

## MATH 4338 Main Problem 3

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*Proof.* Assume  $f(a), h(a)$  are continuous. Assume  $f(x) \leq g(x) \leq h(x)$  for  $|x - a| < k$ . Lastly, assume  $f(a) = h(a)$ .

By the definition of continuity

$$\lim_{x \rightarrow a} f(x) = f(a) \qquad \lim_{x \rightarrow a} h(x) = h(a)$$

Then by the Sandwich Theorem,  $\lim_{x \rightarrow a} g(x) = g(a)$ . Pick  $x = a$  for  $|x - a| < k$ . As this inequality holds for  $x = a$ ,  $f(a) = g(a) = h(a)$ . Furthermore, this means  $g(a)$  exists. By the limit definition of continuity, since  $g(a)$  exists and

$$\lim_{x \rightarrow a} g(x) = g(a)$$

,  $g(x)$  is continuous at  $a$ .

□