1. This problem provides some experience with the Chomsky hierarchy (regular, context-free, context-sensitive, and unrestricted grammars). For each of the following languages (denoted $L$), provide a grammar $G$ occupying the most restrictive level of the hierarchy possible.

   (a) $\Sigma = \{a, b\}$, $L(G) = \{a^n b^n \mid n \in \mathbb{N}\}$.

   (b) $\Sigma = \{a, b\}$, $L(G) = \{\alpha \in \Sigma^* \mid \alpha \text{ is a palindrome}\}$.

   (c) $\Sigma = \{a, b\}$, $L(G) = \{a^n b^m \mid n \in \mathbb{N} \text{ odd}, m \in \mathbb{N} \text{ even}\}$.

   (d) $\Sigma = \{a, b, c\}$, $L(G) = \{a^n b^n c^n \mid n \in \mathbb{N}\}$.

2. Transform the following context-free grammars into Chomsky normal form.

   (a)
   $$\begin{aligned} S &\rightarrow aA \mid bA \mid aE \mid bE \\ A &\rightarrow S \\ E &\rightarrow \epsilon \end{aligned}$$

   (b)
   $$\begin{aligned} S &\rightarrow ab \mid A \\ A &\rightarrow bAaA \mid b \end{aligned}$$

3. In this problem, we study some properties of the Nussinov algorithm for RNA folding. Recall that, given an RNA sequence $R = r_1 \ldots, r_L$, $B(i,j)$ is the maximal number of base-pairings for the substring $r_i \ldots r_j$, and satisfies the recursion,

$$B(i,j) = \max \begin{cases} B(i+1, j-1) + \sigma(r_i, r_j), \\ B(i+1, j), \\ B(i, j-1), \\ \max_{i<k<j-1}\{B(i,k) + B(k+1,j)\}, \end{cases}$$

for $i + 1 < j$, with base cases $B(i,i) = 0$ and $B(i, i+1) = 0$. Here $\sigma(r_i, r_j) = 1$ if $r_i$ and $r_j$ can base pair, 0 otherwise.

(a) (modified from Durbin, question 10.3) What is the minimum length of a hairpin loop in the algorithm above? Modify the Nussinov folding algorithm so that hairpin loops have a minimum length of $h$.

(b) How can you modify the Nussinov folding algorithm to handle *circular* RNA?