

Hash-Based Indexes

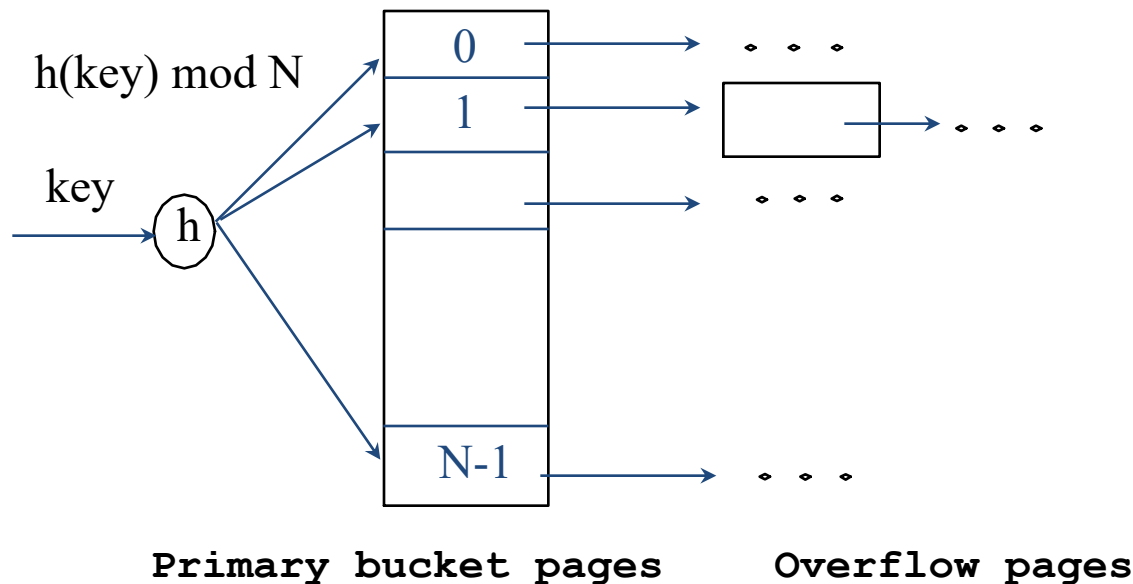
Introduction

As for any index, 3 alternatives for data entries k^ :*

1. Data record with key value k
 2. $\langle k, \text{rid of data record with search key value } k \rangle$
 3. $\langle k, \text{list of rids of data records with search key } k \rangle$
- Choice orthogonal to the *indexing technique*
 - Hash-based indexes are the best for *equality selections*. **Cannot** support range searches.
 - Static and dynamic hashing techniques exist; trade-offs similar to ISAM vs. B+ trees.

Static Hashing

- # primary pages (index data entry pages) fixed, allocated sequentially, never de-allocated; overflow pages if needed.
- $h(k) \bmod M = \text{bucket to which data entry with key } k \text{ belongs. (} M = \# \text{ of buckets)}$



Static Hashing (Contd.)

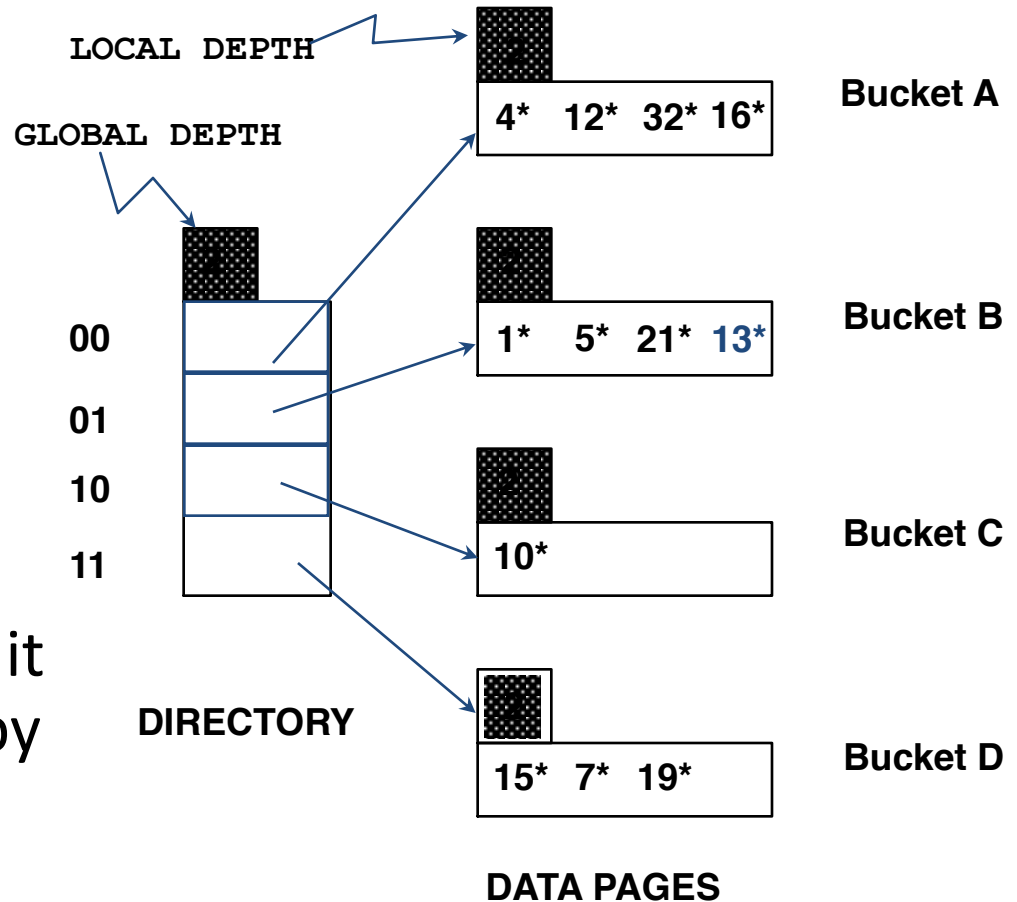
- Buckets contain *data entries*.
- Hash function works on *search key* field of record *r*. Must distribute values over range 0 ... M-1.
 - $h(key) = (a * key + b)$ usually works well.
 - **a** and **b** are constants; lots known about how to tune **h**.
- Long overflow chains can develop and degrade performance.
 - Keep 80% full initially and/or re-hashing
 - *Extendible* and *Linear Hashing*: Dynamic techniques to fix this problem.

Extendible Hashing

- Situation: Bucket (primary page) becomes full. Why not re-organize file by *doubling* # of buckets?
 - Reading and writing all pages is expensive!
 - Idea: Use directory of pointers to buckets, double # of buckets by *doubling the directory*, splitting just the bucket that overflowed!
 - Directory much smaller than file, so doubling it is much cheaper. Only one page of data entries is split. *Ensure no overflow page!*
 - Trick lies in how hash function is adjusted!

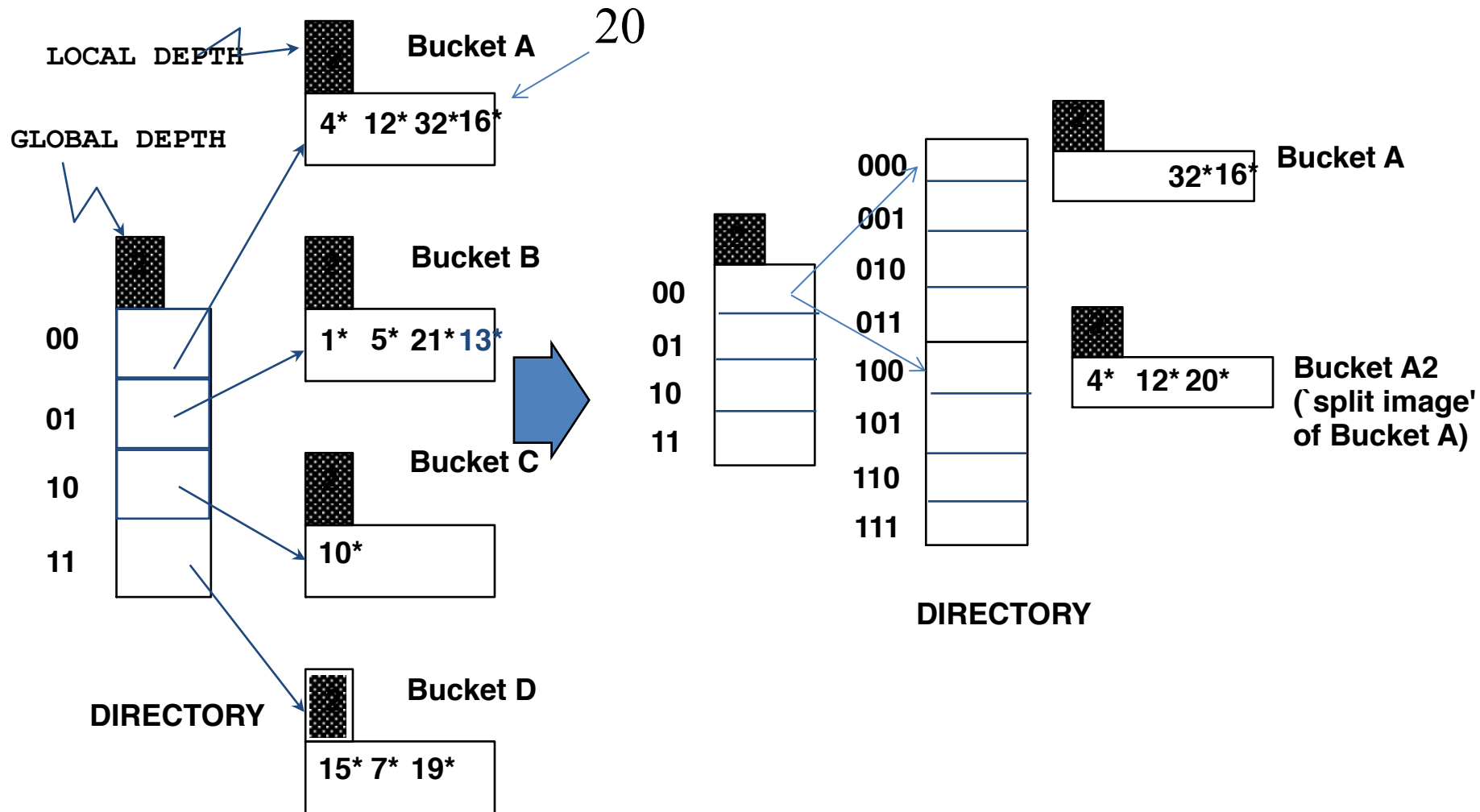
Example

- Directory is array of size 4.
- To find bucket for r , take last '*global depth*' # bits of $h(r)$; we denote r by $h(r)$.
 - If $h(r) = 5 = \text{binary } 101$, it is in bucket pointed to by 01.

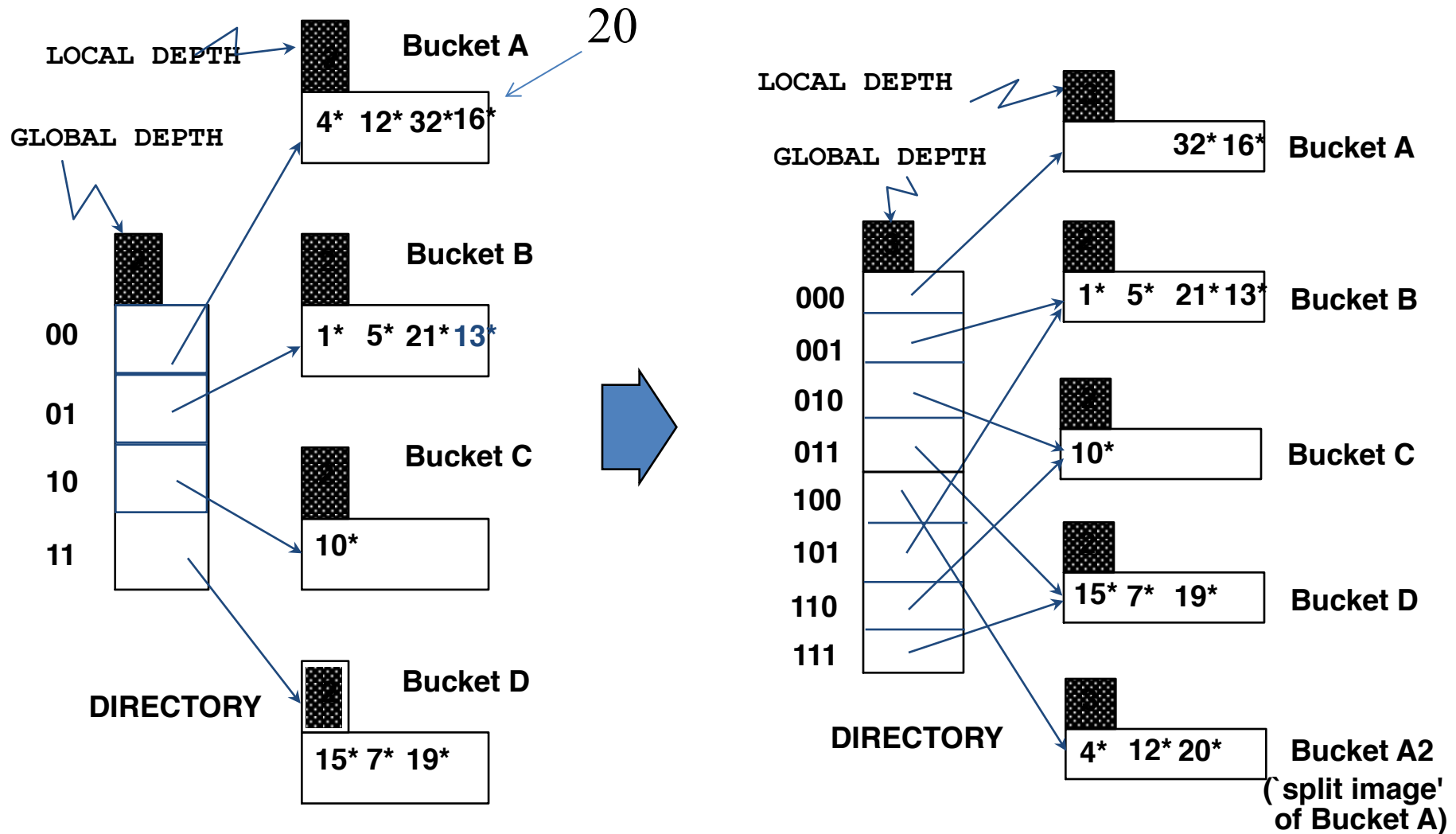


- ❖ **Insert:** If bucket is full, *split* it (allocate new page, re-distribute).
- ❖ *If necessary*, double the directory. (As we will see, splitting a bucket does not always require doubling; we can tell by comparing *global depth* with *local depth* for the split bucket.)

Insert $h(r)=20$ (Causes Doubling)



After inserting $h(r)=20$

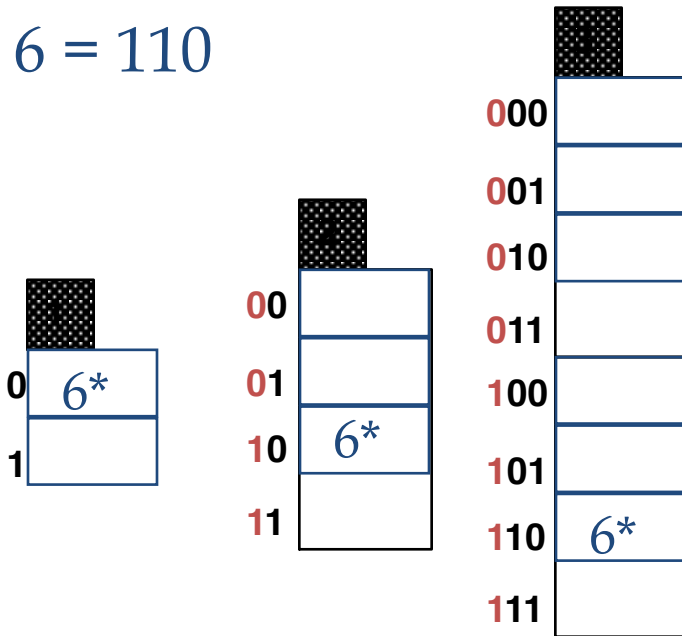


Points to Note

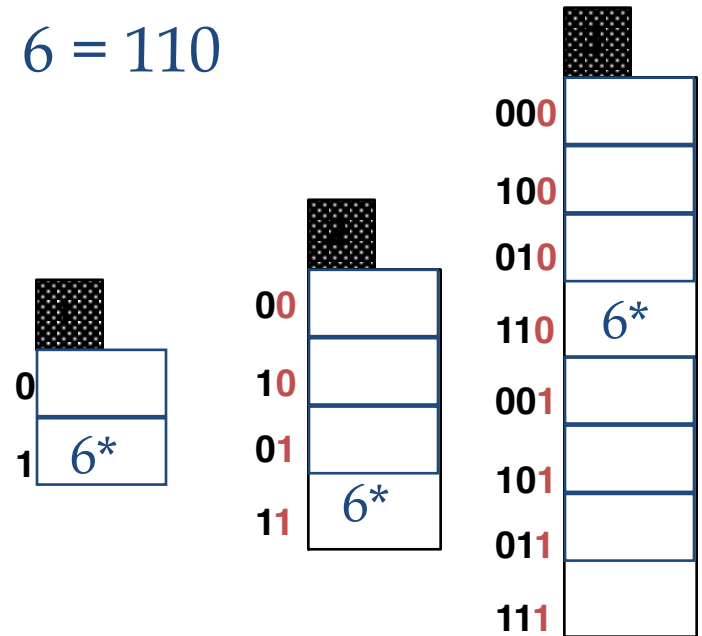
- 20 = binary 10100. Last **2** bits (00) cannot tell us r belongs in A or A2. Last **3** bits can tell us the which bucket.
 - *Global depth of directory*: Max # of bits needed to tell which bucket an entry belongs to.
 - *Local depth of a bucket*: # of bits used to determine if an entry belongs to this bucket.
- When does bucket split cause directory doubling?
 - Before insert, *local depth* of bucket = *global depth*. Insert causes *local depth* to become $>$ *global depth*; directory is doubled by *copying it over* and 'fixing' pointer to split image page. (Use of least significant bits enables efficient doubling via copying of directory!)

Directory Doubling

6 = 110



6 = 110



Least Significant

vs.

Most Significant

Why use least significant bits in directory?

- Hard to decide where to start
- Quite biased in the most significant bids

Comments on Extendible Hashing

- If directory fits in memory, equality search answered with one disk access; else two.
 - 100MB file, 100 bytes/rec, 4K pages contains 1,000,000 records (as data entries) and 25,000 directory elements; chances are high that directory will fit in memory.
 - Directory grows in spurts, and, if the distribution of *hash values* is skewed, directory can grow large.
 - Multiple entries with same hash value cause problems!
- **Delete**: If removal of data entry makes bucket empty, can be merged with 'split image'. If each directory element points to same bucket as its split image, can halve directory.

$$2^N, 2^{N+1}, 2^{N+2}, 2^{N+4}, 2^{N+5}$$

$$N = 100$$

00, 000, 0000,, 000...000,

$$2^{101}$$

Linear Hashing

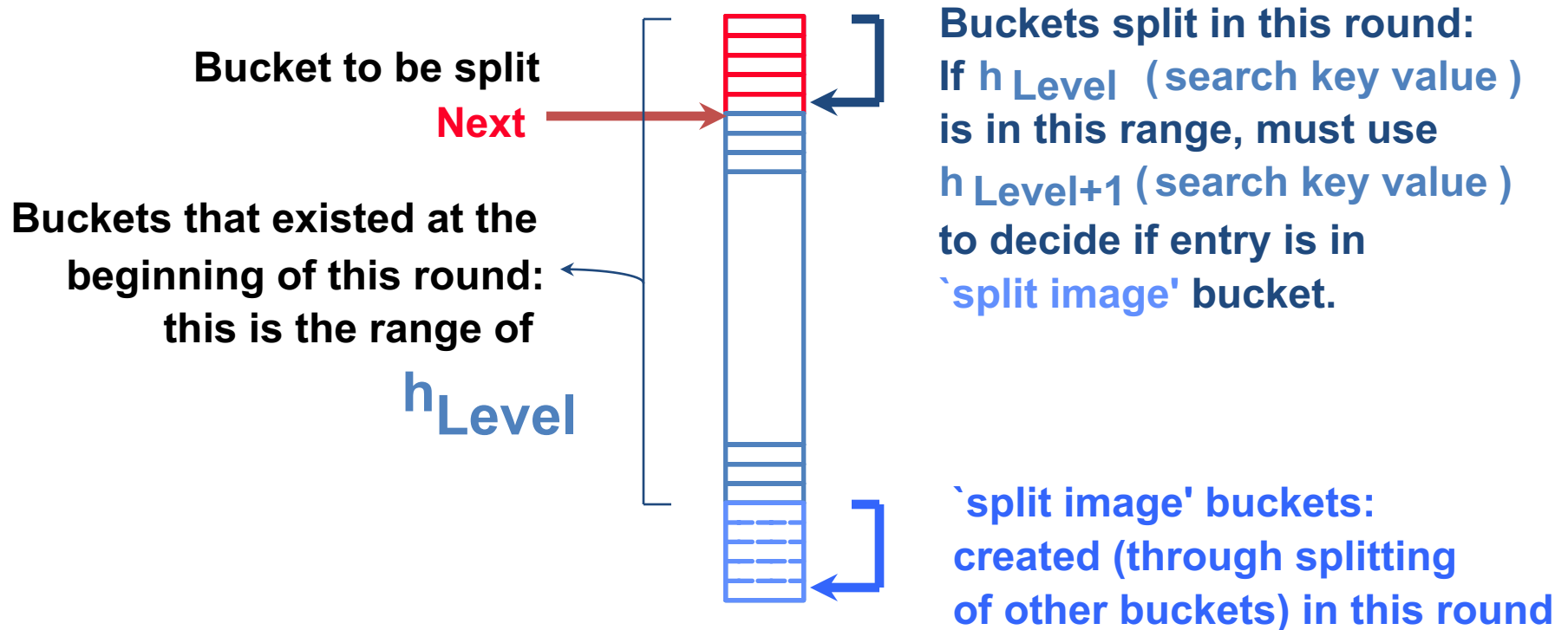
- This is another dynamic hashing scheme, an alternative to Extendible Hashing.
- LH handles the problem of long overflow chains without using a directory, and handles duplicates.
- Idea: Use a family of hash functions $\mathbf{h}_0, \mathbf{h}_1, \mathbf{h}_2, \dots$
 - $\mathbf{h}_i(\text{key}) = \mathbf{h}(\text{key}) \bmod(2^i N)$; N = initial # buckets
 - \mathbf{h} is some hash function (range is *not* 0 to $N-1$)
 - If $N = 2^{d_0}$, for some d_0 , \mathbf{h}_i consists of applying \mathbf{h} and looking at the last d_i bits, where $d_i = d_0 + i$.
 - \mathbf{h}_{i+1} doubles the range of \mathbf{h}_i (similar to directory doubling)

Linear Hashing (Contd.)

- Directory avoided in LH by using overflow pages, and choosing bucket to split round-robin.
 - Splitting proceeds in 'rounds'. Round ends when all N_R initial (for round R) buckets are split. Buckets 0 to *Next-1* have been split; *Next* to N_R yet to be split.
 - Current round number is *Level*.
 - Search: To find bucket for data entry r , find $h_{Level}(r)$:
 - If $h_{Level}(r)$ in range '*Next* to N_R ', r belongs here.
 - Else, r could belong to bucket $h_{Level}(r)$ or bucket $h_{Level}(r) + N_R$; must apply $h_{Level+1}(r)$ to find out.

Overview of LH File

In the middle of a round.



Linear Hashing (Contd.)

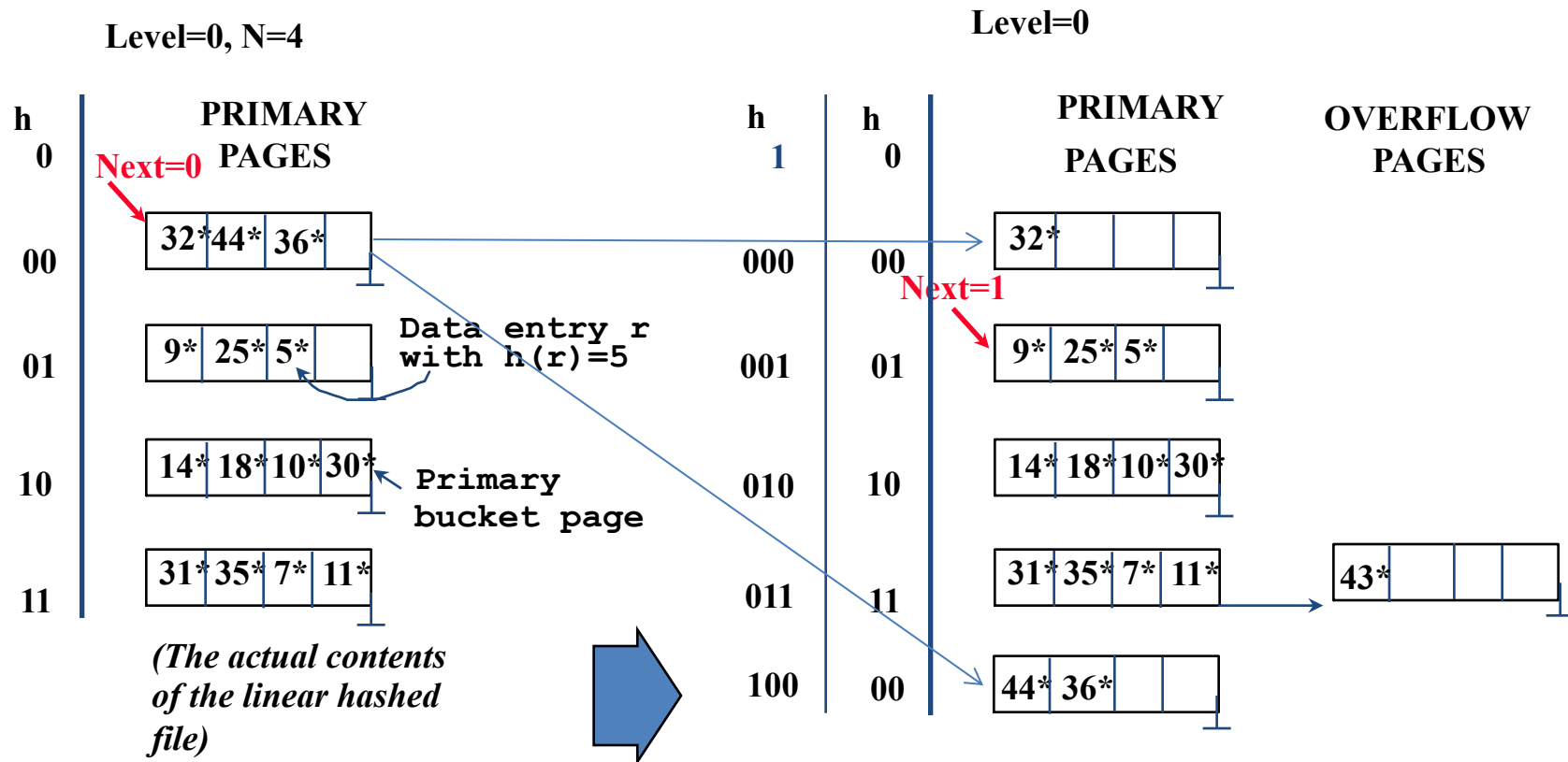
- **Insert:** Find bucket by applying $h_{Level} / h_{Level+1}$:
 - If bucket to insert into is full:
 - Add overflow page and insert data entry.
 - (*Maybe*) Split *Next* bucket and increment *Next*.
- Can choose any criterion to 'trigger' split.
- Since buckets are split round-robin, long overflow chains don't develop!
- Doubling of directory in Extendible Hashing is similar; switching of hash functions is *implicit* in how the # of bits examined is increased.

Usually, when a new overflow page is created.

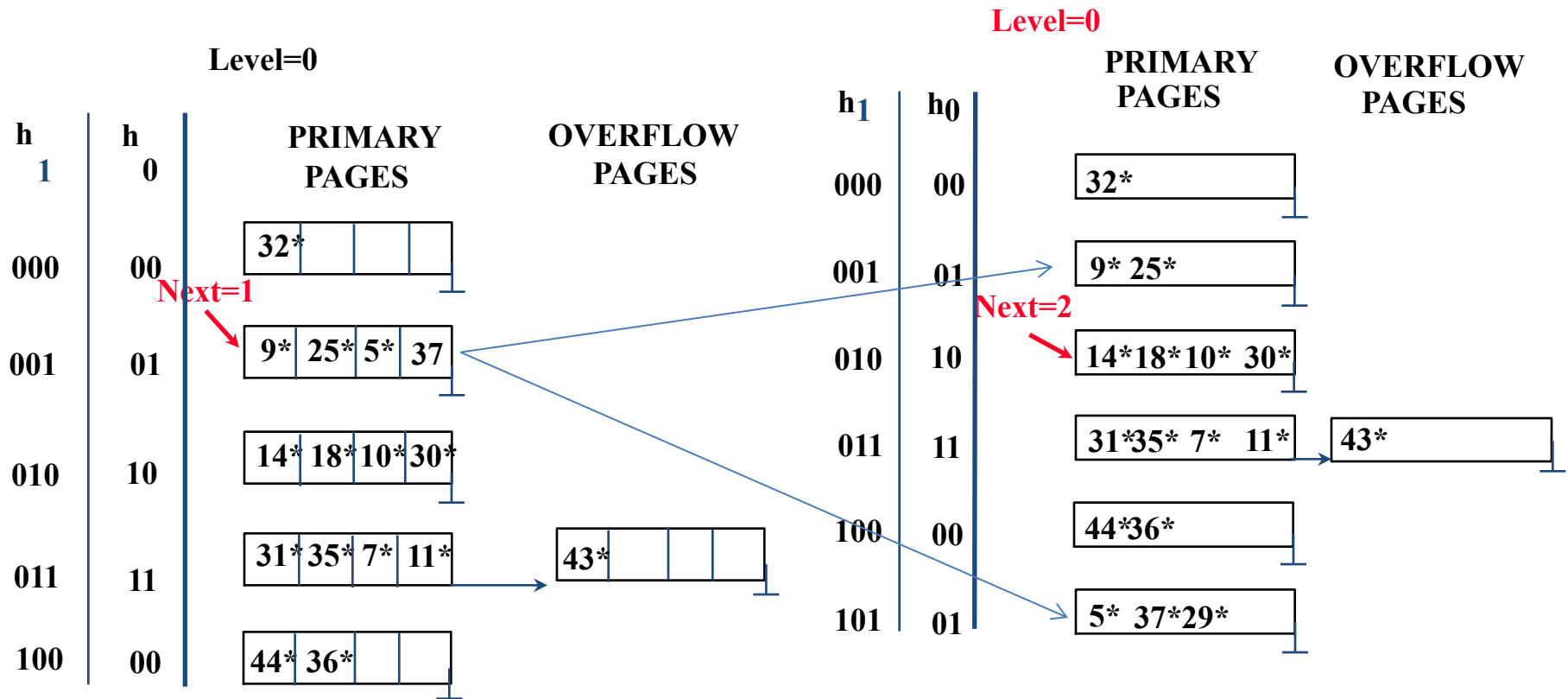


Example of Linear Hashing

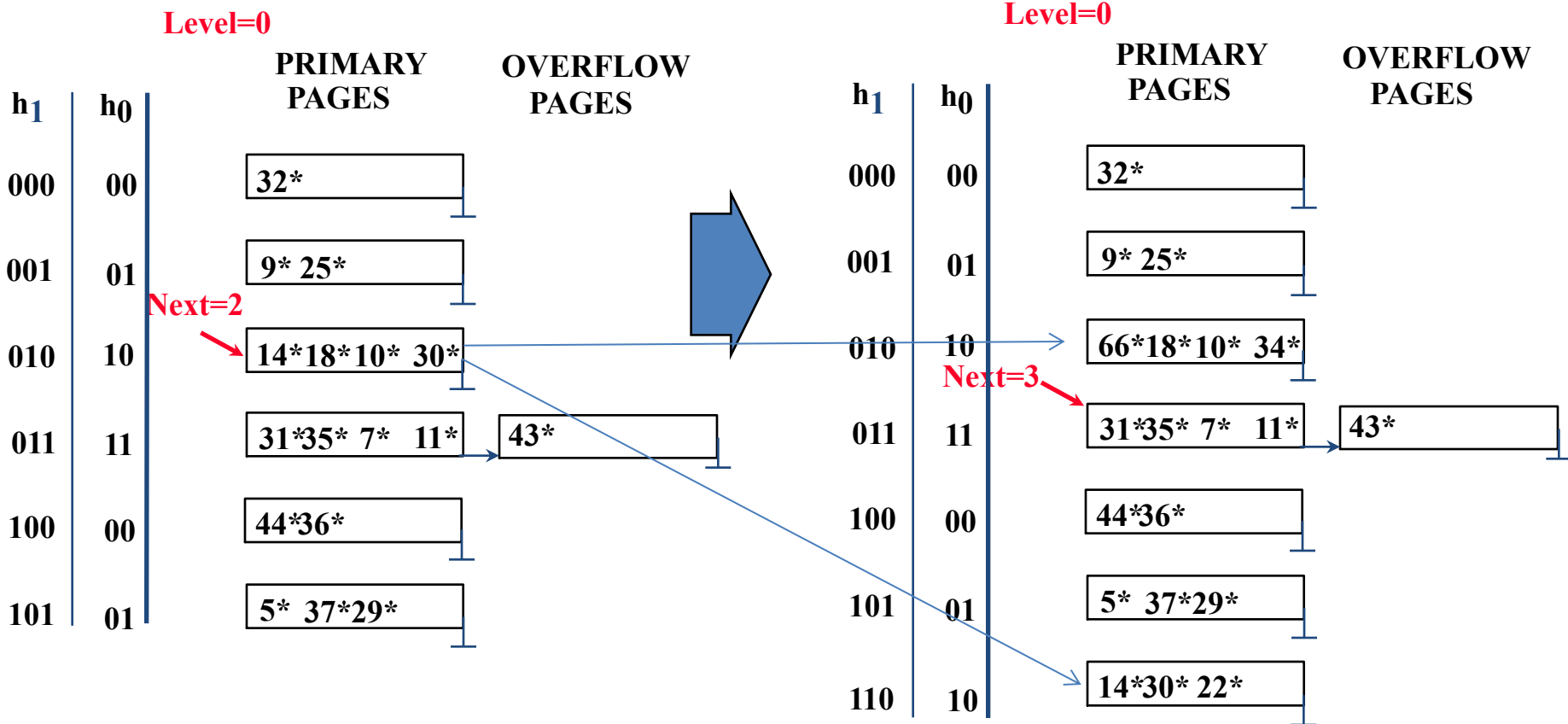
- Insert 43*



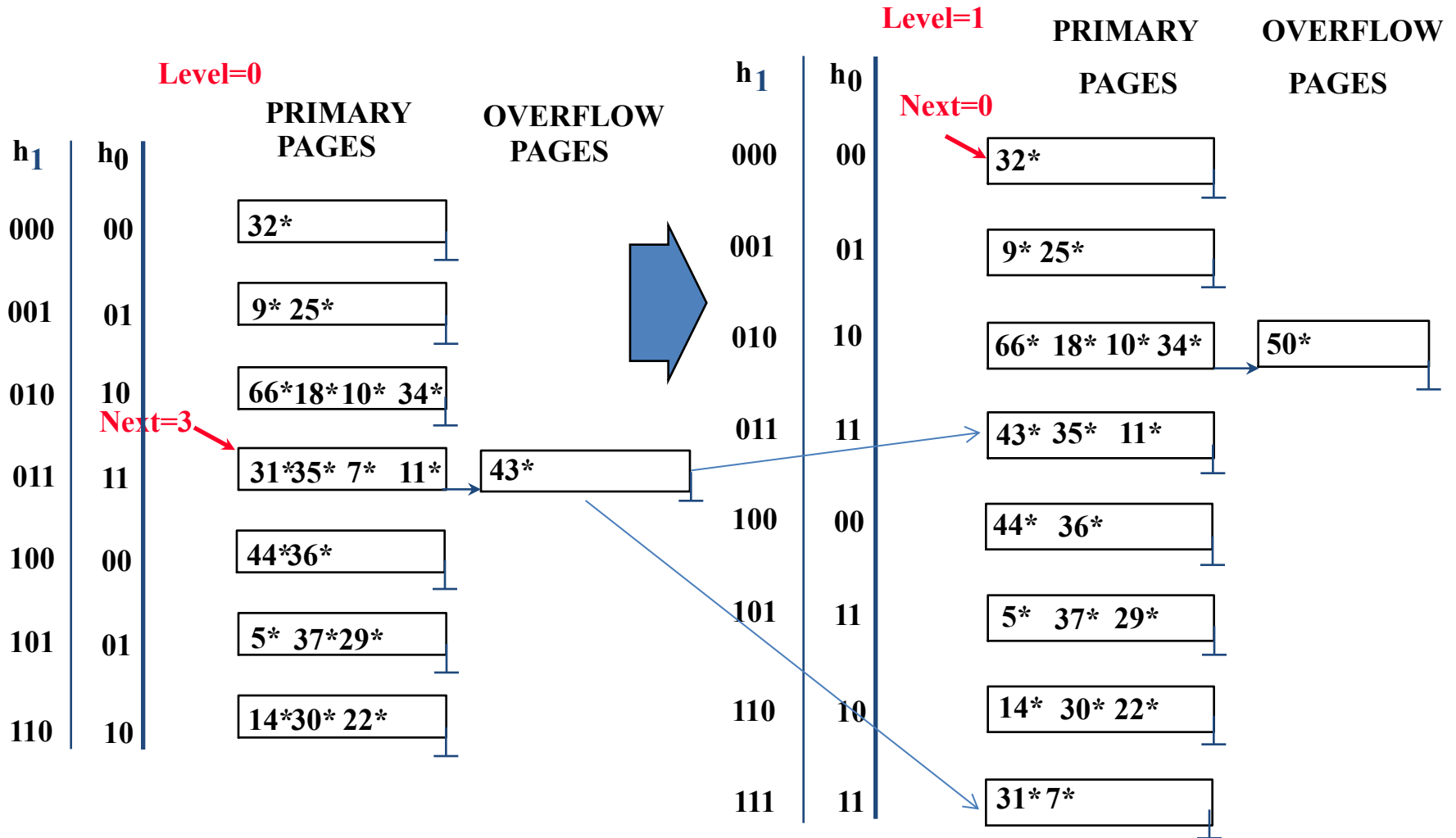
Insert 37 29



Insert 34 66 22



Insert 50



LH Described as a Variant of EH

- The two schemes are actually quite similar:
 - Begin with an EH index where directory has N elements.
 - Use overflow pages, split buckets round-robin.
 - First split is at bucket 0. (Imagine directory being doubled at this point.) But elements $\langle 1, N+1 \rangle$, $\langle 2, N+2 \rangle$, ... are the same. So, need only create directory element N , which differs from 0, now.
 - When bucket 1 splits, create directory element $N+1$, etc.
- So, directory can double gradually. Also, primary bucket pages are created in order. If they are

Summary

- Hash-based indexes: best for equality searches, cannot support range searches.
- Static Hashing can lead to long overflow chains.
- Extendible Hashing avoids overflow pages by splitting a full bucket when a new data entry is to be added to it. (*Duplicates may require overflow pages.*)
 - Directory to keep track of buckets, doubles periodically.
 - Can get large with skewed data; additional I/O if this does not fit in main memory.

Summary (Contd.)

- Linear Hashing avoids directory by splitting buckets round-robin, and using overflow pages.
 - Overflow pages not likely to be long.
 - Duplicates handled easily.
 - Space utilization could be lower than Extendible Hashing, since splits not concentrated on 'dense' data areas.
 - Can tune criterion for triggering splits to trade-off slightly longer chains for better space utilization.
- For hash-based indexes, a *skewed* data distribution is one in which the *hash values* of data entries are not uniformly distributed!