MATH 4338 Main Problem 7

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Proof. Suppose $a \in \mathbb{R}$ and $f'(a)$ exists at a. Then by the definition of derivative,

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

exists. Let it equal L. By Theorem 2.8,

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \frac{\lim_{h \to 0} g(a)}{\lim_{h \to 0} h(a)}$$

, where $g(a) = f(a+h) - f(a)]$ and $h(a) = h$. Thus, the right hand side exists. Further examining $\lim_{h \to 0} g(a)$, by Theorem 2.5,

$$\lim_{h \to 0} g(a) = L_1 + L_2$$

where $L_1 = \lim_{h \to 0} f(a+h)$ and $L_2 = \lim_{h \to 0} f(a)$. As $\lim_{h \to 0} f(a)$ exists, then by Theorem 2.13, the one sided limits exist. Since the derivative is defined at a, a must be in the domain of f. Then by the Corollary to Theorem 2.13, f is continuous at a. $\square$