Analyzing Gravitational Waves through Numerical Simulations

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Contents

Introduction

Gravitational waves can provide a large amount of information about a binary system.

Our method

- Properties of the gravitational waves
- Analysis of the gravitational wave signal
- Relation with the dynamical evolution

Einstein's Field Equation

Einstein's Field Equation is a rather complicated tensor equation

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

- Weak field linearized approximation

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \qquad |h_{\mu\nu}| \ll 1$$

- Trace-reversed perturbation metric

$$\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2} h \eta_{\mu\nu} \qquad \bar{h} = \bar{h}^{\mu\nu} \eta_{\mu\nu} = -h$$

- Lorenz gauge $\partial_\mu h^{\mu\nu} = 0$

Linearized Einstein's field equation

$$\left(\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) \bar{h}_{\mu\nu} = \frac{16\pi G}{c^4} T_{\mu\nu}$$

Linearized Einstein's Field Equation $T_{\mu\nu} = 0$

In vacuum $T_{\mu\nu} = 0$

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)\bar{h}_{\mu\nu} = 0$$

Using the Transverse Traceless gauge

$$h_{0\nu}^{\mathsf{TT}} = 0 \qquad \eta^{\mu\nu} h_{\mu\nu}^{\mathsf{TT}} = 0 \qquad \partial^{\mu} h_{\mu\nu}^{\mathsf{TT}} = 0$$

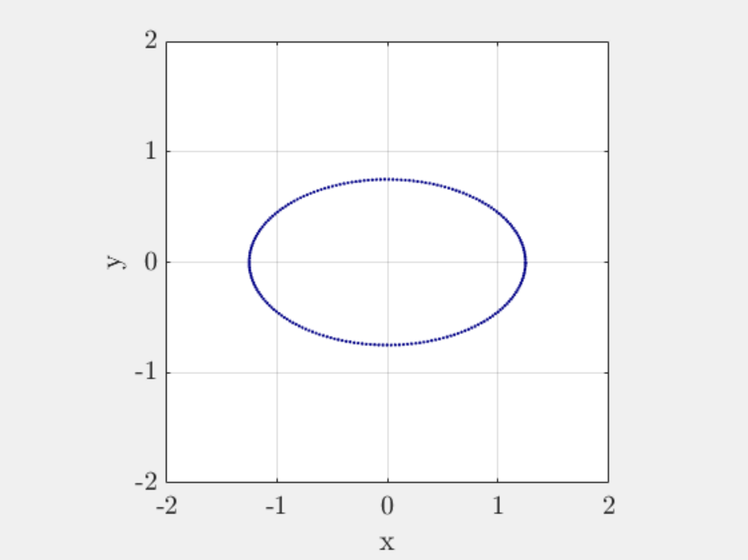
A solution of the above equation is

$$\bar{h}_{\mu\nu}^{\mathsf{TT}} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

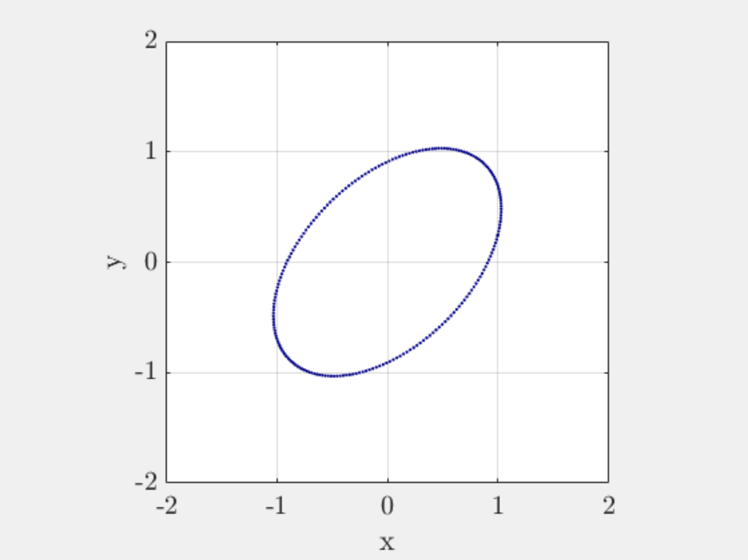
where

$$h_+ = A_{11}\cos\left(\omega(t - z/c)\right) \qquad h_\times = A_{12}\cos\left(\omega(t - z/c)\right)$$

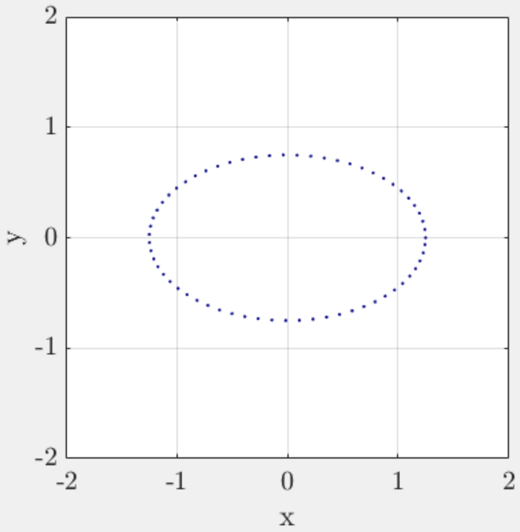
$+$ polarization $h_\times = 0$



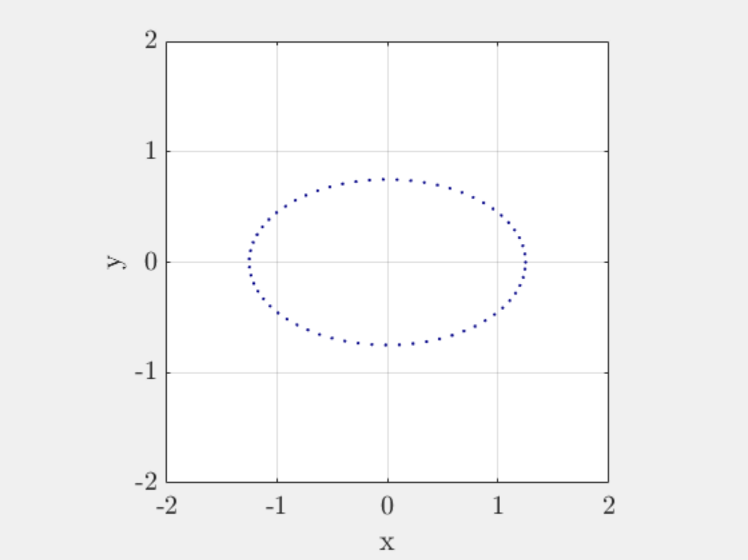
$\times$ polarization $h_+ = 0$



Left-handed polarization



Right-handed polarization



Linearized Einstein's Field Equation $T_{\mu\nu} \neq 0$

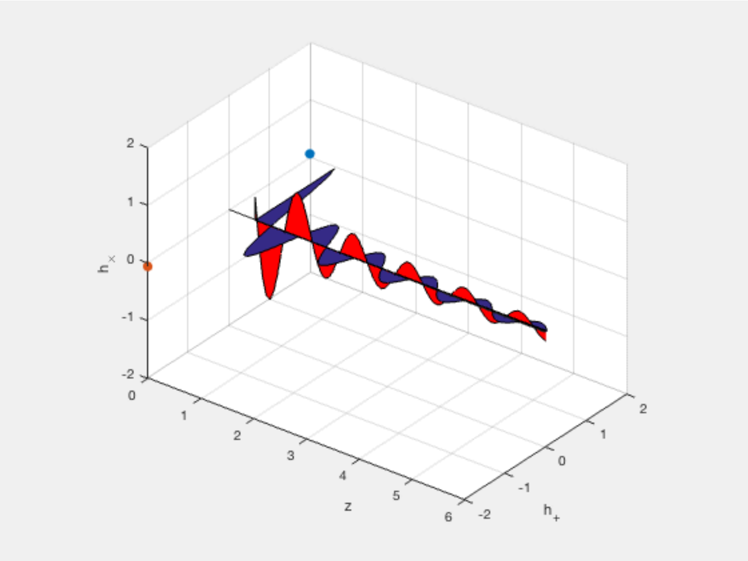
$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)\bar{h}_{\mu\nu} = \frac{16\pi G}{c^4}T_{\mu\nu}$$

- **far field approximation**
- **slowly moving source**
- **isolated system**

For an equal-mass binary system rotating at angular frequency ω and orbital radius R, the solution is

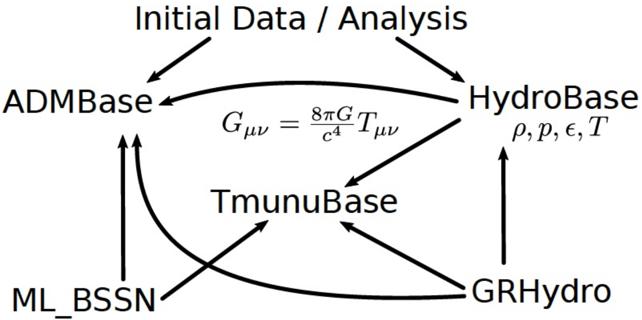
$$h_{ij}^{\mathsf{TT}} = \frac{8\,GM\,R^2\omega^2}{c^4\,r}\begin{bmatrix} -\cos 2\omega t_r & -\sin 2\omega t_r & 0 \\ -\sin 2\omega t_r & \cos 2\omega t_r & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

slowly-moving equal-mass binary system



Numerical Evolution of Compact Binaries

Einstein Toolkit is an open-source computational infrastructure for numerical relativity based on the Cactus Framework.



Quasi-equilibrium initial conditions of compact objects

- **Binary Black Holes (BBH)**

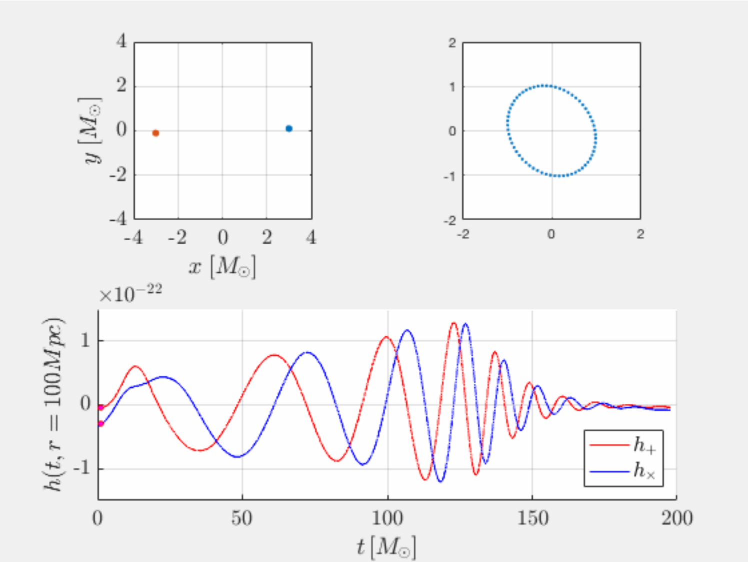
simulation name	par_b	par_m_plus	par_P_plus[1]
BBH-b3	3	0.47656	+0.13808
BBH-b4	4	0.48243	+0.11148
BBH-b5	5	0.48595	+0.095433
BBH-b6	6	0.48830	+0.084541
BBH-b7	7	0.48997	+0.076578
BBH-b10	10	0.49299	+0.061542

- **Binary Neutron Stars (BNS)**

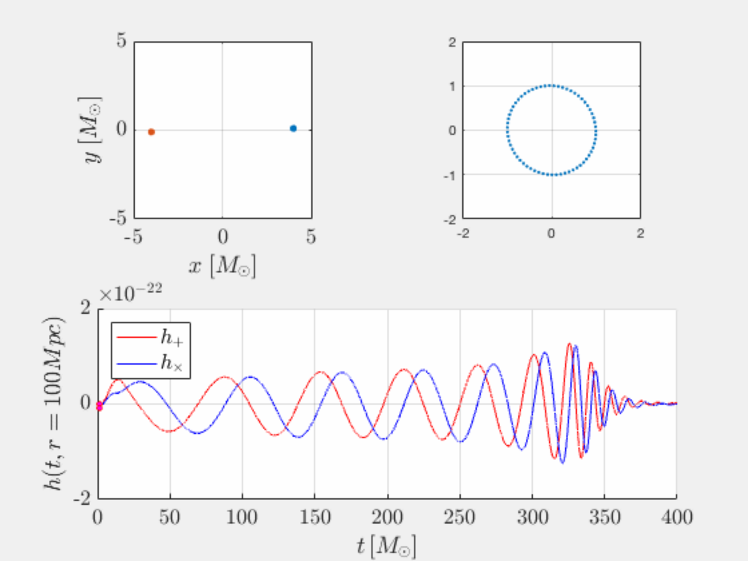
$$M_{\mathsf{TOT}} = 3.251\, M_{\odot} \qquad b = 15.285\, M_{\odot}$$

Ideal fluid approximation

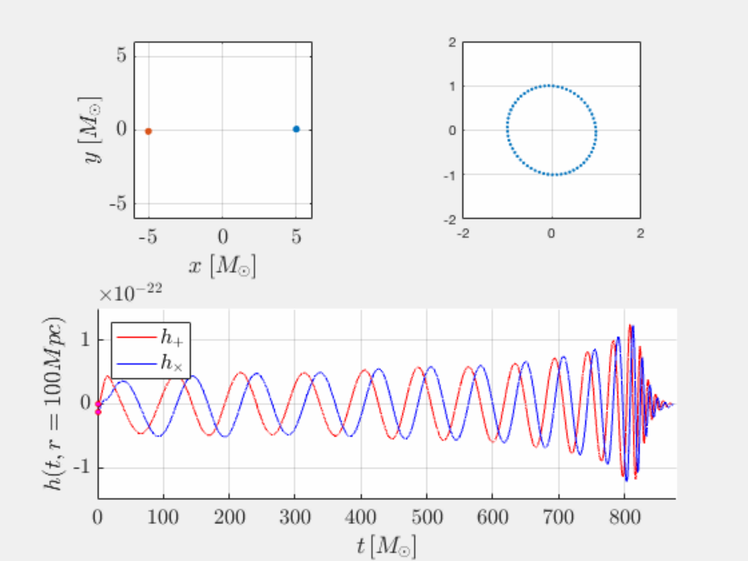
BBH-b3


4
2
0
-2
-4
$y\ [M_\odot]$
$x\ [M_\odot]$
2
1
0
-1
-2
$\times 10^{-22}$
$h(t, r = 100Mpc)$
$t\ [M_\odot]$
0
50
100
150
200
h_+
$h_\times$

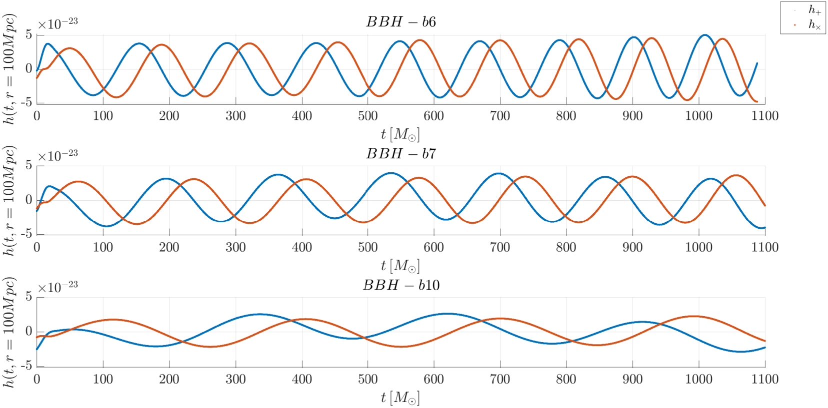
BBH-b4


5
0
-5
-5
0
5
$y\ [M_\odot]$
$x\ [M_\odot]$
2
1
0
-1
-2
-2
0
2
$\times 10^{-22}$
2
0
-2
0
50
100
150
200
250
300
350
400
h_+
$h_\times$
$h(t, r = 100Mpc)$
$t\ [M_\odot]$

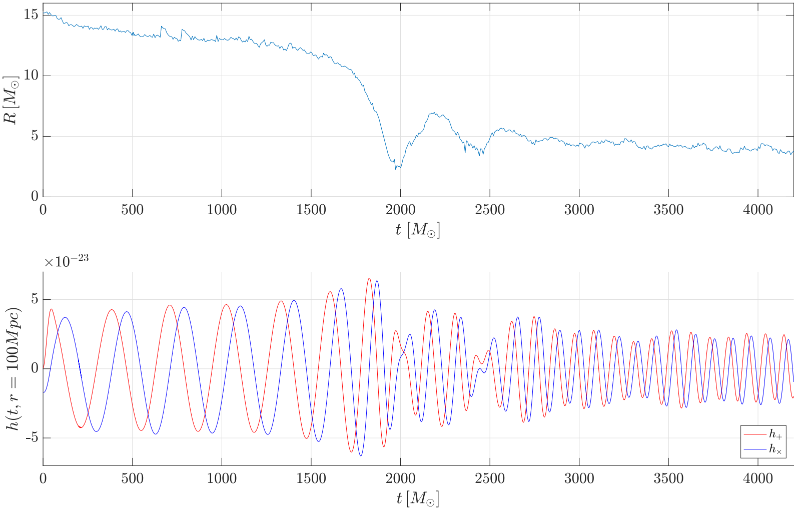
BBH-b5


5
0
-5
$y\ [M_\odot]$
-5 0 5
$x\ [M_\odot]$
2
1
0
-1
-2
-2 0 2
$\times 10^{-22}$
h_+
$h_\times$
1
0
-1
$h(t, r = 100Mpc)$
0 100 200 300 400 500 600 700 800
$t\ [M_\odot]$

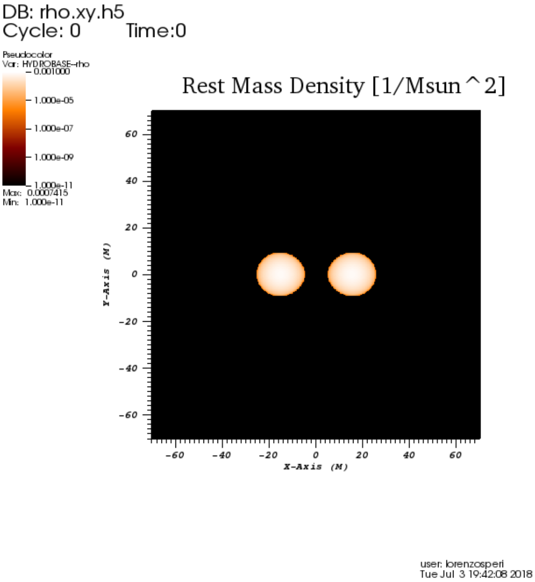
BBH-b6, BBH-b7, BBH-b10



BNS


15
10
5
0
$R\,[M_\odot]$
0
500
1000
1500
2000
2500
3000
3500
4000
$t\,[M_\odot]$
$\times 10^{-23}$
5
0
-5
$h(t, r = 100Mpc)$
h_+
$h_\times$
0
500
1000
1500
2000
2500
3000
3500
4000
$t\,[M_\odot]$

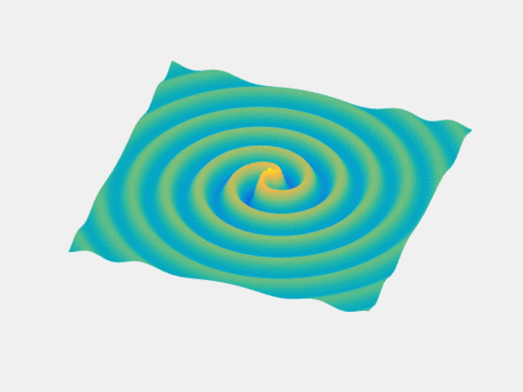
Rest Mass Density Evolution



Conclusions

"Gravitational waves will show us details of the bulk motion of dense concentrations of energy"

Kip S. Thorne



References and Codes

The codes used for the numerical simulations, the scripts for the data analysis, figures and animations are available at the website

`https://github.com/lorenzsp/thesis`