Special Pythagorean Triplet

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1 Euclid's formula

Euclid's formula is a formula for generating Pythagorean triples given an arbitrary pair of integers (m) and (n) with $(m > n > 0)$. The formula states that the integers

$$a = m^2 - n^2, \quad b = 2mn, \quad c = m^2 + n^2$$

form a Pythagorean triple.

2 Calculations

$$\begin{aligned} a + b + c &= 1000 \\ m^2 - n^2 + 2mn + m^2 + n^2 &= 1000 \\ 2m^2 + 2mn &= 1000 \\ m^2 + mn &= 500 \\ m(m + n) &= 500 \end{aligned}$$

We know $23^2 = 529$, therefore $m \leq 22$ and if $m = 22$ and $n = 1$, the lowest value that n can assume, we have:

$$\begin{aligned} m(m + n) &= 22(22 + 1) \\ &= 22 \times 23 \\ &= 506 \end{aligned}$$

Therefore, $m \leq 21$.