Tree-Structured Indexes

Introduction

As for any index, 3 alternatives for data entries **k***:

- Data record with key value **k**
- <**k**, rid of data record with search key value **k**>
- <**k**, list of rids of data records with search key **k**>

- Choice is orthogonal to the *indexing technique* used to locate data entries **k***.
- Tree-structured indexing techniques support both *range searches* and *equality searches*.
- *ISAM*: static structure; *B+ tree*: dynamic, adjusts gracefully under inserts and deletes.

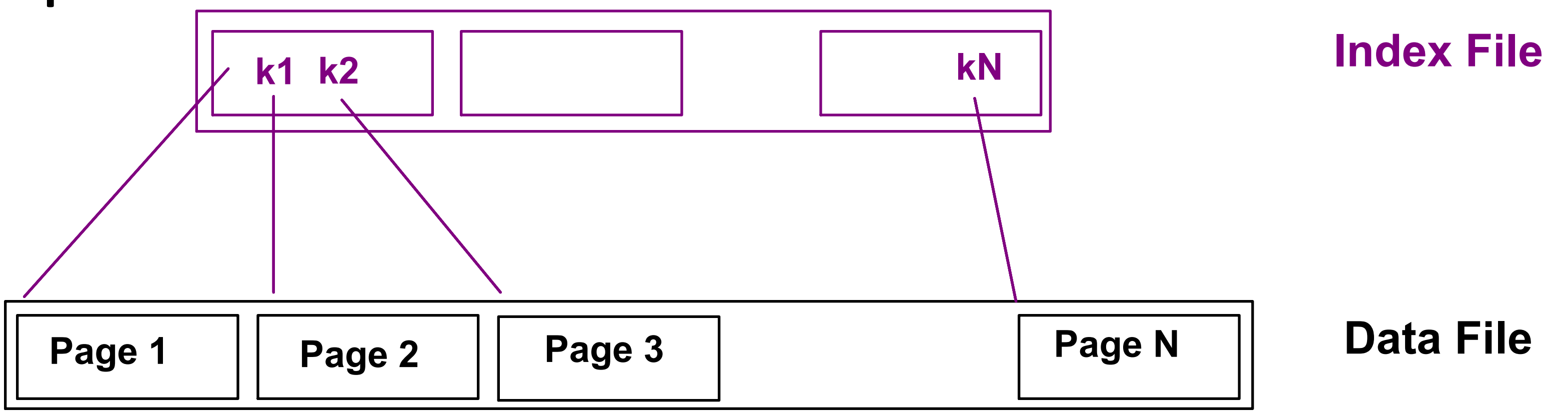
Range Searches

``*Find all students with gpa > 3.0*''

– If data is in sorted file, do binary search to find first such student, then scan to find others.

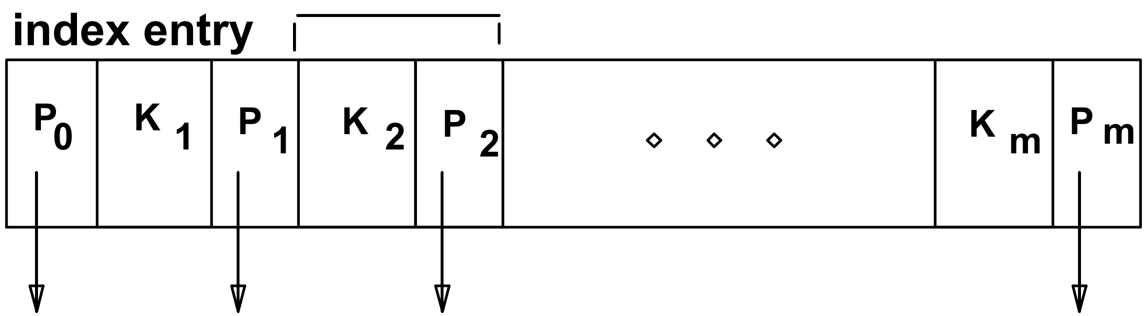
– Cost of binary search can be quite high.

Simple idea: Create an `index' file.

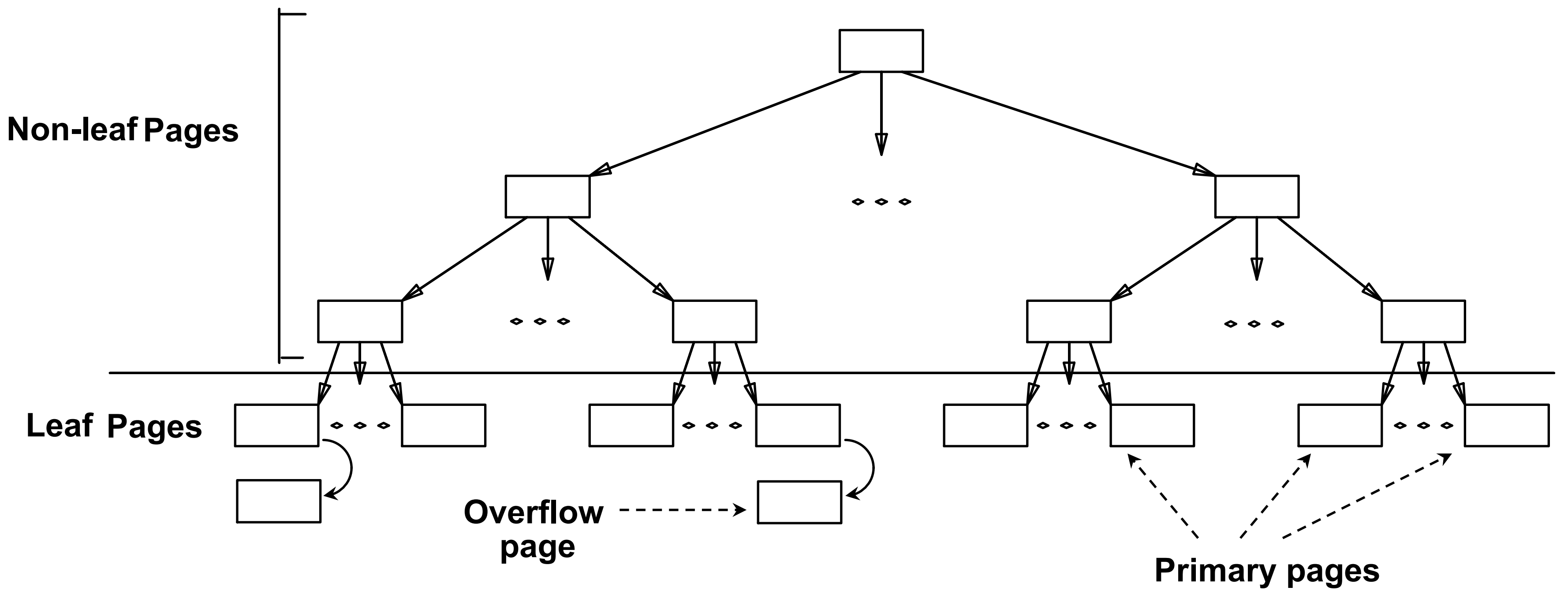


☛ *Can do binary search on (smaller) index file!*

ISAM (Indexed Sequential Access Method)

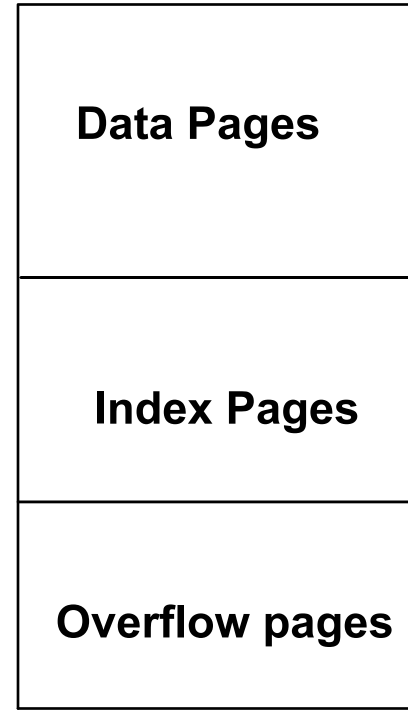


Index file may still be quite large. But we can apply the idea repeatedly!



☛ *Leaf pages contain data entries.*

Comments on ISAM

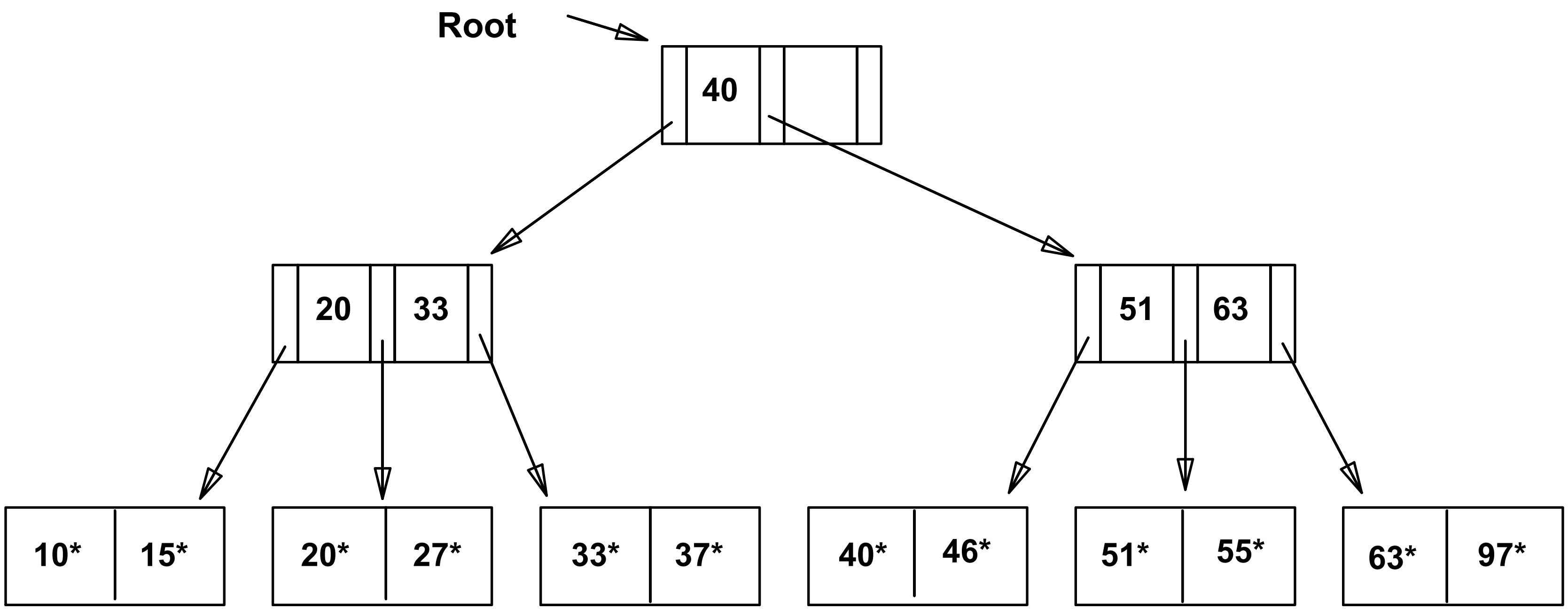


- *File creation*: Leaf (data) pages allocated sequentially, sorted by each key; then index pages allocated, then space for overflow pages.
- *Index entries*: <search key value, page id>; they `direct' search for *data entries*, which are in leaf pages.
- <u>*Search*</u>: Start at root; use key comparisons to go to leaf. Cost = $\log_F N$; L^F = log(F)N $\propto$
- F = # entries/index pg, N = # leaf pgs
- <u>*Insert*</u>: Find leaf data entry belongs to, and put it there.
- <u>*Delete*</u>: Find and remove from leaf; if empty overflow page, de-allocate.

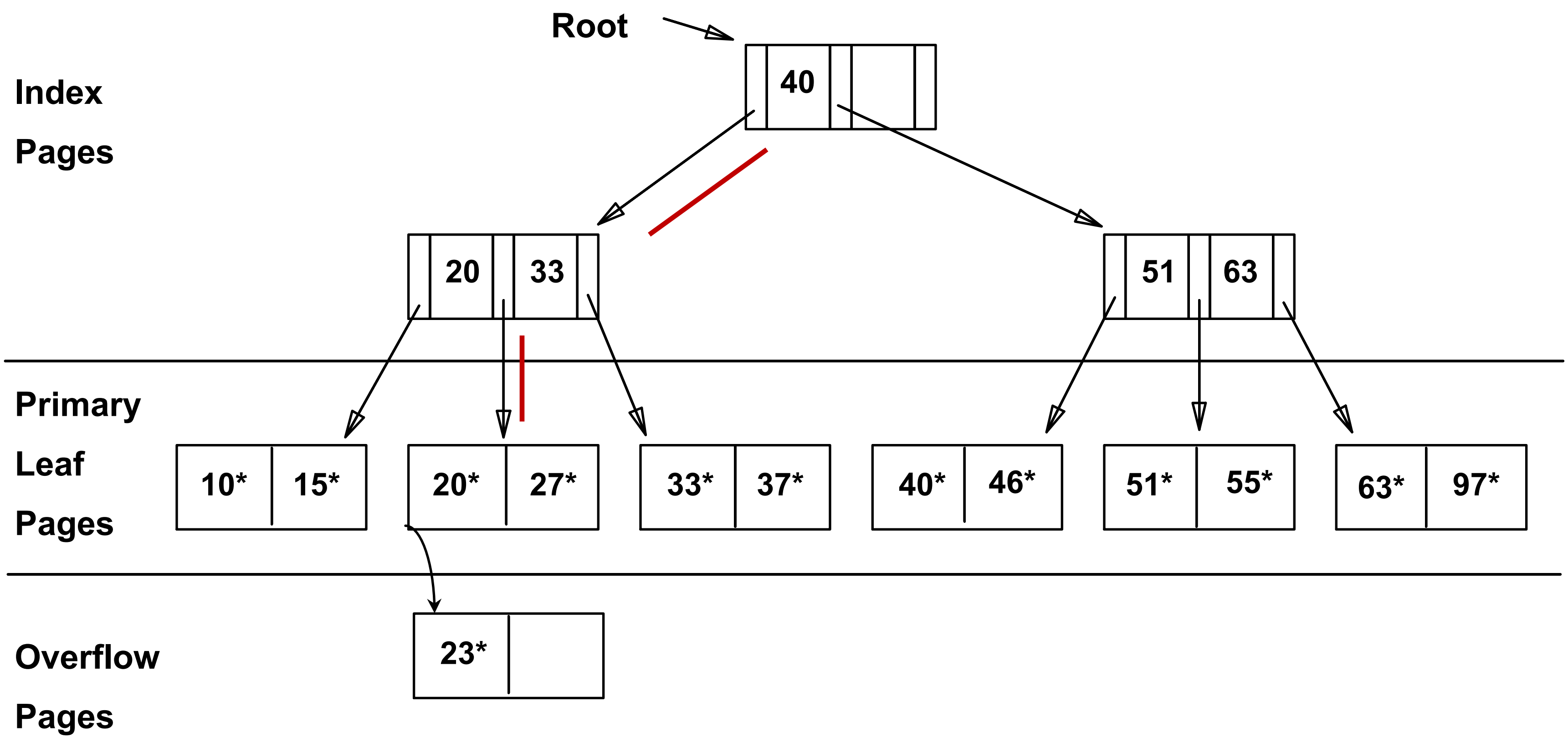
☛ **Static tree structure**: *inserts/deletes affect only leaf pages.*

Example ISAM Tree

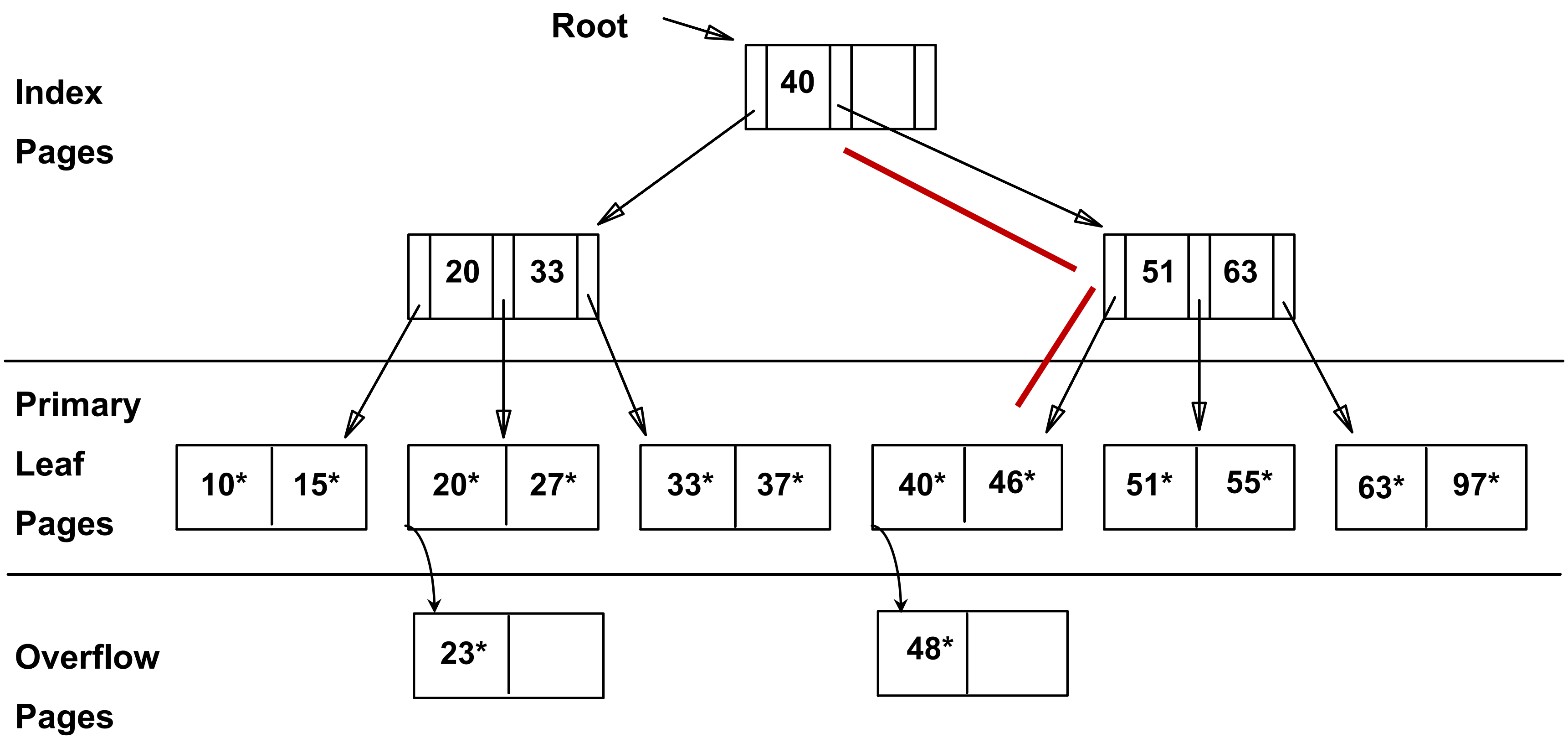
Each node can hold 2 entries; no need for `next-leaf-page' pointers. (Why?)



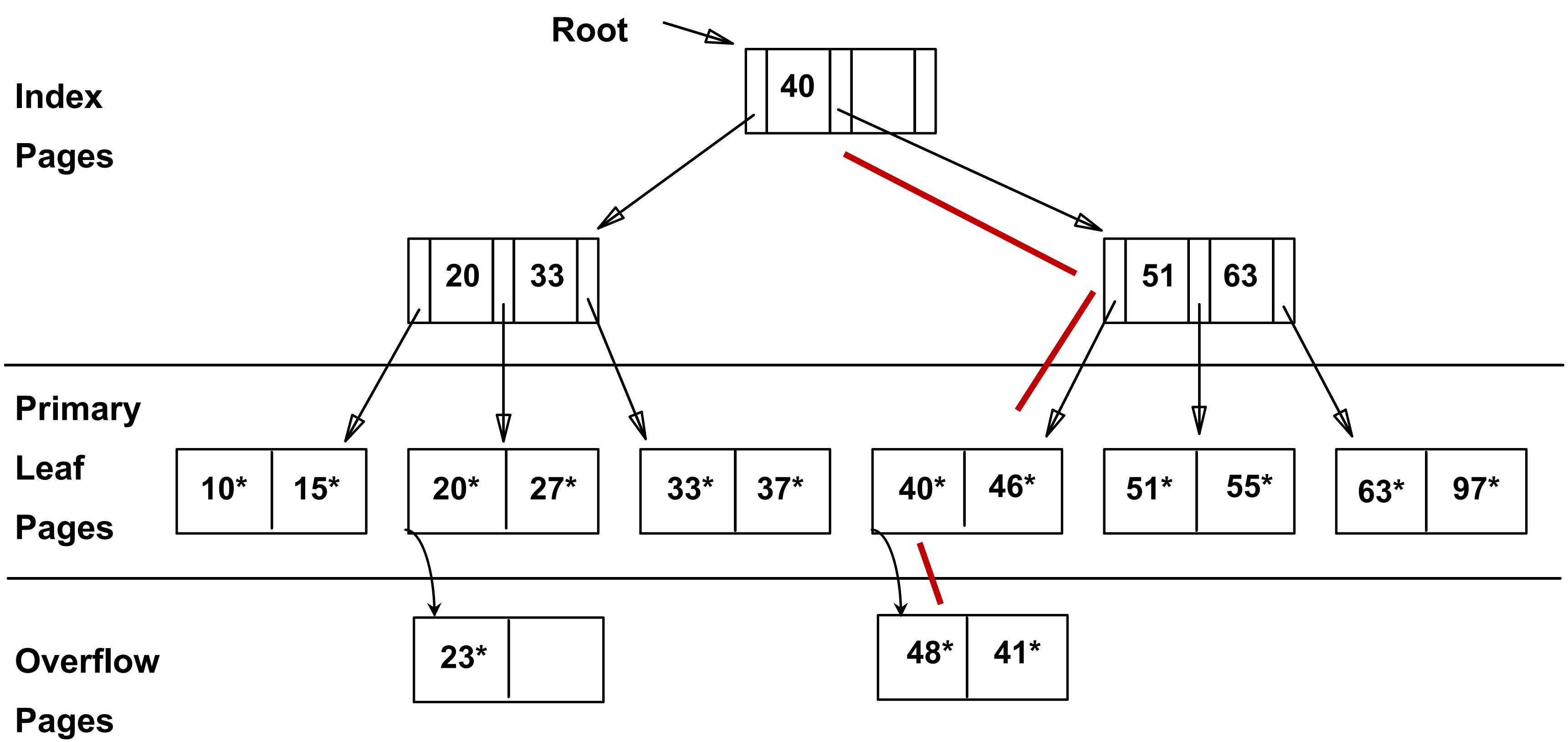
Inserting 23*



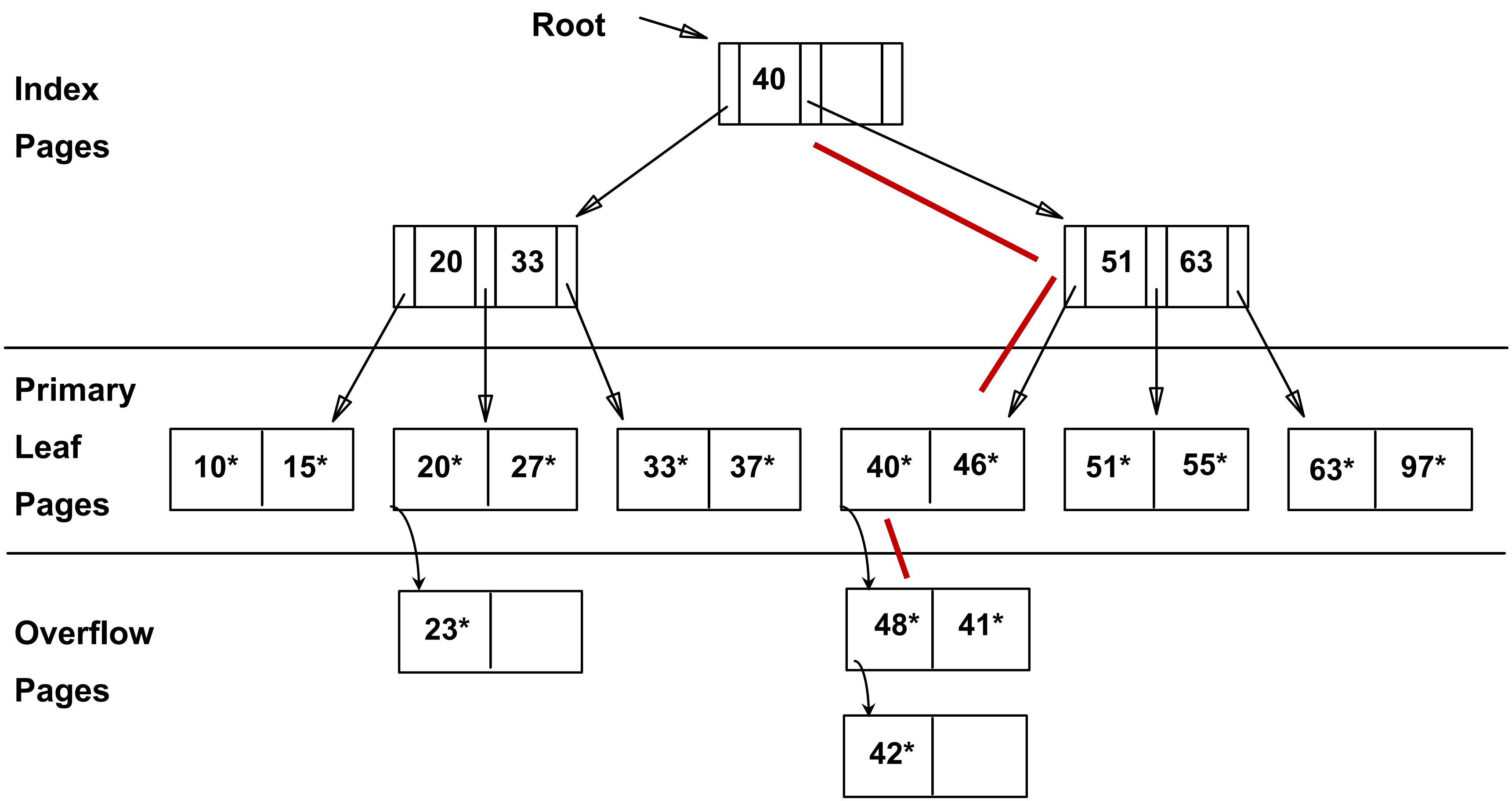
Inserting 48*



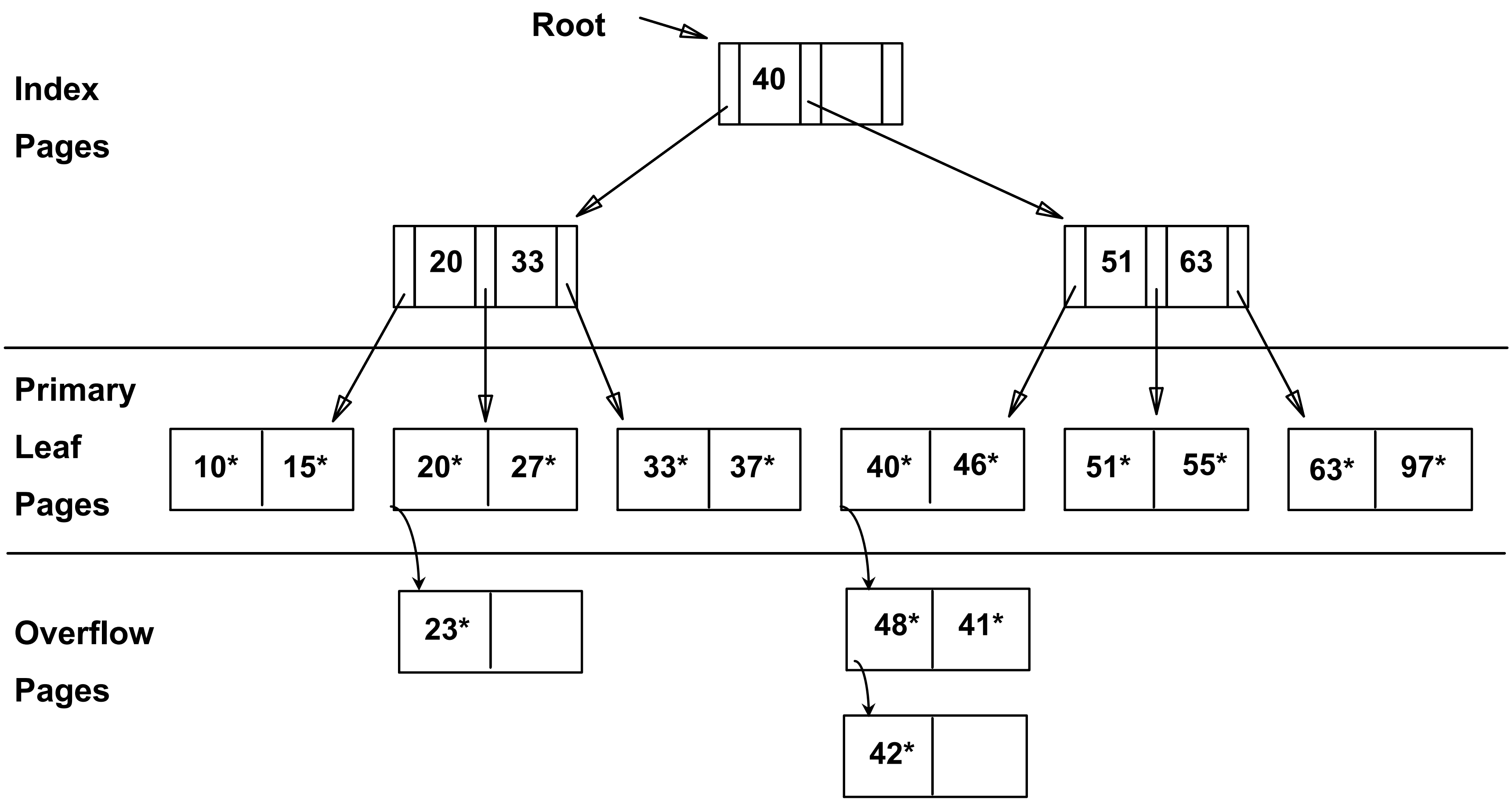
Inserting 41*



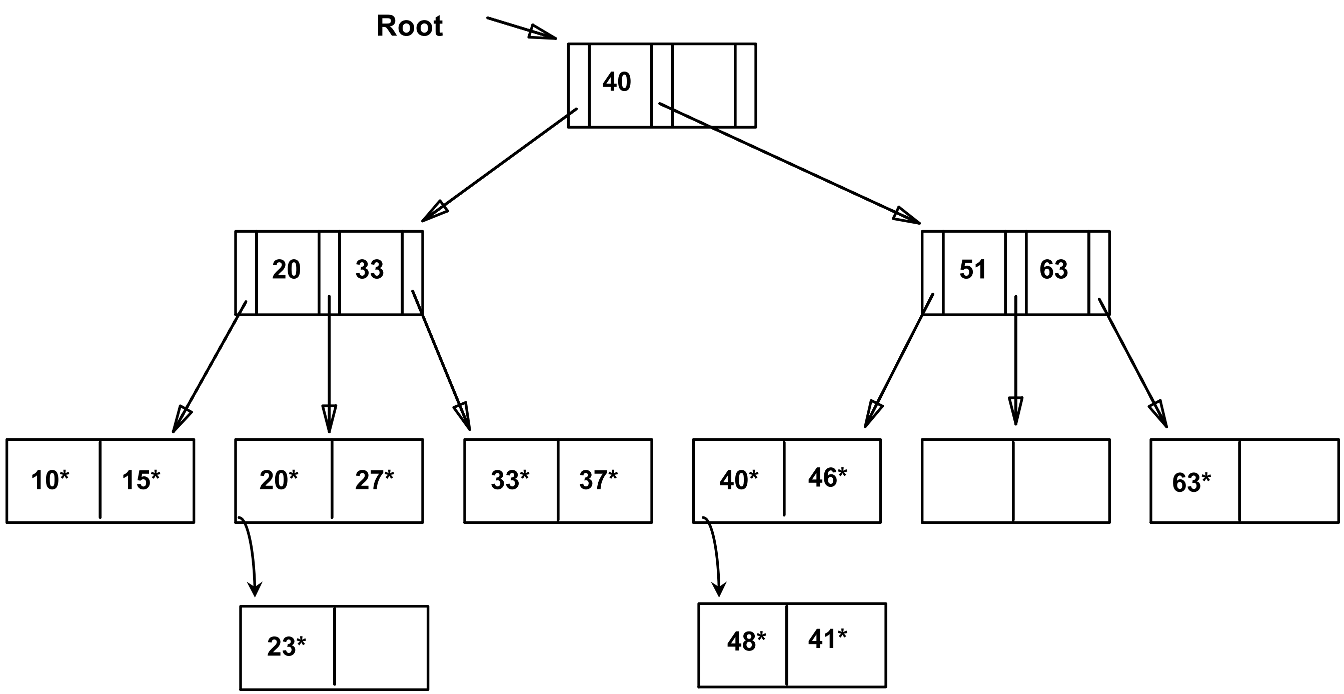
Inserting 42*



Then Deleting 42*, 51*, 97*,55*



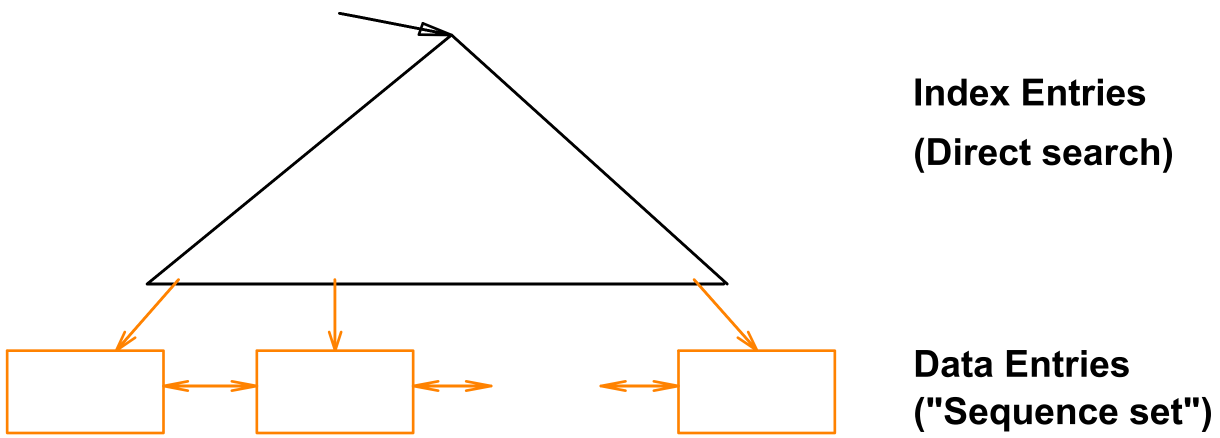
… After Deleting 42*, 51*, 97*, 55*



☛ *Note that 51* appears in index levels, but not in leaf!*

B+ Tree: Most Widely Used Index

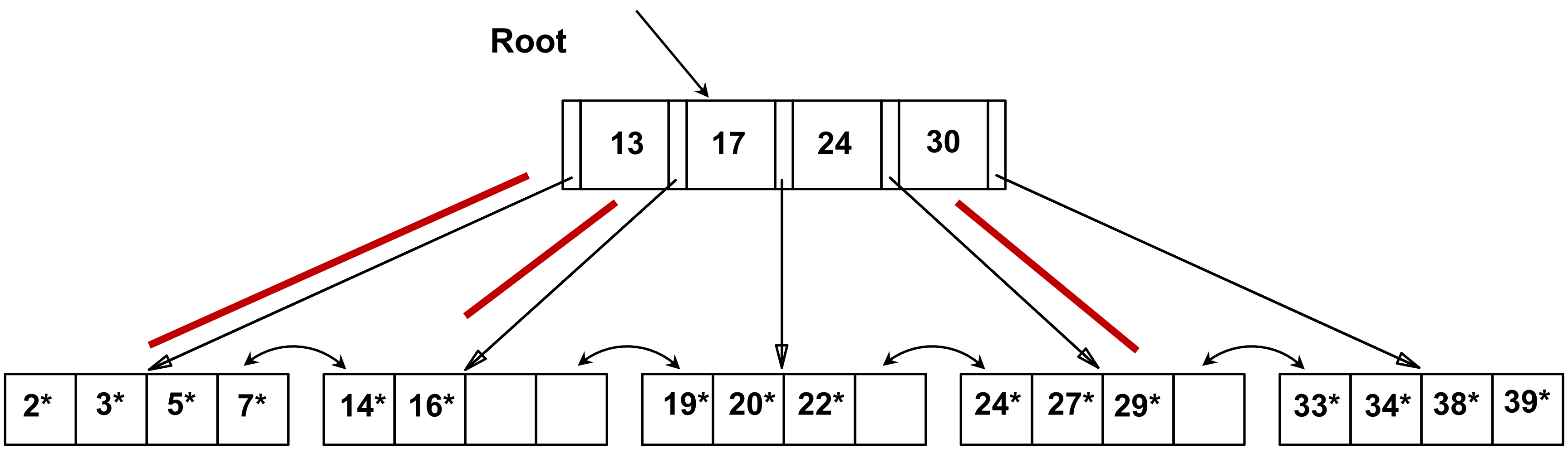
- Insert/delete at log $_F$ N cost; keep tree *height-balanced*. (F = fanout, N = # leaf pages)
- Minimum 50% occupancy (except for root). Each node contains **d** <= $\underline{m}$ <= 2**d** entries.
- The parameter **d** is called the *order* of the tree.
- Supports equality and range-searches efficiently.



Example B+ Tree

Search begins at root, and key comparisons direct it to a leaf.

Search for 5*, 15*, all data entries >= 24* ...



☛ *Based on the search for 15*, we <u>know</u> it is not in the tree!*

B+ Trees in Practice

Typical order: 100. Typical fill-factor: 67%.

- average fanout = 133

Typical capacities:

- Height 4: 133^4 = 312,900,700 records
- Height 3: 133^3 = 2,352,637 records

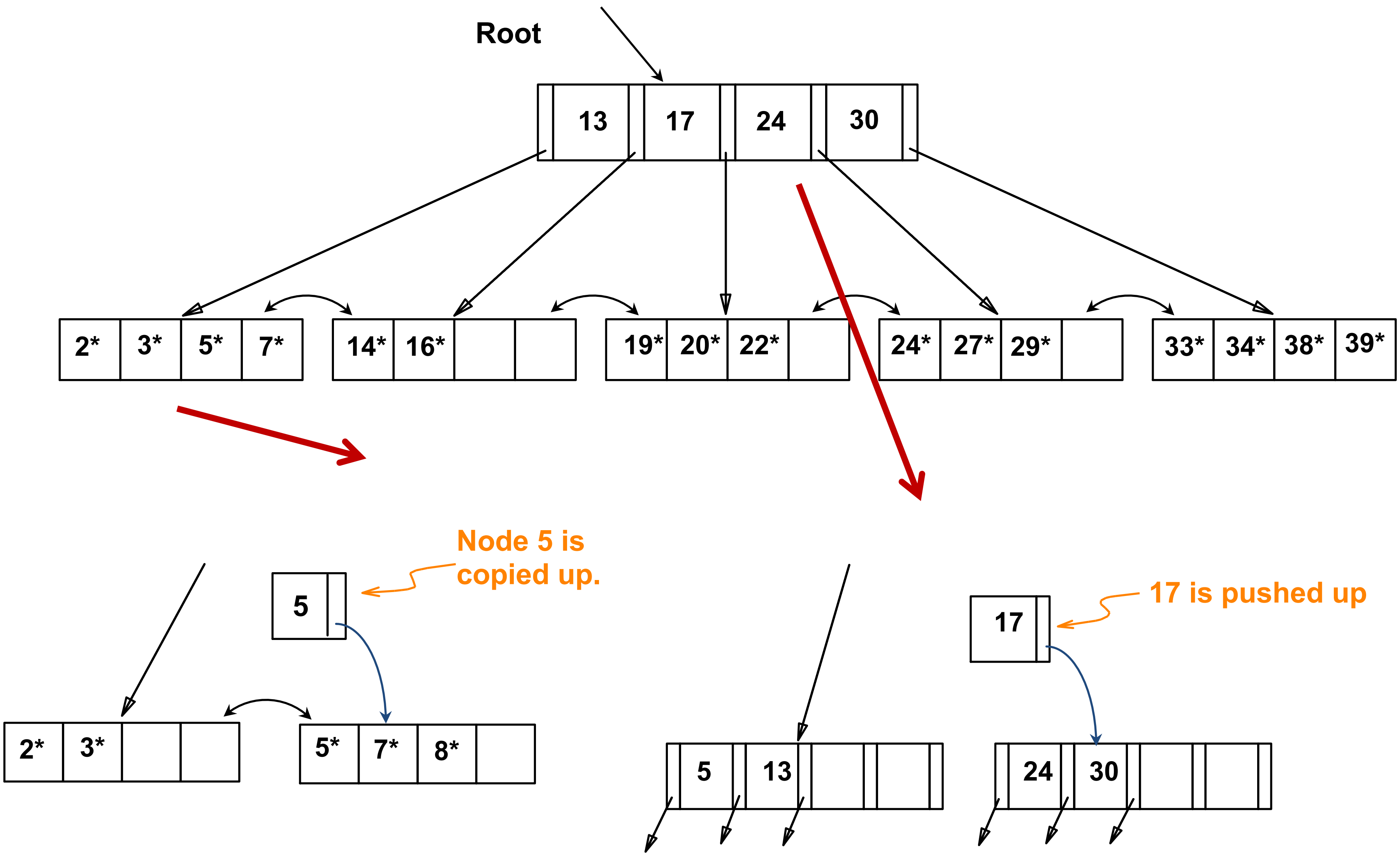
Can often hold top levels in buffer pool:

- Level 1 = 1 page = 8 Kbytes
- Level 2 = 133 pages = 1 Mbyte
- Level 3 = 17,689 pages = 133 MBytes

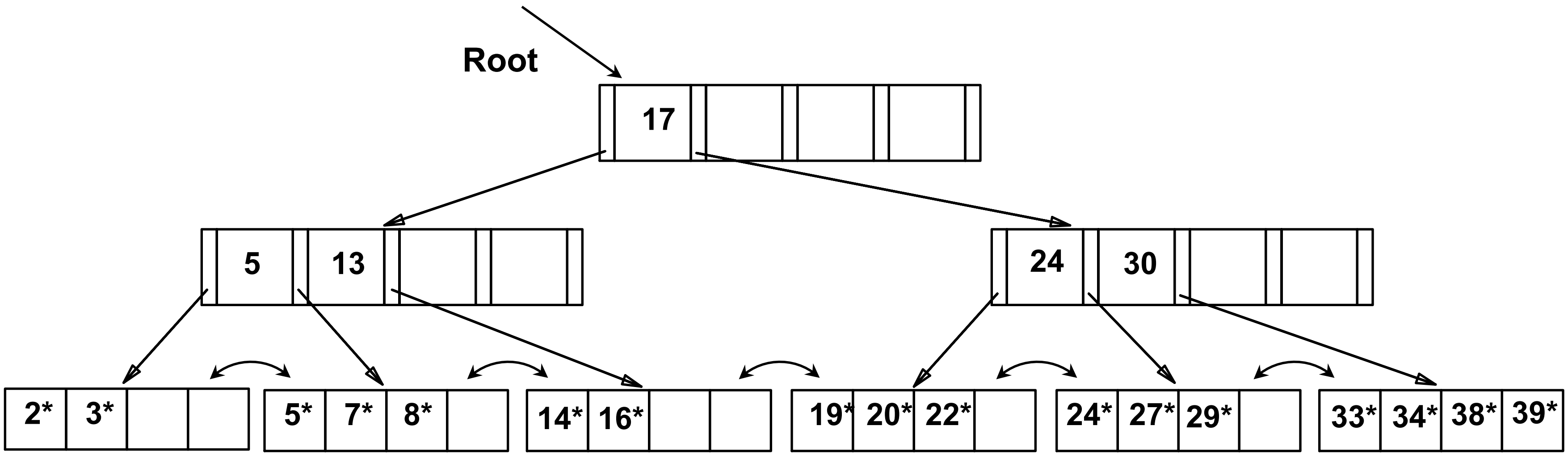
Inserting a Data Entry into a B+ Tree

- Find correct leaf *L.*
- Put data entry onto *L*.
  - If *L* has enough space, *done*!
  - Else, must *split* *L (into L and a new node L2)*
    - Redistribute entries evenly, **copy up** middle key.
    - Insert index entry pointing to *L2* into parent of *L*.
- This can happen recursively
  - To split index node, redistribute entries evenly, but **push up** middle key. (Contrast with leaf splits.)
- Splits "grow" tree; root split increases height.
  - Tree growth: gets *wider* or *one level taller at top.*

Inserting 8*


Root
13
17
24
30
2* 3* 5* 7*
14* 16*
19* 20* 22*
24* 27* 29*
33* 34* 38* 39*
Node 5 is copied up.
5
2* 3*
5* 7* 8*
17 is pushed up
17
5 13
24 30

After Inserting 8*



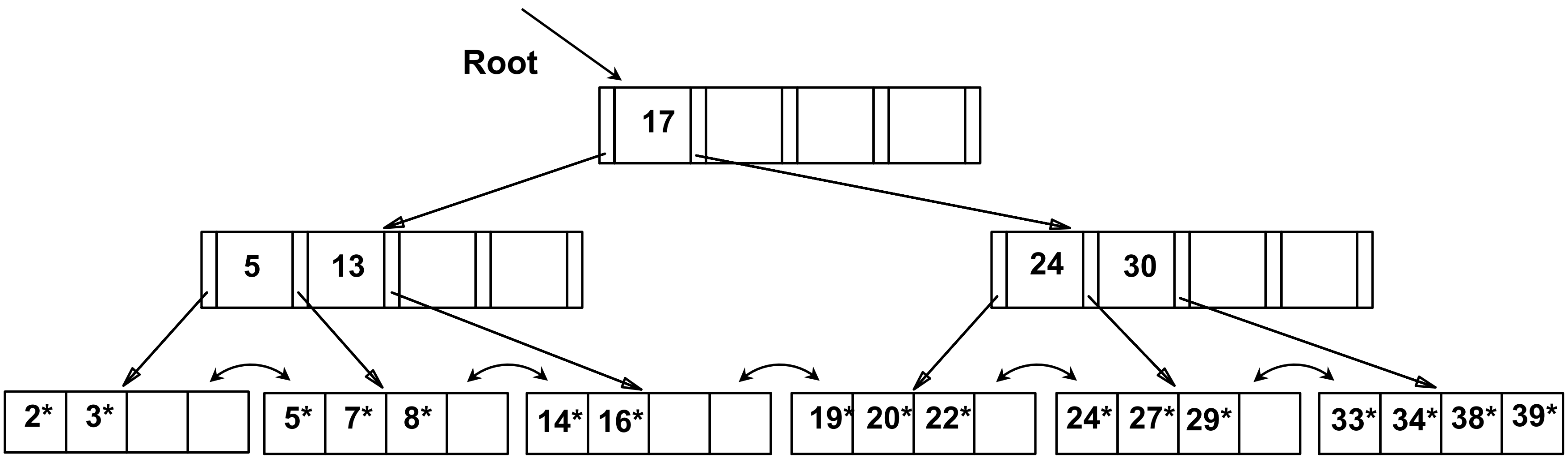
❖ In this example, we can avoid split by re-distributing entries; however, this is usually not done in practice.

- Redistributing I/O costs is not smaller than those of splitting.
- It has a chance that redistributing does not work; thus costs for exploring redistribution are wasted.

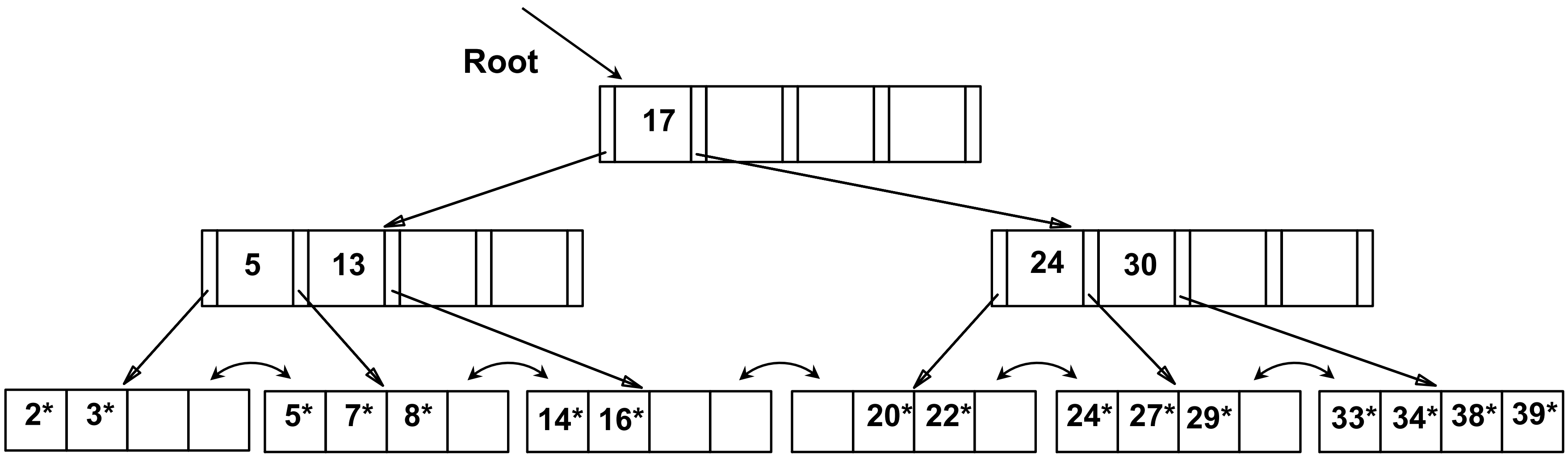
Deleting a Data Entry from a B+ Tree

- Start at root, find leaf *L* where entry belongs.
- Remove the entry.
  - If L is at least half-full, *done!*
  - If L has only **d-1** entries,
    - Try to re-distribute, borrowing from ***sibling*** *(adjacent node with same parent as L)*.
    - If re-distribution fails, ***merge*** *L* and sibling.
- If merge occurred, must delete entry (pointing to *L* or sibling) from parent of *L*.
- Merge could propagate to root, decreasing height.

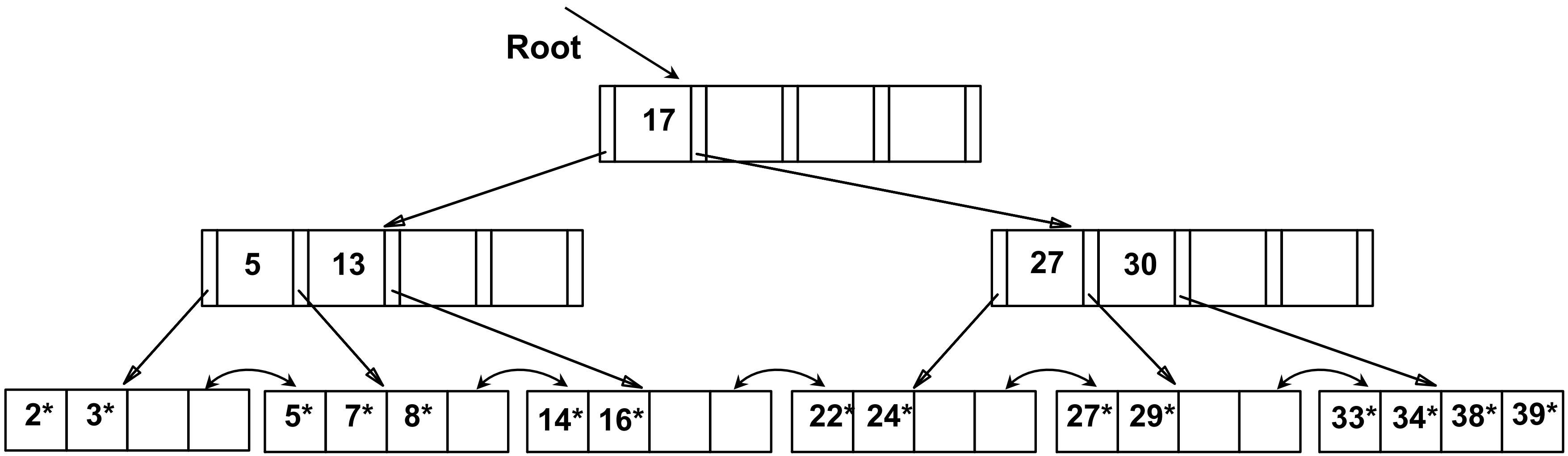
Deleting 19*



Deleting 20*

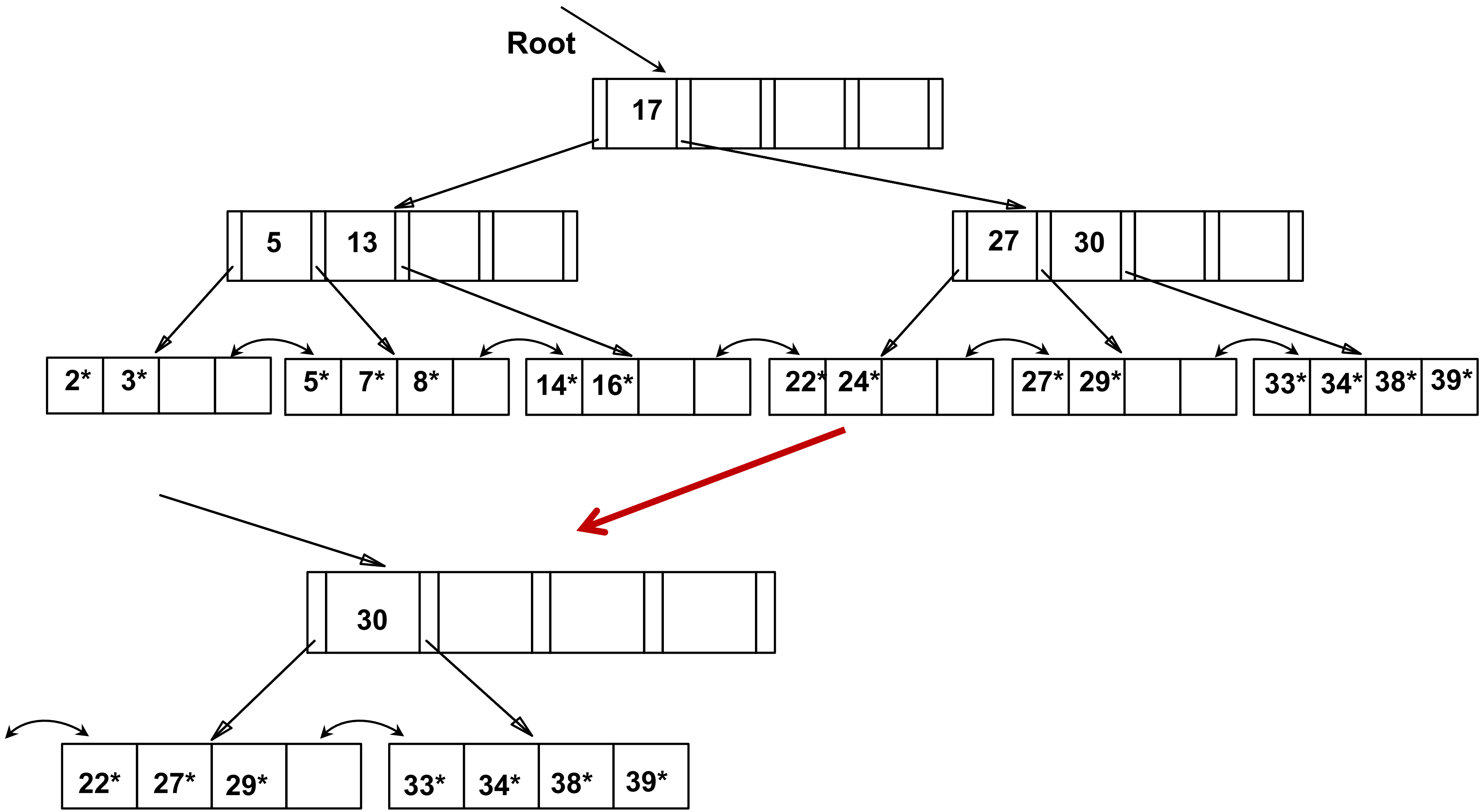


After Deleting 20* ...



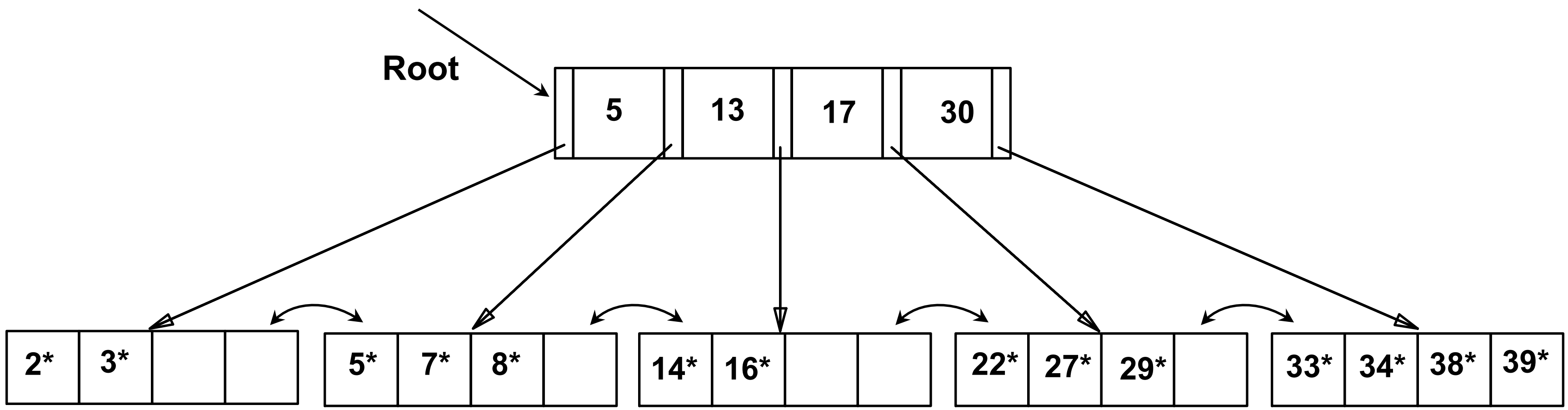
Deleting 20* is done with re-distribution.
Notice how middle key is *copied up*.

Deleting 24* ...

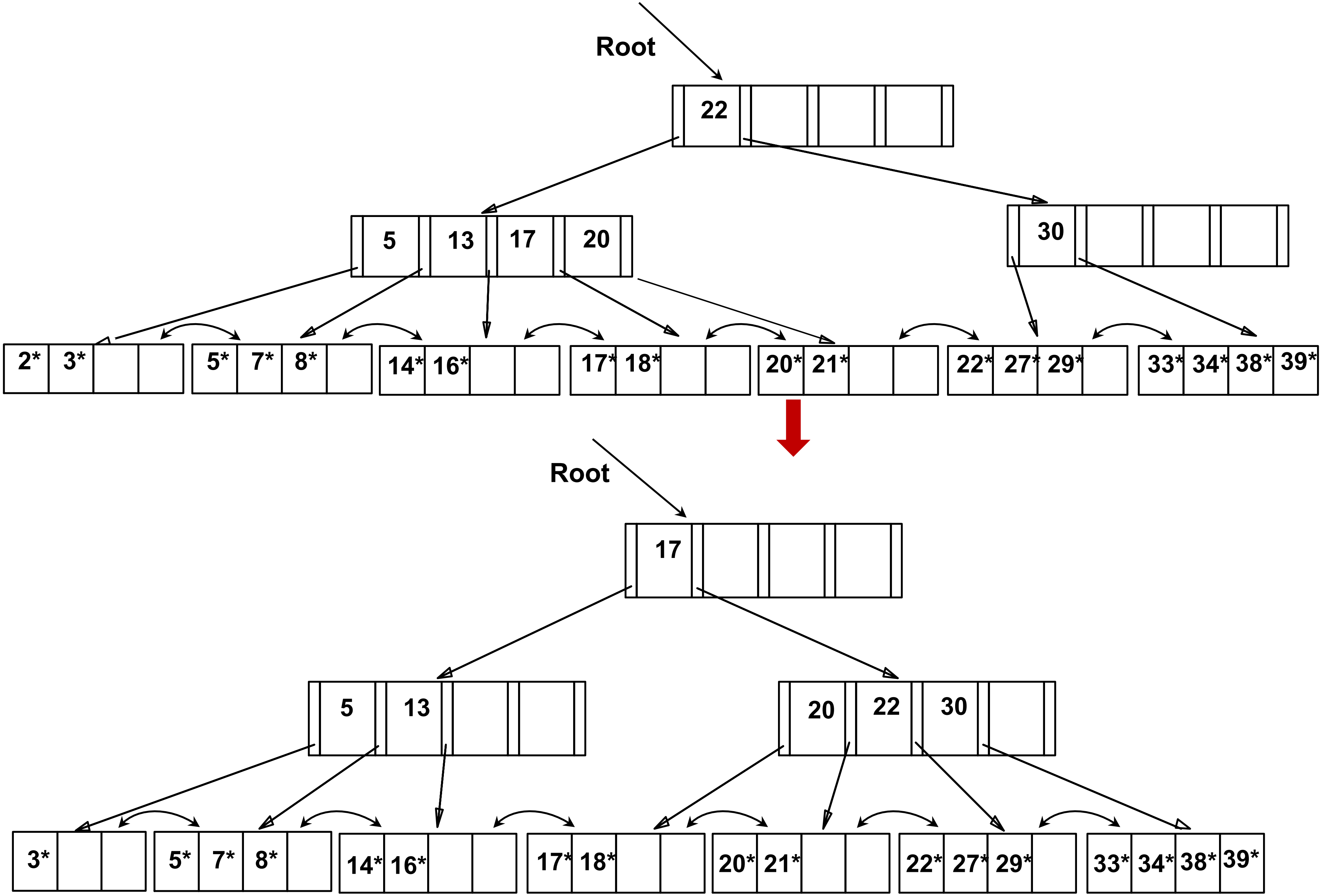


Root
17
5
13
27
30
2*
3*
5*
7*
8*
14*
16*
22*
24*
27*
29*
33*
34*
38*
39*
30
22*
27*
29*
33*
34*
38*
39*

After Deleting 24*

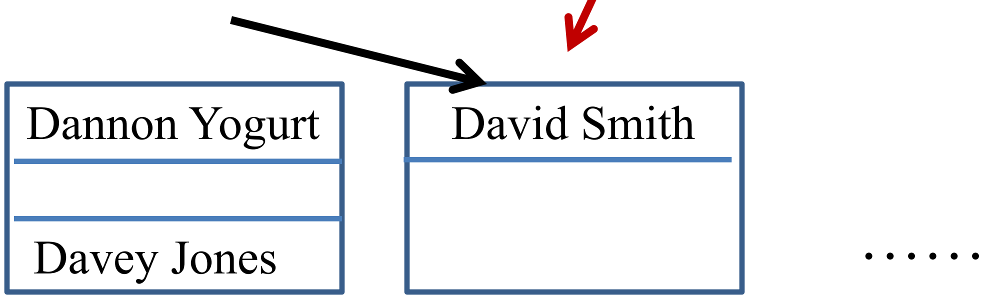


Example of Non-leaf Re-distribution



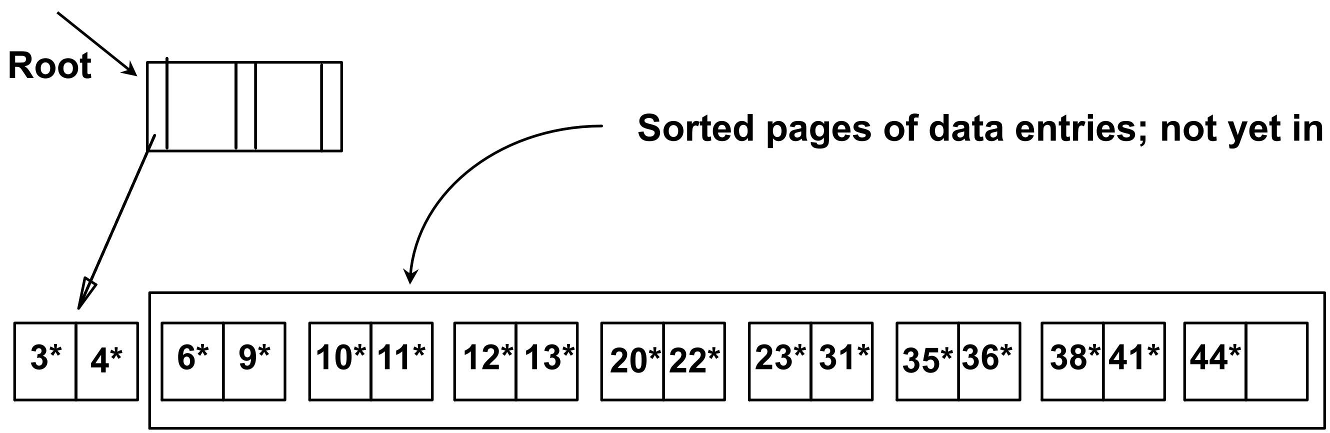
Prefix Key Compression

- Important to increase fan-out. (Why?)
- Key values in index entries only `direct traffic' ; can often compress them.
  - E.g., If we have adjacent index entries with search key values *Dannon Yogurt*, *David Smith* and *Devarakonda Murthy*, we can abbreviate *David Smith* to *Dav*. (The other keys can be compressed too ...)
    - Is this correct? Not quite! What if there is a data entry *Davey Jones*? (Can only compress *David Smith* to *Davi*)
    - In general, while compressing, must leave each index entry greater than every key value (in any subtree) to its left.
- Insert/delete must be suitably modified.



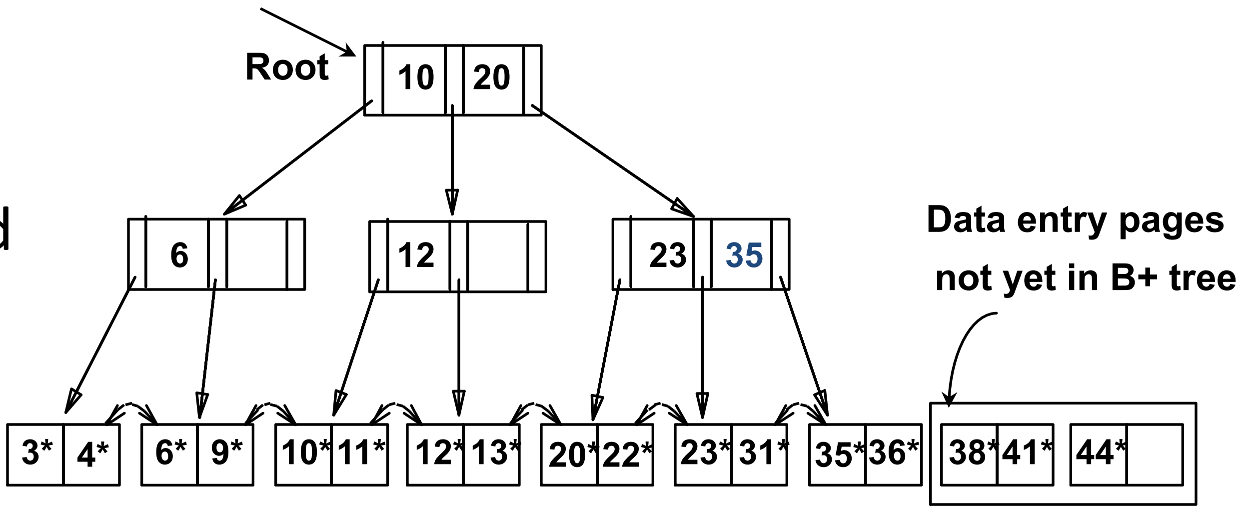
Bulk Loading of a B+ Tree

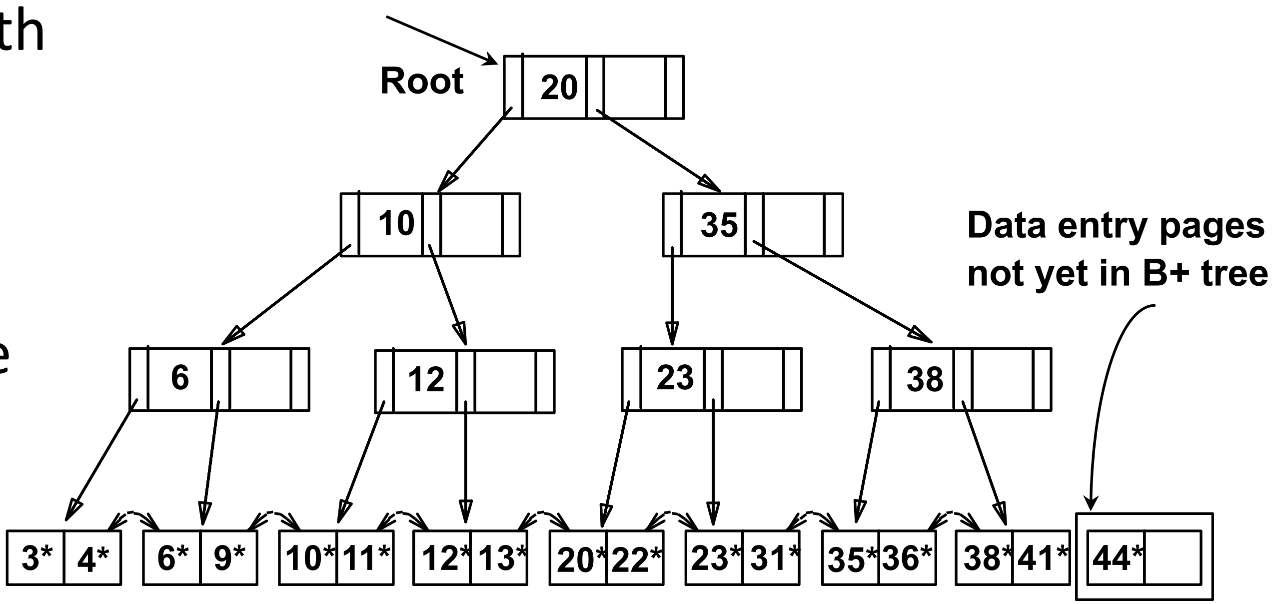
- If we have a large collection of records, and we want to create a B+ tree on some field, doing so by repeatedly inserting records is very slow.
- *Bulk Loading* can be done much more efficiently.
- *Initialization*: Sort all data entries, insert pointer to first (leaf) page in a new (root) page.



Bulk Loading (Contd.)

- Index entries for leaf pages always entered into right-most index page just above leaf level. When this fills up, it splits. (Split may go up right-most path to the root.)
- Much faster than repeated inserts, especially when one considers locking!





Summary of Bulk Loading

Option 1: multiple inserts.

- Slow.
- Does not give sequential storage of leaves.

Option 2: <u>*Bulk Loading*</u>

- Has advantages for concurrency control.
- Fewer I/Os during build.
- Leaves will be stored sequentially (and linked, of course).
- Can control "fill factor" on pages.

A Note on `Order'

Order (**d**) concept replaced by physical space criterion in practice (`*at least half-full*').

- Index pages can typically hold many more entries than leaf pages.
- Variable sized records and search keys mean differnt nodes will contain different numbers of entries.
- Even with fixed length fields, multiple records with the same search key value (*duplicates*) can lead to variable-sized data entries (if we use Alternative (3)).

Summary

Tree-structured indexes are ideal for range-searches, also good for equality searches.

ISAM is a static structure.

- Only leaf pages modified; overflow pages needed.
- Overflow chains can degrade performance unless size of data set and data distribution stay constant.

B+ tree is a dynamic structure.

- Inserts/deletes leave tree height-balanced; $\log_F N$ cost.
- High fanout (**F**) means depth rarely more than 3 or 4.
- Almost always better than maintaining a sorted file.

Summary (Contd.)

  - Typically, 67% occupancy on average.
  - Usually preferable to ISAM, modulo *locking* considerations; adjusts to growth gracefully.
  - If data entries are data records, splits can change rids!
- Key compression increases fanout, reduces height.
- Bulk loading can be much faster than repeated inserts for creating a B+ tree on a large data set.
- Most widely used index in database management systems because of its versatility. One of the most optimized components of a DBMS.