Definition. A *subsequence* of a string is a new string which is formed from the original string by deleting some (can be none) of the characters without disturbing the relative positions of the remaining characters. (ie, "ACE" is a subsequence of "ABCDE" while "AEC" is not).

Optimal Substructure. Let $X = \langle x_1, x_2, \ldots, x_m \rangle$ be subsequence of $Y = \langle y_1, y_2, \ldots, y_n \rangle$

1. if $x_m = y_n$, then X_{m-1} is a subsequence of Y_{n-1}
2. if $x_m \neq y_n$, then X is a subsequence of Y_{n-1}

Recursive Formula. Let us define $c[i, j]$ to be the length of an *longest common subsequence* of X_i and Y_j. If either $i = 0$ or $j = 0$, one of the sequences has length 0, and so LCS has length 0. Finally, if X_m is a subsequence of Y_n, $c[m, n] = m$.

$$c[i,j] = \begin{cases} 0, & \text{if } i = 0 \text{ or } j = 0, \\ c[i-1, j-1], & \text{if } i, j > 0 \text{ and } x_i = y_j, \\ \max(c[i, j-1], c[i-1, j]), & \text{if } i, j > 0 \text{ and } x_i \neq y_j, \end{cases}$$

The following table illustracts the constructed $c[i, j]$ table with X as *"rabbit"* and Y as *"rabbbit"*. The arrows within will be used in the next section of reconstructing the common sequence.

	j	0	1	2	3	4	5	6	7
i		y_j	r	a	b	b	b	i	t
0	x_i	0	0	0	0	0	0	0	0
1	r	0	$\nwarrow$ 1	1	1	1	1	1	1
2	a	0	1	$\nwarrow$ 2	$\leftarrow$ 2	2	2	2	2
3	b	0	1	2	$\nwarrow$ 3	$\nwarrow\leftarrow$ 3	3	3	3
4	b	0	1	2	3	$\nwarrow$ 4	$\nwarrow\leftarrow$ 4	4	4
5	i	0	1	2	3	4	4	$\nwarrow$ 5	5
6	t	0	1	2	3	4	4	5	$\nwarrow$ 6

Reconstruct Solution. A distinct subsequence is a distince path from $c[m, n]$ to $c[0, 0]$. For instance, in our case,

1. $[6, 7] \rightarrow [5, 6] \rightarrow [4, 5] \rightarrow [4, 4] \rightarrow [3, 3] \rightarrow [2, 2] \rightarrow [1, 1] \rightarrow [0, 0]$
2. $[6, 7] \rightarrow [5, 6] \rightarrow [4, 5] \rightarrow [3, 4] \rightarrow [3, 3] \rightarrow [2, 2] \rightarrow [1, 1] \rightarrow [0, 0]$

3. $[6,7] \to [5,6] \to [4,5] \to [3,4] \to [2,3] \to [2,2] \to [1,1] \to [0,0]$