

ECE 5984: Power Distribution System Analysis

Lecture 12: Modeling DERs

Reference: see publications list at the end

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Outline

1) Smart inverters

- IEEE 1547 standard
- control curves
- inverter oversizing
- ride-through curves

2) Energy storage units

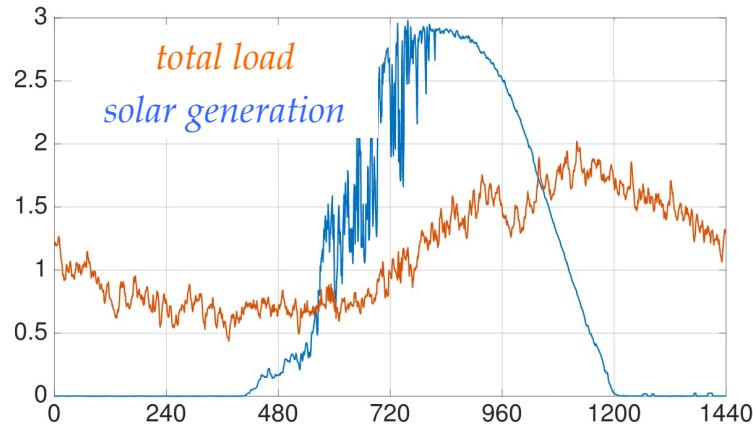
3) Thermostatically-controlled loads

4) Voltage regulators

5) Squared vs. non-squared voltages

Motivation

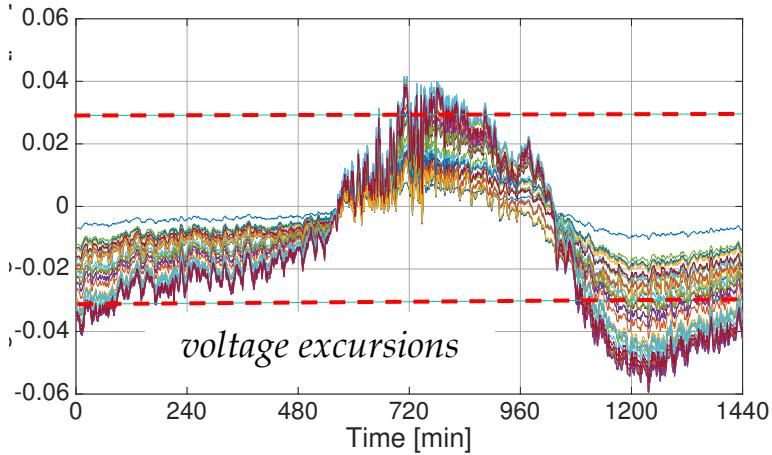
- Voltage fluctuations and transformer overloads due to solar and other DERs



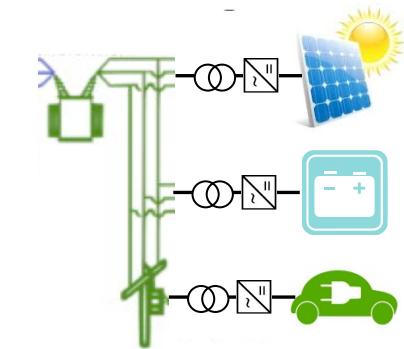
- Inefficiency of voltage control devices



time delays; excessive switching reduces lifetime



- Reactive power control using smart inverters from *Distributed Energy Resources (DERs)*
- *How to decide reactive power injections by DERs?*



Reactive power capability of DERs

- IEEE 1547 specifies DER functionalities
 - requirements per DER type/rating
 - islanding, ride-through, and tripping under abnormal conditions
 - connection and p/q control under normal conditions
 - monitoring capabilities
 - time to respond to commands
 - power quality
- Four modes of reactive power support per IEEE 1547.8 [1]
 - 1) Constant reactive power q
 - 2) Constant power factor $q = \alpha p$
 - 3) Active power-reactive power (Watt/VAR) $q = f(p)$
 - 4) Voltage-reactive power (Volt/VAR) $q = f(v)$

IEEE STANDARDS ASSOCIATION



IEEE Standard for Interconnection and Interoperability of Distributed Energy Resources with Associated Electric Power Systems Interfaces

IEEE Standards Coordinating Committee 21

Sponsored by the
IEEE Standards Coordinating Committee 21 on Fuel Cells, Photovoltaics, Dispersed Generation, and Energy Storage

IEEE
3 Park Avenue
New York, NY 10016-5997
USA

IEEE Std 1547™-2018
(Revision of IEEE Std 1547-2003)

Watt/VAR and Volt/VAR

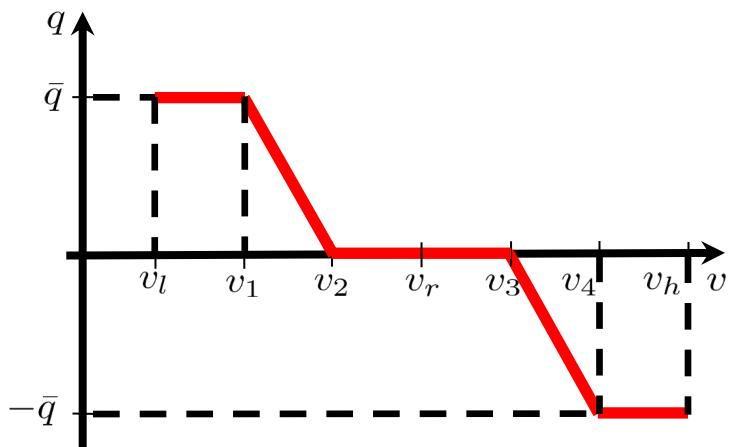
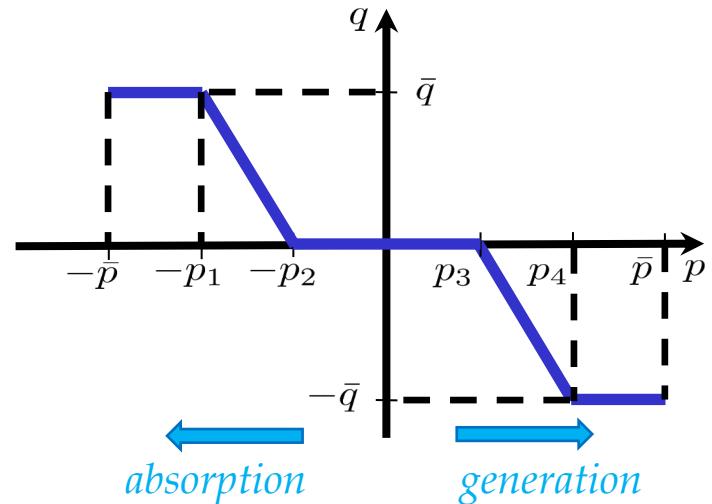
- Control rules are piece-wise linear
- *Deadband* to promote minimal injections
- Design constraints

$$p_2 + 0.1\bar{p} \leq p_1 \leq \bar{p}$$

$$0.4\bar{p} \leq p_2 \leq 0.8\bar{p}$$

$$0.4\bar{p} \leq p_3 \leq 0.8\bar{p}$$

$$p_3 + 0.1\bar{p} \leq p_4 \leq \bar{p}$$

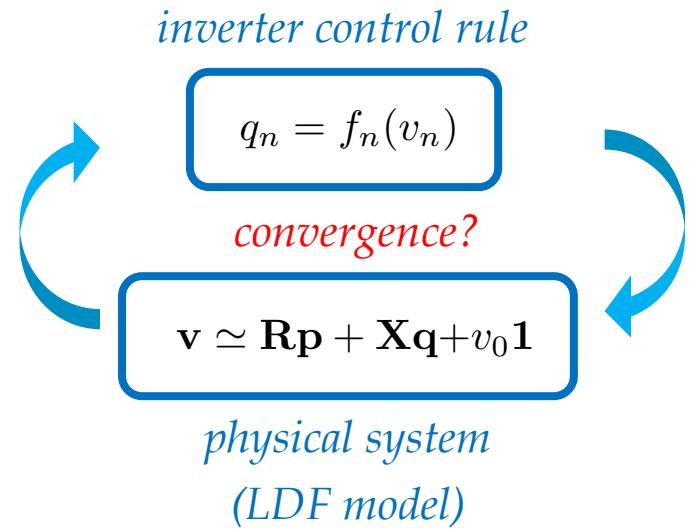
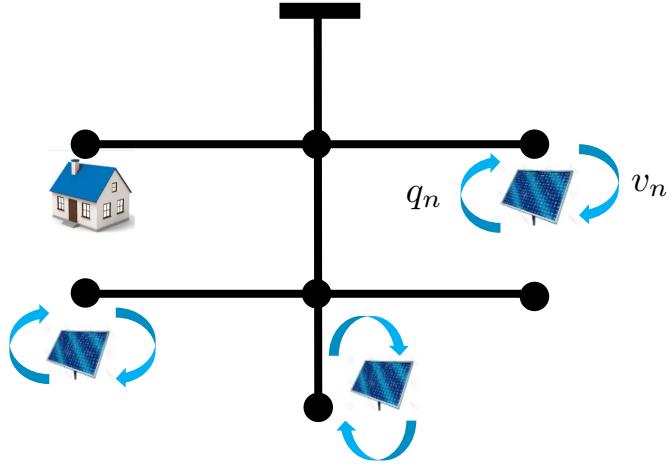


$$\begin{aligned}
 0.95 &\leq v_r \leq 1.05 \\
 v_r - 0.18 &\leq v_1 \leq v_2 - 0.02 \\
 v_r - 0.03 &\leq v_2 \leq v_r \\
 v_r &\leq v_3 \leq v_r + 0.03 \\
 v_3 + 0.02 &\leq v_4 \leq v_r + 0.18
 \end{aligned}$$

- Breakpoints can be optimally designed via mixed-integer linear program (MILP) models

Stability of Volt/VAR control rules

- Controlling \mathbf{q} based on \mathbf{v} forms a *closed-loop system*, which may be **unstable**



- Linearized discrete-time dynamics at equilibrium $\tilde{\mathbf{v}}$

$$\mathbf{v}_{t+1} = \mathbf{Xf}(\mathbf{v}_t) + \mathbf{c} \quad \xrightarrow{\hspace{2cm}} \quad \mathbf{v}_{t+1} = \mathbf{X} \underbrace{\nabla_{\mathbf{v}} \mathbf{f}(\tilde{\mathbf{v}})}_{:= \mathbf{dg}(\mathbf{w})} \mathbf{v}_t + \tilde{\mathbf{c}}$$

locally stable if
 $\|\mathbf{dg}(\mathbf{w})\mathbf{X}\|_2 \leq 1$

- Looser sufficient condition [2]-[4]

- collect maximum absolute slope of the volt/var curves in $\tilde{\mathbf{w}}$

$$\mathbf{dg}(\tilde{\mathbf{w}})\mathbf{X}\mathbf{1} \leq 1$$

designing control curves alongside topology just got more interesting!

Linear control rules

- Control reactive power to minimize *voltage deviations* or *ohmic losses* [2]

Minimize voltage deviations

$$|V_{\pi_n}| - |V_n| \simeq r_n P_n + x_n Q_n \quad \rightarrow \quad p_n^g - p_n^c + \alpha(q_n^g - q_n^c) = 0 \quad \rightarrow \quad q_n^g = \left[q_n^c - \frac{p_n^g - p_n^c}{\alpha} \right]_{q_n}^{\bar{q}_n}$$

where $\alpha = \frac{x_n}{r_n}$

- Projection operator $[x]_{\underline{x}}^{\bar{x}} = \begin{cases} x & , \underline{x} \leq x \leq \bar{x} \\ \underline{x} & , x < \underline{x} \\ \bar{x} & , x > \bar{x} \end{cases}$

Minimize ohmic losses on lines

$$r_n \ell_n = r_n \frac{P_n^2 + Q_n^2}{v_{\pi_n}} \simeq r_n (P_n^2 + Q_n^2) \quad \rightarrow \quad q_n^g = [q_n^c]_{\underline{q}_n}^{\bar{q}_n}$$

*second-order Taylor's series expansion
of losses at flat voltage profile*

- Recall line flows sum up all powers injected downstream
- Bus injections assumed to have similar composition in terms of (p_n^c, q_n^c, p_n^g)

More on ohmic losses

- Ohmic losses on line n $r_n \ell_n = r_n \frac{P_n^2 + Q_n^2}{v_{\pi_n}} \simeq r_n (P_n^2 + Q_n^2)$
- Summing up across all lines

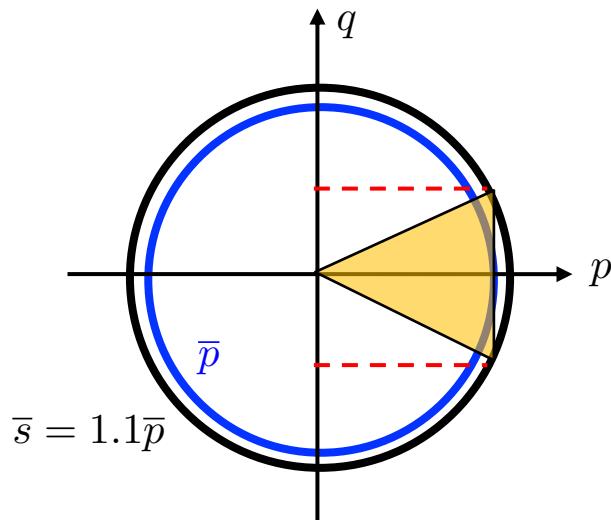
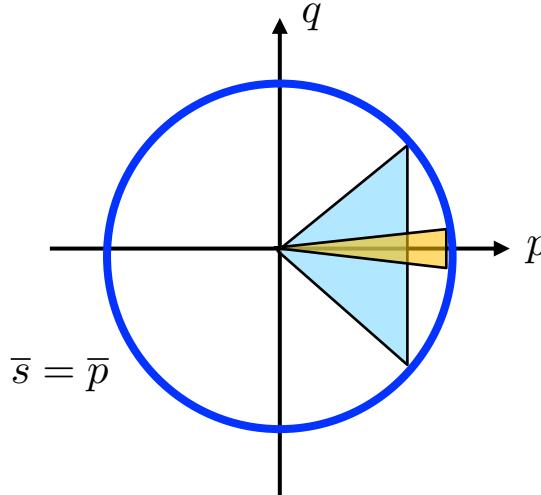
$$\begin{aligned} L &= \sum_{n=1}^N r_n \ell_n \simeq \sum_{n=1}^N r_n (P_n^2 + Q_n^2) && \text{recall that according to LDF} \\ &= \mathbf{P}^\top \text{dg}(\mathbf{r}) \mathbf{P} + \mathbf{Q}^\top \text{dg}(\mathbf{r}) \mathbf{Q} && \mathbf{p} = \mathbf{A}^\top \mathbf{P} \Leftrightarrow \mathbf{P} = \mathbf{F}^\top \mathbf{p} \\ &= \mathbf{p}^\top \mathbf{F} \text{dg}(\mathbf{r}) \mathbf{F}^\top \mathbf{p} + \mathbf{q}^\top \mathbf{F} \text{dg}(\mathbf{r}) \mathbf{F}^\top \mathbf{q} \\ &= \mathbf{p}^\top \mathbf{R} \mathbf{p} + \mathbf{q}^\top \mathbf{R} \mathbf{q} \end{aligned}$$

- This is also the second-order Taylor's series expansion of losses as a function of voltages at the flat voltage profile [7]

Oversizing inverters

- kW rating of solar PV vs. kVA rating of inverter (\bar{p}, \bar{s})
- Reactive power constrained as $p_t^2 + q_t^2 \leq \bar{s}^2 \Rightarrow |q_t| \leq \sqrt{\bar{s}^2 - p_t^2}$

- If $kVA=kW$, there is no room for q at peak solar generation...

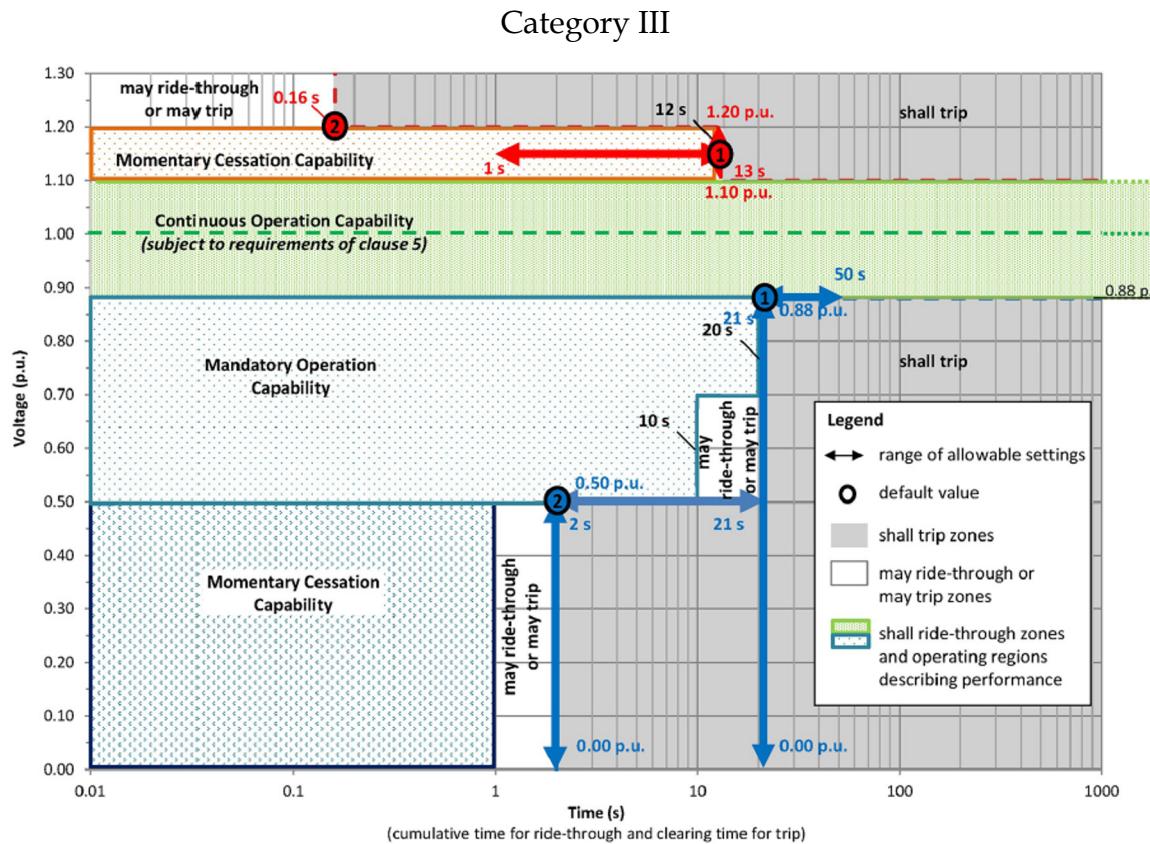


- By oversizing inverters by 10% ($kVA=1.1kW$), p can be compensated by 45% q even at peak solar

$$\bar{q} \leq \sqrt{(1.1\bar{p})^2 - \bar{p}^2} = \sqrt{(1.1^2 - 1)\bar{p}^2} = 0.458\bar{p}$$

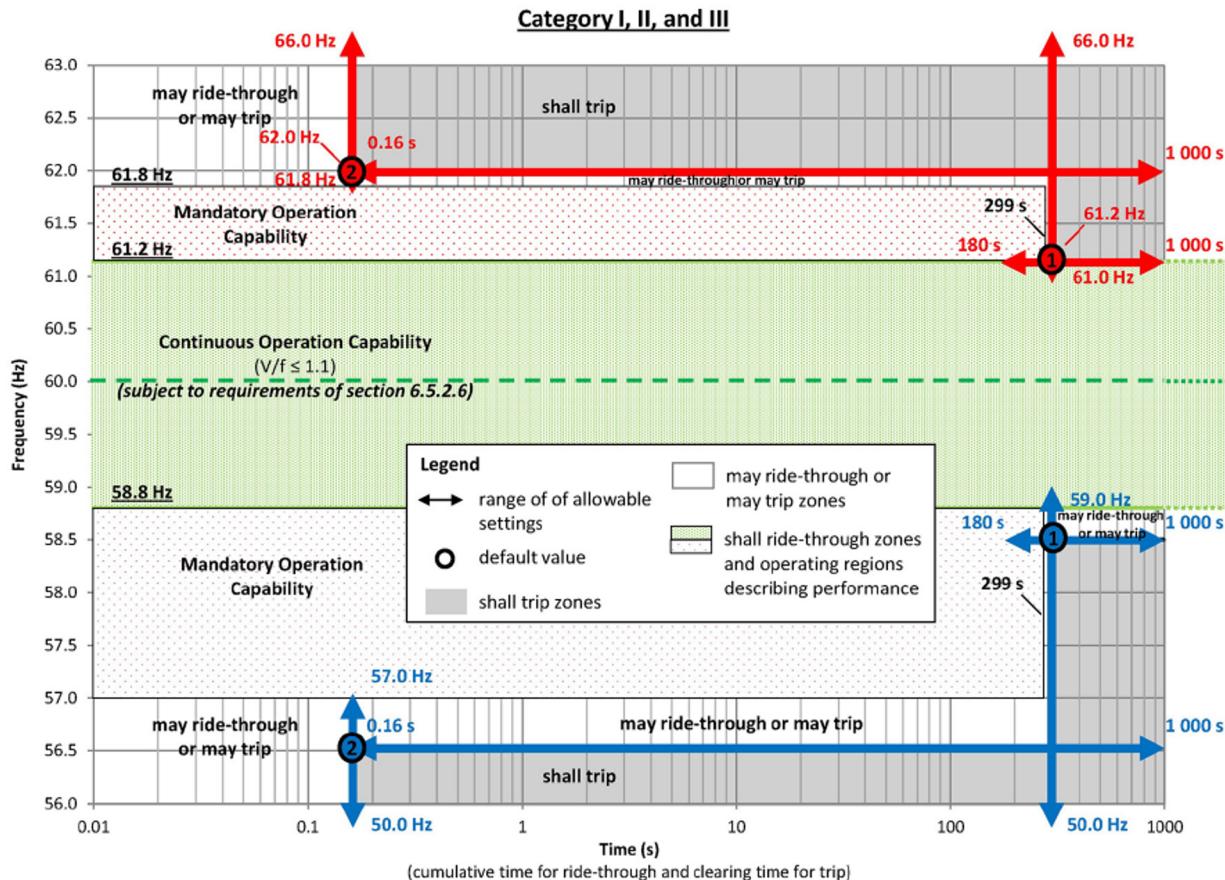
Voltage ride-through

- *Ride-through*: capability of power sources to remain connected during outages [1]



- DERs should trip if over-/under-voltage persists beyond value/time specs
 - Standard designates three DER categories for ride-through depending on type/rating

Frequency ride-through



- Frequency can be controlled by modulating active power within the permitted range
- Voltage ride-through precedes frequency ride-through

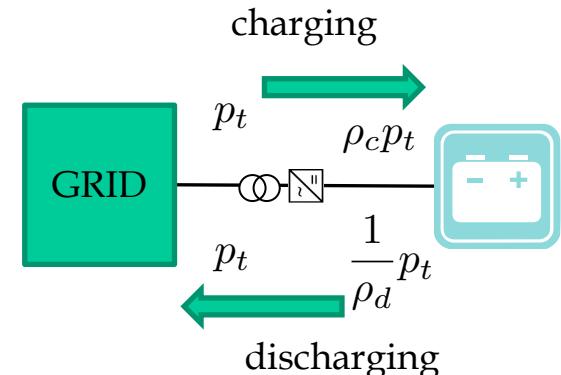
Energy storage units

- State of charge (SoC) of batteries follow a first-order dynamical model

$$\text{charging} \quad s_{t+1} = \lambda s_t + \delta \rho_c p_t \quad (p_t \geq 0)$$

$$\text{discharging} \quad s_{t+1} = \lambda s_t + \delta \frac{1}{\rho_d} p_t \quad (p_t \leq 0)$$

- leakage $\lambda \leq 1$
- duration of control period δ
- power withdrawn from grid p_t
- (dis)-charging efficiencies $\rho_c, \rho_d \leq 1$; round-trip efficiency $\rho_c \rho_d$



- Simplest battery model $(\lambda = \rho = 1)$

$$s_{t+1} = s_t + \delta p_t$$

Energy storage units (cont'd)

- Complete battery operation model

$$\min_{\{p_t\}_{t=1}^T} \sum_{t=1}^T c_t^+ p_t^+ - c_t^- p_t^-$$

subject to $s_{t+1} = \lambda s_t + \delta \rho_c p_t^+ - \delta \frac{1}{\rho_d} p_t^-$

$$p_t = p_t^+ - p_t^-$$

$$p_t^+, p_t^- \geq 0$$

$$p_t^+ p_t^- = 0$$

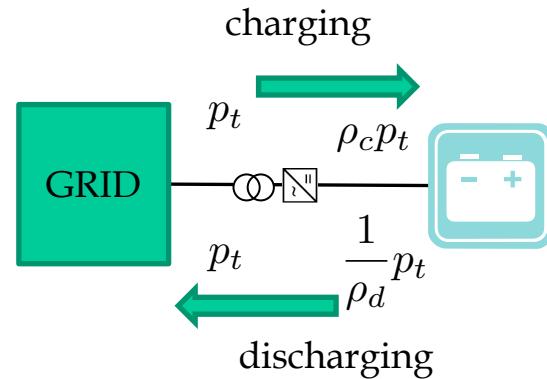
non-convex constraint

$$\underline{s} \leq s_t \leq \bar{s}$$

SoC limits

$$\underline{p} \leq p_t \leq \bar{p}$$

charging limits



- In some battery operation optimization problems, the non-convex constraint is dropped and is still satisfied at optimality
- Non-convex constraint not needed if $c_t^+ = c_t^-$ and $\rho_c = \rho_d = 1$
- Avoid deep discharging by selecting $\underline{s} = 0.1\bar{s}$

Sample datasheet

AC Voltage (Nominal)	120/240 V
Feed-In Type	Split Phase
Grid Frequency	60 Hz
Total Energy	14 kWh
Usable Energy	13.5 kWh
Real Power, max continuous	5 kW (charge and discharge)
Real Power, peak (10 s, off-grid/backup)	7 kW (charge and discharge)
Apparent Power, max continuous	5.8 kVA (charge and discharge)
Apparent Power, peak (10 s, off-grid/backup)	7.2 kVA (charge and discharge)
Maximum Supply Fault Current	10 kA
Maximum Output Fault Current	32 A
Overcurrent Protection Device	30 A
Imbalance for Split-Phase Loads	100%
Power Factor Output Range	+/- 1.0 adjustable
Power Factor Range (full-rated power)	+/- 0.85
Internal Battery DC Voltage	50 V
Round Trip Efficiency^{1,3}	90%
Warranty	10 years



Thermostatically-controlled loads

- Temperature dynamics typically captured by first-order model

cooling
(air-conditioner)

$$\frac{d\theta(t)}{dt} = -\frac{1}{\tau} (\theta(t) - \theta_a(t) + b(t)PR)$$

$\tau = CR$: time constant

C : thermal capacitance

R : thermal resistance

P : thermal energy transfer rate

- $b(t)$: AC status (ON/OFF)

- Discrete dynamics with step size T and $\alpha = T/\tau$

$$\theta_{k+1} = \theta_k + \alpha\theta_k^a - \alpha b_k PR$$

- TCL similar to a *charging-only* battery: 'SoC' improves by smaller temperature

- Goal:* maintain temperature within set range $\underline{\theta} \leq \theta_{k+1} \leq \bar{\theta}$

- Existing TCL automation
(modulo lock-out)

$$b_k = \begin{cases} 1 & , \text{ if } \theta_{k-1} \geq \bar{\theta} \\ 0 & , \text{ if } \theta_{k-1} \leq \underline{\theta} \\ b_{k-1} & , \text{ otherwise.} \end{cases}$$



- Interfere with existing control rule to provide grid services

Voltage regulators (locally-controlled)

- $\{-16,+16\}$ taps scale input voltage within $\pm 10\%$
- Consider locally-controlled VR and ignore LDC

a) If input voltage too low, VR saturates at +16

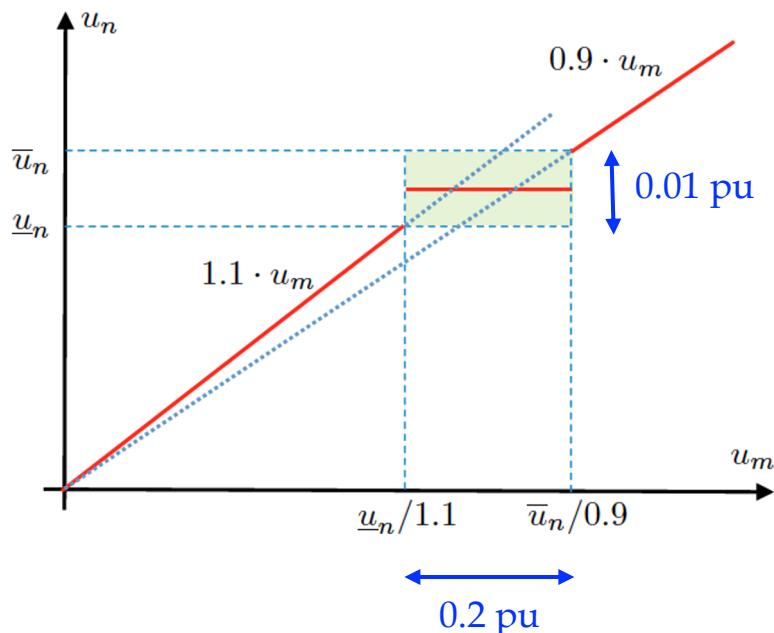
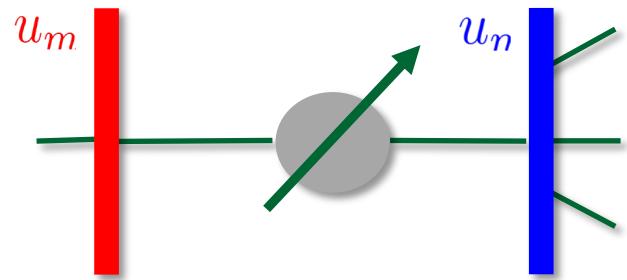
$$u_n = 1.1u_m, \text{ but } 1.1u_m \leq u_n \text{ when } u_m \leq \frac{u_n}{1.1}$$

b) If input voltage too high, VR saturates at -16

$$u_n = 0.9u_m, \text{ but } 0.9u_m \geq \bar{u}_n \text{ when } u_m \geq \frac{\bar{u}_n}{0.9}$$

c) Otherwise, VR maintains u_n within bandwidth

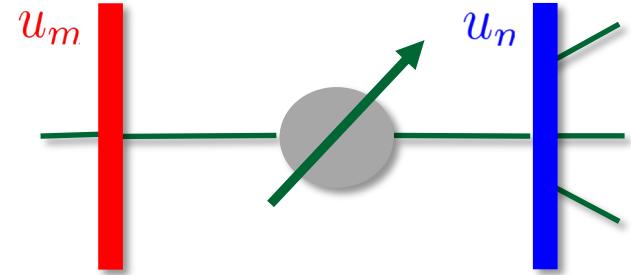
$$\frac{\underline{u}_n}{1.1} \leq u_m \leq \frac{\bar{u}_n}{0.9} \Rightarrow u_n \in [\underline{u}_n, \bar{u}_n]$$



- Exact value of tap and u_n are hard to determine, but uncertainty area is slim
- Approximate model can be captured by *McCormick linearization*

Voltage regulators (remotely-controlled)

- Consider remotely-controlled VRs
- Utility directly controls the tap $t \in \{-16, \dots, +16\}$



Model 1

one binary variable per each of 33 possible states

$$u_n = u_m \sum_{k=1}^{33} b_k t_k$$

$$b_k \in \{0, 1\}, \quad \sum_{k=1}^{33} b_k = 1$$

Model 2: Binary Expansion Model [6]

express state number as a binary number of 6 digits

$$u_n = u_m \left(0.9 + 0.00625 \sum_{i=0}^5 b_i 2^i \right)$$

$$\sum_{i=0}^5 b_i 2^i \leq 33$$

$$b_i \in \{0, 1\}$$

000010	$\rightarrow 2$
001010	$\rightarrow 10$
100001	$\rightarrow 33$
100010	$\rightarrow 34$

- Products between continuous and binary variables handled via *McCormick linearization*

To square or not to square voltages?

- We have seen two linearized power flow models for distribution systems

$$v_{\pi_n} - v_n \simeq 2r_n P_n + 2x_n Q_n \quad \text{versus} \quad |V_{\pi_n}| - |V_n| \simeq \operatorname{Re}\{z_n I_n\} \simeq r_n P_n + x_n Q_n$$

LDF on squared voltage magnitudes *approx. analysis of Chapter 3 on voltage magnitudes*

- Some component models require both, others one of them

- transformer ratios: non-squared voltages
- ZIP loads: squared (constant Z) and non-squared (constant I)
- capacitor banks: squared (constant Z)
- inverter Volt/VAR curves: non-squared

- Consider analysis in per unit wlog

$$\begin{aligned} v_n &= |V_n|^2 \simeq |V_0|^2 + 2|V_0|(|V_n| - |V_0|) \\ &= 1 + 2(|V_n| - 1) = 2|V_n| - 1 \end{aligned}$$

select one set of variables for modeling and substitute for the other

References

- [1] *IEEE 1547 Standard for Interconnecting Distributed Resources with Electric Power Systems*, IEEE Std., 2018. [Online]. Available: http://grouper.ieee.org/groups/scc21/1547/1547_index.html
- [2] M. Farivar, L. Chen, and S. Low, "Equilibrium and dynamics of local voltage control in distribution systems," in *Proc. IEEE Conf. on Decision and Control*, Florence, Italy, Dec. 2013, pp. 4329–4334
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- [4] K. Baker, A. Bernstein, E. Dall'Anese, and C. Zhao, "Network-cognizant voltage droop control for distribution grids," *IEEE Trans. Power Syst.*, vol. 33, no. 2, pp. 2098–2108, Mar. 2018.
- [5] K. Turitsyn, P. Sulc, S. Backhaus and M. Chertkov, "Options for control of reactive power by distributed photovoltaic generators," *Proceedings of the IEEE*, vol. 99, no. 6, pp. 1063-1073, June 2011.
- [6] W. Wu, Z. Tian, and B. Zhang, "An exact linearization method for OLTC transformers in branch flow model," *IEEE Trans. Power Syst.*, vol. 32,no. 3, pp. 2475–2476, May 2017.
- [7] S. Taheri, M. Jalali, V. Kekatos, and L. Tong, "Fast Probabilistic Hosting Capacity Analysis for Active Distribution Systems," *IEEE Trans. on Smart Grid*, (to appear, 2021).