

MATH 4338 Main Problem 5

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Proof. Suppose f is nondecreasing on an interval $I = \{x : a < x < b\}$, where $a, b \in \mathbb{R}$. Suppose $\exists M \in \mathbb{R}$ such that $f(x) \leq M \ \forall x \in I$. By definition, M is an upperbound of $f(x)$. By Theorem 3.5, we can pick $C \in \mathbb{R}$ such that C is, by definition, the least upper bound of $f(x)$. Note, this means $C \leq M$. Then, by the Nested Intervals Theorem, it follows that

$$\lim_{x \rightarrow b} f(x) = C$$

. Since the limit exists, the one sided limit,

$$\lim_{x \rightarrow b^-} f(x) = C$$

must also exist. □