Solutions to Part B of Problem Sheet 4

Solution (4.4)

(a) Given complex numbers $z_1 = a + ib$ and $z_2 = c + id$, we can express the real and imaginary parts of the product $z_3 = z_1 z_2$ as

$$\begin{pmatrix} \mathrm{re}(z_3) \\ \mathrm{im}(z_3) \end{pmatrix} = \begin{pmatrix} a & -b \\ b & a \end{pmatrix} \begin{pmatrix} c \\ d \end{pmatrix}.$$

In the same fashion, a system of equations $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}$, with $\boldsymbol{A}$ and $\boldsymbol{x}$ complex, we can be written as

$$\begin{pmatrix} \mathrm{re}(\boldsymbol{b}) \\ \mathrm{im}(\boldsymbol{b}) \end{pmatrix} = \begin{pmatrix} \mathrm{re}(\boldsymbol{A}) & -\mathrm{im}(\boldsymbol{A}) \\ \mathrm{im}(\boldsymbol{A}) & \mathrm{re}(\boldsymbol{A}) \end{pmatrix} \begin{pmatrix} \mathrm{re}(\boldsymbol{c}) \\ \mathrm{im}(\boldsymbol{c}) \end{pmatrix}.$$

Since we know that the target vector $\boldsymbol{b}$ is real, we only need the upper half of this system. Once this is solved, we can assemble the complex $\boldsymbol{c}$ from it.

(b)+(c) The code could look something like this:

```
% Define signal on interval [0,2\pi]
f = @(x)(1.7*sin((30)*x)+0.5*cos((9)*x)+0.5*sin((6)*x)-...
    1*cos((11)*x)+0.2*sin((13)*x));

% Set up vector f
n = 512;
T = 2*pi/n;
xx = linspace(0,2*pi-T,n)';
yy = f(xx);

% Determine points to subsample from
m = 30;
p = randperm(n);
points = xx(p(1:m));
samples = f(points);

% Plot curve and sample points
subplot(2,1,1);
plot(xx,yy,'LineWidth',2);
hold on;
plot(points,samples,'or','MarkerFaceColor','r');

% Inverse DFT matrix and split into real and imaginary parts
D = ifft(eye(n));
rD = [real(D),-imag(D)];
A = rD(p(1:m),:);

fy = fft(yy);
b = A*[real(fy);imag(fy)];
```

```
cvx_begin
    variable x(2*n);
    minimize ( norm(x,1) );
    subject to
        A*x == b;
cvx_end;

% Test whether what we did worked

newy = real(ifft(x(1:n)+1i*x(n+1:end)));
subplot(2,1,2);
plot(xx,newy,'LineWidth',2);
norm(newy-yy)
```

(d) Repeating the experiments for different values of m and plotting the success probability, we get the following diagram. The interpretation is as follows:

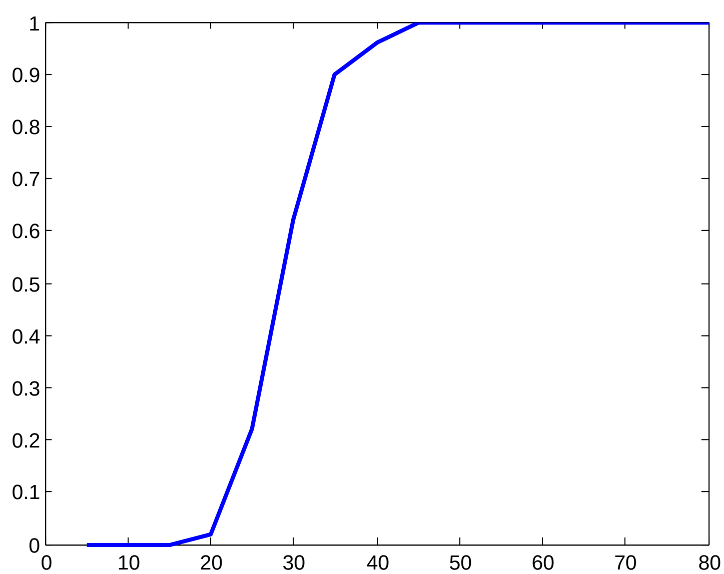

Figure 1: Probability of recovering the whole signal from m samples

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We can recover the values of the specific signal at 512 points from sampling the signal at only about 45 random locations!

(e) The linear programming formulation is

$$
\begin{aligned}
\text{minimize} \quad & \sum_{i=1}^{n} t_i \\
\text{subject to} \quad & -t_i \le c_i \le t_i \\
& t_i \ge 0 \\
& \boldsymbol{D}_I \boldsymbol{c} = \boldsymbol{f}_I .
\end{aligned}
$$