1. Show how to compute the forward probabilities for an HMM in log space.

2. Suppose you are given fixed phylogenetic tree branch lengths $t_1, t_2, \cdots, t_B$ and mutation counts $x = x_1, x_2 \cdots, x_B$ for each branch. Assuming mutations occur as a Poisson process with mutation rate μ, find the MLE (maximum likelihood estimator) for μ. Recall that if a random variable X is Poisson distributed with parameter λ, then the pmf (probability mass function) for X is $\mathbb{P}_\lambda(X = x) = \lambda^x e^{-\lambda}/x!$.

3. The dynamic programming table below shows the fill-in step of Nussinov's RNA folding algorithm on the string S. Fill in the two missing entries, $B(3, 10)$ and $B(1, 12)$. Then perform back-tracing on both entries to find the optimal secondary structure(s) for $S[3 \dots 10]$ and S. Does the number of unique optimal structures equal the number of tracebacks?

S		A	U	C	G	G	A	U	C	G	A	A	C
		1	2	3	4	5	6	7	8	9	10	11	12
A	1	0	0	0	0	1	2	3	3	3	3	3	
U	2		0	0	0	1	2	2	2	2	3	3	3
C	3			0	0	1	1	1	1	2		2	2
G	4				0	0	0	0	1	1	1	1	2
G	5					0	0	0	1	1	1	1	2
A	6						0	0	0	0	1	1	1
U	7							0	0	0	1	1	1
C	8								0	0	0	0	1
G	9									0	0	0	1
A	10										0	0	0
A	11											0	0
C	12												0

4. Suppose a weighted coin, with probability θ of turning up heads is flipped until the first head occurs. Let X represent the total number of flips prior to getting heads. X is said to be geometrically distributed. Compute the pmf for X, $f(x) = \mathbb{P}(X = x)$. Show that $f(x)$ is a proper probability distribution. Also compute the cmf (cumulative mass function) for X, $F(x) = \mathbb{P}(X \leq x)$.

5. The following are two key properties of expectation.

   (a) Let X and Y independent discrete random variables. Prove that

   $$\mathbb{E}[XY] = \mathbb{E}[X]\mathbb{E}[Y].$$

   Does this property imply independence?

   (b) Now let X and Y be arbitrary discrete random variables, and $c, d \in \mathbb{R}$ arbitrary constants. Prove that

   $$\mathbb{E}[cX + dY] = c\mathbb{E}[X] + d\mathbb{E}[Y].$$

   Does this result require independence of X and Y? Show that if $\mathbb{P}(X \leq Y) = 1$ then $\mathbb{E}[X] \leq \mathbb{E}[Y]$.