Homework 2

Algorithm Design 2018-19 - Sapienza

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December 24, 2018

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Let D_t be a random variable that denotes the position at time t. Let $P_n = Pr(H|D_0 = n)$ be the probability Giorgio goes to hospital starting from position n.

$$P_n = \begin{cases} 1 & \text{if } n = -1 \\ p \cdot P_{n-1} + (1-p) \cdot P_{n+1} & \text{if } n \geq 0 \end{cases}$$

Thus we obtain the following recurrence equation:
$(1-p) \cdot P_{n+1} - P_n + p \cdot P_{n-1} = 0$
For $n = -1$ it is true since Giorgio has already touched the highway. Now, be E the event to make a step towards the highway. $P_n = Pr(H|D_0 = n)$
$= Pr(H \cap E|D_0 = n) + Pr(H \cap \neg E|D_0 = n)$
$= Pr(E|D_0 = n) \cdot Pr(H|E \cap D_0 = n) + Pr(\neg E|D_0 = n) \cdot Pr(H|\neg E \cap D_0 = n)$
$= p \cdot Pr(H|D_1 = n-1) + (1-p) \cdot Pr(H|D_1 = n+1)$
$= p \cdot Pr(H|D_0 = n-1) + (1-p) \cdot Pr(H|D_0 = n+1)$
$= p \cdot P_{n-1} + (1-p) \cdot P_{n+1}$
ok!

We can now solve the characteristic equation of the above recurrence equation: $(1-p) \cdot r^2 - r + p = 0$ and finding the roots:

1. $\frac{p}{1-p}$
2. 1

We can thus write: $P_n = A \cdot (\frac{p}{1-p})^n + B \cdot 1^n = A \cdot (\frac{p}{1-p})^n + B$, where A and B are two constants.

Since $P_0 = p = A + B \Rightarrow B = p - A$.

Moreover $P_{-1} = 1 = A \cdot \frac{1-p}{p} + B \Rightarrow A = \frac{p-1}{1-\frac{1-p}{p}}$

Giorgio goes to hospital with probability: $Pr(H) = p - \frac{p-1}{1-\frac{1-p}{p}} + \frac{p-1}{1-\frac{1-p}{p}} \cdot \sum_{n=0}(\frac{p}{1-p})^n$

the series converges for $p \leq \frac{1}{2}$, diverges for $p > \frac{1}{2}$.

diverging here means that the event H always happens after an infinite amount of steps!

if converges, we bound $Pr(H) \leq \frac{1}{2} \Leftrightarrow p \leq \frac{1}{2}$

References