Regole della semantica operazionale

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1 COSTRUTTORI

REGOLA PER `emptyset`

$$\frac{\tau \in \mathtt{Types}}{\rho \triangleright \mathtt{EmptySet}(\tau) \Rightarrow \mathtt{Set}(\tau, \varnothing)}$$

REGOLA PER `singleton`

$$\frac{\tau \in \mathtt{Types} \quad \rho \triangleright e \Rightarrow v \quad \mathtt{check\_from\_ty}(\tau, v)}{\rho \triangleright \mathtt{Singleton}(\tau, e) \Rightarrow \mathtt{Set}(\tau, \{v\})}$$

REGOLA PER `of`

$$\frac{\tau \in \mathtt{Types} \quad \rho \triangleright e \Rightarrow S \quad (\forall v \in S.\ \mathtt{check\_from\_ty}(\tau, v))}{\rho \triangleright \mathtt{Of}(\tau, e) \Rightarrow \mathtt{Set}(\tau, S)}$$

REGOLE PER `set_eval`

$$\frac{}{\rho \triangleright \mathtt{EmptyS} \Rightarrow \varnothing}$$

$$\frac{\rho \triangleright \mathtt{es} \Rightarrow S \quad \rho \triangleright e \Rightarrow v \quad v \in S}{\rho \triangleright \mathtt{Cons}(e, \mathtt{es}) \Rightarrow S}$$

$$\frac{\rho \triangleright \mathtt{es} \Rightarrow S_0 \quad \rho \triangleright e \Rightarrow v \quad v \notin S_0 \quad S \coloneqq (v :: S_0)}{\rho \triangleright \mathtt{Cons}(e, \mathtt{es}) \Rightarrow S}$$

2 OPERAZIONI DI BASE

REGOLE PER `union`

$$\frac{\rho \triangleright e_1 \Rightarrow \mathtt{Set}(\tau_1, S_1) \quad \rho \triangleright e_2 \Rightarrow \mathtt{Set}(\tau_2, S_2) \quad \tau_1 = \tau_2 \quad S \coloneqq S_1 \cup S_2 \quad \tau \coloneqq \tau_1}{\rho \triangleright \mathtt{Union}(e_1, e_2) \Rightarrow \mathtt{Set}(\tau, S)}$$

$$\frac{}{\varnothing \cup S_2 \rightsquigarrow S_2}$$

$$\frac{v \in S_2 \quad S_1 \cup S_2 \rightsquigarrow S}{(v :: S_1) \cup S_2 \rightsquigarrow S}$$

$$\frac{v \in S_2 \quad S_1 \cup S_2 \rightsquigarrow S' \quad S \coloneqq (v :: S')}{(v :: S_1) \cup S_2 \rightsquigarrow S}$$

REGOLE PER `inters`

$$\frac{\rho \triangleright e_1 \Rightarrow \texttt{Set}(\tau_1, S_1) \quad \rho \triangleright e_2 \Rightarrow \texttt{Set}(\tau_2, S_2) \quad \tau_1 = \tau_2 \quad S \coloneqq S_1 \cap S_2 \quad \tau \coloneqq \tau_1}{\rho \triangleright \texttt{Inters}(e_1, e_2) \Rightarrow \texttt{Set}(\tau, S)}$$

$$\frac{}{\varnothing \cap S_2 \rightsquigarrow \varnothing}$$

$$\frac{v \in S_2 \quad S_1 \cap S_2 \rightsquigarrow S' \quad S \coloneqq (v :: S')}{(v :: S_1) \cap S_2 \rightsquigarrow S}$$

$$\frac{v \notin S_2 \quad S_1 \cap S_2 \rightsquigarrow S}{(v :: S_1) \cap S_2 \rightsquigarrow S}$$

REGOLE PER `setdiff`

$$\frac{\rho \triangleright e_1 \Rightarrow \texttt{Set}(\tau_1, S_1) \quad \rho \triangleright e_2 \Rightarrow \texttt{Set}(\tau_2, S_2) \quad \tau_1 = \tau_2 \quad S \coloneqq S_1 \setminus S_2 \quad \tau \coloneqq \tau_1}{\rho \triangleright \texttt{SetDiff}(e_1, e_2) \Rightarrow \texttt{Set}(\tau, S)}$$

$$\frac{}{\varnothing \setminus S_2 \rightsquigarrow \varnothing}$$

$$\frac{v \in S_2 \quad S_1 \setminus S_2 \rightsquigarrow S}{(v :: S_1) \setminus S_2 \rightsquigarrow S}$$

$$\frac{v \notin S_2 \quad S_1 \setminus S_2 \rightsquigarrow S' \quad S \coloneqq (v :: S')}{(v :: S_1) \setminus S_2 \rightsquigarrow S}$$

REGOLE PER `insert`

$$\frac{\rho \triangleright e_1 \Rightarrow v \quad \rho \triangleright e_2 \Rightarrow \texttt{Set}(\tau, S) \quad \texttt{check\_from\_ty}(\tau, v) \quad S' \coloneqq \texttt{set\_insert}(v, S)}{\rho \triangleright \texttt{Insert}(e_1, e_2) \Rightarrow \texttt{Set}(\tau, S')}$$

$$\frac{}{\texttt{set\_insert}(v, \varnothing) \rightsquigarrow \{v\}}$$

$$\frac{v = v'}{\texttt{set\_insert}(v, v' :: S') \rightsquigarrow (v' :: S')}$$

$$\frac{v \neq v' \quad \texttt{set\_insert}(v, S') \rightsquigarrow S_0 \quad S \coloneqq (v' :: S_0)}{\texttt{set\_insert}(v, v' :: S') \rightsquigarrow S}$$

REGOLE PER `remove`

$$\frac{\rho \triangleright e_1 \Rightarrow v \quad \rho \triangleright e_2 \Rightarrow \texttt{Set}(\tau, S) \quad \texttt{check\_from\_ty}(\tau, v) \quad S' \coloneqq \texttt{set\_remove}(v, S)}{\rho \triangleright \texttt{Remove}(e_1, e_2) \Rightarrow \texttt{Set}(\tau, S')}$$

$$\frac{}{\texttt{set\_remove}(v, \varnothing) \rightsquigarrow \varnothing}$$

$$\frac{v = v'}{\texttt{set\_remove}(v, v' :: S') \rightsquigarrow S'}$$

$$\frac{v \neq v' \quad \texttt{set\_remove}(v, S') \rightsquigarrow S_0 \quad S \coloneqq (v :: S_0)}{\texttt{set\_remove}(v, v' :: S') \rightsquigarrow S}$$

REGOLE PER `contains`

$$\frac{\rho \triangleright e_1 \Rightarrow v \quad \rho \triangleright e_2 \Rightarrow \texttt{Set}(\tau, S) \quad \texttt{check\_from\_ty}(\tau, v) \quad b \coloneqq v \in S}{\rho \triangleright \texttt{Contains}(e_1, e_2) \Rightarrow b}$$

$$\frac{}{v \in \varnothing \rightsquigarrow \texttt{Bool False}}$$

$$\frac{v = v'}{v \in (v' :: S') \rightsquigarrow \texttt{Bool True}}$$

$$\frac{v \neq v' \quad v \in S' \rightsquigarrow b}{v \in (v' :: S') \rightsquigarrow b}$$

REGOLE PER `subset`

$$\frac{\rho \triangleright e_1 \Rightarrow \texttt{Set}(\tau_1, S_1) \quad \rho \triangleright e_2 \Rightarrow \texttt{Set}(\tau_2, S_2) \quad \tau_1 = \tau_2 \quad b \coloneqq S_1 \subseteq S_2}{\rho \triangleright \texttt{Subset}(e_1, e_2) \Rightarrow b}$$

$$\frac{}{\varnothing \subseteq S_2 \rightsquigarrow \texttt{Bool True}}$$

$$\frac{v \notin S_2}{(v :: S_1) \subseteq S_2 \rightsquigarrow \texttt{Bool False}}$$

$$\frac{v \in S_2 \quad S_1 \subseteq S_2 \rightsquigarrow b}{(v :: S_1) \subseteq S_2 \rightsquigarrow b}$$

REGOLE PER `minof`

$$\frac{\rho \triangleright e \Rightarrow \texttt{Set}(\tau, S) \quad \tau \in \{\texttt{IntTy}, \texttt{StrTy}, \texttt{BoolTy}\} \quad v \coloneqq \min S}{\rho \triangleright \texttt{MinOf}(e) \Rightarrow v}$$

$$\frac{}{\min\{v\} \rightsquigarrow v}$$

$$\frac{\min S \rightsquigarrow m \quad v < m}{\min(v :: S) \rightsquigarrow v}$$

$$\frac{\min S \rightsquigarrow m \quad m \leqslant v}{\min(v :: S) \rightsquigarrow m}$$

REGOLE PER `maxof`

$$\frac{\rho \triangleright e \Rightarrow \texttt{Set}(\tau, S) \quad \tau \in \{\texttt{IntTy}, \texttt{StrTy}, \texttt{BoolTy}\} \quad v \coloneqq \max S}{\rho \triangleright \texttt{MaxOf}(e) \Rightarrow v}$$

$$\frac{}{\max\{v\} \rightsquigarrow v}$$

$$\frac{\max S \rightsquigarrow m \quad v > m}{\max(v :: S) \rightsquigarrow v}$$

$$\frac{\max S \rightsquigarrow m \quad m \geqslant v}{\max(v :: S) \rightsquigarrow m}$$

3 OPERATORI FUNZIONALI

REGOLE PER forall

$$\frac{\rho \triangleright f \Rightarrow \texttt{Closure}(\texttt{id}, \texttt{body}, \texttt{funDeclEnv}, \tau, \texttt{Bool}) \quad \rho \triangleright e \Rightarrow \texttt{Set}(\tau, S) \quad b := (\forall v \in S.\ f(v))}{\rho \triangleright \texttt{ForAll}(f, e) \Rightarrow b}$$

$$\frac{}{(\forall v \in \varnothing.\ f(v)) \rightsquigarrow \texttt{Bool True}}$$

$$\frac{\texttt{funDeclEnv}[\texttt{id} := v'] \triangleright \texttt{body} \Rightarrow \texttt{Bool False}}{(\forall v \in (v' :: S).\ f(v)) \rightsquigarrow \texttt{Bool False}}$$

$$\frac{\texttt{funDeclEnv}[\texttt{id} := v'] \triangleright \texttt{body} \Rightarrow \texttt{Bool True} \quad (\forall v \in S.\ f(v)) \rightsquigarrow b}{(\forall v \in (v' :: S).\ f(v)) \rightsquigarrow b}$$

REGOLE PER exists

$$\frac{\rho \triangleright f \Rightarrow \texttt{Closure}(\texttt{id}, \texttt{body}, \texttt{funDeclEnv}, \tau, \texttt{Bool}) \quad \rho \triangleright e \Rightarrow \texttt{Set}(\tau, S) \quad b := (\exists v \in S.\ f(v))}{\rho \triangleright \texttt{Exists}(f, e) \Rightarrow b}$$

$$\frac{}{(\exists v \in \varnothing.\ f(v)) \rightsquigarrow \texttt{Bool False}}$$

$$\frac{\texttt{funDeclEnv}[\texttt{id} := v'] \triangleright \texttt{body} \Rightarrow \texttt{Bool True}}{(\exists v \in (v' :: S).\ f(v)) \rightsquigarrow \texttt{Bool True}}$$

$$\frac{\texttt{funDeclEnv}[\texttt{id} := v'] \triangleright \texttt{body} \Rightarrow \texttt{Bool False} \quad (\exists v \in S.\ f(v)) \rightsquigarrow b}{(\exists v \in (v' :: S).\ f(v)) \rightsquigarrow b}$$

REGOLE PER filter

$$\frac{\rho \triangleright f \Rightarrow \texttt{closure} \quad \rho \triangleright e \Rightarrow \texttt{Set}(\tau, S) \quad \texttt{closure} := \texttt{Closure}(\texttt{id}, \texttt{body}, \texttt{funDeclEnv}, \tau, \texttt{Bool}) \quad b := \texttt{set\_filter}(\texttt{closure}, S)}{\rho \triangleright \texttt{Filter}(f, e) \Rightarrow b}$$

$$\frac{}{\texttt{set\_filter}(\texttt{closure}, \varnothing) \rightsquigarrow \varnothing}$$

$$\frac{\texttt{funDeclEnv}[\texttt{id} := v] \triangleright \texttt{body} \Rightarrow \texttt{Bool False} \quad \texttt{set\_filter}(\texttt{closure}, S) \rightsquigarrow S'}{\texttt{set\_filter}(\texttt{closure}, (v :: S)) \rightsquigarrow S'}$$

$$\frac{\texttt{funDeclEnv}[\texttt{id} := v] \triangleright \texttt{body} \Rightarrow \texttt{Bool True} \quad \texttt{set\_filter}(\texttt{closure}, S) \rightsquigarrow S_0 \quad S' := (v :: S_0)}{\texttt{set\_filter}(\texttt{closure}, (v :: S)) \rightsquigarrow S'}$$

REGOLE PER map

$$\frac{\rho \triangleright f \Rightarrow \texttt{closure} \quad \rho \triangleright e \Rightarrow \texttt{Set}(\tau_1, S_1) \quad \texttt{closure} := \texttt{Closure}(\texttt{id}, \texttt{body}, \texttt{funDeclEnv}, \tau_1, \tau_2) \quad S_2 := \texttt{set\_map}(\texttt{closure}, S_1)}{\rho \triangleright \texttt{Map}(f, e) \Rightarrow Set(\tau_2, S_2)}$$

$$\frac{}{\texttt{set\_map}(\texttt{closure}, \varnothing) \rightsquigarrow \varnothing}$$

$$\frac{\texttt{funDeclEnv}[\texttt{id} := v] \triangleright \texttt{body} \Rightarrow v' \quad \texttt{set\_map}(\texttt{closure}, S) \rightsquigarrow S' \quad S'' := (v' :: S')}{\texttt{set\_map}(\texttt{closure}, (v :: S)) \rightsquigarrow S''}$$