1. Ecuația unei drepte

$$ax + by + c = 0 \quad , a,b,c \in \mathbb{R} \text{ și } (a,b) \neq (0,0)$$

2. Ec. unei drepte când cunoaștem 2 pct.

$$M(x_M, y_M) \quad N(x_N, y_N)$$

$$\frac{x - x_M}{x_N - x_M} = \frac{y - y_M}{y_N - y_M}$$

$$y(x_N - y_M) - y_M(x_N - x_M) = x(y_N - y_M) - x_M(y_N - y_M)$$

Repr. parametrică

$$\frac{y - y_M}{y_N - y_M} = t \qquad \frac{x - x_M}{x_N - x_M} = t$$

$$\begin{cases} x = x_M + t(x_N - x_M) \\ y = y_M + t(y_N - y_M) \end{cases}$$

$$\begin{cases} x = (1-t)\,x_M + t\,x_N \\ y = (1-t)\,y_M + t\,y_N \end{cases}$$

Interpretare

1. Un pct. $P \in \mathbb{R}^2$ aparține dreptei MN det. de pct. M, N cu $M \neq N \iff$

$\exists\, t \in \mathbb{R}$ a.î.

$$P = (1-t) M + t \cdot N$$

2. Un pct. $P \in \mathbb{R}^2$ aparține segm MN det. de pct. $\iff \exists\, t \in [0,1]$ a.î.

$$P = (1-t) M + t N$$

Det. pct. de $\cap$ a două segmente folosind ec. lor

Segm. AB și CD se $\cap \iff \exists\, s_0, t_0 \in [0,1]$ a.î.

$(1-t_0) A + t_0 \cdot B = (1-s_0) C + s_0 \cdot D$.

$$\Rightarrow \begin{cases} (1-t_0) x_A + t_0 x_B = (1-s_0) x_C + s_0 \cdot x_D \\ (1-t_0) \cdot y_A + t_0 \cdot y_B = (1-s_0) \cdot y_C + s_0 \cdot y_D \end{cases}$$

$$x_A - t_0 \cdot x_A + t_0 \cdot x_B - x_C + s_0 \cdot x_C - s_0 \cdot x_D = 0$$

$$t_0 (x_B - x_A) + s_0 (x_C - x_D) = x_C - x_A$$

EX. TEST

Fie $A(0,4)$, $B(4,0)$, $C(-1,-1)$, $D(3,7)$. Det. dacă $AB \cap CD$. Dacă da, aflați $P = ?$

$$\begin{cases} AB: 4t_0 - 4s_0 = -1 \\ CD: \underline{-4t_0 - 8s_0 = -5} \end{cases} \quad (+) \quad \Rightarrow -12 s_0 = -6 \Rightarrow s_0 = \frac{1}{2} \in [0,1]$$

$$\Rightarrow 4t_0 - 2 = -1 \Rightarrow t_0 = \frac{1}{4} \in [0,1]$$

$$\Rightarrow \exists\, t_0, s_0 \in [0,1] \Rightarrow [AB] \cap [CD] \text{ (A)}$$

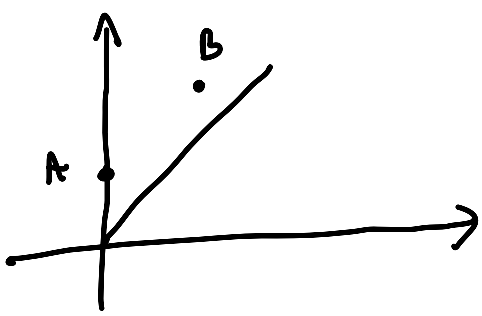
$$\begin{cases} \frac{3}{4} x_A + \frac{1}{4} x_B = \frac{1}{2} x_C + \frac{1}{2} x_D \\ \frac{3}{4} y_A + \frac{1}{4} y_B = \frac{1}{2} y_C + \frac{1}{2} y_D \end{cases} \Rightarrow$$

$$\left.\begin{cases} 0 + 1 = 1 \\ 3 + 0 = \frac{6}{2} = 3 \end{cases}\right\} \Rightarrow P(1,3)$$

Regulă:

Două pct. M, N sunt de o parte și de alta a unei drepte de ecuație $f(x,y) = ax + by + c \Leftrightarrow$

$$f(M) \cdot f(N) < 0$$



$f(x,y) = x - y$

$A(0,3)$, $B(4,7)$

$f(A) = 0 - 3 = -3$
$f(B) = -3$ $\Rightarrow$

$f(A) \cdot f(B) \geq 0 \Rightarrow$ nu sunt de o parte și de alta a dreptei

Produsul scalar

$a = < a_1, a_2, a_3 >$

$b = < b_1, b_2, b_3 >$

$a \cdot b = a_1 \cdot b_1 + a_2 \cdot b_2 + a_3 \cdot b_3$

Produsul vectorial

$a = \langle a_1, a_2, a_3 \rangle$

$b = \langle b_1, b_2, b_3 \rangle$

$$a \times b = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} =$$

$$= \vec{i} \cdot (-1)^2 \cdot \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} + \vec{j} \cdot (-1)^3 \cdot \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} +$$

$$\vec{k} \cdot (-1)^4 \cdot \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$$

EXERCIȚII

1. Ec. dreptei det. de $A(2,3)$ și $B(-5,4)$
2. Calc. prod. scalar $a \cdot b$, $b \cdot c$, $a \cdot c$ pt.

   $a = \langle 4, -2, 5 \rangle$

   $b = \langle -1, 3, -6 \rangle$

   $c = \langle 7, -5, 1 \rangle$

3. a) $v \times w$ $\quad v = \langle 1,2,0 \rangle$

$w = \langle -3,1,0 \rangle$

b) $v \times w$ $\quad v = \langle 2,4,1 \rangle$

$w = \langle -1,-1,1 \rangle$

4. $A(0,8)$ $\quad B(8,0)$ $\quad C(-2,-2)$ $\quad D(6,14)$

$[AB] \cap [CD]$ $\quad P = ?$

1. $$\frac{x-2}{-5-2} = \frac{y-3}{4-3} \Rightarrow x-2 = -7y+21$$

$$x+7y-23=0$$

2. $a \cdot b = 4\cdot(-1) + (-2)\cdot 3 + 5\cdot(-6) = -40$

$b \cdot c = (-1)\cdot 7 + 3\cdot(-5) + (-6)\cdot 1 = -28$

$a \cdot c = 4\cdot 7 + (-2)\cdot(-5) + 5\cdot 1 = 43$

3. a) $$v \times w = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 0 \\ -3 & 1 & 0 \end{vmatrix} = \vec{i} \cdot \begin{vmatrix} 2 & 0 \\ 1 & 0 \end{vmatrix} -$$

$$- \vec{j} \begin{vmatrix} 1 & 0 \\ -3 & 0 \end{vmatrix} + k \cdot \begin{vmatrix} 1 & 2 \\ -3 & 1 \end{vmatrix} = 0\cdot\vec{i} - 0\cdot\vec{j} + 7\cdot\vec{k}$$

$\Rightarrow \langle 0,0,7 \rangle$

b) $\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 4 & 1 \\ -1 & -1 & 1 \end{vmatrix} = \vec{i} \cdot \begin{vmatrix} 4 & 1 \\ -1 & 1 \end{vmatrix} - \vec{j} \cdot \begin{vmatrix} 2 & 1 \\ -1 & 1 \end{vmatrix} +$

$\vec{k} \cdot \begin{vmatrix} 2 & 4 \\ -1 & -1 \end{vmatrix} = 5\vec{i} - 3\vec{j} + 2\vec{k} \Rightarrow \langle 5, -3, 2 \rangle$

4. $A(\overset{x_A}{0}, \overset{y_A}{8})$

$B(\overset{x_B}{8}, \overset{y_B}{0})$

$C(\overset{x_C}{-2}, \overset{y_C}{-2})$

$D(\overset{x_D}{6}, \overset{y_D}{14})$

$$\begin{cases} t_0(x_B - x_A) + s_0(x_C - x_D) = x_C - x_A \\ t_0(y_B - y_A) + s_0(y_C - y_D) = y_C - y_A \end{cases} \Rightarrow$$

$$\begin{cases} 8t_0 - 8s_0 = -2 \\ -8t_0 - 16s_0 = -10 \end{cases} \Rightarrow -24s_0 = -12 \Rightarrow$$

$s_0 = \frac{1}{2} \in [0,1]$

$t_0 = \frac{1}{4} \in [0,1]$

$s_0, t_0 \in [0,1]$ deci $AB \cap CD \neq \emptyset$

$$\left.\begin{array}{l} x_P = \frac{1}{2} \cdot (-2) + \frac{1}{2} \cdot 6 = 2 \\ y_P = \quad \dots \end{array}\right\} \Rightarrow P(2,6)$$