Generating Accurate Pseudo-labels in Semi-Supervised Learning and Avoiding Overconfident Predictions via Hermite Polynomial Activations

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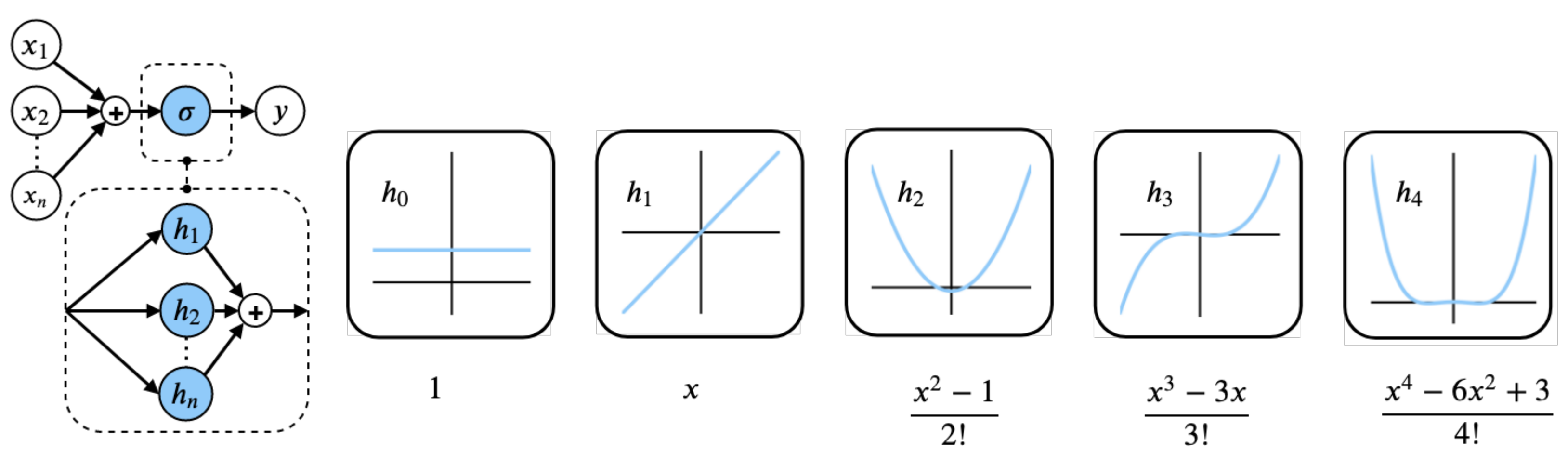
Hermite Polynomials

Hermite Polynomials as activations

- The lower order terms in the Hermite polynomial series expansion of ReLU is used as an **activation function** with **the coefficients as trainable parameters**.
- Optionally, a SoftSign function is added to handle large numerical values attained by the polynomials.



Gap in the Literature

- Ge et al. [3] showed that for one hidden layer network, one could **avoid spurious local minima** by utilizing an orthogonal basis expansion for ReLUs.

Conclusion: Theoretical results not empirically investigated on regular computer vision architectures

- Nar et al. [4] showed that **smoother landscapes** enable the use of a larger range of step sizes in order for the gradient descent algorithm to converge.

Conclusion: Smoothness of the objective helps in convergence

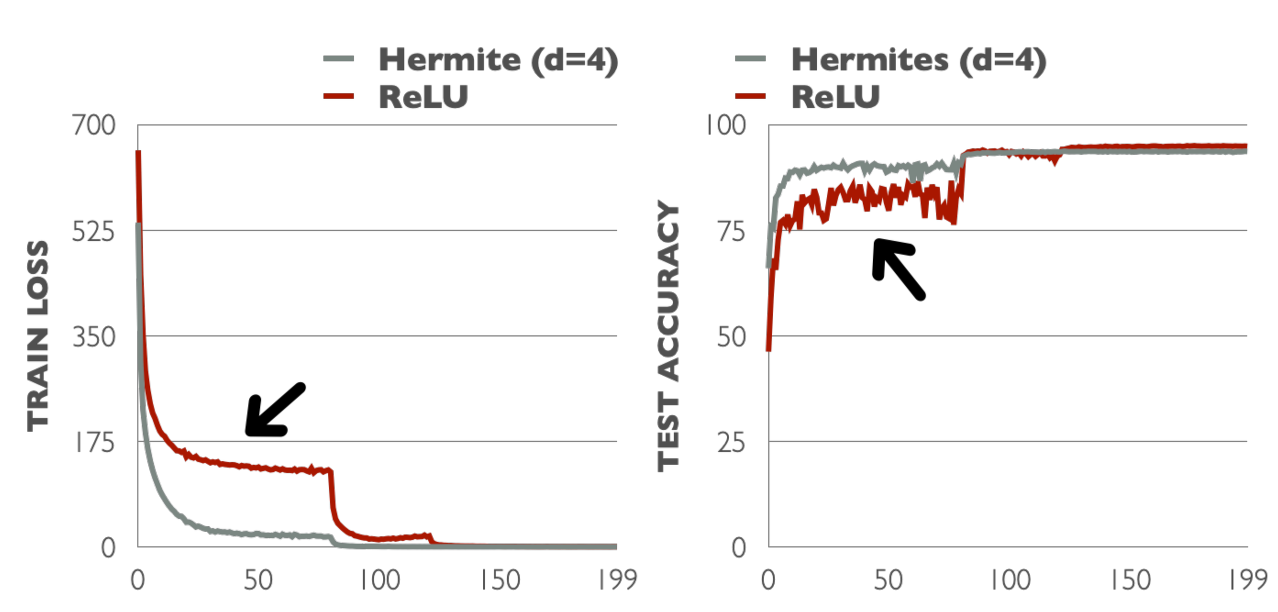
Hermite Polynomials in ResNet 152

- Test accuracy for hermite model converges in **less than half the number of epochs**.

Dataset CIFAR10	Number of Trainable Parameters	Best Test Accuracy	Epochs to reach 90% Test Accuracy
Hermite	58,145,574	95.48%	30
ReLU	58,144,842	94.5%	80

Hermite Polynomials in ResNet 18

- Hermites have **faster convergence in test accuracies over the initial epochs** but ReLU has the higher test accuracy at the end of training.



Hermite Polynomials make conscious classifications

- Unlike ReLU [5], when the test data is different from the training data, Hermite networks consciously make **(approximately) random predictions** unlike ReLU networks

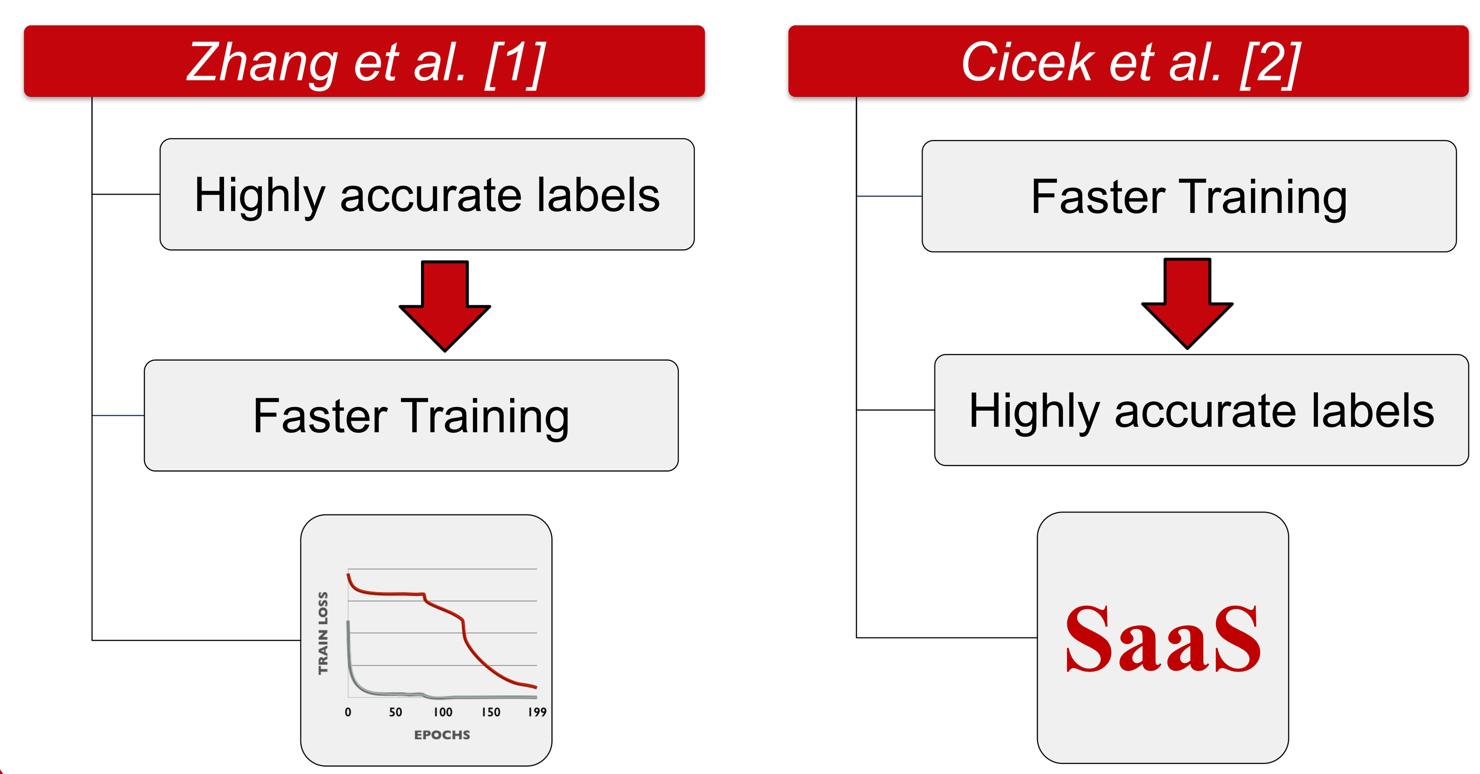
	Train Similar to Test	Train **Different From** Test
	Train Test	Train Test
ReLU	High Confidence Predictions	High Confidence Predictions
Hermite	High Confidence Predictions	**Low** Confidence Predictions

Theorem: When the training data is zero mean (mean normalized) then for a K class classification problem we have,

$$||x_{test}|| > \log\frac{1}{\epsilon} \implies \frac{1}{K} - \epsilon \le \frac{e^{f_k(x)}}{\sum_{l=1}^{K} f_l(x)} \le \frac{1}{K} + \epsilon$$

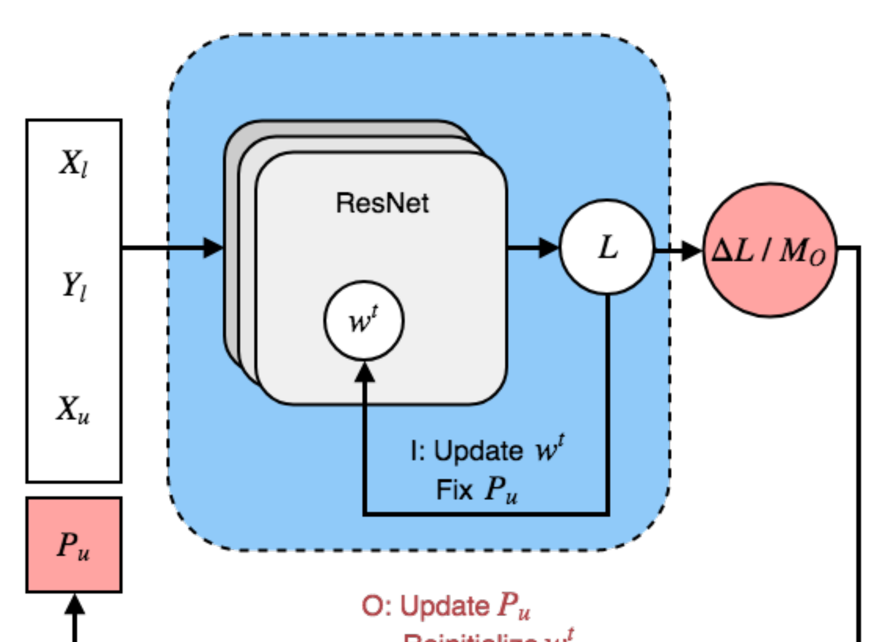
The Semi-supervised Learning Setup

- SaaS: Find labels with least training time



Speed as a Supervisor (SaaS)

- SaaS seeks to find a set of pseudolabels that maximizes the decrease in the loss function over a small number of epochs.

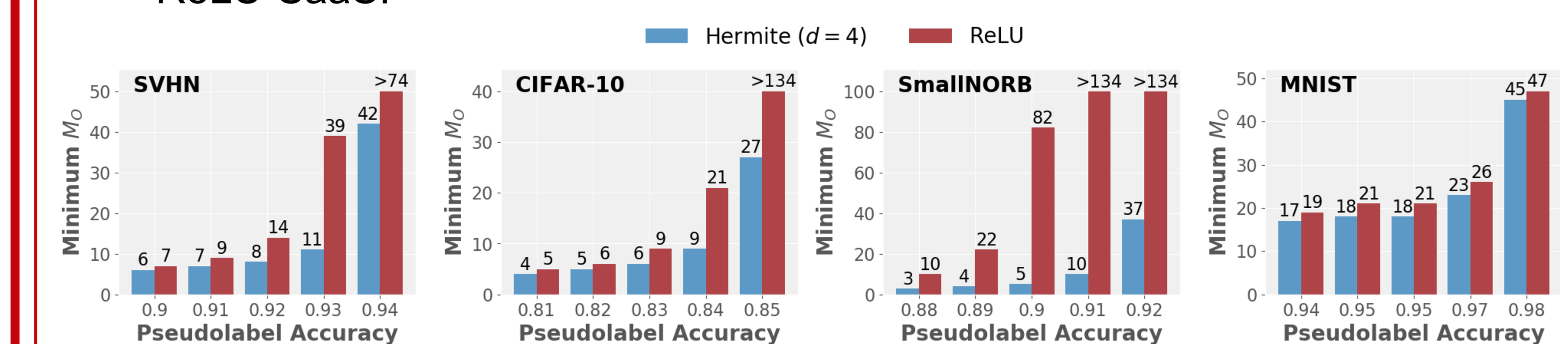


Conclusion: Smoothness of the objective helps in training SaaS faster

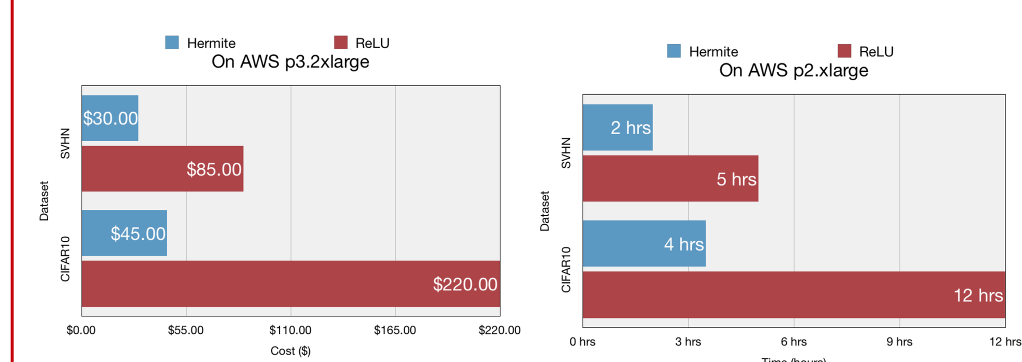
Computational Benefits

Hermite-SaaS trains faster

- Hermite-SaaS trains faster. The minimum number of epochs M_O to reach a given value of pseudolabel accuracy is **lower for Hermite-SaaS** than ReLU-SaaS.

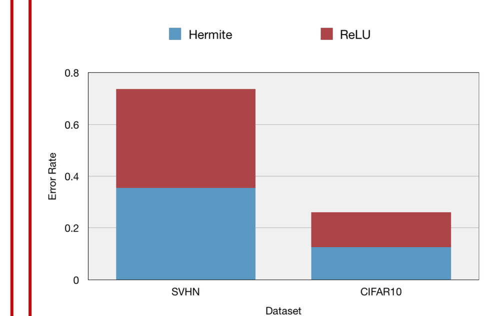


Hermite-SaaS saves time and money



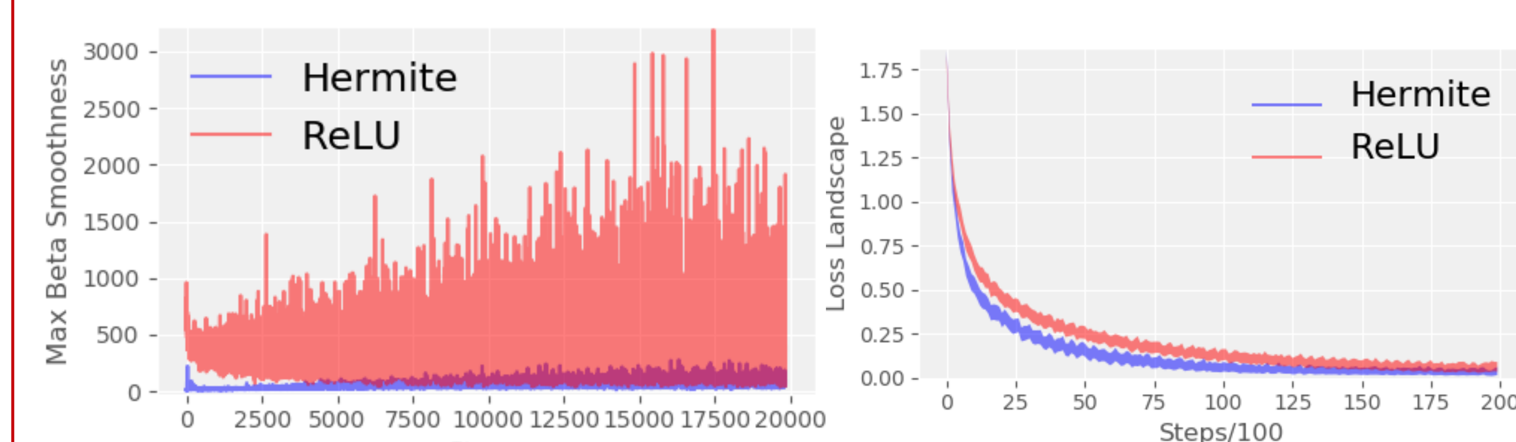
- **(Left)** Hermite-SaaS is saving $$ on AWS p3.2xlarge. **(Right)** Hermite-SaaS is saving compute time on AWS p2.xlarge.

Hermite-SaaS generalizes better



- Lower generalization error in the semi-supervised learning setup.

Hermite generates smoother landscape than ReLU



- Gradients are more stable on Hermite loss landscape: lower maximum beta smoothness.
- Magnitude of loss and its variation is lower for Hermites.

References

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