Numerical Methods for the Solution of PDEs

Laboratory with deal.II — www.dealii.org

Triangulation, DoFHandler, FiniteElement

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Aims for this module

- Gain familiarity with three core classes
  - **Triangulation**
  - **DoFHandler**
  - **FiniteElement**
- Create and interrogate meshes
- Create and interrogate sparsity patterns

Reference material

- Main page
  https://dealii.org/current/doxygen/deal.II/index.html

- Tutorials

  - Step-1
    https://dealii.org/current/doxygen/deal.II/step_1.html

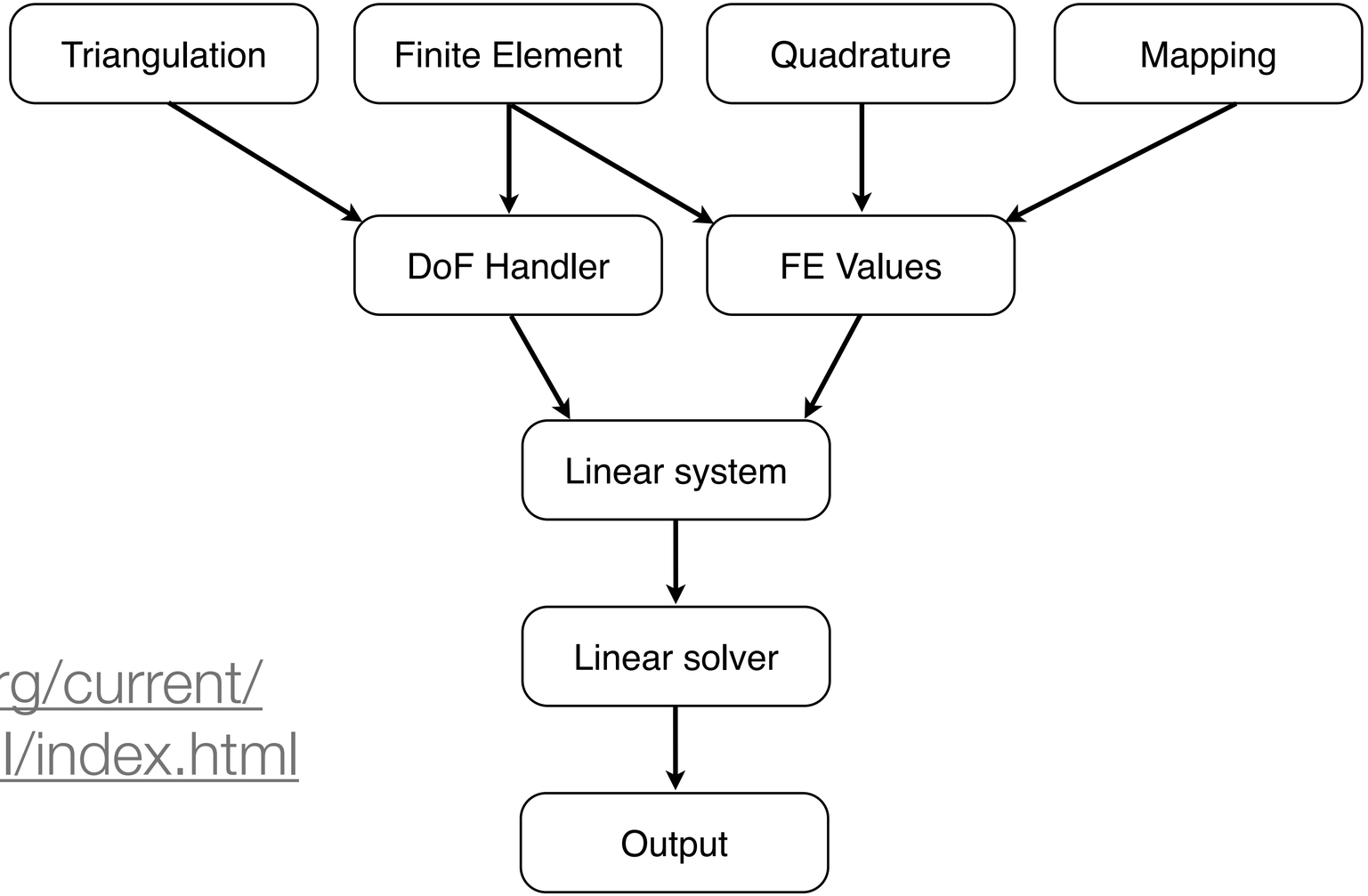
  - Step-49
    https://dealii.org/current/doxygen/deal.II/step_49.html

  - Step-2
    https://dealii.org/current/doxygen/deal.II/step_2.html

First and **BIGGEST** tip

- Program **defensively**
  - Program and test in **debug** mode
    - Additional compiler warnings
    - Add assertions
  - Perform studies in **release** mode

Structure of a prototypical FE problem



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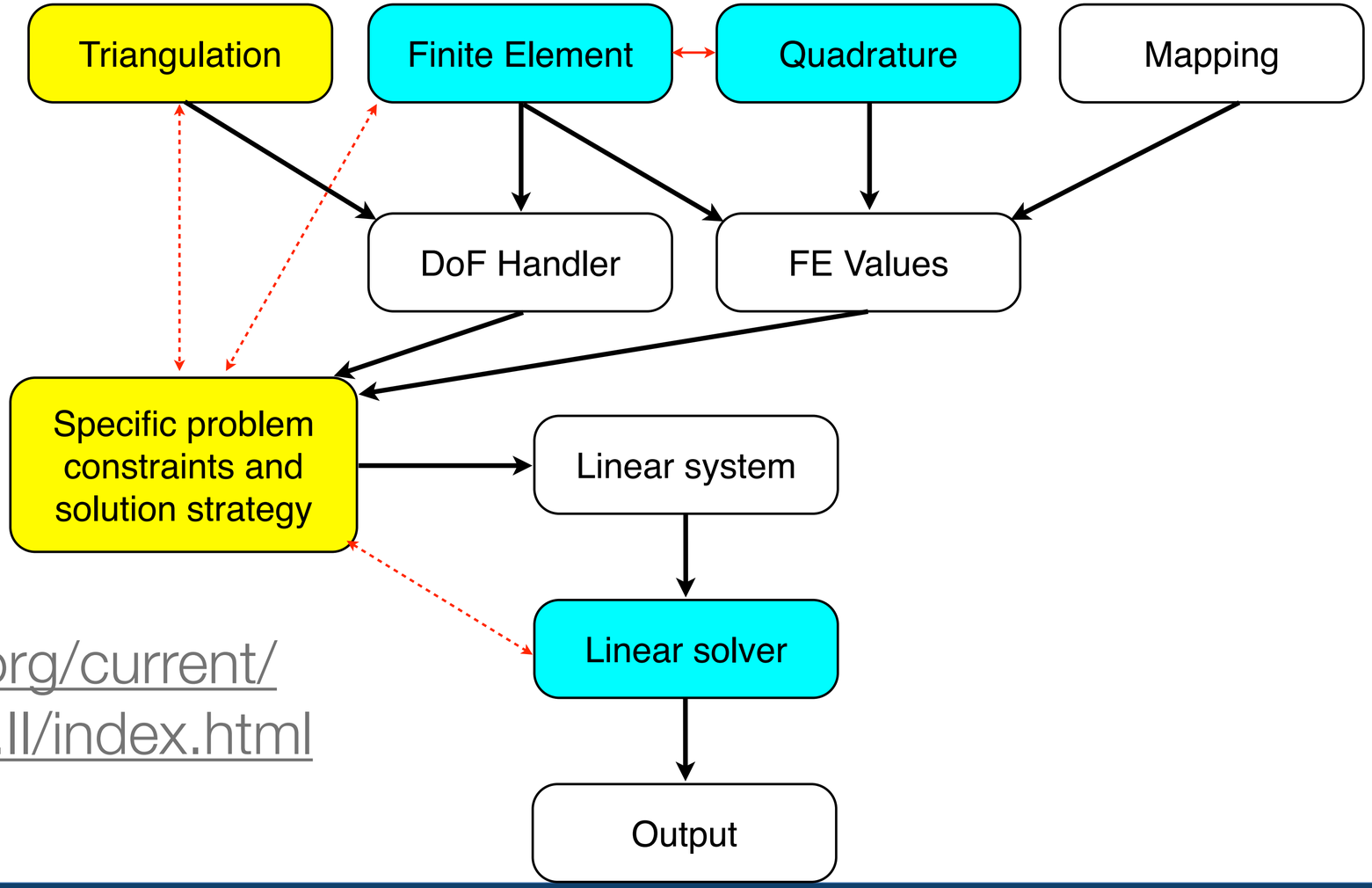

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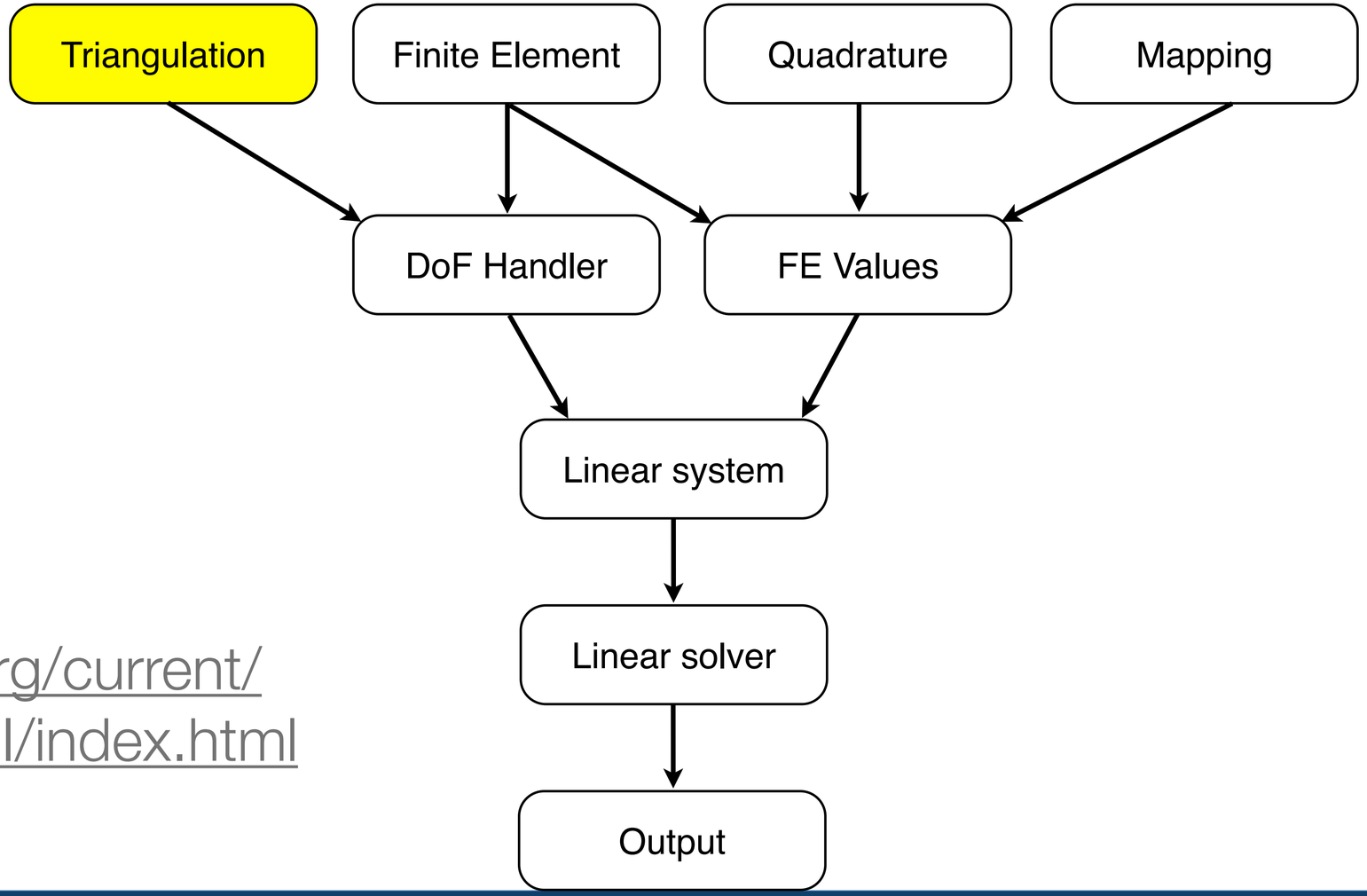

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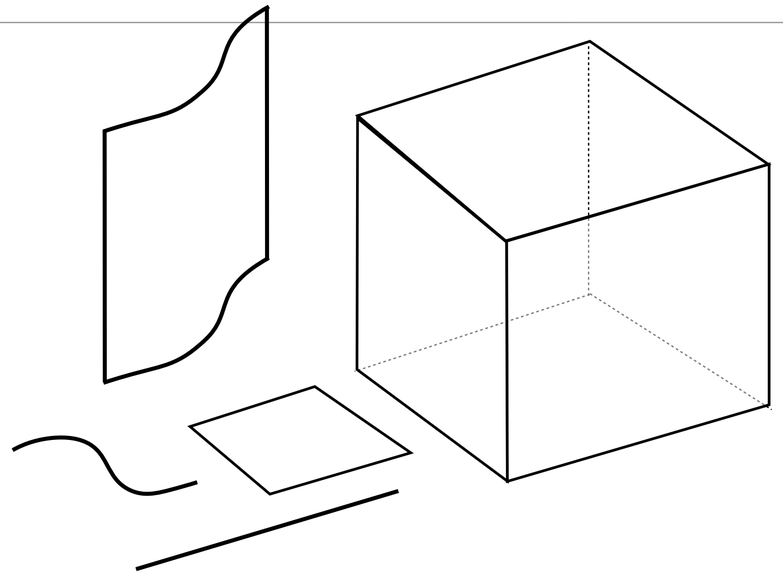
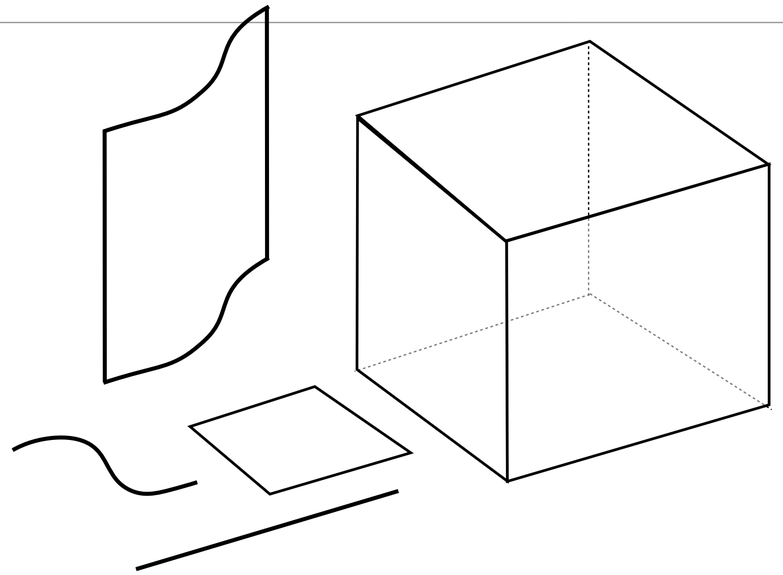
Interaction with geometry: the Triangulation class

- Describes problem geometry
  - Support for lines, quad, hex elements
  - Conceptually even higher order!
  - Structured/unstructured meshes
  - Co-dimension 1 or 2 case
- Grid creation
  - Built-in basic grid generation and manipulation tools
  - Can read in grids



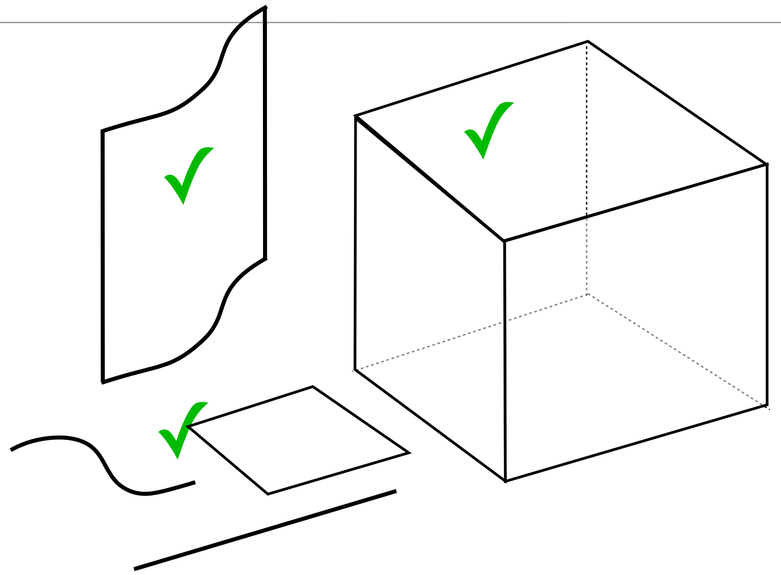
Interaction with geometry: the Triangulation class

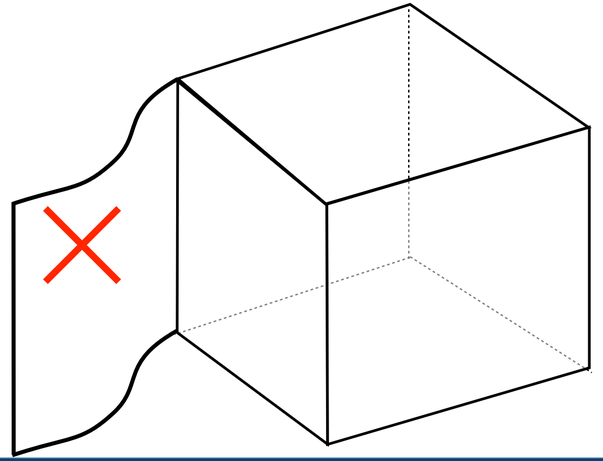
- Assign helper ID’s
  - Materials
  - Boundaries
  - Manifolds
- Allows storage of custom data-structure attached to each cell/face
- Cells know about neighbour cells
  - Useful for DG methods



Interaction with geometry: the Triangulation class

- Can enforce topologies
  - Manifolds on boundary
  - Internal manifolds
- Disadvantage
  - Cannot mix triangulation types
  - e.g. Volumetric body with extended manifold surface

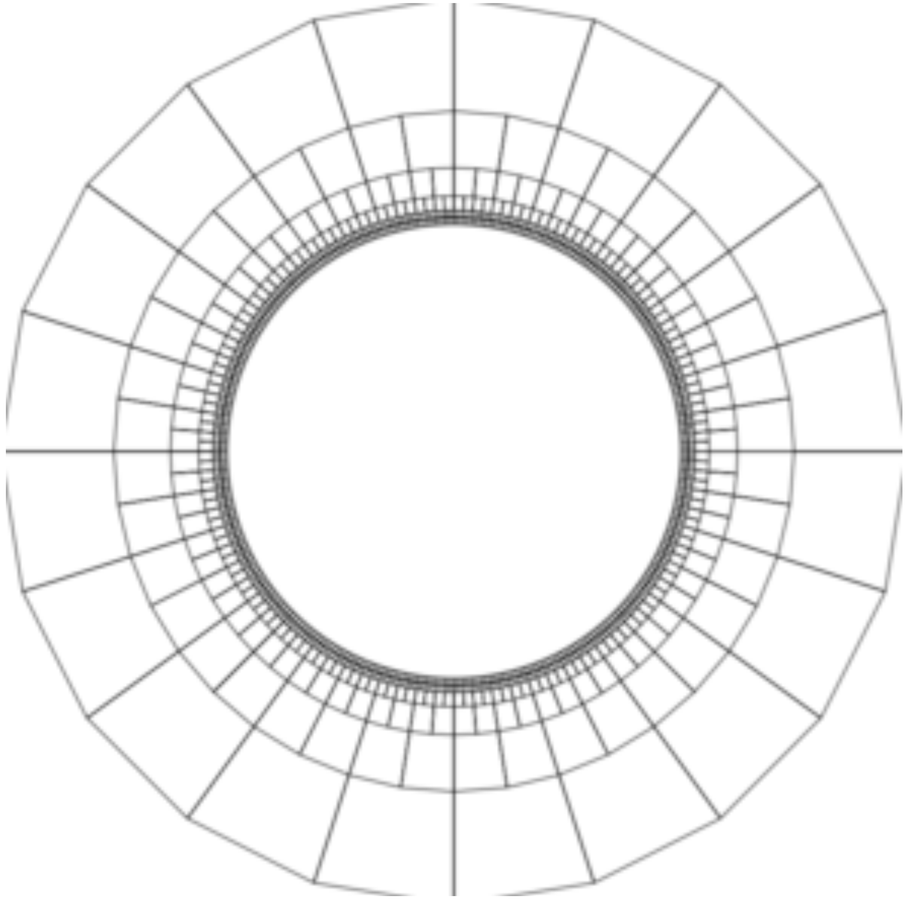




Interaction with geometry: the Triangulation class

- Demonstration: Step-1, step-49
  https://www.dealii.org/current/doxygen/deal.II/step_1.html
  https://www.dealii.org/current/doxygen/deal.II/step_49.html
  http://www.math.colostate.edu/~bangerth/videos.676.5.html
  http://www.math.colostate.edu/~bangerth/videos.676.6.html

- Key points
  - deal.II headers
  - Creating a triangulation
  - Boundary topology
  - Traversing a triangulation
  - Querying geometric information
  - Manipulating a triangulation
  - Aspects of grid refinement
  - Visualising a triangulation



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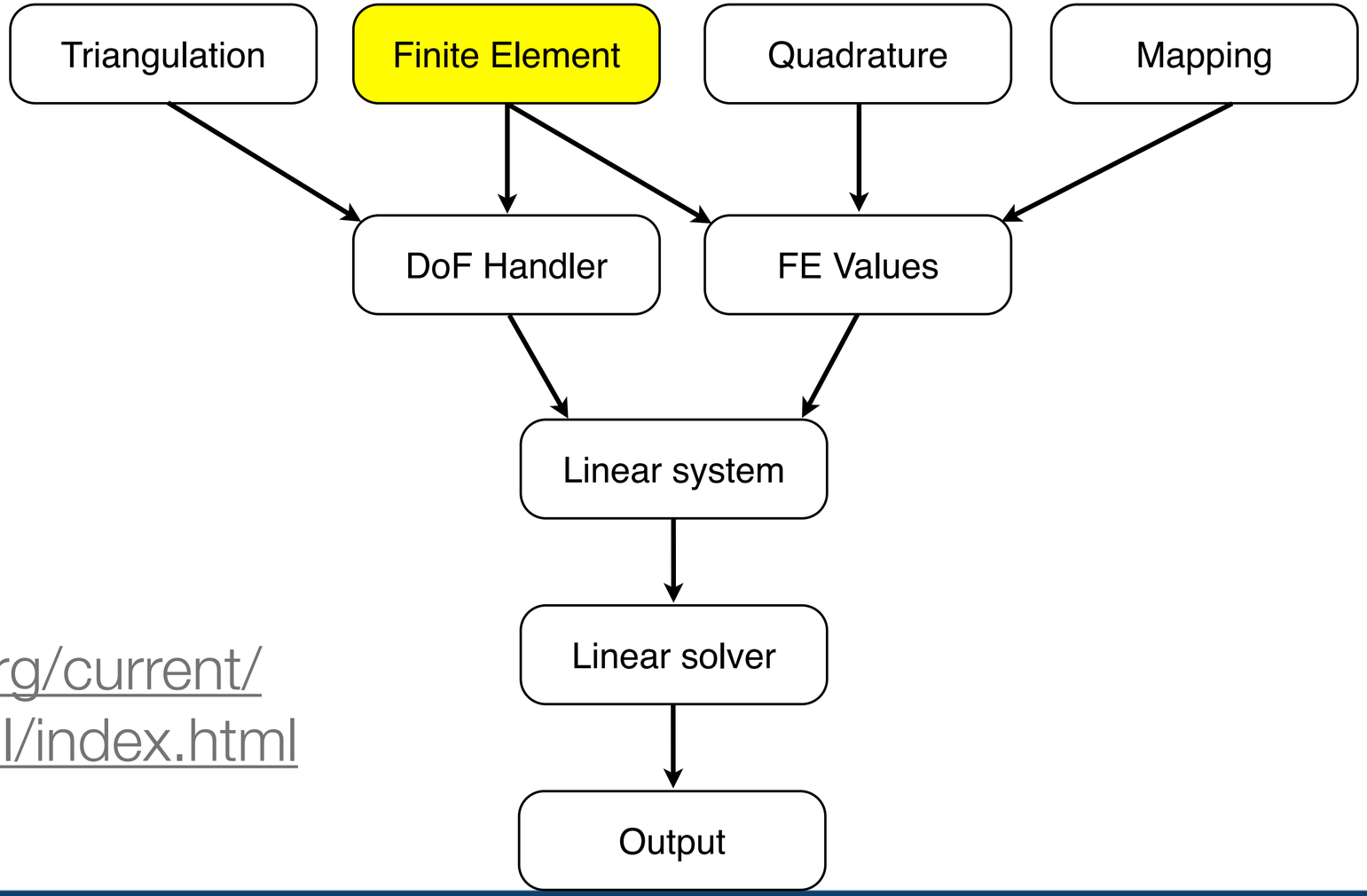

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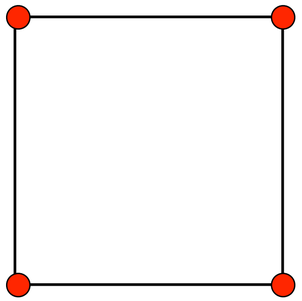
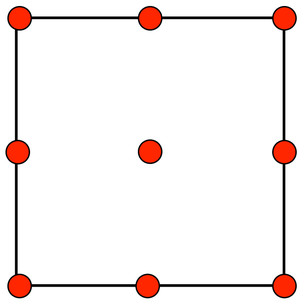
Assigning degrees-of-freedom: the FiniteElement classes

- Built in Finite Elements
  - Continuous
    - Piecewise Lagrange polynomials
  - Discontinuous
    - Monomials
    - Legendre polynomials
  - Vector-valued
    - Nedelec (H^{curl}, C/Dc)
    - Raviart-Thomas (H^{div}, C/Dc)
- A few more...
- Can develop finite elements from scratch
  - Specialisation for FE's derived by polynomial expansions
  - Enhanced/bubble elements

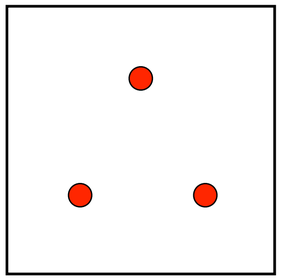
`FE_Q<2>(1)`



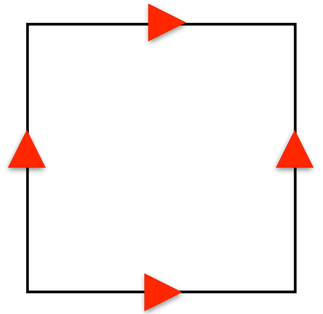
`FE_Q<2>(2)`



`FE_DGPMonomial<2>(1)`



`FE_Nedelec<2>(0)`



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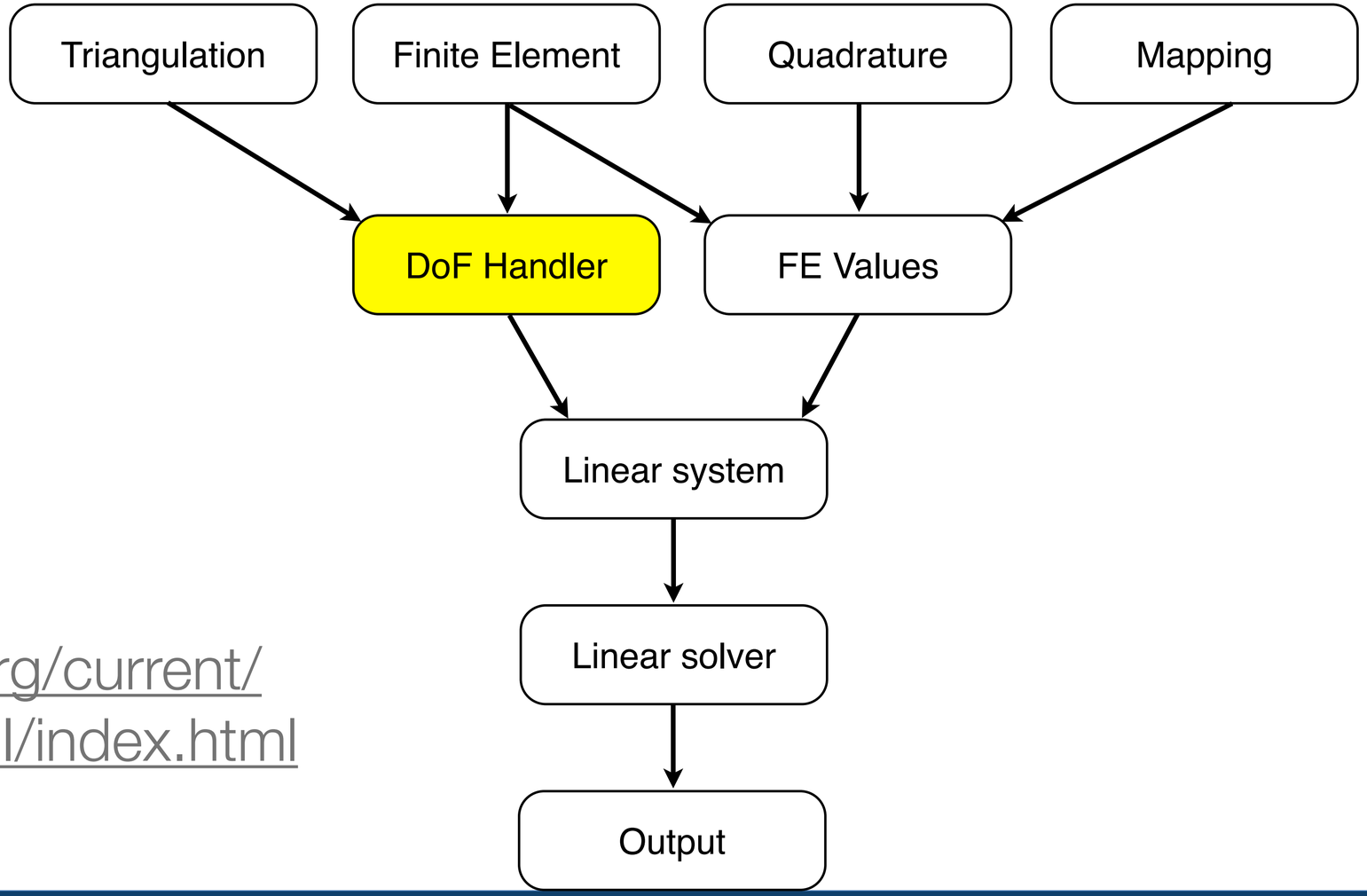

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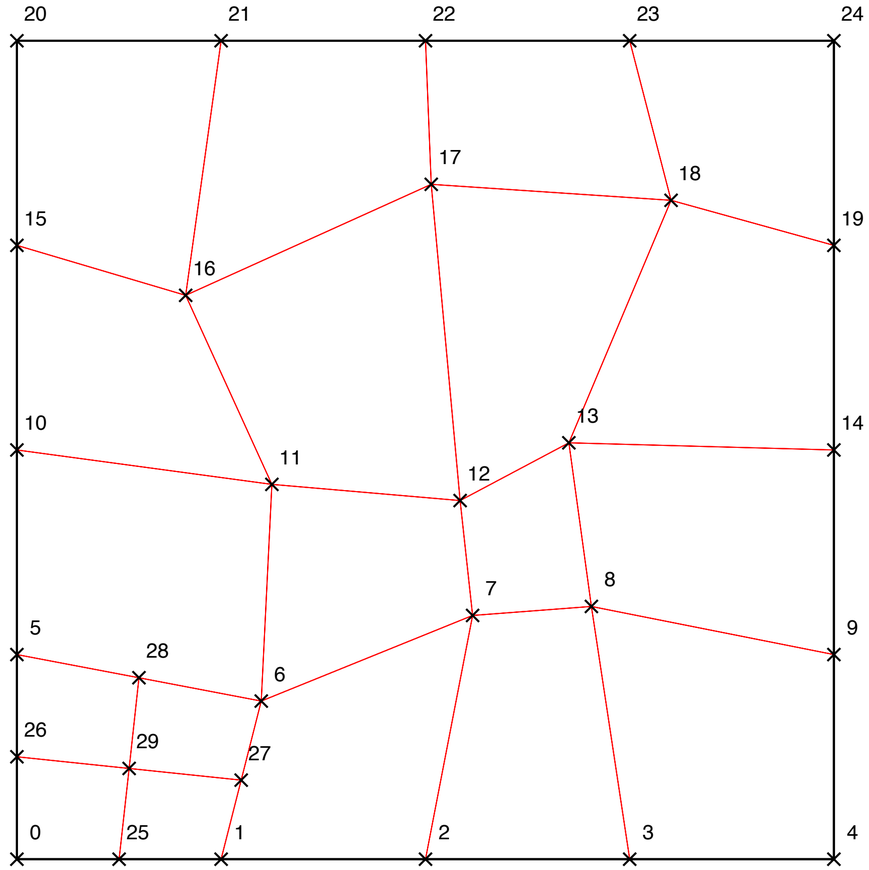
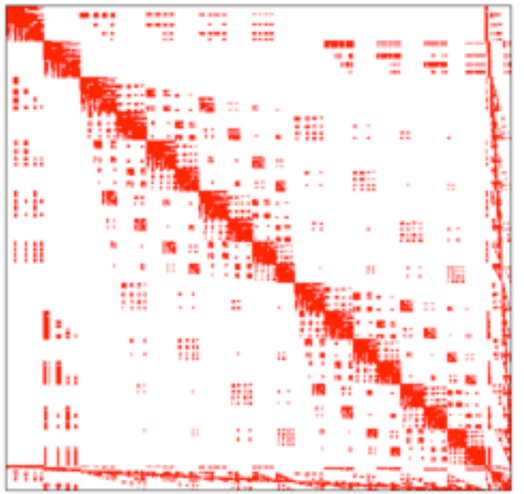
Assigning degrees-of-freedom: the DoFHandler class

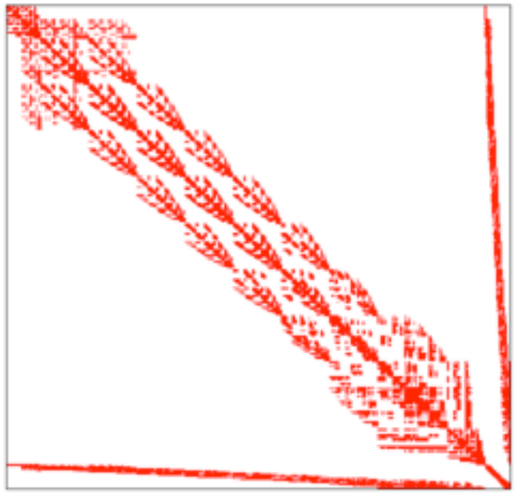
- DoFHandler assigns DoF's to grid
  - Important: separate from Triangulation!
- Unified way to access DoF's, regardless of FE used
  - e.g. Discontinuous elements: support points not necessarily at vertices
- Fast access and grid traversal
  - STL-type cell iterators
  - Access to faces, edges through these



Assigning degrees-of-freedom: the DoFRenumbering namespace

- Renumbering schemes
  - Cuthill McKee
  - King
  - Downwind
- Reduce bandwidth
- Collect like-components
- Induce block-structure
- Directional (fluid flow)
- MPI subdomain





Assigning degrees-of-freedom: the FiniteElement and DoFHandler classes

- Demonstration: Step-2
  https://www.dealii.org/current/doxygen/deal.II/step_2.html
  http://www.math.colostate.edu/~bangerth/videos.676.9.html

- Key points
  - Choosing a Finite Element
  - Distributing degrees-of-freedom on a mesh
  - Renumbering degrees-of-freedom
  - Visualising sparsity patterns