MATH 4338 Main Problem 8

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Proof. Verify

$$g'(x_0) = \frac{1}{f'[g(x_0)]}$$

Computing $f'(x)$:

Note: Computing $a'(x)$ and $b'(x)$:

$$a'(x) = 3x^{3-1} = 3x^2 \qquad \text{[by Theorem 4.7]}$$

$$b'(x) = 1x^{1-1} + 0 = 1 \qquad \text{[by Theorem 4.3, 4.7, and 4.1]}$$

$$\begin{aligned} f'(x) &= a'(b(x)) \cdot b'(x) && \text{[by the chain rule and} \\ & && \text{where } a(x) = x^3 \text{ and } b(x) = x - 1] \\ &= (3(x-1)^2)(1) && \text{[substituting in } a'(x), b(x), b'(x)] \\ &= 3(x-1)^2 \end{aligned}$$

Computing $g'(x)$:

$$\begin{aligned} x =& (y-1)^3 \\ \sqrt[3]{x} =& (y-1) \\ \sqrt[3]{x} + 1 =& y \\ \sqrt[3]{x} + 1 =& g(x) \end{aligned}$$

Let $c(x) = \sqrt[3]{x}$ and $d(x) = 1$. Then by Theorem 4.9,4.7, and 4.1, $c'(x) = 3^{-1}(x)^{3^{-1}-1}(1) = \frac{x^{-2/3}}{3} = \frac{1}{3x^{2/3}}$. Also by Theorem 4.1, $d'(x) = 0$. Thus,

$$\begin{aligned} g'(x) &= c'(x) + d'(x) && \text{[by Theorem 4.3]} \\ g'(x) &= \frac{1}{3x^{2/3}} + 0 && \text{[by substitution]} \\ g'(x) &= \frac{1}{3x^{2/3}} \end{aligned}$$

Now we verify this is Formula 4.13 of the Inverse Differentiation Theorem.

$$
\begin{aligned}
\frac{1}{f'[g(x_0)]} &= \\
&= \frac{1}{3(g(x_0)-1)^2} && \text{[by substitution]} \\
&= \frac{1}{3(\sqrt[3]{x_0}+1-1)^2} && \text{[by substitution]} \\
&= \frac{1}{3(\sqrt[3]{x_0})^2} && \text{[combine like terms]} \\
&= \frac{1}{3((x_0)^{1/3})^2} && \text{[square root property]} \\
&= \frac{1}{3(x_0)^{2/3}} && \text{[exponent property]} \\
&= g(x_0) && \text{[definition of } g'(x)\text{]}
\end{aligned}
$$

Thus, Formula 4.13 of the Inverse Differentiation Theorem is verified. $\square$