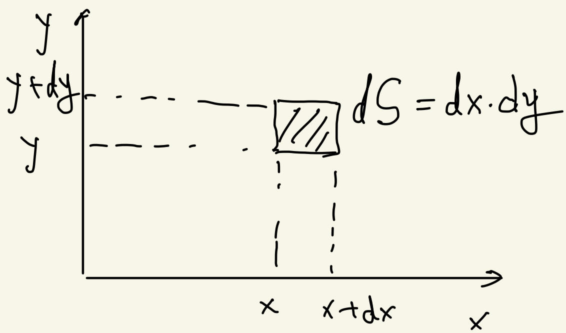
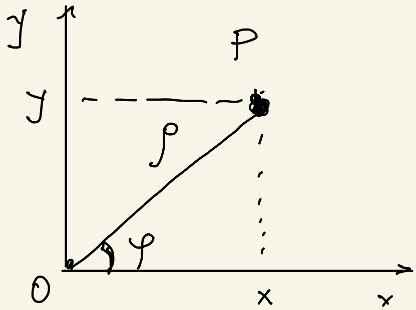
$$\begin{cases} E_{P_x} = \int dE_{P,x} = \int \dfrac{k\sigma\, dS'(-x')}{(x'^2+y'^2+z^2)^{3/2}} \\ E_{P_y} = \int dE_{P,y} = \int \dfrac{k\sigma\cdot dS'(-y')}{(x'^2+y'^2+z^2)^{3/2}} \\ E_{P_z} = \int dE_{P,z} = \int \dfrac{k\sigma\cdot dS'\cdot z}{(x'^2+y'^2+z^2)^{3/2}} \end{cases}$$

$$\begin{cases} E_{P_x} = -k\sigma \int \dfrac{x'\, dS'}{(x'^2+y'^2+z^2)^{3/2}} \\ E_{P_y} = -k\sigma \int \dfrac{y'\, dS'}{(x'^2+y'^2+z^2)^{3/2}} \\ E_{P_z} = k\sigma z \cdot \int \dfrac{dS'}{(x'^2+y'^2+z^2)^{3/2}} \end{cases}$$



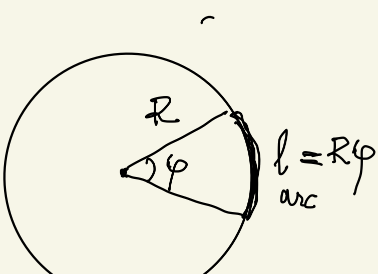
coordonate carteziene

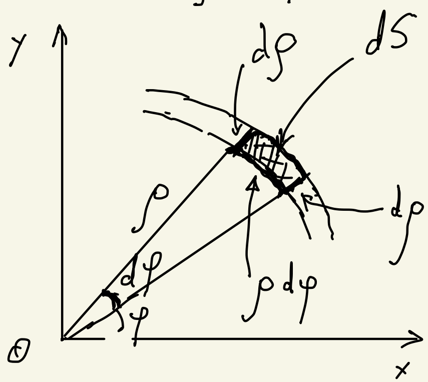


coordonate polare

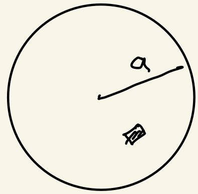
$$dS = dx\, dy.$$

$$x = \rho \cos\varphi$$
$$y = \rho \sin\varphi.$$





$$dS' = \rho \, d\rho \, d\varphi$$



$$\rho \in [0, a]$$
$$\varphi \in [0, 2\pi]$$

$$x' = \rho \cos\varphi$$
$$y' = \rho \sin\varphi$$

$$\begin{cases} E_{p_x} = -k\sigma \displaystyle\int \frac{\rho\cos\varphi \cdot \rho \, d\rho \, d\varphi}{(\rho^2\cos^2\varphi + \rho^2\sin^2\varphi + z^2)^{3/2}} \\[2ex] E_{p_y} = -k\sigma \displaystyle\int \frac{\rho\sin\varphi \; \rho \, d\rho \, d\varphi}{(\rho^2\cos^2\varphi + \rho^2\sin^2\varphi + z^2)^{3/2}} \\[2ex] E_{p_z} = k\sigma z \cdot \displaystyle\int \frac{\rho \, d\rho \, d\varphi}{(\rho^2\cos^2\varphi + \rho^2\sin^2\varphi + z^2)^{3/2}} \end{cases}$$

$$\begin{cases} E_{p_x} = -k\sigma \displaystyle\int \frac{\rho^2 \, d\rho \, d\varphi \, \cos\varphi}{(\rho^2 + z^2)^{3/2}} \\[2ex] E_{p_y} = -k\sigma \displaystyle\int \frac{\rho^2 \, d\rho \, d\varphi \, \sin\varphi}{(\rho^2 + z^2)^{3/2}} \\[2ex] E_{p_z} = k\sigma z \cdot \displaystyle\int \frac{\rho \, d\rho \, d\varphi}{(\rho^2 + z^2)^{3/2}} \end{cases}$$

$$\rho \in [0, a]$$
$$\varphi \in [0, 2\pi]$$

$$\sum_{i=1}^{3}\sum_{j=1}^{2} a_i b_j = \sum_{i=1}^{3}\left(a_i b_1 + a_i b_2\right) =$$

$$= a_1 b_1 + a_1 b_2 + a_2 b_1 + a_2 b_2 + a_3 b_1 + a_3 b_2 =$$

$$= a_1\left(b_1+b_2\right) + a_2\left(b_1+b_2\right) + a_3\left(b_1+b_2\right) =$$

$$= \left(b_1+b_2\right)\left(a_1+a_2+a_3\right) = \left(\sum_{j=1}^{2} b_i\right)\left(\sum_{i=1}^{3} a_i\right)$$

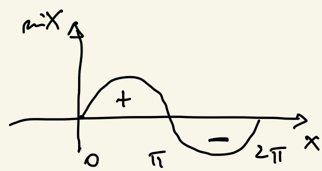
$$\sum_{i=1}^{3}\sum_{j=1}^{2} a_i b_j = \left(\sum_{i=1}^{3} a_i\right)\left(\sum_{j=1}^{2} b_i\right)$$

$$\sum\sum() \longrightarrow \left[\sum()\right]\left[\sum()\right]$$

$$\sum \longrightarrow \int$$

$$\int\int (1+2) = \left(\int (1)\right)\cdot\left(\int (2)\right)$$

$$\begin{cases} E_{p_x} = -k\sigma \cdot \displaystyle\int_0^a \frac{\rho^2 d\rho}{(\rho^2+z^2)^{3/2}} \cdot \int_0^{2\pi} \cos\varphi \, d\varphi \\ E_{p_y} = -k\sigma \cdot \displaystyle\int_0^a \frac{\rho^2 d\rho}{(\rho^2+z^2)^{3/2}} \cdot \int_0^{2\pi} \sin\varphi \, d\varphi \\ E_{p_z} = \cancel{k}\sigma z \cdot \displaystyle\int_0^a \frac{\rho \, d\rho}{(\rho^2+z^2)^{3/2}} \cdot \int_0^{2\pi} d\varphi \end{cases}$$



$$\int_0^{2\pi} \sin\varphi \cdot d\varphi = \int_0^{2\pi} \cos\varphi \, d\varphi = 0$$

$$\begin{cases} E_{p_x} = 0 \\ E_{p_y} = 0 \\ E_{p_z} = k\sigma z \cdot \int_0^a \frac{\rho\, d\rho}{(\rho^2+z^2)^{3/2}} \cdot \int_0^{2\pi} d\varphi \end{cases}$$

$$E_{p_z} = 2\pi k \sigma z \cdot \int_0^a \frac{\rho\, d\rho}{(\rho^2+z^2)^{3/2}}$$

$$\int \frac{x\, dx}{(x^2+3)^{3/2}} \qquad \int u^n u' = \frac{u^{n+1}}{n+1}$$

$$E_{p_z} = 2\pi k \sigma z \cdot \frac{1}{2} \int_0^a \frac{2\rho\, d\rho}{(\rho^2+z^2)^{3/2}} = \pi k \sigma z \int_0^a (\rho^2+z^2)^{-3/2} (\rho^2+z^2)' d\rho$$

$$= \pi k \sigma z \cdot \frac{(\rho^2+z^2)^{-\frac{3}{2}+1}}{-\frac{3}{2}+1} \Bigg|_0^a = -2\pi k \sigma z \cdot (\rho^2+z^2)^{-\frac{1}{2}} \Bigg|_0^a$$

$$= -2\pi k \sigma z \cdot \frac{1}{\sqrt{\rho^2+z^2}} \Bigg|_0^a = -2\pi k \sigma z \cdot \left(\frac{1}{\sqrt{a^2+z^2}} - \frac{1}{\sqrt{z^2}} \right)$$

$$= -2\pi k\sigma z\cdot\left(\frac{1}{\sqrt{z^2+a^2}} - \frac{1}{|z|}\right) = 2\pi k\sigma z\cdot\left(\frac{1}{|z|} - \frac{1}{\sqrt{z^2+a^2}}\right)$$

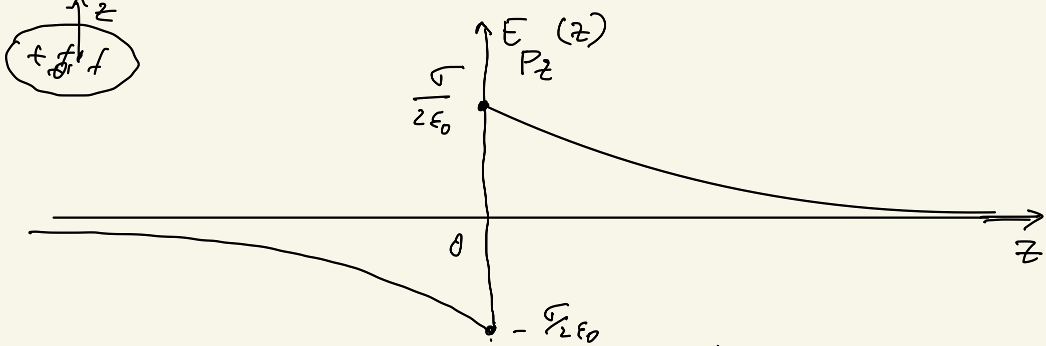
$$E_{Pz} = 2\pi k\sigma\left(\frac{z}{|z|} - \frac{z}{\sqrt{z^2+a^2}}\right)$$

$$|z| = \begin{cases} z, & z>0 \\ -z, & z<0 \end{cases}$$

$$E_{Pz}(z) = \begin{cases} 2\pi k\sigma\left(-1 - \frac{z}{\sqrt{z^2+a^2}}\right), & z<0 \\ 2\pi k\sigma\left(1 - \frac{z}{\sqrt{z^2+a^2}}\right), & z>0 \end{cases}$$

$$2\pi k\sigma = 2\pi\cdot\frac{1}{4\pi\varepsilon_0}\sigma = \frac{\sigma}{2\varepsilon_0}$$

$$E_{Pz}(z) = \begin{cases} \frac{\sigma}{2\varepsilon_0}\left(-1 - \frac{z}{\sqrt{z^2+a^2}}\right), & z<0 \\ \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2+a^2}}\right), & z>0 \end{cases}$$



$$\lim_{z\to\infty} E_{P_z}(z) = \lim_{z\to\infty} \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2+a^2}}\right) = \lim_{z\to\infty} \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{z\sqrt{1+\frac{a^2}{z^2}}}\right)$$

$$= 0$$

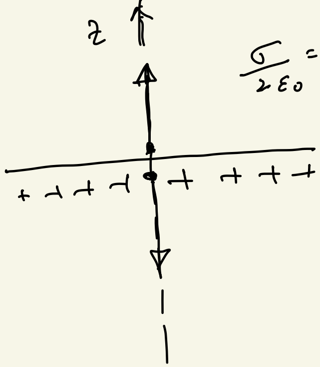
$$\lim_{\substack{z\to 0\\ z>0}} E_{P_z}(z) = \frac{\sigma}{2\varepsilon_0}$$

$$\lim_{z\to -\infty} E_{P_z}(z) = \lim_{z\to -\infty} \frac{\sigma}{2\varepsilon_0}\left(-1 - \frac{z}{\sqrt{z^2+a^2}}\right) =$$

$$\lim_{z\to -\infty} \frac{\sigma}{2\varepsilon_0}\left(-1 - \frac{z}{\sqrt{z^2\left(1+\frac{a^2}{z^2}\right)}}\right) =$$

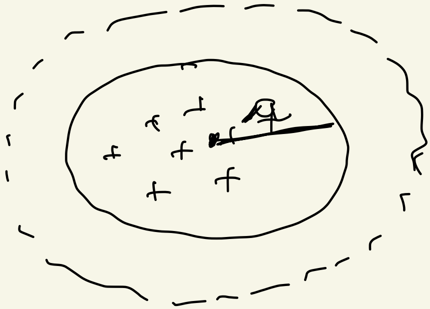
$$\lim_{z\to -\infty} \frac{\sigma}{2\varepsilon_0}\left(-1 - \frac{z}{-z\cdot\sqrt{1+\frac{a^2}{z^2}}}\right) = 0$$

$$\lim_{\substack{z\to 0\\ z<0}} E_{P_z}(z) = -\frac{\sigma}{2\varepsilon_0}$$

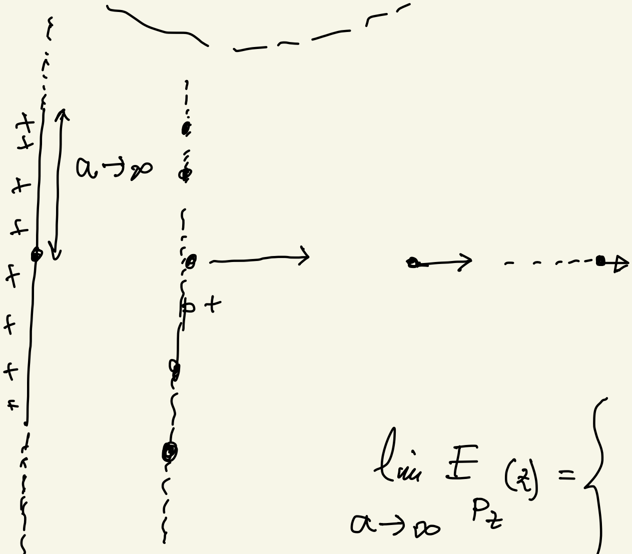


$$\frac{\sigma}{2\varepsilon_0} = |\vec{E}|$$

<u>Obs.</u>

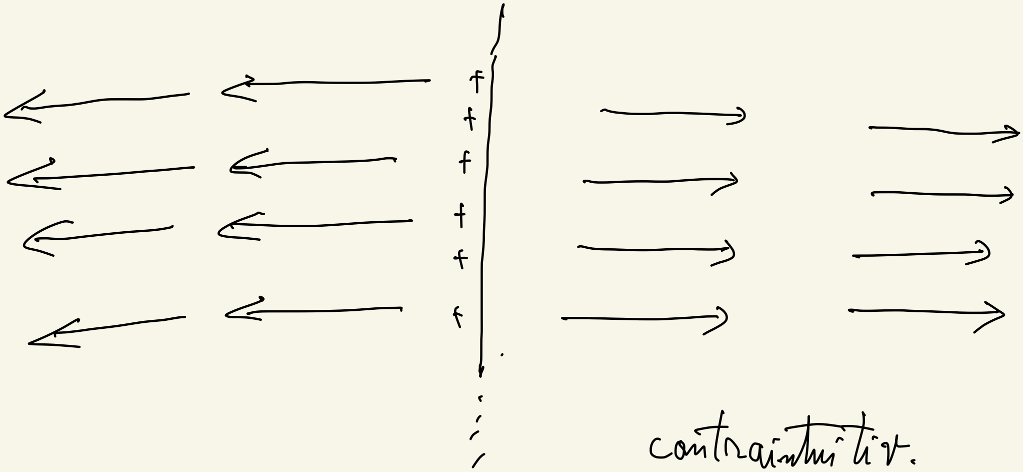


$a \to \infty$



plan

$$\lim_{a\to\infty} E_{P_z}(z) = \begin{cases} -\dfrac{\sigma}{2\varepsilon_0}, & z<0 \\[2ex] \dfrac{\sigma}{2\varepsilon_0}, & z>0 \end{cases}$$



contraintuitiv.