Given a set of integers, S, return all possible subsets.

Notation and terminology

We say that $S = \langle x_1, x_2, \ldots, x_n \rangle$ is a *set*, in which all x_i are integers and all element placed in non-decending order, that is, $x_1 \leq x_2 \leq \cdots \leq x_n$.

We say that S' is a *subset* of S, when $\forall x \in S', x \in S$. In another word,

$S' = \langle x_{i_1}, x_{i_2}, \ldots, x_{i_m} \rangle$, in which, $i_1 < i_2 < \cdots < i_m \leq n$.

1 S Without Duplicate Elements

When S is a set without duplication elements, all elements are in ascending order:

$$x_1 < x_2 < \cdots < x_n$$

The following defines a new set with all possible sets containing only the first k elements of S.

SUBSETS$(S, k) = \{\sigma | \sigma = \langle x_{i_1}, x_{i_2}, \ldots, x_{i_m} \rangle$, in which $i_1 < i_2 < \cdots < i_m \leq k \leq n\}$

Lemma 1. *SUBSETS$(S, k+1)$ - SUBSETS(S, k)* $= \{\sigma' | \sigma' = \sigma \cup \{x_{k+1}\}, \sigma \in$ *SUBSETS(S, k)*$\}$

Proof. For $\forall \gamma \in$ SUBSETS$(S, k+1)$, $\gamma = \langle x_{i_1}, x_{i_2}, \ldots, x_{i_m} \rangle$, in which $i_1 < i_2 < \cdots < i_m \leq k+1 \leq n$.

If $i_m \neq k+1$, then $x_{i_m} \neq x_{k+1}$. Because $x_k \neq x_{k+1}$, $\gamma \in$ SUBSETS(S, k).

If $i_m = k+1$, then $x_{i_m} = x_{k+1}$. Because $x_{i_{m-1}} \neq x_{k+1}$, then $\langle x_{i_1}, x_{i_2}, \ldots, x_{i_{m-1}} \rangle \in$ SUBSETS(S, k).

□

2 S With Duplicate Elements

Recursive Formula. *Let us define* $c[i, j]$ *to be the length of an* longest common subsequence *of* X_i *and* Y_j*. If either* $i = 0$ *or* $j = 0$*, one of the sequences has length 0, and so LCS has length 0. Finally, if* X_m *is a subsequence of* Y_n*,* $c[m, n] = m$*.*

$$c[i,j] = \begin{cases} 0, & \textit{if } i = 0 \textit{ or } j = 0, \\ c[i-1, j-1], & \textit{if } i,j > 0 \textit{ and } x_i = y_j, \\ max(c[i, j-1], c[i-1, j]), & \textit{if } i,j > 0 \textit{ and } x_i \neq y_j, \end{cases}$$

The following table illustracts the constructed $c[i, j]$ *table with* X *as* "rabbit" *and* Y *as* "rabbbit"*. The arrows within will be used in the next section of reconstructing the common sequence.*

	j	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*
i		y_j	r	a	b	b	b	i	t
0	x_i	0	0	0	0	0	0	0	0
1	r	0	$\nwarrow$ 1	1	1	1	1	1	1
2	a	0	1	$\nwarrow$ 2	$\leftarrow$ 2	2	2	2	2
3	b	0	1	2	$\nwarrow$ 3	$\nwarrow$ $\leftarrow$ 3	3	3	3
4	b	0	1	2	3	$\nwarrow$ 4	$\nwarrow$ $\leftarrow$ 4	*4*	4
5	i	0	1	2	3	4	4	$\nwarrow$ 5	5
6	t	0	1	2	3	4	4	5	$\nwarrow$ 6

Reconstruct Solution. *A distinct subsequence is a distince path from* $c[m, n]$ *to* $c[0, 0]$*. For instance, in our case,*

1. $[6, 7] \to [5, 6] \to [4, 5] \to [4, 4] \to [3, 3] \to [2, 2] \to [1, 1] \to [0, 0]$
2. $[6, 7] \to [5, 6] \to [4, 5] \to [3, 4] \to [3, 3] \to [2, 2] \to [1, 1] \to [0, 0]$
3. $[6, 7] \to [5, 6] \to [4, 5] \to [3, 4] \to [2, 3] \to [2, 2] \to [1, 1] \to [0, 0]$