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# Lab Report: Differentiator and Integrator Circuits

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**Abstract**

In the realm of electronics and signal processing, a profound understanding of circuit behavior is paramount for designing high-efficiency systems. The primary objective of this experiment was to delve into the intricacies of differentiator and integrator circuits, shedding light on their frequency responses. To achieve this, custom-tailored circuits were meticulously constructed to fulfill the distinct roles of measuring rate of change and cumulative input. Through comprehensive frequency sweeps that encompassed a spectrum from low to high frequencies, intricate data regarding gain and phase behaviors were collected and systematically scrutinized, revealing distinct patterns and behaviors across different frequency ranges.

# Contents

# 1 Theoretical background

This section provides an overview of the fundamental principles underlying the experience.

## 1.1 Feedback

A feedback loop is created when all or some portion of the output is fed back to the input. Negative feedback occurs when the fed-back output signal has a relative phase of $\pi$ with respect to the input signal, while positive feedback occurs when the signals are in phase.

## 1.2 Operational amplifier

An operational amplifier, often abbreviated as op-amp, is a voltage-controlled voltage generator. The op-amp's primary function is to amplify the difference in voltage between its two input terminals, usually labeled as the inverting input (-) and the non-inverting input (+). It has a high input impedance, meaning it draws very little current from the input sources, and a low output impedance, enabling it to drive other circuit components effectively. In its ideal form, an op-amp has infinite voltage gain, however, real-world op-amps have limitations due to factors like power supply voltage, noise, and frequency response. Op-amps are often used in circuits with negative feedback.

## 1.3 Virtual ground

The concept of a virtual ground is a fundamental aspect of negative feedback operational amplifier (op-amp) circuits. In these circuits, a virtual ground is created at the inverting input terminal of the op-amp, even though it is not physically connected to the ground reference. This virtual ground serves as a reference point for analyzing and designing circuits, allowing simplified calculations and intuitive understanding. In fact, negative feedback attempts to drive the inverting input to a voltage that matches the non-inverting input, which can result in $V^+ - V^- = 0\,\mathrm{V}$ and, consequently, $I^+ = I^-$. An op-amp is a voltage-controlled voltage generator, so it does not absorb current at the input (behaves like an open circuit), hence $I^+ = I^- = 0\,\mathrm{A}$.

## 1.4 Frequency response

The frequency response of a circuit is the quantitative description of how its output responds to its input signal, within a range of different frequencies. The frequency response characterizes systems in the frequency domain, just as the impulse response characterizes systems in the time domain. In fact, the frequency response is the Fourier transform of the impulse response.
For a signal $x(t)$, the Fourier transform is defined as:

$$X(\nu) = \int_{-\infty}^{\infty} dt\; x(t)e^{-j2\pi\nu t}$$

where $\nu$ is the frequency and $j$ is the imaginary unit.

The frequency response of a system is defined as:

$$H = \frac{X_{out}(\nu)}{X_{in}(\nu)}$$

where $X_{in}(\nu)$ and $X_{out}(\nu)$ are the Fourier transforms of the input and output signals, respectively. The frequency response $H$ lies in the complex field as it employs a complex extension of Ohm's law to articulate the relationship between voltage and current in a circuit:

$$V = Z \cdot I$$

where $Z$ is the complex impedance (measured in $\Omega$).
Consequently, to describe the frequency response of a circuit, it is necessary to consider both the magnitude and the phase of the output signal with respect to the input signal as polar representation of $H$ in the complex plane.

## 1.5 Differentiator circuit

The differentiator circuit's functionality is firmly grounded in the mathematical concept of differentiation, a fundamental calculus concept. Analogous to how differentiation calculates the rate of change of a mathematical function concerning its independent variable, the differentiator circuit accentuates swift variations in input signal amplitude.

The differentiator circuit's design involves the integration of passive and active electronic components: capacitors and resistors play pivotal roles in shaping the circuit's frequency response, allowing it to differentiate signal components with different frequencies; while the operational amplifier is employed to achieve high gain and accurate differentiation by ensuring that the inverting input adheres to a voltage level analogous to ground potential.
The schematic diagram of the differentiator circuit is illustrated in Figure 1.

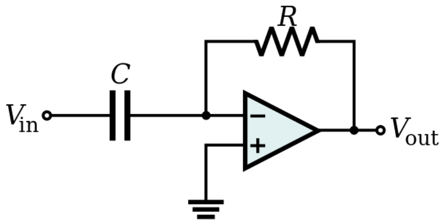


Figure 1: Differentiator circuit schematic.

Applying the Kirchhoff's circuit laws at the op-amp's inverting input terminal, which is held at ground potential due to the virtual ground concept, it is possible to establish that the current flowing in the capacitor is equal to the current through the resistor, $i_C(t) = i_R(t), \forall t$, and further more:

$$C \cdot \frac{d}{dt}(V_{in}(t) - V^-) = \frac{V^- - V_{out}(t)}{R}$$

Remembering that $V^- = 0\,\mathrm{V}$, by solving the equation for $V_{out}$, it is shown that the output of the differentiator circuit is proportional to the rate of change of the input signal with respect to time:

$$V_{out}(t) = -RC \cdot \frac{d}{dt} V_{in}(t)$$

Using Fourier's transform and its property, it is possible to further simplify the first order differential equation to obtain the frequency response of the differentiator circuit:

$$V_{out}(\nu) = -RC \cdot j2\pi\nu V_{in}(\nu)$$

Therefore, the frequency response of the differentiator circuit is directily proportional to the input signal's frequency. Since all the constants are real and positive, the frequency response lies in the complex field's negative imaginary axis, it is expected to find a phase of $-\frac{\pi}{2}$.

## 1.6 Integrator circuit

The integrator circuit can be easily obtained from a differentiator circuit by simply swapping the resistor and the capacitor. The schematic diagram of the circuit is illustrated in Figure 2.

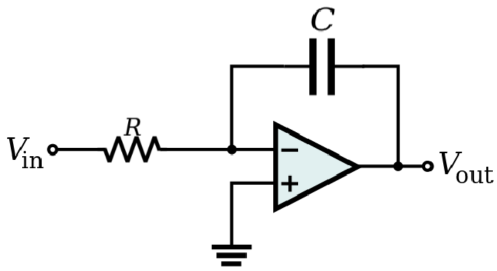


Figure 2: Integrator circuit schematic.

Similar to the approach taken with the differentiator circuit, Kirchhoff's circuit laws can be employed at the virtual ground node to obtain the equation that characterizes the integrator circuit's behavior:

$$\frac{V_{in}(t)}{R} = -C \cdot \frac{d}{dt} V_{out}(t) \quad \Longrightarrow \quad V_{out}(t) = -\frac{1}{RC} \cdot \int_{0\,\mathrm{s}}^{t} d\tau \; V_{in}(\tau) + V(0\,\mathrm{s})$$

Notably, the output voltage is proportional to the integral of the input voltage over time, with adjustments related to the time constant and the initial conditions of the circuit.
The term $V(0\,\mathrm{s})$ represents the initial condition of the output voltage at $t = 0\,\mathrm{s}$ and introduces an offset or a foundational voltage level to the integrated output signal. This offset can serve as a calibration factor, facilitating the alignment of the output signal with a desired reference level.
Using Fourier's transform and its property, it is possible to further simplify the integral equation to obtain the frequency response of the circuit:

$$V_{out}(\nu) = \frac{j}{RC \cdot 2\pi\nu} \cdot V_{in}(\nu)$$

The integrator circuit's frequency response exhibits an inverse relationship with the input signal's frequency and is situated along the positive imaginary axis within the complex domain. Therefore it is expected to measure a phase of $\frac{\pi}{2}$.
The limitations of an ideal integrator can be minimized by adding resistor in parallel with the capacitor, which avoids op-amp going into open loop configuration at low frequencies. For instance, the initial measurements demonstrated an unexpected shift, it became imperative to introduce a robust resistor ($R = 0.98\,\mathrm{M\Omega}$) to the circuit. This strategic inclusion rectified the observed shifts in measurements.

## 2 Equipment

The experimental setup consists of the following components:

- *Bread-board*
- Probe and coaxial cables
- BNC T-splitter
- Resistor ($9.94\,\mathrm{k\Omega}$)
- Capacitor ($2.27\,\mathrm{nF}$)
- Operational Amplifier (op-amp uA741): used to amplify the voltage difference between its input terminals.
- Power supplies: provide the necessary voltage levels for the op-amp.
- Signal generator Agilent 33220A: provides the input voltage signal.
- Oscilloscope Tektronix TDS2012C: used to measure and visualize the input and output waveforms and their time shift.

## 3 Procedure

The following steps were undertaken to conduct the experiment:

1. **Frequency Setup**: Begin by setting the desired frequency of the input signal using the function generator. It is advised to follow a logarithmic scale.
2. **Input Signal Configuration**: Configure the input signal to align with the chosen frequency and amplitude specifications. Opt for a sinusoidal waveform as input signal. Using the BNC T-splitter establish the connection between the configured signal and the circuit's input and oscilloscope.
3. **Output Signal Measurement**: Employ an oscilloscope to measure the output signal of the circuit. Connect the oscilloscope to the circuit's output to visualize its response to the input signal.
4. **Voltage Measurement Across Resistor (Differentiator Circuit)**: For the differentiator circuit, measure the voltage across the resistor using the oscilloscope.
5. **Voltage Measurement Across Capacitor (Integrator Circuit)**: For the integrator circuit, introduce an additional resistor in parallel with the capacitor to incorporate an offset voltage and measure the voltage across the capacitor using the oscilloscope.
6. **Time Difference Measurement**: Further analyze the phase shift between the input and output signals by measuring the time difference between corresponding points on the two signals using the oscilloscope's cursors.
7. **Gain Calculation**: Calculate the gain of the circuit in decibels (dB) using the formula:

   $$\text{Gain}_{\text{dB}} = 20\log_{10}\left(\frac{V_{\text{out}}}{V_{\text{in}}}\right)$$

   where $V_{\text{out}}$ represents the measured output voltage and $V_{\text{in}}$ is the input voltage.
8. **Phase Shift Calculation**: From the time difference measured, determine the phase shift between the input and output signals in radians through the formula:

   $$\text{Phase shift (radians)} = 2\pi \cdot \text{Time difference} \cdot \text{Frequency}$$

   The phase shift is positive when the output signal crosses zero before the input signal; and negative when the output signal crosses zero after the input signal.

# 4 Discussion

The experimental investigation of the differentiator and integrator circuits' frequency response characteristics is presented in this section.

## 4.1 Bode Plots

In the endeavor to comprehend the behaviors of these circuits across various frequency ranges, Bode plots were utilized as graphical representations, revealing gain and phase shift patterns across a spectrum of frequencies.
Figures 3 and 4 showcase the Bode plots of the differentiator and integrator circuits, respectively. To generate these plots, data was extracted from a text file containing recorded frequency values along with their corresponding circuit gain (expressed in decibels) and phase shift (in radians). The dataset used for each Bode plot is also included in the Appendix (Tables 3 and 4).

A logarithmic scale is employed on the x-axis to cover the range of frequencies effectively and to visualize the frequency response of the circuits in a more intuitive manner, as it allows to represent with lines both directily and inversely proportional relationships between the frequency response and the input signal's frequency.
The incorporation of dual y-axes facilitates the visual comprehension of the frequency response traits exhibited by the circuits: the left y-axis is dedicated to displaying gain, while the right y-axis portrays phase shift. To enhance clarity, data is displayed with different color markers: blue for the gain and yellow for the phase shift.

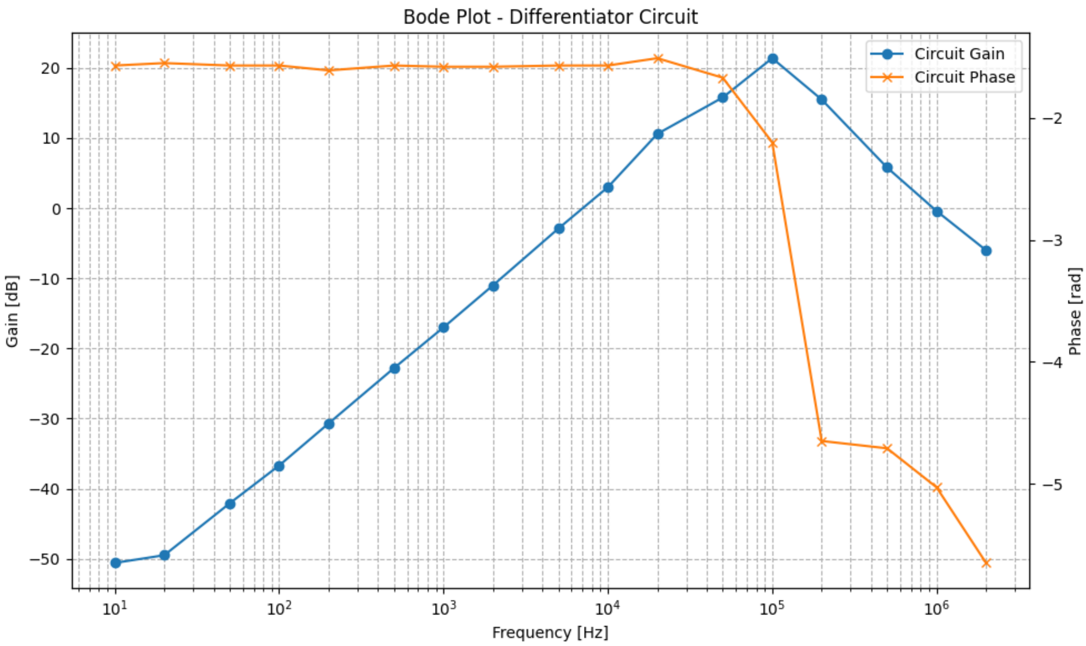


Figure 3: Bode plot for the differentiator circuit.

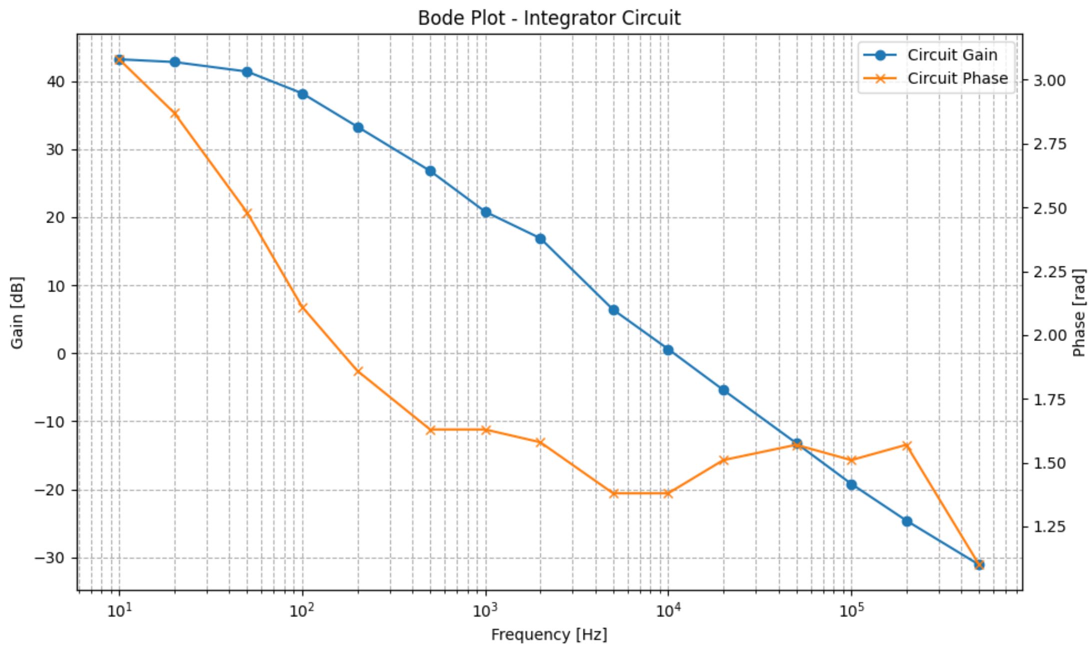


Figure 4: Bode plot for the integrator circuit.

For a wide range of frequencies, the differentiator circuit's gain is linearly proportional to the input signal's frequency and the phase shift is constant and equal to $-\frac{\pi}{2}$, as theoretically predicted. However, when a specific frequency is attained, the circuit experiences a resonance effect, leading to a peak in gain and a corresponding drop in phase shift. At higher frequencies beyond this resonance point, the gain decreases due to non-ideal behavior of the op-amp.
In the integrator circuit, at low frequencies the gain is almost constant and the phase is $\pi$ because the capacitor effectively acts as a short circuit in this regime. As the frequency increases, the gain decreases linearly and the phase shift decreases until it reaches a point of inflection with horizontal tangent at $\frac{\pi}{2}$, as theoretically predicted. At higher frequencies, the circuit stops behaving as an integrator due to non-ideal behavior of the op-amp.

## 4.2 Simulated results

To delve deeper into the analysis of our experimental findings, the differentiator and integrator circuits were simulated using Ngspice, a renowned simulation tool based on the Berkeley SPICE software. This simulation process aimed to replicate the behavior of the electronic circuits under investigation. The net-lists employed for simulating these circuits can be found in the Appendix for reference.

Figures 5 and 6 provide a platform for comparing the experimental data with the corresponding simulated results for the differentiator circuit. Similarly, figures 7 and 8 present a comparative analysis between experimental and simulated data for the integrator circuit.

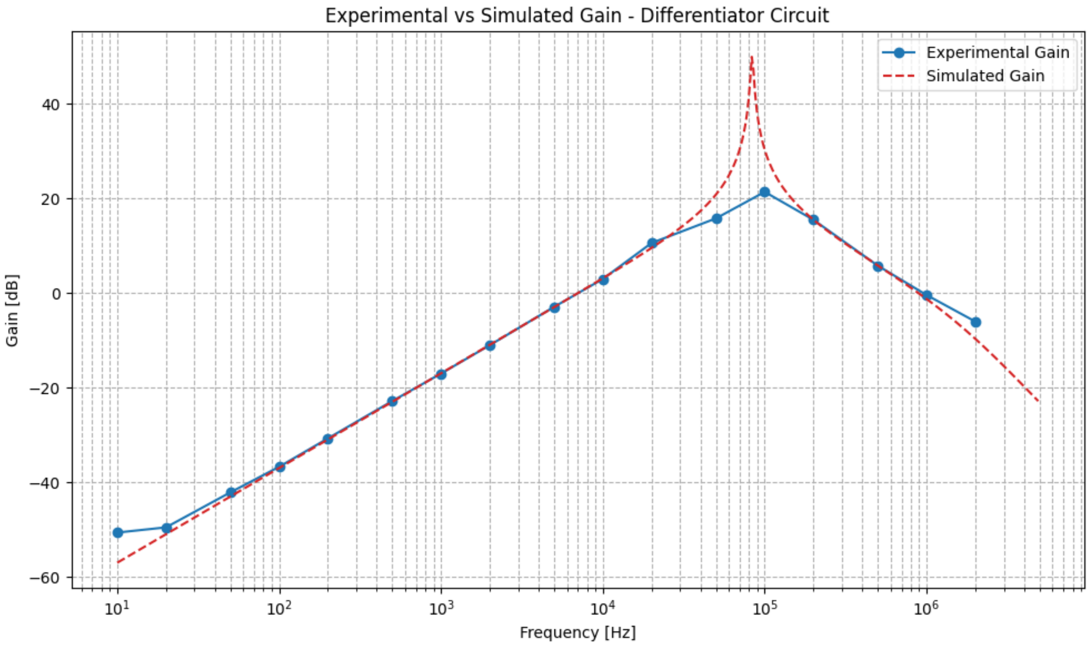


Figure 5: Gain comparison between experimental and simulated data for the differentiator circuit.

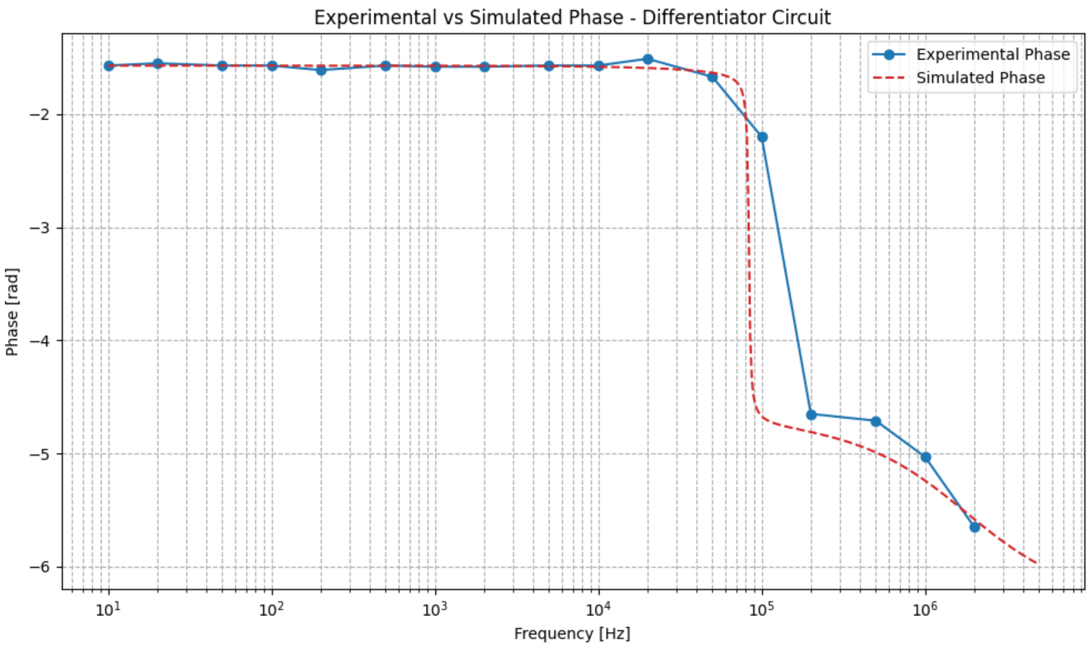


Figure 6: Phase comparison between experimental and simulated data for the differentiator circuit.

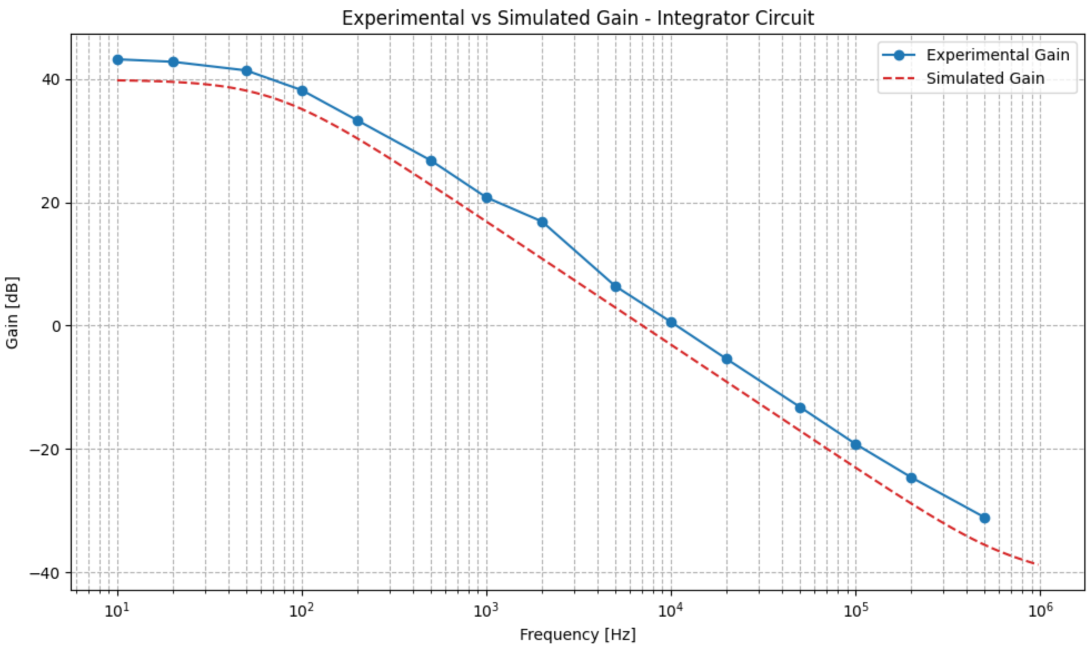


Figure 7: Gain comparison between experimental and simulated data for the integrator circuit.

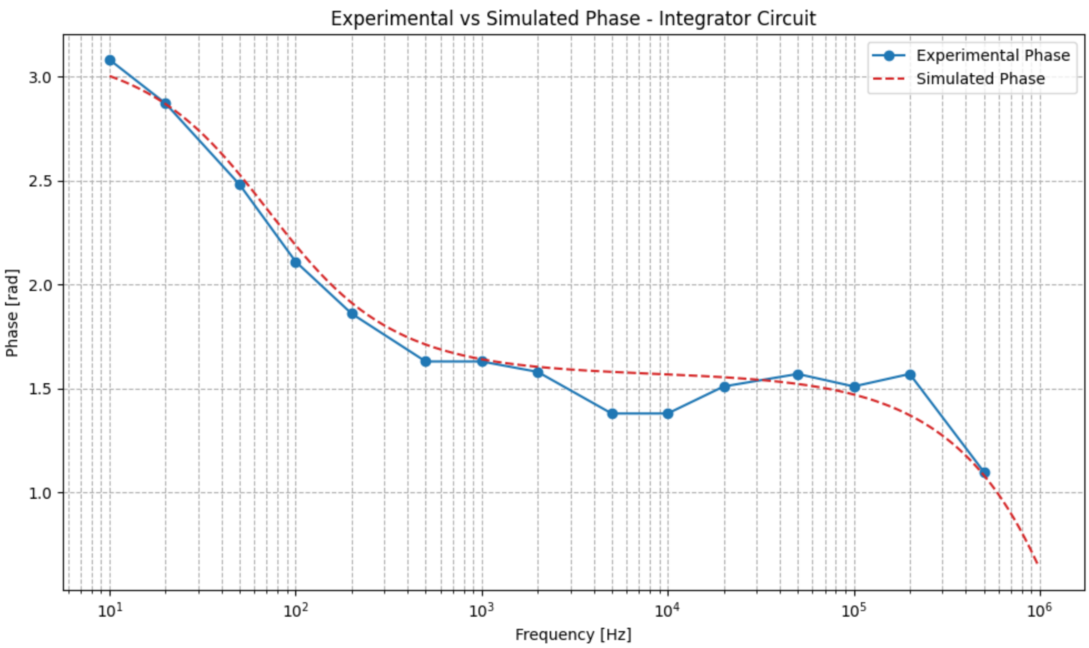


Figure 8: Phase comparison between experimental and simulated data for the integrator circuit.

The alignment between our real-world tests and simulated results demonstrates the harmony between our theoretical predictions and observed outcomes, underscoring the accuracy of our hands-on methods. However, it's noteworthy that a systematic error is apparent in the integrator circuit's gain.

### 4.3 Gain slope and Cutoff Frequency

It was previously stated that the gain of the differentiator circuit is directily proportional to the frequency of the input signal, while the integrator's is inversely proportional. It is possible to calculate the gain slope by doing a linear regression of the data. Then it will be possible to find the cutoff frequency ($f_c$), at which the output voltage is equal to the input voltage and consequently the circuit's gain is equal to 0 dB. It is defined as

$$f_c = \frac{1}{2\pi RC}$$

where $R$ and $C$ are the circuit's resistance and capacitance.
Due to the op-amp's not ideal behavior, let just consider a frequency range where the gain is clearly linearly proportional to the frequency: from $1 \times 10^2$ Hz to $1 \times 10^4$ Hz for the differentiator circuit and from $5 \times 10^2$ Hz to $1 \times 10^5$ Hz for the integrator circuit.

The data presented in Tables 1 and 2 exhibits a consistent alignment between theoretical, experimental, and simulated results for both the differentiator and integrator circuits.
The gain slope of the integrator circuit is not as close to the theoretical value as the differentiator's, but it is still within the error margin. This leads to a higher cutoff frequency, which is also within the error margin, but confirms the prediction of a systematic error in the gain of the integrator circuit.

Table 1: Gain slope comparison between theoretical, experimental and simulated data.

| **Circuit** | **Theoretical** | **Experimental** | **Simulated** |
|---|---|---|---|
| Differentiator | 20.0 dB/decade | 19.86 dB/decade | 20.03 dB/decade |
| Integrator | −20.0 dB/decade | −20.25 dB/decade | −19.97 dB/decade |

Table 2: Cutoff frequency comparison between theoretical, experimental and simulated data.

| **Circuit** | **Theoretical** | **Experimental** | **Simulated** |
|---|---|---|---|
| Differentiator | 7053.5 Hz | 7070.4 Hz | 7020.3 Hz |
| Integrator | 7053.5 Hz | 11 143.9 Hz | 7002.0 Hz |

Except for the integrator's experimental data, the cutoff frequencies are within a 1% error margin from the theoretical value.

# Appendix

## Experimental datasets

Table 3: Differentiator Circuit Data

| $\nu$ [kHz] | $V_{in}$ [V] | $V_{out}$ [V] | G [dB] | $\tau$ [ms] | $\phi$ [rad] |
|---|---|---|---|---|---|
| 0.01 | 5.08 | 0.015 | $-50.6$ | $-25.0$ | $-1.57$ |
| 0.02 | 5.08 | 0.017 | $-49.5$ | $-12.3$ | $-1.55$ |
| 0.05 | 5.08 | 0.04 | $-42.1$ | $-5.0$ | $-1.57$ |
| 0.1 | 5.08 | 0.074 | $-36.7$ | $-2.5$ | $-1.57$ |
| 0.2 | 5.08 | 0.15 | $-30.7$ | $-1.28$ | $-1.61$ |
| 0.5 | 5.04 | 0.36 | $-22.8$ | $-0.5$ | $-1.57$ |
| 1 | 5.08 | 0.72 | $-17.0$ | $-0.252$ | $-1.58$ |
| 2 | 5.08 | 1.43 | $-11.0$ | $-0.126$ | $-1.58$ |
| 5 | 5.08 | 3.64 | $-2.9$ | $-0.05$ | $-1.57$ |
| 10 | 5.08 | 7.20 | 3.0 | $-0.025$ | $-1.57$ |
| 20 | 5.08 | 17.20 | 10.6 | $-0.012$ | $-1.51$ |
| 50 | 0.10 | 0.62 | 15.8 | $-0.0053$ | $-1.67$ |
| 100 | 0.10 | 1.20 | 21.4 | $-0.0035$ | $-2.20$ |
| 200 | 0.10 | 0.61 | 15.5 | $-0.0037$ | $-4.65$ |
| 500 | 0.10 | 0.20 | 5.8 | $-0.0015$ | $-4.71$ |
| 1000 | 0.10 | 0.096 | $-0.4$ | $-0.0008$ | $-5.03$ |
| 2000 | 0.10 | 0.05 | $-6.0$ | $-0.000\,45$ | $-5.65$ |

Table 4: Integrator Circuit Data

| $\nu$ [kHz] | $V_{in}$ [V] | $V_{out}$ [V] | G [dB] | $\tau$ [ms] | $\phi$ [rad] |
|---|---|---|---|---|---|
| 0.01 | 0.1 | 14.4 | 43.2 | 49.0 | 3.08 |
| 0.02 | 0.1 | 13.8 | 42.8 | 22.8 | 2.87 |
| 0.05 | 0.1 | 11.8 | 41.4 | 7.9 | 2.48 |
| 0.1 | 0.1 | 8.16 | 38.2 | 3.36 | 2.11 |
| 0.2 | 0.1 | 4.6 | 33.3 | 1.48 | 1.86 |
| 0.5 | 0.1 | 2.2 | 26.8 | 0.52 | 1.63 |
| 1 | 0.1 | 1.1 | 20.8 | 0.26 | 1.63 |
| 2 | 0.2 | 1.4 | 16.9 | 0.126 | 1.58 |
| 5 | 0.2 | 0.42 | 6.4 | 0.044 | 1.38 |
| 10 | 0.5 | 0.54 | 0.6 | 0.022 | 1.38 |
| 20 | 0.5 | 0.27 | $-5.4$ | 0.012 | 1.51 |
| 50 | 0.5 | 0.11 | $-13.2$ | 0.005 | 1.57 |
| 100 | 5.1 | 0.56 | $-19.2$ | 0.0024 | 1.51 |
| 200 | 5.1 | 0.3 | $-24.6$ | $0.001\,25$ | 1.57 |
| 500 | 10.0 | 0.28 | $-31.1$ | $0.000\,35$ | 1.10 |

## NGSPICE Op Amp Model and Circuit Net-lists

### Op Amp Model (uA741)

```
* Model for uA741 Op Amp (from EVAL library in PSpice)
```

```
* connections: non-inverting input
*              | inverting input
*              | | positive power supply
*              | | | negative power supply
*              | | | | output
*              | | | | |
.subckt uA741  1 2 3 4 5

c1 11 12 8.661E-12
c2 6 7 30.00E-12
dc 5 53 dy
de 54 5 dy
dlp 90 91 dx
dln 92 90 dx
dp 4 3 dx
egnd 99 0 poly(2),(3,0),(4,0) 0 .5 .5
fb 7 99 poly(5) vb vc ve vlp vln 0 10.61E6 -1E3 1E3 10E6 -10E6
ga 6 0 11 12 188.5E-6
gcm 0 6 10 99 5.961E-9
iee 10 4 dc 15.16E-6
hlim 90 0 vlim 1K
q1 11 2 13 qx
q2 12 1 14 qx
r2 6 9 100.0E3
rc1 3 11 5.305E3
rc2 3 12 5.305E3
re1 13 10 1.836E3
re2 14 10 1.836E3
ree 10 99 13.19E6
ro1 8 5 50
ro2 7 99 100
rp 3 4 18.16E3
vb 9 0 dc 0
vc 3 53 dc 1
ve 54 4 dc 1
vlim 7 8 dc 0
vlp 91 0 dc 40
vln 0 92 dc 40
.model dx D(Is=800.0E-18 Rs=1)
.model dy D(Is=800.00E-18 Rs=1m Cjo=10p)
.model qx NPN(Is=800.0E-18 Bf=93.75)
.ends
```

### Differentiator Circuit

```
.include UA741.SPI

Vin 1 0 DC 0 AC 1
Rgen 0 1 50
C 1 2 2.27e-9
R 2 3 9.94K
XOA 0 2 10 11 3 UA741
VSP 10 0 12V
VSN 11 0 -12V

.control
ac dec 100 10 5e6
```

### Integrator Circuit

```
.include UA741.SPI

Vin 1 0 DC 0 AC 1
Rgen 0 1 50
R 1 2 9.94K
Roff 2 3 0.98e6
C 2 3 2.27e-9
XOA 0 2 10 11 3 UA741
VSP 10 0 12V
VSN 11 0 -12V

.control
ac dec 100 10 1e6
```