Given a set of integers, S, return all possible subsets.

Notation and terminology

We say that $S = \langle x_1, x_2, \dots, x_m \rangle$ is a *set*, in which all x_i are integers and all element placed in non-decending order, that is, $\langle x_1 \leq x_2 \leq \cdots \leq x_m \rangle$.

We say that S' is a *subset* of S, when $\forall x \in S', x \in S$.

1 Without Duplication

2 With Duplication

Problem. *Given a set of distinct integers, S, return all possible subsets.*

Note *Elements in a subset must be in non-descending order.*

The solution set must not contain duplicate sets.

Optimal Substructure. *Let $X = \langle x_1, x_2, \dots, x_m \rangle$ be subsequence of $Y = \langle y_1, y_2, \dots, y_n \rangle$*

1. *if $x_m = y_n$, then X_{m-1} is a subsequence of Y_{n-1}*
2. *if $x_m \neq y_n$, then X is a subsequence of Y_{n-1}*

Recursive Formula. *Let us define* c$[i,j]$ *to be the length of an* longest common subsequence *of X_i and Y_j. If either $i = 0$ or $j = 0$, one of the sequences has length 0, and so LCS has length 0. Finally, if X_m is a subsequence of Y_n, $c[m,n] = m$.*

$$c[i,j] = \begin{cases} 0, & \textit{if } i = 0 \textit{ or } j = 0, \\ c[i-1, j-1], & \textit{if } i,j > 0 \textit{ and } x_i = y_j, \\ max(c[i,j-1], c[i-1,j]), & \textit{if } i,j > 0 \textit{ and } x_i \neq y_j, \end{cases}$$

The following table illustracts the constructed c$[i,j]$ *table with X as* “rabbit” *and Y as* “rabbbit”*. The arrows within will be used in the next section of reconstructing the common sequence.*

	j	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*
i		y_j	r	a	b	b	b	i	t
0	x_i	0	0	0	0	0	0	0	0
1	r	0	↖ 1	1	1	1	1	1	1
2	a	0	1	↖ 2	← 2	2	2	2	2
3	b	0	1	2	↖ 3	↖ ← 3	3	3	3
4	b	0	1	2	3	↖ 4	↖ ← 4	*4*	4
5	i	0	1	2	3	4	4	↖ 5	5
6	t	0	1	2	3	4	4	5	↖ 6

Reconstruct Solution. *A distinct subsequence is a distince path from* $c[m, n]$ *to* $c[0, 0]$. *For instance, in our case,*

1. $[6, 7] \to [5, 6] \to [4, 5] \to [4, 4] \to [3, 3] \to [2, 2] \to [1, 1] \to [0, 0]$

2. $[6, 7] \to [5, 6] \to [4, 5] \to [3, 4] \to [3, 3] \to [2, 2] \to [1, 1] \to [0, 0]$

3. $[6, 7] \to [5, 6] \to [4, 5] \to [3, 4] \to [2, 3] \to [2, 2] \to [1, 1] \to [0, 0]$