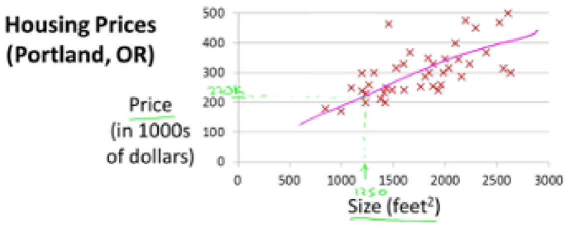
1 Linear regression (for regression)

Model a scalar target with one or more quantitative features.
Although regression computes a linear combination, features can be transformed by nonlinear functions if relationships are known or can be guessed.



1.1 Univariate linear regression

- Description

  Simple linear regression is a statistical method that studies the relationship between two variables:
  - x: the predictor, explanatory, independent variable,
  - y: the response, outcome, dependent variable.

- Model's hypothesis

  $h(x) = \theta_0 + \theta_1 * x = \hat{y}$

- Model's parameters (# 2)

  - θ_0, θ_1

- Cost function (in this case, the squared error function)

  $J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(\hat{y}^i - y^{(i)}\right)^2$

- Goal

  $\min_{\theta_0, \theta_1} J(\theta_0, \theta_1)$

- Algorithm

  - notations

    - x: the independent variable
    - y: the dependent variable
    - m: the number of training examples
    - $x^{(i)}$: the input value of the i^{th} training example
    - $y^{(i)}$: the target value of the i^{th} training example
    - $\hat{y}^{(i)}$: the prediction made on the i^{th} training example by the current hypothesis function
    - α: the learning rate; determines how big steps we take when updating the θ parameters

  - gradient descent

    - start with some initial values for θ_0 and θ_1 (usually zero)

    - keep changing θ_0 and θ_1 to reduce $J(\theta_0, \theta_1)$

      - $\theta_0 = \theta_0 - \alpha \frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1) = \theta_0 - \alpha \left(\frac{1}{m} \sum_{i=1}^{m} \left(\hat{y}^i - y^{(i)}\right) \right)$

      - $\theta_1 = \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1) = \theta_1 - \alpha \left(\frac{1}{m} \sum_{i=1}^{m} \left(\hat{y}^i - y^{(i)}\right) \cdot x^{(i)} \right)$

- directly use the following **formulas**
    - $\theta_1 = \frac{\sum_{i=1}^{m}(x^{(i)}-\bar{x})(y^{(i)}-\bar{y})}{\sum_{i=1}^{m}(x^{(i)}-\bar{x})^2}$
    - $\theta_0 = \bar{y} - \theta_1\bar{x}$
- hyperparameters
    - α: if too small, slow gradient descent; if too large, gradient descent may fail to converge

1.2 Multivariate linear regression

- Description

  Multivariate linear regression is a statistical method that studies the relationship between multiple variables:
  - n variables $x = \{x_1, x_2, ..., x_n\}$: the predictor, explanatory, independent variables,
  - one y variable: the response, outcome, dependent variable.

- Model's hypothesis

  $h(x) = \theta_0 + \theta_1 * x_1 + \theta_2 * x_2 + ... + \theta_n * x_n = \hat{y}$

- Model's parameters (# $n + 1$)

  $\theta = \{\theta_0, \theta_1, ..., \theta_n\}$

- Cost function (in this case, the squared error function)

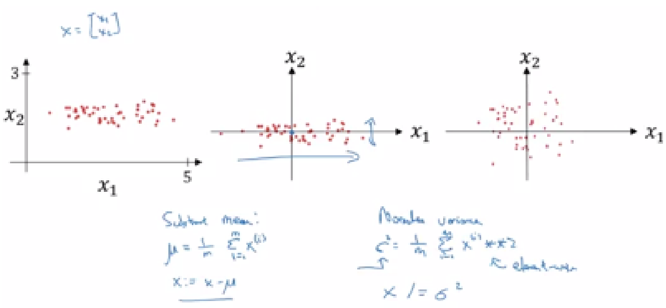
  $J(\theta) = \frac{1}{2m}\sum_{i=1}^{m}\left(\hat{y}^{(i)} - y^{(i)}\right)^2$

- Goal

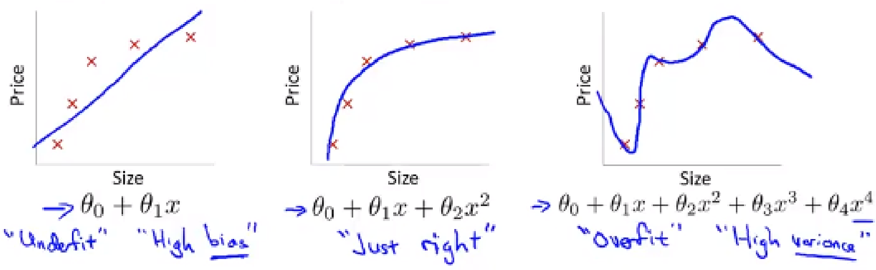
  $\min\theta J(\theta)$

- Algorithm
  - notations
    - x: the independent variables
    - y: the dependent variable
    - m: the number of training examples
    - n: the number of features representing each training example
    - $x^{(i)}$: the input values of the i^{th} training example
    - $x_j^{(i)}$: the value of the j^{th} feature of the i^{th} training example
    - $y^{(i)}$: the target value of the i^{th} training example
    - $\hat{y}^{(i)}$: the prediction made on the i^{th} training example by the current hypothesis function
    - α: the learning rate; determines how big steps we take when updating the θ parameters
  - gradient descent
    - start with some initial values for θ_0, θ_1, ..., θ_n (usually 0)
    - keep changing θs to reduce $J(\theta)$
        - $\theta_0 = \theta_0 - \alpha\frac{\partial J}{\partial\theta_0} = \theta_0 - \alpha\left(\frac{1}{m}\sum_{i=1}^{m}\left(\hat{y}^i - y^{(i)}\right)\right)$
        - $\theta_1 = \theta_1 - \alpha\frac{\partial J}{\partial\theta_1} = \theta_1 - \alpha\left(\frac{1}{m}\sum_{i=1}^{m}\left(\hat{y}^i - y^{(i)}\right) \cdot x_1^{(i)}\right)$
        - ...

- - - $\theta_n = \theta_n - \alpha \frac{\partial J}{\partial \theta_n} = \theta_n - \alpha \left(\frac{1}{m} \sum_{i=1}^{m} \left(\hat{y}^i - y^{(i)} \right) \cdot x_n^{(i)} \right)$
  - hyperparameters
    - α
- Performance evaluation
  - R^2 regression score: $R^2 = 1 - \sum_{i=1}^{n} \frac{\hat{y}_i - y_i}{y_i - \bar{y}}$
- Problems
  - make sure features are on similar scales; gradient descent may be slow otherwise
    - feature scaling: $x' = \frac{x - min(x)}{max(x) - min(x)}$
    - feature scaling with mean normalization: $x' = \frac{x - mean(x)}{max(x) - min(x)}$
    - feature scaling with standardization: $x' = \frac{x - mean(x)}{std(x)}$



  - make sure the gradient descent is working correctly
    - plot the value of the cost function J over the number of iterations (# *epochs*)
    - for a sufficiently small α, $J(\theta)$ should decrease on every iteration
    - if α is too small, the gradient descent can be slow to converge
    - if α is too large, $J(\theta)$ may not decrease on every iteration; may not converge
    - try values $\alpha \in \{\ldots, 0.001, 0.003, 0.01, 0.03, 0.1, 0.3, 1, \ldots\}$
  - check model performance



    - in case of **underfitting**
      - try adding new features
        - e.g, use a polynomial regression

$$\theta_0 + \theta_1 * x \quad \Rightarrow \quad \theta_0 + \theta_1 * x + \theta_2 * x^2 + \theta_3 * x^3 + \ldots$$

    - in case of **overfitting**
      - get more training examples
      - try reducing the number of features

    - manually select which features to keep
    - use a model-selection algorithm

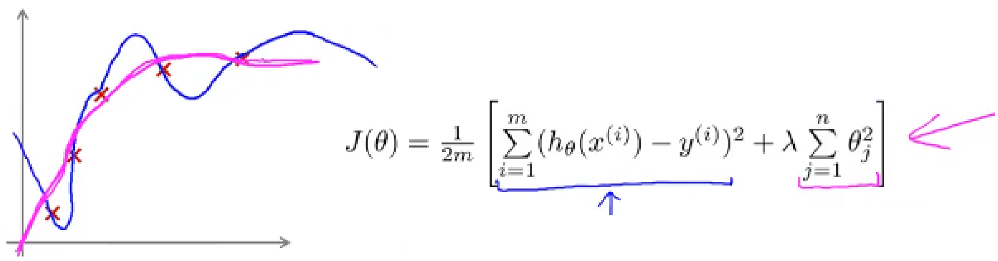
- try getting additional features

- try adding polynomial features

- use regularization

    - keep all the features, but reduce the magnitude of parameters θ;
      works well when working with a lot of features, each of which contributes a bit to predicting y

    - intuition: *shrink* model parameters in order to *smooth out* the decision boundary (generate a *simpler* hypothesis)



    - cost function with the *regularization term*

      $$J(\theta) = \frac{1}{2m}\sum_{i=1}^{m}\left(\hat{y}^{(i)} - y^{(i)}\right)^2 + \frac{\lambda}{2m}\sum_{j=1}^{n}\theta_j^2$$

    - parameter update in gradient descent (for $j \in \{1, 2, \ldots, n\}$)

      $$\theta_j = \theta_j - \alpha\frac{\partial J}{\partial \theta_j} = \theta_j - \alpha\left(\frac{1}{m}\sum_{i=1}^{m}\left(\hat{y}^{i} - y^{(i)}\right)\cdot x_j^{(i)} + \frac{\lambda}{m}\theta_j\right) = \theta_j\left(1 - \alpha\frac{\lambda}{m}\right) - \alpha\frac{1}{m}\sum_{i=1}^{m}\left(\hat{y}^{i} - y^{(i)}\right)\cdot x_j^{(i)}$$

    - λ is the *regularization parameter* and needs to be tuned;
      it controls the trade-off between the goal of fitting the training data well and the goal of keeping the parameters small

- when using regularization, try increasing or decreasing λ