Problem Set 7

This problem set is due on Friday, March 20, by 5pm. Please submit your solution online using bcourses, as a pdf file.

You can type your solution, or handwrite it. If you handwrite it, then either scan it or take a good resolution picture of each page and then collate the pictures and export them to a *single* pdf file.

Problem 1: Kolmogrov Complexity

Let $L = \{(x, y) : K(x) > K(y)\}$. Prove that L is not decidable.

Problem 2: More Kolmogrov Complexity

Let $L = \{(x, y, n) : \max(K(x), K(y)) > n\}$. Prove that L is not recognizable.

Problem 3: Yet More Kolmogrov Complexity

Let $f : \Sigma^* \to \mathbb{Z}$ be a computable function. Show that if $\forall x : f(x) \leq K(x)$, then f must be bounded. That is, there must be some c such that $\forall x : f(x) \leq c$.