{pmatrix}$$

$$\xleftrightarrow{C_3 \leftrightarrow C_1} \begin{array}{c} \begin{pmatrix} -3 & 1 & 1 & -3 \\ -2 & -1 & 1 & 7 \\ -1 & 2 & 1 & 12 \end{pmatrix} \\ \begin{array}{cccc} z & y & x & \ \end{array} \end{array} \sim \begin{array}{c} \begin{pmatrix} 1 & -\frac{1}{3} & -\frac{1}{3} & 1 \\ -2 & -1 & 1 & 7 \\ -1 & 2 & 1 & 12 \end{pmatrix} \\ \begin{array}{cccc} z & y & x & \ \end{array} \end{array} \xleftrightarrow{L_2+2L_1} \begin{array}{c} \begin{pmatrix} 1 & -\frac{1}{3} & -\frac{1}{3} & 1 \\ 0 & -\frac{5}{3} & \frac{1}{3} & 9 \\ -1 & 2 & 1 & 12 \end{pmatrix} \\ \begin{array}{cccc} z & y & x & \ \end{array} \end{array}$$

$$\xrightarrow{L_3+L_1} \begin{array}{c} \begin{pmatrix} 1 & -\frac{1}{3} & -\frac{1}{3} & 1 \\ 0 & -\frac{5}{3} & \frac{1}{3} & 9 \\ 0 & \frac{5}{3} & \frac{2}{3} & 13 \end{pmatrix} \\ \begin{array}{cccc} z & y & x & \ \end{array} \end{array} \sim \begin{array}{c} \begin{pmatrix} 1 & -\frac{1}{3} & -\frac{1}{3} & 1 \\ 0 & 1 & -\frac{1}{5} & -\frac{27}{5} \\ 0 & \frac{5}{3} & \frac{2}{3} & 13 \end{pmatrix} \\ \begin{array}{cccc} z & y & x & \ \end{array} \end{array} \underset{L_3-\frac{5}{3}L_2}{\sim} \begin{array}{c} \begin{pmatrix} 1 & -\frac{1}{3} & -\frac{1}{3} & 1 \\ 0 & 1 & -\frac{1}{5} & -\frac{27}{5} \\ 0 & 0 & 1 & 22 \end{pmatrix} \\ \begin{array}{cccc} z & y & x & \ \end{array} \end{array}$$

$\Rightarrow x=22$

$y-\frac{1}{5}x=-\frac{27}{5} \Rightarrow y=-1$

$z-\frac{1}{3}y-\frac{1}{3}x=1 \Rightarrow z-\frac{1}{3}\cdot(-1)-\frac{22}{3}=1 \Rightarrow z=1+\frac{21}{3}$

$z=8$

$\Rightarrow \begin{cases} x=22 \\ y=-1 \\ z=8 \end{cases}$

2) $A = \begin{bmatrix} -3 & 0 & 1 \\ 6 & 1 & 2 \\ 1 & 4 & 1 \end{bmatrix}$ LU cu GFP

$m_{21} = \frac{a_{21}}{a_{11}} = \frac{6}{-3} = -2$

$m_{31} = \frac{a_{31}}{a_{11}} = \frac{1}{-3}$

$\Rightarrow \begin{pmatrix} -3 & 0 & 1 \\ 6 & 1 & 2 \\ 1 & 4 & 1 \end{pmatrix} \xrightarrow[L_3 + \frac{1}{3} L_1]{L_2 + 2L_1} \begin{pmatrix} -3 & 0 & 1 \\ 0 & 1 & 4 \\ 0 & 4 & \frac{4}{3} \end{pmatrix} \xrightarrow{L_3 - 4L_2} \begin{pmatrix} -3 & 0 & 1 \\ 0 & 1 & 4 \\ 0 & 0 & -\frac{44}{3} \end{pmatrix}$

$m_{32} = \frac{a_{32}}{a_{22}} = \frac{4}{1} = 4$

$L = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -\frac{1}{3} & 4 & 1 \end{pmatrix} \quad U = \begin{pmatrix} -3 & 0 & 1 \\ 0 & 1 & 4 \\ 0 & 0 & -\frac{44}{3} \end{pmatrix}$

3) $f(x) = 4^x + 3 \cdot 3^x - 11 \cdot x^2$ $\quad f:[0,5] \to R$ Newton diferențe divizate

$x_1 = 0$

$x_2 = 3$

$x_3 = 5$

$P_2(x) = f[x_1] + f[x_1, x_2] \cdot (x - x_1) + f[x_1, x_2, x_3](x - x_1)(x - x_2)$

$= f[0] + f[0,3] \cdot (x - 0) + f[0,3,5](x-0)(x-3)$

x_i	$f[x_i]$	$f[x_i, x_{i+1}]$	$f[x_i, x_{i+1}, x_{i+2}]$
0	$f[0] = 4$		
3	$f[3] = 46$	$f[0,3] = \frac{42}{3}$	
5	$f[5] = 1478$	$f[3,5] = 716$	$f[0,3,5] = \frac{702}{5}$

$f[0] = f(0) = 4$

$f[0,3] = \frac{f[3] - f[0]}{3 - 0} = \frac{42}{3}$

$f[3,5] = \frac{f[5] - f[3]}{5 - 3} = \frac{1478 - 46}{2} = 716$

$f[0,3,5] = \frac{f[3,5] - f[0,3]}{5 - 0} = \frac{716 - \frac{42}{3}}{5} = \frac{702}{5}$

$$\Rightarrow P_2(x) = 4 + \frac{42}{3}x + \frac{702}{5}x(x-3) =$$

$$= 4 + \frac{42}{3}x + \frac{702}{5}x^2 - \frac{3 \cdot 702}{5}x =$$

$$= \frac{702}{5}x^2 + \cancel{\frac{42}{3}x} - \frac{2036}{5}x + 4$$

4) $$I = \int_1^4 \left(3 \cdot x^6 + 6 \cdot x^5 + \frac{3}{2}x^3 + 9 \cdot x^2\right) dx \qquad m = 3 : h = \frac{4-1}{3} = 1$$

a)

$x_1 = 1 + 0 \cdot h = 1$

$x_2 = 1 + 1 \cdot h = 2$

$x_3 = 1 + 2 \cdot h = 3 \qquad x_4 = 1 + 3h = 4$

~~$f(a) = f(x_1) =$~~

$$I^3_{trapez} = \frac{1}{2}\left(f(1) + 2f(2) + 2f(3) + f(4)\right) = \frac{1}{2}\left(\frac{39}{2} + 2 \cdot 432 + 2 \cdot \frac{7533}{2} + 18672\right) =$$

$$= \frac{1}{2}\left(\frac{54177}{2}\right) = \frac{54177}{4} = 13544{,}25$$

b) $$\int_1^4 3x^6 + 6x^5 + \frac{3}{2}x^3 + 9x^2\, dx = \left.\frac{3x^7}{7} + x^6 + \frac{3x^4}{8} + 3x^3\right|_1^4 = \frac{79840}{7} - \frac{269}{56} =$$

$$= \frac{638451}{56} = 11400{,}91$$

$$R = |13544{,}25 - 11400{,}91| = 2143{,}34$$