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(2). $f_n(x) = \dfrac{nx^2+4}{nx+4}$ $\quad f_n : [0,\infty) \to \mathbb{R}$

CS, CU pe $[0,1]$ și $(1;\infty)$

Fie $x \in [0,1]$ fixat

$$\lim_{n\to\infty} f_n(x) = \lim_{n\to\infty} \frac{nx^2+4}{nx+4} = \lim_{n\to\infty} \frac{n(x^2+\frac{4}{n})}{n(x+\frac{4}{n})} = x$$

$$\Rightarrow \text{pe } [0,1] \; f_n \xrightarrow[{[0,1]}]{CS} x$$

$$\lim_{n\to\infty} \sup_{x\in[0,1]} |f_n(x) - f(x)| = \lim_{n\to\infty} \sup_{x\in[0,1]} \left| \frac{nx^2+4}{nx+4} - x \right| =$$

$$= \lim_{n\to\infty} \sup_{x\in[0,1]} \left| \frac{nx^2+4-nx^2-4x}{nx+4} \right| = \lim_{n\to\infty} \sup_{x\in[0,1]} \left| \frac{-4x+4}{nx+4} \right| =$$

$$= 0$$

$$\Rightarrow f_n \xrightarrow[{[0,1]}]{C.U} x$$

Fie $x \in (1,\infty)$ fixat

$$\lim_{n\to\infty} f_n(x) = \lim_{n\to\infty} \frac{nx^2+4}{nx+4} = \lim \frac{n(x^2+\frac{4}{n})}{n(x+\frac{4}{n})} = x$$

$$\Rightarrow f_n \xrightarrow[{(1,\infty)}]{CS} x$$

$$\lim_{n\to\infty} \sup_{x\in(1,\infty)} |f_n(x) - f(x)| = \lim_{n\to\infty} \sup_{x\in(1,\infty)} \left| \frac{nx^2+4-nx^2-4x}{nx+4} \right| =$$

$$= \lim_{n\to\infty} \sup_{x\in(1,\infty)} \left| \frac{-4x+4}{nx+4} \right| = 0$$

$$\Rightarrow f_n \xrightarrow[{(1,\infty)}]{C.U} x$$

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Examen Analiză

① $\sum_{n\geq 1} \frac{(n!)^4}{(4n)!} x^{3n}$

Fie C mulțimea de convergență a seriei

$(-R, R) \subseteq C \subseteq [-R, R]$

$\frac{1}{R} = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{((n+1)!)^4}{(4(n+1))!} \cdot \frac{(4n)!}{(n!)^4}\right| =$

$= \lim_{n\to\infty} \left|\frac{(n! \cdot (n+1))^4}{(4n+4)!} \cdot \frac{(4n)!}{(n!)^4}\right| = \lim_{n\to\infty} \left|\frac{(n!)^4 \cdot (n+1)^4}{(4n)! \cdot (4(n+1))(4n+2)(4n+3)(4n+4)} \cdot \frac{(4n)!}{(n!)^4}\right| = \lim_{n\to\infty} \left|\frac{(n+1)^4}{(4n+1)(4n+2)(4n+3)(4n+4)}\right| = \lim_{n\to\infty} \frac{(n+1)^4}{(4n+1)(4n+2)(4n+3)(4n+4)}$

$= \lim_{n\to\infty} \frac{n^4(1+\frac{1}{n})^4}{4^4 n^4(\ldots\ldots)} = \frac{1}{4^4} = \frac{1}{256}$

$\frac{1}{R} = \frac{1}{256} \Rightarrow R = 256 = 4^4$

~~Seria converge pe $(-256, 256)$ $R_x = \sqrt[3]{4^4}$~~

$R_x = \sqrt[3]{4^4} = 4^{\frac{4}{3}}$

Seria converge pe $\left(-4^{\frac{4}{3}}, 4^{\frac{4}{3}}\right)$

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3. $f(x,y) = xy + \frac{40}{x} + \frac{10}{y}$ $f: \mathbb{R}^2 \to \mathbb{R}$

$$\begin{cases} \frac{\partial f}{\partial x}(x,y) = 0 \\ \frac{\partial f}{\partial y}(x,y) = 0 \end{cases} \Rightarrow \begin{cases} y - \frac{40}{x^2} = 0 \\ x - \frac{10}{y^2} = 0 \end{cases}$$