1. (**Markov chains**) Suppose that each day, the weather is either *sun*, *rain*, or *hail*. The probability of *sun* is 0.7, of *rain* is 0.2, and of *hail* is 0.1. Suppose that the weather is a Markov process, so the weather today only depends on what the weather was yesterday. If it is *hail* today, it will always be *sun* tomorrow, and if it is *sun* today, it will never *hail* tomorrow. Assume that if it is *sun* today, then it is 4 times more likely to be *sun* tomorrow than *rain*.

   (a) Draw a graphical model to represent this Markov chain.

   (b) What is the period of each state? Is this Markov chain periodic?

   (c) Is is irreducible? Why or why not?

   (d) Find the matrix of transition probabilities.

2. (**Perfect phylogeny**) The table below shows a collection of 2-state full characters for $X = \{a, b, c, d, e\}$, one for each of 4 loci.

	1	2	3	4
a	0	0	1	0
b	0	0	1	0
c	0	1	1	1
d	1	0	1	1
e	1	1	0	0

   Does a perfect phylogeny exist for this dataset? If yes, what is it? If no, why not?

3. (**Ross, Ch. 4, #32**) Each of two switches is either on or off during a day. On day n, each switch will independently be on with probability

$$[1 + \text{number of on switches during day } (n-1)]/4$$

   For instance, if both switches are on during day $n-1$, then each will independently be on during day n with probability $3/4$. What fraction of days are both switches on? What fraction are both off?