The best of two worlds: Forecasting High Frequency Volatility for cryptocurrencies and traditional currencies wit....

Article *in* Expert Systems with Applications · December 2017

DOI: 10.1016/j.eswa.2017.12.004

Yaohao Peng

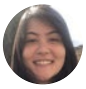
University of Brasília

Pedro Albuquerque

University of Brasília

Jader Martins Camboim de Sá

University of Brasília



Ana Julia Padula

University of Brasília

Expert Systems With Applications 97 (2018) 177–192

Contents lists available at ScienceDirect

Expert Systems With Applications

journal homepage: www.elsevier.com/locate/eswa

The best of two worlds: Forecasting high frequency volatility for cryptocurrencies and traditional currencies with Support Vector Regression

Yaohao Peng, Pedro Henrique Melo Albuquerque*, Jader Martins Camboim de Sá, Ana Julia Akaishi Padula, Mariana Rosa Montenegro

University of Brasilia, Campus Darcy Ribeiro, Brasilia, Distrito Federal 70910-900, Brazil

ARTICLE INFO

Article history:
Received 11 September 2017
Revised 1 December 2017
Accepted 2 December 2017
Available online 6 December 2017

Keywords:
Volatility forecasting
Machine learning
GARCH
Cryptocurrency
Support vector machines
Exchange rates

ABSTRACT

This paper provides an evaluation of the predictive performance of the volatility of three cryptocurrencies and three currencies with recognized stores of value using daily and hourly frequency data. We combined the traditional GARCH model with the machine learning approach to volatility estimation, estimating the mean and volatility equations using Support Vector Regression (SVR) and comparing to GARCH family models. Furthermore, the models' predictive ability was evaluated using Diebold-Mariano test and Hansen's Model Confidence Set. The analysis was reiterated for both low and high frequency data. Results showed that SVR-GARCH models managed to outperform GARCH, EGARCH and GJR-GARCH models with Normal, Student's *t* and Skewed Student's *t* distributions. For all variables and both time frequencies, the SVR-GARCH model exhibited statistical significance towards its superiority over GARCH and its extensions.

1. Introduction

Some controversy among financial economists lean on the highest precision on volatility estimation. Merton (1980) and Nelson (1992) noticed that the volatility forecasting does not need a huge amount of historical data, instead, a short period of observation is enough to make such analysis (Andersen, Bollerslev, Diebold, & Labys, 1999).

It was also observed that, with an arbitrarily short span of data, it is possible to get an accurate volatility estimation (Poon & Granger, 2003). For this reason, the progress of volatility studies are related to the use of higher frequency data. Regarding that, this work provides an evaluation of the predictive performance of the volatility of three cryptocurrencies and three traditional currency pairs, using daily and hourly frequency data.

In order to estimate volatility, researchers most use the GARCH model. However, nowadays the Support Vector Regression (SVR) emerged as a strong and robust method, capable of covering multivariate and dynamic characteristic of financial series. This method is rooted in the Structural Risk Minimization (SRM) process, which aims to estimate the nonlinear data generating process trough a risk minimization and a regularization term to achieve the minimal unknown populational risk.

In this context, we proposed the application of this study using the cryptocurrency market. These new assets have a new combination of characteristics that makes them so unique in operation and transaction, been unable to relate completely with others markets for several reasons: first, compared to the commodities market, they do not have a great historical background and no future market to be the benchmark, but despite that, we were able to achieve an interesting result in the volatility forecast. Second, the use of cryptocash is different from the traditional cash; such that even if a country adopt it as an official digital currency, the transactions were designed to be done directly between economic agents without the need of an intermediary institution, monetary control or accountability system. Third, the cryptocurrencies value and distribution is based on a P2P-network, it has no physical representation to handle it, only a string is necessary, which is called a wallet, and its password, that is used to send and receive cryptocurrencies. For those reasons it is important to do a specific research in the cryptocurrencies market, using traditional and novel methodologies to create a model capable of understand the unique characteristics and dynamics provided by this new asset.

* Corresponding author.
E-mail addresses: peng.yaohao@ipea.gov.br (Y. Peng), pedroa@unb.br (P.H.M. Albuquerque), jader.martins@ipea.gov.br (J.M. Camboim de Sá), lamfo@unb.br (A.J.A. Padula), lamfo@unb.br (M.R. Montenegro).
URL: http://www.lamfo.unb.br (M.R. Montenegro)

https://doi.org/10.1016/j.eswa.2017.12.004
0957-4174/

This research presents a combination of SVR approach with a GARCH model (SVR-GARCH) and tested it against several traditional GARCH models and well known extensions. Furthermore, the machine learning based model was tested for both cryptocurrencies and traditional currencies, in order to check whether this approach yields significant boosts in predicting ability, as well as investigating potential similarities and differences between cryptocash and traditional money. Moreover, we replicated the tests for low and high data periodicities and for different time periods, in order to verify whether SVR-GARCH model's predictive performance is satisfactory over the whole time extension of the series, using Diebold and Mariano (1995) test for predictive accuracy and Hansen, Lunde, and Nason (2011) Model Confidence Set.

This paper's contributions consists in joining two emerging research agendas in finance; the study of cryptocurrencies and the use of machine learning forecasting techniques: while numerous papers presented applications of machine learning based models in various research agendas in finance, works that link this approach to cryptocurrencies remain scarce. Specifically, cryptocurrencies' volatility levels are usually much higher than traditional currencies (Yermack, 2013), making prediction associated to this segment a potentially even more challenging task than to commonly addressed variables, such as stocks indexes or exchange rates. Therefore, we compared a machine learning based model to well established models in financial econometrics for both traditional and cryptocurrencies, and verify the relative performance of the machine learning paradigm in both "worlds".

This paper is structured as follows: Section 2 presents key features of cryptocurrencies, discussing their relevance in finance and comparing them to traditional currencies. Section 3 describes the theoretical background of volatility estimating methods in high frequency data, as well as the benchmark models and the methodology used to estimate volatility in this paper. Section 4 addresses the empirical analysis of the daily and hourly volatility estimation using bitcoin, ethereum and dash prices (in US dollars) and the spot exchange rate between US Dollar and Euro, British Pound and Japanese Yen between January 4th 2016 and July 31th 2017. Section 5 presents the forecasting results and discuss their implication in view of the financial theory. Finally, Section 6 presents conclusions and remarks, showing limitations to this approach and recommendations to future researches.

2. Theoretical background

2.1. Cryptocurrencies

Ferguson (2008) presents a brief chronology of the evolution of the concept of "wealth": in the medieval era, wealth was associated to having the means to conquer and pillage, so that it was regarded as a consequence of power; with the rise of mercantilism and capitalism afterwards, wealth began to be interpreted as the possession of material goods (such as precious metals) and the means to generate production and trade it for more material goods; thus, money was increasingly viewed as the cause of power. As the capitalist system consolidated in the western society, the main indicator of wealth became the possession of money, since its high liquidity allows it to be converted into any other asset. Nowadays, the hard core of the world's wealth is concentrated in financial assets rather than real assets: at this stage, a successful businessman's fortune is mainly evaluated based on his company's stock value, instead of his yacht or his luxurious car.

While a consolidate model is still in discussion, in 2009 a new kind of asset appeared in the market, the bitcoin, leading the world to analyze this newcomer and try to figure out its place and dynamics, and whether the rise of cryptocurrencies can change the concept of "wealth" once more. Recent works like Vigna and Casey (2016) argue that the new ideas and technologies that come alongside with cryptocurrencies, such as the blockchain and the decentralization of money, have the potential to lead the world into a "new economy": while a "revolution" on the foundations of social life is unlikely to occur, the introduction of decentralized cybermoney tend to push the traditional ways of economic transactions even further into the digital realm, nerfing the cost of global scale transactions, which in turn provides viability for individuals to work for companies in different countries, and makes even well-established companies to adapt their corporate strategies into this new reality.

Bitcoin wasn't the first digital currency: e-Gold (G&SR, 1998), eCash (Chaum, 1983), Beenz (Cohen, 1998) and Flooz (Levitan, 1999) were previous attempts of a purely virtual way to make transactions, with e-gold being the most successful among them. Created by Gold & Silver Reserve Inc. in 1996, e-Gold was an anonymous service that allowed the possibility to make instant transfers of value – ownership of gold and other precious metals – to other e-gold accounts (G&SR, 2006a), transaction flows reached a peak of U$2 billion per year envolving over a million users (G&SR, 2006b). E-Gold attracted great attention, but with the possibility of user-anonymity, cybercriminals, money-launderers and other kinds of criminals developed interests for that service and a series of prosecutions occurred against it. In 2008, the CEO of Gold & Silver was sentenced and all e-Gold's accounts were frozen, the company had to close a few months later and the end of e-Gold was declared. Other digital currencies had a similar end, by similar causes, but an alternative would be born sooner.

Nakamoto (2008) proposed bitcoin in order to be an easy way to make transactions over the Internet, which works globally, faster, independent from an institution to operate it and limited to 21 million units, such that only the deflation is expected to occur (Darlington, 2014), a feature that may be used as a vanish point for citizens "running away" from economies that suffer great levels of inflation (Vasquez, 2017). Bitcoin represented a breakthrough over the previously cited digital currencies for solving cryptographic problems like the "Trusted Third Party" (TTP) (Andrychowicz, Dziembowski, Malinowski, & Mazurek, 2014) and the "Double-spending" (Rosenfeld, 2014). Thus, bitcoin would solve the failure points of older attempts, giving the market a much more solid solution to the market aspirations for digital currencies.

Bitcoin is based on a peer-to-peer ledger that is governed by mathematical restrictions, this ledger only allows real transactions to be written in it. Every new transactions go to a pool of "unconfirmed transactions", while miners take these transactions and write them into the longest chain of verified blocks. Every new block linked to the block of a mined transaction is called a "confirmation"; the suggested number of confirmations is 6 (Community, 2017). If a criminal intends to fake 6 confirmations, it will cost him more then half a million dollars in bitcoins reward for mining the block (as of November 2017), apart from the huge computation power required to this. So, the faking process would not yield economic gains for transactions lower than this "faking cost". Moreover, the attempt of writing fake transactions must outdo the computational power of half the miners to write the longest blockchain, which is impracticable both economically and computationally. In fact, since its inception, bitcoin never registered a single falsification with more than 1 confirmation. Every transaction and the agents involved are registered, any change over the bitcoin data nullify its uses in future transactions. Until now the average number of transactions registered are 288,155 per day (Blockchain, 2017a; CoinDesk, 2017a), a growth of 40% compared to the same period of 2016, in which registered around 205,246 transactions per day.

Many key concepts of bitcoin are relatively new to financial theory: bitcoin has no association with any authority, has no physical representation, is infinitely divisible and is built using the most sophisticated mathematics and computation techniques. For those reasons, cryptocurrencies can give fear to any specialist that is not used to this kind of "money" and is widely astonishing to the market world. Different from the traditional currencies that operate in the market, the bitcoins and other cryptocurrencies that follow similar logic don't have their value based on any country economy or in some physical and tangible asset, like the gold-dollar parity under the Bretton Woods system. Instead, the conception of value has it origin in the security of the algorithm, traceability of the transactions and the precedence of each bitcoin. Also, the exact number of circulating bitcoins in the market is known, so this asset's money supply is constant, providing a incentive for bitcoin owners to simply keep them still without trading them for goods or services, which in fact happen at a high proportion; unlike in traditional centralized currencies, in which the boost on money supply would affect the relative value of banknotes (Kristoufek, 2015).

Due its nature, bitcoins assume a dual feature as a medium of exchange and as an asset of investment. Authors like Polasik, Piotrowska, Wisniewski, Kotkowski, and Lightfoot (2015) found both characteristics in bitcoins evident in different time windows and various market investors' profiles; others such as Evans (2014) and Segendorf (2014) discuss about bitcoin's appliance, although there are still some critics about how this asset will deal with legal matters and taxes, as pointed by Polasik et al. (2015). Evans (2014) present the concern with the bitcoin's volatility and in what extent it would affect the payments transactions made with it, and states that the market would stabilize with time or upon the formation of big wallets that would assume the role as value guarantee. According to Segendorf (2014), the analysis of bitcoin's volatility is based on the market reality and the risk that bitcoin present if used as an official digital currency, the security of information and transactions still discussed as an apprehension topic in contrast with the Swedish case as an example of functionality, making bitcoin's implications in the financial market a issue of interest for investors and scholars alike.

The debate about bitcoin's taxonomy has by itself aroused great academic interest. While the exponential increase of bitcoin's prices resembles a bubble behavior, it might not be purely related to speculative aspects, as indicated by recent academic studies: Gandal and Halaburda (2014) state that the inclusion of bitcoin into a diversified portfolio significantly increases its risk-adjusted returns, due to both bitcoin's high average returns and low correlation with other financial assets. Bouri, Gupta, Tiwari, Roubaud et al. (2017) present evidences that indicate that bitcoin can be indeed used as a hedge to market specific risk. Dyhrberg (2016a) analyzes the volatility of bitcoin in comparison to US Dollar and Gold – traditionally regarded as "safe" value reserves – using GARCH (1,1) and EGARCH (1,1). The paper concluded that bitcoin bears significant similarities to both assets, specially concerning hedging capabilities and volatility reaction to news, suggesting that bitcoin can be a useful tool for portfolio management, risk analysis and market sentiment analysis. As evidence of the recent acknowledgment of the hedge propriety of bitcoin, economic agents already invested a total of 19 billion dollars in the cryptocurrency until March of 2017, suggesting an increase of the usage, popularization and trust in the bitcoin (Blockchain, 2017b; CoinDesk, 2017b); Dyhrberg (2016a) also points out that the bitcoin reactions may be quicker than gold and Dollar, thus substantiating the analysis of both high and low data frequencies in this paper. The author replicates the study using TGARCH(1,1) and find similar conclusions (Dyhrberg, 2016b).

The other financial line of bitcoin studies is the speculation factor that comes with the high volatility, making the arbitrage possible to investors. Yermack (2013) for example criticizes the bitcoin as a currency and a hedge asset, pointing the obstacles to make it a functional digital currency, since its value and liquidity do not behave as other real currencies do, pointing out the bitcoin's speculative potential due its limited amount available in the market. Another supporter of the speculative characteristic is Kristoufek (2015): while the author recognizes the similarities between bitcoin and traditional currencies, bitcoin is shown to have a more dynamic and unstable value over time that drives its nature to be a speculative asset more than a currency.

When it comes to comprehending the market and the economic agents involved, bitcoin uses are still very restricted to some countries and activities. Usually, the one's that are open to this new asset are technological or innovation centers, which are able to understand the potential and advantages of the cryptomoney. Estonia, United Sates, Denmark, Sweden, South Korea, Netherlands, Finland, Canada, United Kingdom and Australia are ten countries friendly to the uses of bitcoin. The main application of cryptocurrencies in those countries was on the on-line marketing, known as e-commerce, in exchange for products and services. However, due the increase of bitcoin's market value, its exchange nature has been relatively put aside.

The asset flexibility in transactions (since there are few regularizations of this new market) and the high level of the cryptography gives the coin enough trust to be used instead of cash, like in Denmark. Even with the importance and its uses, the worldwide acceptance is still far from happening and the impact over the cryptocurrency dynamics in the present is inevitable. Yet, there is not enough literature review that discusses or presents efficient methodologies to estimate the new bitcoin market behavior around the world. Some authors already conducted studies about this new market, (Bouri, Azzi, & Haubo Dyhrberg, 2016; Dyhrberg, 2016a; Xiong, Bao, & Hu, 2014; Yermack, 2013), but they are still restricted to the unknown aspects of the behavior and/or acceptance of bitcoin.

Bitcoin is not the only cryptocurrency, as other types of digital currency were created, called altcoins. Therefore, in addition to bitcoin, we chose another two relevant cryptocurrencies, ethereum and dashcoin, using market cap and price as criteria, to proceed with the same volatility analysis with the SVR models

Ethereum is a branch from the original bitcoin project, so it inherits his principal concepts, but ethereum was not designed to be a rival of bitcoin, it actually used bitcoin principles to produce more technology by including smart contracts, creating a new world of possibilities, like implementing a voting system without any third party to trust, with his virtual machine an a programmable system this coins has not only promise for a revolution in the cash system but a revolution in any contractual system. Ethereum runs with no chance of censorship, fraud, third party interference or downtime, since it functions precisely as programmed (Kim, 2016).

Since Ethereums "smart contracts" are built with a part of an enforcing mechanism, they are considered more flexible. The major difference of Ethereum from other cryptocurrency is that it automatically enforce the clauses of an agreement if one of the participants disrespect it. It is important to mention that all the counterparty risks cannot be mitigated with these contracts and that it is complex to enforce it in certain cases (Balta, Creeze, Ezzra, and Sidokhine,).

Another important cryptocurrency is Dash, which comes from Digital Cash. It deals with instant transactions and is characterized as a privacy-centric digital currency cryptography, ensuring total anonymity. Also, by using a two tier network, Dash improves bitcoin system. In addition, Dash apply an anonymity technology that precludes the acknowledgment of who made the block chain, which is a similar for bitcoins. This technology uses a protocol mix utilizing an innovative decentralized network of servers, this advance in the system is known as Masternodes. This provide a more trustworthy system (Kim, 2016).

2.2. Cryptocurrencies and traditional currencies

The classic definition of “money” requires an asset to be usable as a medium of exchange, a unit of account and a store of value. Researches like Urquhart (2016) indicate that cryptocurrencies such as bitcoin still present informational inefficiency, even though it has been showing a trend towards efficiency. Thus, separating the volatility analysis of bitcoin prices considering different time horizons may provide a better understanding regarding this finding. Additionally, studies like Yermack (2013) and Dowd (2014) states that cryptocurrencies are susceptible to speculative bubbles that can mine its fundamental value, and their behavior characterizes them as a speculative investment rather than a currency.

On the other hand, according to authors like Lo and Wang (2014), cryptocurrencies like bitcoin has satisfied all three functions of money, emphasizing the similarities between virtual and real coins. Based on this argument and the discussion regarding the proportion of their fundamentalist value in relation to the speculative component, it is relevant to analyze the dynamics of their volatility over time. Dyhrberg (2016a) also argues that bitcoin can be classified as an asset between a “pure medium of exchange”, like US Dollar, and a “pure store of value”, like gold. While bitcoin is not a currency *per se*, it combines advantages of both Gold and Dollar and has the potential of being an important instrument in financial markets and portfolio management.

The reason for choosing cryptocurrencies is centered in the innovative and potential that these assets have in the current global economy. The method of machine learning itself has already presented good results in diverse situations, cases and problems, since it is possible to find a pattern and estimate decision guide lines, but for cryptocurrencies there is the differential of unknown and unimaginable future. Cryptocurrencies bring along themselves not only potentially profitable assets, but also new concepts that differ greatly from the traditional monetary economy, like direct transaction, intangible value base, absence of a central institution to assure its store of value, as well as propelling advances in security and information storage at global scale.

As seen in the worldwide spillover effects of financial crisis in late 1990s (Belke & Setzer, 2004), high levels of volatility can bring over a herd behavior and financial contagion that can lead to unpredictable and large turnovers in the financial market. Thus, analyzing whether the volatility patterns of cryptocurrencies behave similarly with traditional currencies can be a important framework to better understand cryptocurrencies’ role in nowadays finance and investigate whether there are evidences that the “virtual world” currencies are ready to merge into the “real world”.

Furthermore, issues concerning the heterogeneity among different cryptocurrencies such as the potential influence on the operating system of different cryptocurrencies on their market behavior or volatility level remains unexplored in the finance literature. Indeed, we observed that the volatility patterns of the three major cryptocurrencies we picked (bitcoin, Ethereum and Dash) differ significantly among themselves in both daily and hourly time frequencies, as seen in Figs. 1–4. The overall volatility levels of cryptocurrencies is much higher than the exchange rates, in both daily and hourly frequencies; the only notable peak of volatility in traditional currencies is the dollar-pound quotation in mid-2016, coinciding with the “Brexit” referendum. Over the whole analyzed perior, the volatility levels of bitcoin was significantly higher than the three exchange rates (in they were plotted at the same scale, the exchange rates volatility would resemble a straight line), but at the same time significantly lower than the other cryptocurren-

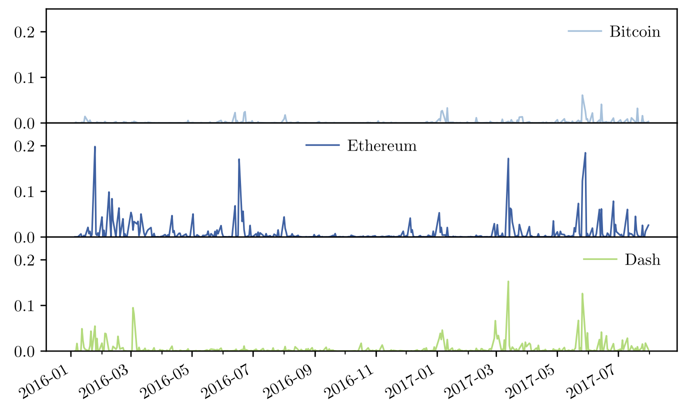


Fig. 1. Cryptocurrencies daily volatility.

Fig. 2. Exchange rates daily volatility.

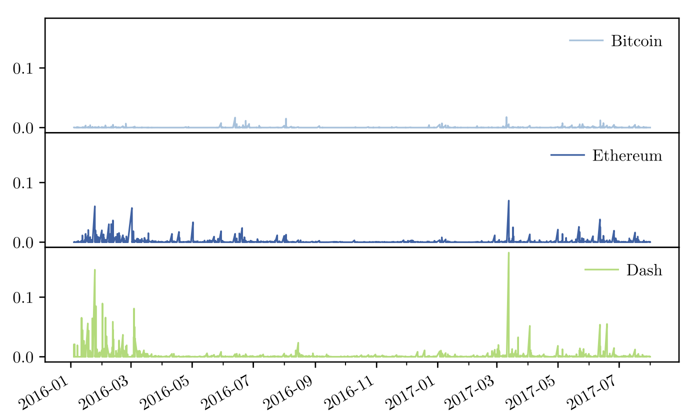


Fig. 3. Cryptocurrencies hourly volatility.

Fig. 4. Exchange rates hourly volatility.

cies. The heterogeneity between different cryptocurrencies motivated the inclusion of the volatility estimation of different cryptocurrencies so that the predictive performance of both traditional econometric models and the machine learning approach can be observed in different cryptocurrencies, generating further evidences of their quality in various contexts.

Since the cryptocurrencies market, notably bitcoin, operates without a supervising organization or entity to ensure it's value or conduct the transactions, as we see in stock exchanges institutions, the economic agents may find difficult to predict this asset, since there is no history or methodology established in the academic or business environment. The construction of a machine, capable to forecast the risk variable, interpreted as volatility, represents an improvement in the studies and business operation using this kind of currency. Furthermore, it may be the first step in the construction of pricing models for the cryptocurrencies and possibles derivatives of it. Therefore, the application of machine learning methods seems very attractive to capture underlying patterns regarding those issues, thus providing a more comprehensive and accurate view of this new agenda.

Analyzing the present techniques and concepts used in the scientific literature for cryptocurrencies estimation, there are few studies that use the machine learning approach in forecasting its volatility, even with well known methodologies such as the GARCH model (Li & Wang, 2017). In special, most applications of machine learning methods in finance compare predictive performance based on error metrics like directional accuracy, mean squared error and mean absolute error, instead of using a statistical test as criteria: this procedure is recurrent in papers that apply state-of-art machine learning methods in finance, as seen in Evans, Pappas, and Xhafa (2013), Sermpinis, Stasinakis, Theofilatos, and Karathanasopoulos (2015) and Shen, Chao, and Zhao (2015) . Concerning the superiority of the SVR model over GARCH, many recent works (Chen, Jeong, & Härdle, 2008; Gavrishchaka & Banerjee, 2006; Gavrishchaka & Ganguli, 2003; Premanode & Toumazou, 2013; Santamaría-Bonfil, Frausto-Solís, & Vázquez-Rodarte, 2015) present favorable evidences towards SVR's superiority, but without enunciating it based on a stronger statistical criteria. Bearing in mind those issues related to financial aspects of cryptocurrencies, this paper combined the machine learning approach with volatility forecasting, splitting the analysis into datasets of high and low frequencies and evaluating the predictive performance of the models using hypothesis tests in order to check in what extent SVR's superiority really holds.

3. Method

3.1. Volatility estimation

Within the field of financial study, the learning and analysis of financial time series has risen much interest among researchers until today. Technology advance allowed the expansion of financial market and, consequently, of trading operations, which increased the availability of information, quantity of transactions carried out during the day and, mainly, the track of real time assets prices. Regarding financial series analysis, volatility forecasting bears a huge importance, as it has decisive impacts on risk management and derivatives pricing. One of the main stylized facts of this literature states that financial series conditional variance is typically non-constant. According to Deboeck (1994), Abu-Mostafa and Atiya (1996) and Cao and Tay (2003) these series presents dynamisms and the distribution also shows great variations over time, without exhibiting an apparent and constant pattern in its' disposal. In the financial time series analysis, one can divide the data according to their frequency over time: (1) monthly frequency are usually classified as low frequency. They present a more extensive analysis on macroeconomic variables and analysis of resource allocation and on investment evaluation (Easley, López de Prado, & OHara, 2012); (2) data with appearance in minutes or seconds are commonly classified as high frequency. They are strongly influenced by recent events or the availability of market information, as discussed by Reboredo, Matías, and Garcia-Rubio (2012); (3) daily frequency data is the usual periodicity for financial data analysis, such as stock market prediction and volatility estimation, but in recent years, the literature has been moving towards increasingly higher time frequencies or even a volume based paradigm, so that the daily data is used as a low frequency baseline to which the higher frequency data are compared (Easley et al., 2012). In this paper, we considered daily data as low frequency and used the hourly periodicity as high frequency.

Due to the importance of fresh news, regarding assets price, Andersen and Bollerslev (1998) explain that financial series present an extremely volatility behavior since they incorporate expectations and reactions of economic agents in the face of events. Currently, market asset volatility forecast and estimation are highly relevant in the composition of derivatives prices, in the portfolio risk analysis and in the investment risk analysis itself. So, the development of methods that help decision making arouses great interest among investors.

Particularly, the high frequency data analysis has caught the attention of many scholars and financial market agents, given the sharp increase in worldwide financial transaction flows, which makes high frequency trading a relevant paradigm for nowadays finance, as discussed in Easley et al. (2012). With respect to volatility estimation, studies like Li and Wang (2017) indicate that exchange rates and cryptocurrencies intra-day volatility tend to be very high, motivating its analysis using high frequency data, as seen in Çelik and Ergin (2014) and Baruník and Křehlík (2016).

The standard model used by the academy for volatility estimation is the GARCH model (Bollerslev, 1986), which was derived from the ARCH model (Engle, 1982). The GARCH model main advantage is its ability to generalize an ARCH(∞), making it a parsimonious and efficient model to deal with many typical behaviors of financial time series volatility, as highlighted by Marcucci et al. (2005). Furthermore, this model is broadly studied and used by financial analysts, for instance Hansen and Lunde (2005) compared 330 ARCH-type models in terms of their ability to describe the conditional variance using Diebold and Mariano (1995) predictive accuracy test and found no evidence that a GARCH(1,1) was outperformed by more sophisticated models in their exchange rates analysis, but they concluded that GARCH(1,1) was inferior to models that can accommodate a leverage effect in the stock's market analysis.

Recently, other techniques to predict volatility have been discussed, a strong and consistent method used is the Support Vector Regression (SVR), which covers the nonlinearity and dynamic characteristic of the financial series. Gavrishchaka and Ganguli (2003), Gavrishchaka and Banerjee (2006) and Chen et al. (2008) have already presented empirical results regarding the efficiency and superior predictability of volatility using SVR when compared to the GARCH benchmark and other techniques, such as neural networks and technical analysis (Baruník & Křehlík, 2016).

In the last years, there has been a great interest in using traditional methodologies, such as the GARCH model and new ones, like SVR and neural networks, to estimate new assets volatility in the stock or commodities market. As pointed out in Hsu, Lessmann, Sung, Ma, and Johnson (2016), the machine learning approach has been consistently outperforming traditional econometric models in many research fields inside the finance literature, making such class of methods hugely popular in recent works.

3.2. GARCH models

Given P_t the observed price at time $t = 1, \ldots, T,$ the GARCH(1,1) (Generalized Auto-regressive Conditional Heteroskedasticity) model can be summarized as follows:

$$r_t = \mu_t + \epsilon_t \tag{1}$$

where $r_t = \log(\frac{P_t}{P_{t-1}})$ is the log return and ϵ_t is a stochastic term with zero mean. The mean equation of the return in Eq. (1) is defined by an AR(1) model:

$$\mu_t = \gamma_0 + \gamma_1 r_{t-1} \tag{2}$$

and the volatility equation is given by:

$$h_t = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta_1 h_{t-1} \tag{3}$$

such that $\mathbb{V}(\epsilon_t) = h_t$.

Since the actual volatility is not directly obtained, an *ex-post* proxy volatility is needed to estimate the volatility through SVR. Following Chen, Härdle, and Jeong (2010) and Bezerra and Albuquerque (2017), we defined the proxy volatility as:

$$\tilde{h}_t = (r_t - \bar{r})^2 \tag{4}$$

where $\bar{r} = \sum_{t=1}^{T} \frac{r_t}{N}$ is the arithmetic mean of the log returns over the N periods of the in-sample data.

The GARCH (1,1) is one of the main models regarding volatility estimation in finance, given its easy estimation, low number of parameters and ability to capture volatility clusters and conditional variance's non-constant behavior (Hansen & Lunde, 2005). Through a visual analysis of the volatility graphs displayed in Figs. 1–4, the cryptocurrencies volatilities exhibit a clustering behavior in both low and high frequencies, similarly to the traditional currencies exchange rate pattern, in a much higher magnitude nevertheless, such that we can conclude that the volatility clustering stylized fact seems to hold for the dataset we analyzed, which in turn justifies the use of GARCH models to estimate their volatility.

Even when ϵ_t is assumed to be normally distributed, GARCH presents a fat-tailed behavior in comparison to the Gaussian distribution, even though still not quite incorporating the financial data's stylized facts (Cont, 2001). Thus, it is common to assume that ϵ_t follows non-Gaussian distributions, in order to fit better to the financial data. In this paper, we estimated the GARCH (1,1) assuming three conditional distributions for ϵ_t: Normal, Student's t and Skewed Student's t. Despite the fact that literature proposed a wide variety of distributions to be assumed in the GARCH model, authors like Sun and Zhou (2014) argue that the Student's t is enough to give a good fit to the financial data's heavy tail behavior, while innovations concerning GARCH conditional distributions does not seem robust.

As discussed by studies like Awartani and Corradi (2005), the asymmetric volatility (also known as the "leverage effect") is a well know stylized fact in financial markets: typically, a negative shock in t tend to make a higher impact on the volatility at $t+1$ than positive shocks. Thus, even though the literature presented many positive evidences towards GARCH (1,1) model over a great number of conditional heteroskedasticity models (Hansen & Lunde, 2005), studies like Awartani and Corradi (2005), Wang (2009) and Laurent, Rombouts, and Violante (2012) present evidences that GARCH extensions that incorporate the effects of asymmetry in financial series' volatility, such as EGARCH and GJR-GARCH, yielded smaller out-of-sample prediction errors in comparison to the standard GARCH. Thus, we incorporated EGARCH (1,1) and GJR-GARCH (1,1) as benchmarks as well.

The volatility equation for EGARCH (1,1) (Nelson, 1991) is given by:

$$\log(h_t) = \alpha_0 + g(z_{t-1}) + \beta_1 \log(h_{t-1}) \tag{5}$$

where $\epsilon_t = h_t z_t$ and $g(z_t) = \alpha_1 z_t + \gamma[|z_t| - \mathbb{E}(|z_t|)]$ and $z_t \sim N(0, 1)$. And the volatility equation of GJR-GARCH (1,1) (Glosten, Jagannathan, & Runkle, 1993) is given by:

$$h_t = \alpha_0 + (\alpha_1 + \gamma_1 I_{t-1})\epsilon_{t-1}^2 + \beta_1 h_{t-1}$$
$$I_t = \begin{cases} 0, & \epsilon_t \geq 0, \\ 1, & \epsilon_t < 0. \end{cases} \tag{6}$$

3.3. Support Vector Regression

Despite its popularity, GARCH (1,1) still considers linear functional forms in its estimation, which motivates the introduction of nonlinear structural forms. That is one of the main contributions of machine learning and kernel methods, as many studies showed (Li & Suohai, 2013; Shen et al., 2015) that the introduction of nonlinear interactions can significantly boost the explanatory power and the forecasting ability of many models applied to financial contexts, including volatility estimation (Chen et al., 2010; Premanode & Toumazou, 2013; Santamaría-Bonfil et al., 2015). Regarding high frequency forecasting, that issue is also noted (Santos, da Costa, & dos Santos Coelho, 2007).

Therefore, we used Support Vector Regression to estimate of the GARCH's mean and volatility equations – described in Eqs. (2) and (3). Instead of using a standard linear regression, we introduced nonlinearities, in order to provide a better fit to the data. Concerning the high volatility of bitcoin data – specially in high frequency transactions - this approach seems particularly attractive.

The Support Vector Regression (SVR) (Drucker, Burges, Kaufman, Smola, & Vapnik, 1997; Vapnik, 1995) is a regression model that aims to find a decision function which is the best approximation of a set of observations, bearing in mind the middle ground between a good power of generalization and an overall stable behavior, in order to make good out-of-sample inferences. Associated with these two desirable features, there are two corresponding problems in regression models, constituting the so called "bias-variance dilemma". To perform the regularization of the decision function, two parameters are added: a band of tolerance δ[1], to avoid over-fitting; and a penalty C to the objective function, for points that lie outside this confidence interval for an amount $\boldsymbol{\xi} > 0$.

Therefore, the predicted values $f(\mathbf{x_i})$, such that $|y_i - f(\mathbf{x_i})| \leq \delta$ and $f(.)$ is the SVR decision function, are considered to be statistically equal to y. The SVR defined from the addition of these two parameters is known as ε-SVR. The loss function implied in the construction of the ε-SVR is the ε-insensitive loss function (Vapnik, 1995), $L_\varepsilon[y_i, f(\mathbf{x_i})]$, given by:

$$L_\varepsilon[y_i, f(\mathbf{x_i})] = \begin{cases} |y_i - f(\mathbf{x_i})| - \delta, & |y_i - f(\mathbf{x_i})| > \delta, \\ 0, & |y_i - f(\mathbf{x_i})| \leq \delta. \end{cases} \tag{7}$$

It is worth noting that the ε-insensitive loss function is not the only possible way to define penalties for SVR; extensions that include different penalty structures includes the ν-SVR (Chang & Lin, 2002). The ε-SVR formulation was chosen for this paper because it is the most commonly used form in the finance forecasting literature, and requires lesser computational time to perform the optimization.

In order to introduce nonlinear interactions in the regression estimation, a mapping φ is applied, such that the objective function to be optimized for ε-SVR is formulated as follows:

$$\begin{aligned} \text{Minimize}: \quad & \frac{1}{2}\mathbf{w}^\top \mathbf{w} + C\boldsymbol{\xi}^\top \mathbf{1} + C\boldsymbol{\xi}^{*\top} \mathbf{1} \\ \text{Subject to}: \quad & \Phi \mathbf{w} + w_0 - \mathbf{y} \leq \delta \mathbf{1} + \boldsymbol{\xi} \end{aligned}$$

[1] ε is the usual symbol used in SVR models for the confidence band. In this paper, we changed ε to δ to avoid ambiguities with the GARCH model error term ϵ.

Table 1
Search intervals used for the parameters' training.

Parameter	Search interval
δ	$[0.05, 0.1, \ldots, 0.95, 1]$
C	$[0.5, 1, \ldots, 4.5, 5]$
σ	$[0.05, 0.1, \ldots, 1.95, 2]$

$$\begin{aligned} & \mathbf{y} - \Phi\mathbf{w} - w_0 \leq \delta\mathbf{1} + \boldsymbol{\xi}^* \\ \text{with :} \quad & \boldsymbol{\xi}, \boldsymbol{\xi}^* \geq 0 \end{aligned} \tag{8}$$

where Φ is a $T \times q$ matrix created by the Feature Space, i.e., the original explanatory variables $\mathbf{X}_{(T\times p)}$ is mapped through the $\varphi(\mathbf{x})$ function, $\mathbf{w}$ is a vector of parameters to be estimated, C and δ are hyper-parameters and $\boldsymbol{\xi}, \boldsymbol{\xi}^*$ are slack variables in the Quadratic Programming Problem.

In other words, $\mathbf{w}_{(q\times 1)}$ is the vector of the angular coefficients of the decision hyperplane in $\mathbb{R}^q$; $w_0 \in \mathbb{R}$ is the linear coefficient (intercept) of the decision hyperplane in $\mathbb{R}^q$; $\Phi_{(T\times q)}$ is the augmented matrix of observations, after the original data being transformed by φ; $\mathbf{y}_{(T\times 1)}$ is the vector that provides the dependent variable values of the observed points; $C \in \mathbb{R}$ is the cost of error; $\delta > 0$ is the tolerance band that defines the confidence interval for which there is no penalty; $\boldsymbol{\xi}^*(T \times 1)$ is the vector concerning points above the tolerance band; and $\boldsymbol{\xi}_{(T\times 1)}$ is the vector concerning points below the tolerance band.

After some algebraic manipulations, it can be shown that the decision function of ε-SVR can be written as:

$$f(\mathbf{x_i}) = \mathbf{w}^\top \varphi(\mathbf{x}) - w_0 = \sum_{t=1}^{T} \kappa(\mathbf{x_i}, \mathbf{x_j})(\lambda_j^* - \lambda_j) - w_0 \tag{9}$$

where $\kappa(\mathbf{x_i}, \mathbf{x_j}) = \varphi(\mathbf{x_i}) \cdot \varphi(\mathbf{x_j}) \in \mathbb{R}, i, j = 1, 2, 3, \ldots, T$ is the kernel function. Since φ transforms the original data to a higher dimension, which can even be infinite, the use of the kernel function prevents the need to explicitly compute the functional form of $\varphi(\mathbf{x})$; instead, κ computes the inner product of φ, a term that appears in SVR's dual formulation (Drucker et al., 1997). This is known as the *kernel trick*. In this paper, we used the Gaussian Kernel as κ, whose expression is given by:

$$\kappa(\mathbf{x_i}, \mathbf{x_j}) = exp\left(-\frac{||\mathbf{x_i} - \mathbf{x_j}||^2}{2\sigma^2}\right), \sigma > 0 \tag{10}$$

The Gaussian Kernel is the most widely used Kernel in machine learning literature. It enjoys huge popularity in various knowledge fields since this function is able to induce an infinite dimensional feature space while depending on only one scattering parameter σ.

3.4. SVR-GARCH

The SVR-GARCH is the joining result of the GARCH model structure and the nonlinearities introduced by the kernel function via SVR. Santamaría-Bonfil et al. (2015) presented empirical evidences that SVR-GARCH managed to outperform standard GARCH's predictions, showing better ability to approximate the nonlinear behavior of financial data and stylized facts, such as heavy tails and volatility clusters. The specification of SVR-GARCH (1,1) is the same of the conventional GARCH (1,1), but the mean and volatility equations were estimated via SVR, such that:

$$r_t = f_m(r_{t-1}) + \epsilon_t \tag{11}$$

$$h_t = f_v(h_{t-1}, \epsilon_{t-1}^2) \tag{12}$$

where $f_m(.)$ is the SVR decision function for the mean Eq. (1), and $f_v(.)$ is the SVR decision function for the volatility Eq. (3).

Depending of which parameters δ, C and σ are set to the SVR formulation, a different decision function is obtained. In order to decide the "better" combination of parameters, we did a

Table 2
Forecasting performance for low frequency test set data.

Training set: January 4th 2016–October 31st 2016

Validation set: November 1st 2016–February 28th 2017

Test set: March 1st 2017–July 31st 2017

Model	Bitcoin		Ethereum		Dash	
	RMSE	MAE	RMSE	MAE	RMSE	MAE
Normal GARCH (1,1)	0.11262520	0.11025030	0.26447120	0.24894240	0.19964301	0.16836890
Student's *t* GARCH (1,1)	0.15802830	0.15560480	0.33843840	0.33294630	0.20517540	0.18140470
Skewed Student's *t* GARCH (1,1)	0.15603930	0.15356490	0.60801570	0.59901870	0.20540960	0.18172430
Normal EGARCH (1,1)	0.14851058	0.06021389	0.24741551	0.12332978	0.18845499	0.10440545
Student's *t* EGARCH (1,1)	0.15294444	0.13761023	0.29789732	0.27103321	0.19264046	0.11003185
Skewed Student's *t* EGARCH (1,1)	0.15437027	0.13844548	0.86978330	0.83615954	0.19297362	0.11041533
Normal GJR-GARCH (1,1)	0.15878742	0.06073869	0.24829644	0.18500229	**0.18631002**	**0.10449338**
Student's *t* GJR-GARCH (1,1)	0.15060326	0.13426739	0.29438251	0.26876020	0.19277590	0.11011040
Skewed Student's *t* GJR-GARCH (1,1)	0.14354459	0.17748075	0.61748915	0.58003697	0.19363469	0.11046644
SVR-GARCH (1,1)	**0.03133926**	**0.01315455**	**0.20422370**	**0.08627904**	**0.15282070**	**0.04538759**
Model	Euro		British Pound		Japanese Yen	
	RMSE	MAE	RMSE	MAE	RMSE	MAE
Normal GARCH (1,1)	0.01163661	0.01162828	0.00936268	0.00936006	0.01028482	0.01028177
Student's *t* GARCH (1,1)	0.02416139	0.02374070	0.01008274	0.01007267	0.01103492	0.01102153
Skewed Student's *t* GARCH (1,1)	0.02411368	0.02369469	0.01009985	0.01008939	0.01098031	0.01096716
Normal EGARCH (1,1)	0.00557848	0.00473396	0.00688289	0.00539079	0.00724395	0.00592201
Student's *t* EGARCH (1,1)	0.00556659	0.00467683	0.00691821	0.00542716	0.00745208	0.00608476
Skewed Student's *t* EGARCH (1,1)	0.00585727	0.00498910	0.00691028	0.00541879	0.00766734	0.00631347
Normal GJR-GARCH (1,1)	0.00580383	0.00499980	0.00680241	0.00537477	0.00716407	0.00590922
Student's *t* GJR-GARCH (1,1)	0.00580788	0.00500314	0.00674019	0.00531741	0.00715457	0.00589986
Skewed Student's *t* GJR-GARCH (1,1)	0.00580250	0.00498450	0.00675847	0.00533939	0.00715230	0.00589715
SVR-GARCH (1,1)	**0.00030316**	**0.00011757**	**0.00023233**	**0.00008382**	**0.00022602**	**0.00014921**

Hansen et al.'s (2011) superior set models are highlighted in bold.

Table 3
Forecasting performance for high frequency test set data: Period 1.

Training set: January 4th 2016–May 31st 2016

Validation set: June 1st 2016–July 31st 2016

Test set: August 1st 2016–September 30th 2016

Model	Bitcoin		Ethereum		Dash	
	RMSE	MAE	RMSE	MAE	RMSE	MAE
Normal GARCH (1,1)	0.00759729	0.00569709	0.01080670	0.00972066	0.01514854	0.01404126
Student's *t* GARCH (1,1)	0.00817163	0.00600172	0.01155509	0.00999820	0.01446214	0.01363025
Skewed Student's *t* GARCH (1,1)	0.00797165	0.00580175	0.01156629	0.01000044	0.01442460	0.01359423
Normal EGARCH (1,1)	0.00660961	0.00546262	0.01066027	0.00972147	0.01490163	0.01395768
Student's *t* EGARCH (1,1)	0.00560289	0.00486634	0.01124901	0.00997496	0.01427962	0.01358828
Skewed Student's *t* EGARCH (1,1)	0.00563103	0.00487856	0.01125544	0.00998051	0.01428608	0.01358456
Normal GJR-GARCH (1,1)	0.00774527	0.00570928	0.01082471	0.00968486	0.01529941	0.01407232
Student's *t* GJR-GARCH (1,1)	0.00564167	0.00491907	0.01158647	0.00998025	0.01457111	0.01367225
Skewed Student's *t* GJR-GARCH (1,1)	0.00555230	0.00483404	0.01159366	0.00998519	0.01456874	0.01366403
SVR-GARCH (1,1)	**0.00059805**	**0.00026484**	**0.00068952**	**0.00021251**	**0.00092300**	**0.00020908**
Model	Euro		British Pound		Japanese Yen	
	RMSE	MAE	RMSE	MAE	RMSE	MAE
Normal GARCH (1,1)	0.00082456	0.00082383	0.00118071	0.00118059	0.00132147	0.00131698
Student's *t* GARCH (1,1)	0.00080980	0.00080745	0.00130299	0.00129675	0.00138082	0.00136957
Skewed Student's *t* GARCH (1,1)	0.00089016	0.00088014	0.00126084	0.00125772	0.00129791	0.00128906
Normal EGARCH (1,1)	0.00081491	0.00081073	0.00119775	0.00119498	0.00132418	0.00130711
Student's *t* EGARCH (1,1)	0.00089333	0.00081018	0.00130253	0.00123891	0.00134704	0.00133007
Skewed Student's *t* EGARCH (1,1)	0.00089234	0.00081048	0.00130001	0.00123844	0.00134781	0.00133081
Normal GJR-GARCH (1,1)	0.00084235	0.00083376	0.00124946	0.00123813	0.00137471	0.00135760
Student's *t* GJR-GARCH (1,1)	0.00077467	0.00076647	0.00119304	0.00118418	0.00125635	0.00124371
Skewed Student's *t* GJR-GARCH (1,1)	0.00077294	0.00076013	0.00128225	0.00127839	0.00131223	0.00130378
SVR-GARCH (1,1)	**0.00024853**	**0.00009478**	**0.00072465**	**0.00050004**	**0.00005872**	**0.00002306**

Hansen et al.'s (2011) superior set models are highlighted in bold.

grid search for those three parameters for both mean and volatility equations, and evaluated the Root Mean Square Error (RMSE) of each decision function. The model is first estimated with a subset of the data (known as training dataset) and then the estimated model is used to forecast both mean and volatility in another subset (validation dataset). The combination that minimizes the RMSE for the validation dataset was chosen as the best one. This combination of parameters was applied to yield the forecasts of the out-of-sample data, known as the test set. The search intervals for each parameter of the SVR-GARCH are displayed in Table 1.

Table 4
Forecasting performance for high frequency test set data: Period 2.

Training set: March 1st 2016–July 31st 2016

Validation set: August 1st 2016–September 30th 2016

Test set: October 1st 2016–November 30th 2016

Model	Bitcoin		Ethereum		Dash	
	RMSE	MAE	RMSE	MAE	RMSE	MAE
Normal GARCH (1,1)	0.00460814	0.00440171	0.00775727	0.00770857	0.01163329	0.01151671
Student's *t* GARCH (1,1)	0.00479175	0.00467229	0.00782142	0.00765811	0.01191285	0.01171240
Skewed Student's *t* GARCH (1,1)	0.00414653	0.00392619	0.00780785	0.00764424	0.01191912	0.01171801
Normal EGARCH (1,1)	0.00449658	0.00436728	0.00782352	0.00775327	0.01162451	0.01149187
Student's *t* EGARCH (1,1)	0.00430255	0.00418052	0.00770778	0.00758103	0.01190195	0.01170119
Skewed Student's *t* EGARCH (1,1)	0.00431631	0.00419205	0.00770736	0.00758063	0.01191014	0.01170721
Normal GJR-GARCH (1,1)	0.00465698	0.00438188	0.00483759	0.00371794	0.01165126	0.01152582
Student's *t* GJR-GARCH (1,1)	0.00441468	0.00422006	0.00762143	0.00748573	0.01193339	0.01171407
Skewed Student's *t* GJR-GARCH (1,1)	0.00441076	0.00421270	0.00779723	0.00763222	0.01194239	0.01172274
SVR-GARCH (1,1)	**0.00077868**	**0.00020442**	**0.00177067**	**0.00056101**	**0.00307538**	**0.00127483**
Model	Euro		British Pound		Japanese Yen	
	RMSE	MAE	RMSE	MAE	RMSE	MAE
Normal GARCH (1,1)	0.00111576	0.00101530	0.00161297	0.00155571	0.00174503	0.00147923
Student's *t* GARCH (1,1)	0.00120193	0.00106713	0.00167885	0.00162099	0.00153819	0.00141659
Skewed Student's *t* GARCH (1,1)	0.00115309	0.00103867	0.00169058	0.00162774	0.00154086	0.00142141
Normal EGARCH (1,1)	0.00109249	0.00102655	0.00157519	0.00153497	0.00149050	0.00139378
Student's *t* EGARCH (1,1)	0.00115206	0.00107847	0.00199462	0.00171057	0.00152148	0.00141757
Skewed Student's *t* EGARCH (1,1)	0.00115383	0.00108003	0.00199594	0.00171036	0.00152152	0.00141599
Normal GJR-GARCH (1,1)	0.00114574	0.00104284	**0.00013351**	**0.00000491**	0.00154525	0.00139834
Student's *t* GJR-GARCH (1,1)	0.00117087	0.00108371	0.00165860	0.00159177	0.00159770	0.00145711
Skewed Student's *t* GJR-GARCH (1,1)	0.00114196	0.00105452	0.00164845	0.00159493	0.00007137	0.00008873
SVR-GARCH (1,1)	**0.00050881**	**0.00015161**	**0.00011788**	**0.00002368**	**0.00010146**	**0.00005867**

Hansen et al.'s (2011) superior set models are highlighted in bold.

Table 5
Forecasting performance for high frequency test set data: Period 3.

Training set: May 1st 2016–September 30th 2016						
Validation set: October 1st 2016–November 30th 2016						
Test set: December 1st 2016–January 31st 2017						
Model	Bitcoin		Ethereum		Dash	
	RMSE	MAE	RMSE	MAE	RMSE	MAE
Normal GARCH (1,1)	0.00913742	0.00749972	0.01270315	0.01127455	0.01415568	0.01325402
Student's *t* GARCH (1,1)	0.00925140	0.00722405	0.01294731	0.01139057	0.01564308	0.01437991
Skewed Student's *t* GARCH (1,1)	0.00925199	0.00722519	0.01294729	0.01138956	0.01564466	0.01437814
Normal EGARCH (1,1)	0.00955980	0.00785587	0.01294379	0.01147051	0.01340430	0.01302949
Student's *t* EGARCH (1,1)	0.01113715	0.00888592	0.01284171	0.01138972	0.01519257	0.01426543
Skewed Student's *t* EGARCH (1,1)	0.01119126	0.00892786	0.01284247	0.01139012	0.01538706	0.01444303
Normal GJR-GARCH (1,1)	0.00869412	0.00727078	0.01277006	0.01130760	0.01392639	0.01348153
Student's *t* GJR-GARCH (1,1)	0.00931768	0.00722120	0.01299166	0.01139946	0.01539582	0.01413736
Skewed Student's *t* GJR-GARCH (1,1)	0.00932389	0.00722400	0.01299131	0.01139830	0.01544307	0.01416606
SVR-GARCH (1,1)	**0.00042599**	**0.00008641**	**0.00050119**	**0.00030067**	**0.00076065**	**0.00020646**
Model	Euro		British Pound		Japanese Yen	
	RMSE	MAE	RMSE	MAE	RMSE	MAE
Normal GARCH (1,1)	0.00133953	0.00131835	0.00151323	0.00148351	0.00161222	0.00158059
Student's *t* GARCH (1,1)	0.00136766	0.00129345	0.00157570	0.00156285	0.00159663	0.00156250
Skewed Student's *t* GARCH (1,1)	0.00154868	0.00146029	0.00162702	0.00160678	0.00157617	0.00154267
Normal EGARCH (1,1)	0.00134860	0.00132980	0.00158497	0.00144133	0.00159121	0.00155684
Student's *t* EGARCH (1,1)	0.00153668	0.00144513	0.00157096	0.00150291	0.00162717	0.00159317
Skewed Student's *t* EGARCH (1,1)	0.00156194	0.00146674	0.00156715	0.00149893	0.00163267	0.00159789
Normal GJR-GARCH (1,1)	0.00134752	0.00133283	0.00154829	0.00149573	0.00163351	0.00159968
Student's *t* GJR-GARCH (1,1)	0.00135133	0.00131171	0.00155834	0.00154596	0.00160787	0.00156749
Skewed Student's *t* GJR-GARCH (1,1)	0.00135251	0.00130687	0.00155696	0.00153681	0.00157856	0.00153916
SVR-GARCH (1,1)	**0.00070290**	**0.00020679**	**0.00080172**	**0.00022069**	**0.00062456**	**0.00022606**

Hansen et al.'s (2011) superior set models are highlighted in bold.

4. Empirical analysis

For the empirical test, we used data between January 4th 2016 and July 31st 2017 of three cryptocurrencies: bitcoin, ethereum and dash market price (in US dollars); and three traditional currencies: euro, british pound and japanese yen (in US dollars).The data was collected from Altcoin Charts (http://alt19.com) and Forex Historical Data (http://fxhistoricaldata.com). Since FOREX data are not available for weekends, we collected only the weekdays in all variables to assure that the models were fitted in the same days. We

Table 6
Forecasting performance for high frequency test set data: Period 4.

Training set: July 1st 2016–November 30th 2016						
Validation set: December 1st 2016–January 31st 2017						
Test set: February 1st 2017–March 31st 2017						
Model	Bitcoin		Ethereum		Dash	
	RMSE	MAE	RMSE	MAE	RMSE	MAE
Normal GARCH (1,1)	0.01028286	0.00902283	0.01793080	0.01505344	0.02825603	0.02666399
Student's *t* GARCH (1,1)	0.01032863	0.00883706	0.01710276	0.01338831	0.02649039	0.02170007
Skewed Student's *t* GARCH (1,1)	0.01033805	0.00884552	**0.01710271**	**0.01338835**	0.02649035	0.02170456
Normal EGARCH (1,1)	0.01016503	0.00900445	0.01862030	0.01599543	0.02905840	0.02581053
Student's *t* EGARCH (1,1)	0.01036353	0.00897735	0.01890077	0.01521900	0.03421714	0.02878764
Skewed Student's *t* EGARCH (1,1)	0.01045986	0.00898360	0.01890052	0.01521885	0.03429000	0.02884370
Normal GJR-GARCH (1,1)	0.01034703	0.00900930	0.01960318	0.01560942	0.02827459	0.02584707
Student's *t* GJR-GARCH (1,1)	0.01084610	0.00903140	0.01749649	0.01346106	0.02661699	0.02150698
Skewed Student's *t* GJR-GARCH (1,1)	0.01106700	0.00915978	0.01749658	0.01346125	0.02661630	0.02150540
SVR-GARCH (1,1)	**0.00063104**	**0.00017862**	**0.00252631**	**0.00099954**	**0.00597564**	**0.00120334**
Model	Euro		British Pound		Japanese Yen	
	RMSE	MAE	RMSE	MAE	RMSE	MAE
Normal GARCH (1,1)	0.00092297	0.00091336	0.00114754	0.00114037	0.00116164	0.00115678
Student's *t* GARCH (1,1)	0.00096412	0.00095584	0.00133873	0.00133610	0.00116044	0.00114760
Skewed Student's *t* GARCH (1,1)	0.00090031	0.00089295	0.00117397	0.00116803	0.00116685	0.00115889
Normal EGARCH (1,1)	0.00089818	0.00088116	0.00116322	0.00111639	0.00114097	0.00113461
Student's *t* EGARCH (1,1)	0.00094176	0.00092113	0.00131765	0.00127703	0.00114329	0.00113341
Skewed Student's *t* EGARCH (1,1)	0.00094128	0.00092054	0.00131757	0.00127717	0.00114347	0.00113358
Normal GJR-GARCH (1,1)	0.00096640	0.00095289	0.00117486	0.00115431	0.00117479	0.00116757
Student's *t* GJR-GARCH (1,1)	0.00088538	0.00087678	0.00125185	0.00124700	0.00113320	0.00112218
Skewed Student's *t* GJR-GARCH (1,1)	0.00093148	0.00092358	0.00120794	0.00120456	0.00110560	0.00109304
SVR-GARCH (1,1)	**0.00002209**	**0.00001039**	**0.00043386**	**0.00014066**	**0.00041692**	**0.00016176**

Hansen et al.'s (2011) superior set models are highlighted in bold.

Table 7
Forecasting performance for high frequency test set data: Period 5.

Training set: September 1st 2016–January 31st 2017

Validation set: February 1st 2017–March 31st 2017

Test set: April 1st 2017–May 31st 2017

Model	Bitcoin		Ethereum		Dash	
	RMSE	MAE	RMSE	MAE	RMSE	MAE
Normal GARCH (1,1)	0.01140054	0.00964473	0.01968106	0.01760813	0.02057793	0.01910974
Student's *t* GARCH (1,1)	0.01148559	0.00971204	0.01873198	0.01621085	0.02052849	0.01852332
Skewed Student's *t* GARCH (1,1)	0.01148641	0.00971607	0.01872158	0.01622386	0.02049204	0.01850511
Normal EGARCH (1,1)	0.01166953	0.00979897	0.01852513	0.01699511	0.02003365	0.01874298
Student's *t* EGARCH (1,1)	0.01137208	0.00976844	0.01990472	0.01741331	0.02128344	0.01936220
Skewed Student's *t* EGARCH (1,1)	0.01139669	0.00978829	0.01970050	0.01726016	0.02127683	0.01935668
Normal GJR-GARCH (1,1)	0.01176680	0.00973092	0.01984240	0.01763048	0.02076537	0.01916559
Student's *t* GJR-GARCH (1,1)	0.01174897	0.00977212	0.01873072	0.01608512	0.02041031	0.01828486
Skewed Student's *t* GJR-GARCH (1,1)	0.01177109	0.00978936	0.01871247	0.01609221	0.02039492	0.01827536
SVR-GARCH (1,1)	**0.00050642**	**0.00021148**	**0.00173549**	**0.00124855**	**0.00157584**	**0.00118765**
Model	Euro		British Pound		Japanese Yen	
	RMSE	MAE	RMSE	MAE	RMSE	MAE
Normal GARCH (1,1)	0.00103368	0.00089070	0.00092294	0.00090855	0.00113294	0.00112792
Student's *t* GARCH (1,1)	0.00092145	0.00087629	0.00090351	0.00089595	0.00110693	0.00108632
Skewed Student's *t* GARCH (1,1)	0.00099263	0.00097098	0.00093310	0.00092156	0.00114681	0.00112563
Normal EGARCH (1,1)	**0.00145672**	**0.00090461**	0.00091872	0.00089352	0.00111866	0.00110740
Student's *t* EGARCH (1,1)	0.00107954	0.00087316	0.00098508	0.00094400	0.00111971	0.00111020
Skewed Student's *t* EGARCH (1,1)	0.00108103	0.00872119	0.00098502	0.00094325	**0.00111423**	**0.00110502**
Normal GJR-GARCH (1,1)	0.00093251	0.00089027	0.00091505	0.00090916	0.00115428	0.00113325
Student's *t* GJR-GARCH (1,1)	**0.00091850**	**0.00089133**	0.00091570	0.00089692	0.00072788	0.00040702
Skewed Student's *t* GJR-GARCH (1,1)	0.00091459	0.00089133	0.00091284	0.00089180	**0.00110604**	**0.00109003**
SVR-GARCH (1,1)	**0.00008139**	**0.00000166**	**0.00018515**	**0.00008813**	**0.00050841**	**0.00013768**

Hansen et al.'s (2011) superior set models are highlighted in gray.

used the daily basis for low frequency analysis (411 observations) and the hour periodicity for high frequency estimation (9742 observations).

Both databases were partitioned into three mutually exclusive datasets: training set, validation set and test set. The purpose of this segmentation is to allow the machine learning algorithm to test its predictive performance on data that were not priorly used, in order to better evaluate the real explanatory power of the found decision function when dealing with new data. The training and validation sets constitute the in-sample data for GARCH models

Table 8
Forecasting performance for high frequency test set data: Period 6.

Training set: November 1st 2016–March 31st 2017

Validation set: April 1st 2017–May 31st 2017

Test set: June 1st 2017–July 31st 2017

Model	Bitcoin		Ethereum		Dash	
	RMSE	MAE	RMSE	MAE	RMSE	MAE
Normal GARCH (1,1)	0.01341529	0.01289053	0.02267557	0.02171070	0.01540930	0.01384991
Student's *t* GARCH (1,1)	0.01418239	0.01304112	0.02367832	0.02093723	0.02074012	0.01945195
Skewed Student's *t* GARCH (1,1)	0.01416461	0.01302035	0.02366414	0.02091874	0.02088385	0.01954755
Normal EGARCH (1,1)	0.01323332	0.01278953	0.02251908	0.02172502	0.02110245	0.02097898
Student's *t* EGARCH (1,1)	0.01423269	0.01304949	0.02552701	0.02223959	0.01993591	0.01909908
Skewed Student's *t* EGARCH (1,1)	0.01449978	0.01328147	0.02565733	0.02230468	0.02025959	0.01937293
Normal GJR-GARCH (1,1)	0.01359696	0.01288202	0.02281586	0.02170132	0.02119260	0.02107908
Student's *t* GJR-GARCH (1,1)	0.01443705	0.01306728	0.02374665	0.02080738	0.02046007	0.01938376
Skewed Student's *t* GJR-GARCH (1,1)	0.01463602	0.01322887	0.02386151	0.02088308	0.02082588	0.01968665
SVR-GARCH (1,1)	**0.00058801**	**0.00020410**	**0.00193367**	**0.00066853**	**0.00259729**	**0.00053860**
Model	Euro		British Pound		Japanese Yen	
	RMSE	MAE	RMSE	MAE	RMSE	MAE
Normal GARCH (1,1)	0.00090206	0.00089563	0.00113611	0.00111291	0.00101607	0.00101388
Student's *t* GARCH (1,1)	0.00085591	0.00083969	0.00114517	0.00111681	0.00106887	0.00105437
Skewed Student's *t* GARCH (1,1)	0.00087847	0.00086434	0.00109577	0.00106716	0.00108514	0.00107198
Normal EGARCH (1,1)	0.00102635	0.00089134	0.00121050	0.00110969	0.00103235	0.00102242
Student's *t* EGARCH (1,1)	0.00099420	0.00093139	0.00087107	0.00117713	0.00104719	0.00102476
Skewed Student's *t* EGARCH (1,1)	0.00099030	0.00092624	0.00088244	0.00118472	0.00104756	0.00102427
Normal GJR-GARCH (1,1)	0.00093917	0.00091280	**0.00123846**	**0.00000336**	0.00102764	0.00102210
Student's *t* GJR-GARCH (1,1)	0.00087293	0.00085482	0.00014394	0.00108982	0.00101661	0.00100684
Skewed Student's *t* GJR-GARCH (1,1)	0.00084019	0.00082184	0.00115170	0.00111520	0.00100928	0.00099567
SVR-GARCH (1,1)	**0.00003128**	**0.00001030**	**0.00060816**	**0.00003409**	**0.00036032**	**0.00001103**

Hansen et al.'s (2011) superior set models are highlighted in gray.

Table 9
Diebold-Mariano test statistic and *p*-value for low frequency data.

Model	Bitcoin		Ethereum		Dash	
	Test statistic	*P*-value	Test statistic	*P*-value	Test statistic	*P*-value
Normal GARCH (1,1)	3.1076	.001006*	2.4027	.008867*	1.3705	.086480
Student's *t* GARCH (1,1)	3.5682	.000194*	5.7427	1.56E−07*	1.4245	.078380
Skewed Student's *t* GARCH (1,1)	3.5106	.000230*	8.8443	2.21E−16*	1.4316	.077370
Normal EGARCH (1,1)	2.4695	.007483*	2.1036	.018791	1.7968	.037465
Student's *t* EGARCH (1,1)	3.1232	.001136*	2.8926	.002275*	2.2311	.013787
Skewed Student's *t* EGARCH (1,1)	3.4270	.000430*	19.6920	2.21E−16*	2.2586	.012871
Normal GJR-GARCH (1,1)	2.5124	.006672*	1.5564	.061071	1.6434	.051486
Student's *t* GJR-GARCH (1,1)	2.8048	.002945*	2.7916	.003064*	2.2425	.013481
Skewed Student's *t* GJR-GARCH (1,1)	4.5249	.000007*	14.2278	2.21E−16*	2.2909	.011872
Model	Period 4		Period 5		Period 6	
	Test statistic	*P*-value	Test statistic	*P*-value	Test statistic	*P*-value
Normal GARCH (1,1)	3.2808	.000550*	3.2708	.000692*	3.2790	.000674*
Student's *t* GARCH (1,1)	2.7967	.003069*	2.2321	.013690	5.7872	2.67E− 08*
Skewed Student's *t* GARCH (1,1)	2.7077	.003667*	3.6659	.000181*	1.7026	.003977*
Normal EGARCH (1,1)	2.4996	.006918*	3.2151	.000849*	3.3026	.000634*
Student's *t* EGARCH (1,1)	2.4279	.008354*	3.2834	.000682*	3.5453	.000289*
Skewed Student's *t* EGARCH (1,1)	3.2622	.000720*	3.2699	.000715*	3.8388	.000126*
Normal GJR-GARCH (1,1)	3.2095	.000863*	3.1958	.000898*	3.2832	.000685*
Student's *t* GJR-GARCH (1,1)	3.2195	.000834*	3.1372	.001087*	3.2660	.000713*
Skewed Student's *t* GJR-GARCH (1,1)	3.1933	.000992*	3.1671	.000980*	3.2615	.000725*

*Denotes statistical significance at 1% level.

Table 10
Diebold-Mariano test statistic and *p*-value for high frequency data: Bitcoin.

Model	Period 1		Period 2		Period 3	
	Test statistic	*P*-value	Test statistic	*P*-value	Test statistic	*P*-value
Normal GARCH (1,1)	2.0473	.020400	2.5388	.005635*	2.8350	.002338*
Student's *t* GARCH (1,1)	2.6767	.009817*	2.7224	.003295*	2.2863	.011220
Skewed Student's *t* GARCH (1,1)	2.4195	.012017	2.2822	.011341	2.9503	.001624*
Normal EGARCH (1,1)	1.9534	.025525	3.7374	.000094*	5.3781	4.69E− 08*
Student's *t* EGARCH (1,1)	1.4983	.067175	3.7529	.000093*	7.7801	8.99E− 15*
Skewed Student's *t* EGARCH (1,1)	1.4592	.072446	3.8885	.000054*	7.8593	4.97E− 15*
Normal GJR-GARCH (1,1)	2.2098	.013661	3.4254	.000312*	2.7161	.003361*
Student's *t* GJR-GARCH (1,1)	1.5683	.058559	2.9747	.001501*	3.6485	.000147*
Skewed Student's *t* GJR-GARCH (1,1)	1.4026	.080523	3.0699	.001099*	3.6576	.000132*
Model	Period 4		Period 5		Period 6	
	Test statistic	*P*-value	Test statistic	*P*-value	Test statistic	*P*-value
Normal GARCH (1,1)	2.9716	.001516*	5.1006	2.02E− 07*	2.9249	.001761*
Student's *t* GARCH (1,1)	2.9823	.001464*	4.1741	.000016*	3.5042	.000239*
Skewed Student's *t* GARCH (1,1)	2.9883	.001436*	4.1763	.000016*	3.4895	.000252*
Normal EGARCH (1,1)	2.8469	.002253*	5.4736	2.77E− 08*	2.2613	.011977
Student's *t* EGARCH (1,1)	2.5737	.005106*	5.0936	2.09E− 07*	4.7987	9.16E− 07*
Skewed Student's *t* EGARCH (1,1)	2.4634	.006963*	5.1527	1.54E− 07*	5.4796	2.68E− 08*
Normal GJR-GARCH (1,1)	2.9632	.001562*	5.5041	2.34E− 08*	3.1871	.000745*
Student's *t* GJR-GARCH (1,1)	3.1437	.000867*	5.4798	2.68E− 08*	5.0355	2.81E− 07*
Skewed Student's *t* GJR-GARCH (1,1)	3.2937	.000510*	5.5261	2.08E− 08*	5.4944	2.47E− 08*

*Denotes statistical significance at 1% level.

and the test set constitute the out-of-sample data in which the predictions were made. The horizon of the forecasts was one step ahead (one day for low frequency and one hour for high frequency data).

For low frequency data, we allocated 10 months for training (January 2016–October 2016 – 216 days), 4 months for validation (November 2016–February 2017 – 84 days) and 5 months for test (March 2017–July 2017 – 119 days).

In order to verify the robustness of SVR-GARCH model's predictive performance over time, we split the high frequency dataset with a moving window over the whole period, defining smaller time periods with 9 months each. In each period, we allocated the first 5 months for training, the next 2 months for validation and the last 2 months for test. For the following time intervals, the time periods were shifted two months forward until the end of the dataset extension, totaling six time periods of hourly data. Inside the subsets, we did not apply rolling windows for the estimations.

Therefore, the time periods for high frequency data were defined as follows:

- Period 1: January 4th 2016–September 30th 2016.
- Period 2: March 1st 2016–November 30th 2016.
- Period 3: May 2nd 2016–January 31st 2017.
- Period 4: July 1st 2016–March 31st 2017.
- Period 5: September 1st 2016–May 31st 2017.
- Period 6: November 1st 2016–July 31st 2017.

Firstly, the optimization of the SVR algorithm was applied to each training set, by performing a grid search for each one of the

Table 11
Diebold-Mariano test statistic and *p*-value for high frequency data: Ethereum.

Model	Period 1		Period 2		Period 3	
	Test statistic	*P*-value	Test statistic	*P*-value	Test statistic	*P*-value
Normal GARCH (1,1)	2.4904	.006455*	5.1662	1.43E− 07*	2.2184	.013390
Student's *t* GARCH (1,1)	3.1436	.000859*	5.2612	8.70E− 08*	2.4427	.007375*
Skewed Student's *t* GARCH (1,1)	3.1527	.000831*	5.2353	9.97E− 08*	2.4426	.007376*
Normal EGARCH (1,1)	2.4252	.007731*	5.6387	1.11E− 08*	2.8385	.002313*
Student's *t* EGARCH (1,1)	3.6946	.000116*	4.7779	.000001*	2.5150	.006032*
Skewed Student's *t* EGARCH (1,1)	3.7096	.000114*	4.7754	.00000*1	2.5169	.000624*
Normal GJR-GARCH (1,1)	2.4415	.007392*	2.3807	.008736*	2.3923	.008468*
Student's *t* GJR-GARCH (1,1)	3.8997	.000051*	5.1359	1.69E− 07*	2.8706	.002094*
Skewed Student's *t* GJR-GARCH (1,1)	3.9118	.000049*	5.2056	1.17E− 07*	2.8692	.002881*
Model	**Period 4**		**Period 5**		**Period 6**	
	Test statistic	*P*-value	Test statistic	*P*-value	Test statistic	*P*-value
Normal GARCH (1,1)	2.0124	.022220	2.9990	.001387*	4.2444	.000012*
Student's *t* GARCH (1,1)	1.6976	.044941	2.3534	.009394*	4.7414	.000001*
Skewed Student's *t* GARCH (1,1)	1.6328	.051418	2.3480	.009531*	4.7334	.000001*
Normal EGARCH (1,1)	2.9144	.001825*	1.7739	.038192	4.1469	.000018*
Student's *t* EGARCH (1,1)	2.5334	.005725*	3.2161	.000637*	6.0355	1.10E− 09*
Skewed Student's *t* EGARCH (1,1)	2.5332	.005734*	2.8446	.002271*	6.0151	1.25E− 09*
Normal GJR-GARCH (1,1)	3.2028	.000716*	2.9896	.001436*	4.3739	.000006*
Student's *t* GJR-GARCH (1,1)	2.7264	.003250*	2.2146	.002027*	4.8830	6.05E− 07*
Skewed Student's *t* GJR-GARCH (1,1)	2.3974	.008356*	2.5355	.005691*	5.0172	3.09E− 07*

*Denotes statistical significance at 1% level.

Table 12
Diebold-Mariano test statistic and *p*-value for high frequency data: Dashcoin.

Model	Period 1		Period 2		Period 3	
	Test statistic	*P*-value	Test statistic	*P*-value	Test statistic	*P*-value
Normal GARCH (1,1)	2.7245	.003273*	4.2093	.000014*	2.2012	.013980
Student's *t* GARCH (1,1)	2.3434	.009646*	3.9008	000051*	3.2967	.000506*
Skewed Student's *t* GARCH (1,1)	2.3229	.010190	4.5353	.000003*	3.2975	.000505*
Normal EGARCH (1,1)	2.0813	.018825	4.0735	.000025*	4.2449	.000012*
Student's *t* EGARCH (1,1)	2.0574	.019944	5.2817	7.81E− 08*	4.8814	6.13E− 07*
Skewed Student's *t* EGARCH (1,1)	2.0735	.019238	5.3128	6.61E− 08*	5.3664	4.99E− 08*
Normal GJR-GARCH (1,1)	2.9399	.001677*	4.1774	.000016*	1.7125	.043657
Student's *t* GJR-GARCH (1,1)	2.3527	.009408*	5.3468	5.51E− 08*	4.8508	7.13E− 07*
Skewed Student's *t* GJR-GARCH (1,1)	2.3424	.009671*	5.3854	4.48E− 08*	4.9205	5.04E− 07*
Model	**Period 4**		**Period 5**		**Period 6**	
	Test statistic	*P*-value	Test statistic	*P*-value	Test statistic	*P*-value
Normal GARCH (1,1)	3.0978	.001001*	4.1358	.000019*	1.8299	.033780
Student's *t* GARCH (1,1)	2.4952	.006373*	4.0277	.000030*	2.3599	.009234*
Skewed Student's *t* GARCH (1,1)	2.4955	.006368*	3.9982	.000034*	2.5752	.005079*
Normal EGARCH (1,1)	3.3524	.001825*	1.7739	.038198	4.3126	.000008*
Student's *t* EGARCH (1,1)	6.0762	8.65E− 10*	4.9447	4.45E− 07*	5.4134	3.85E− 08*
Skewed Student's *t* EGARCH (1,1)	6.1052	7.26E− 10*	4.9337	4.70E− 07*	4.6512	.000002*
Normal GJR-GARCH (1,1)	3.2028	.000701*	2.9896	.001438*	9.0311	2.21E− 16*
Student's *t* GJR-GARCH (1,1)	2.4603	.007026*	3.2242	.000652*	5.4175	3.76E− 08*
Skewed Student's *t* GJR-GARCH (1,1)	2.4598	.007039*	3.2027	.000702*	6.1086	7.12E− 10*

*Denotes statistical significance at 1% level.

associated parameters for both mean and volatility equations in low and high frequencies. The search ranges for the parameters δ, C and σ are listed in Table 1.

Based on each combination of parameters applied to the training set, the accuracy of each optimal obtained decision function was checked for the validation set using the error metric RMSE, defined as:

$$RMSE = \sqrt{\frac{\sum_{t=1}^{T} (\tilde{h}_t - \widehat{h}_t)^2}{T}} \tag{13}$$

where $\tilde{h}_t$ is the proxy volatility as defined in 4 and $\tilde{h}_t$ is the predicted volatility.

Each decision function obtained in the training set was fed with data from the validation set to compute the prediction of the dependent variable for these data. This forecast was then confronted with the actual values observed in the validation set, then the RMSE between predicted and observed values was calculated. Repeating the process for every parameter combination, the optimal combination is the one that minimizes the RMSE associated with its prediction. For the GARCH models, the training and validation sets were jointly used as in-sample data to estimate the respective coefficients.

Subsequently, the optimal parameters were applied to fit the model for the test set, and then compared with the results generated by GARCH models, obtaining the one-step ahead volatility estimation for the time periods of each test set, totaling seven sets of forecasts (one for daily data and six for hourly data). For this step, we considered the error metrics RMSE, defined in Eq. (13);

Table 13
Diebold-Mariano test statistic and *p*-value for high frequency data: Euro.

Model	Period 1		Period 2		Period 3	
	Test statistic	*P*-value	Test statistic	*P*-value	Test statistic	*P*-value
Normal GARCH (1,1)	3.1210	.000926*	3.5607	.000193*	4.1963	.000015*
Student's *t* GARCH (1,1)	3.0174	.001306*	3.7950	.000078*	4.3448	.000008*
Skewed Student's *t* GARCH (1,1)	3.3358	.000441*	3.4473	.000295*	5.9567	1.78E− 09*
Normal EGARCH (1,1)	2.7713	.002846*	2.7242	.003278*	5.7828	4.93E− 09*
Student's *t* EGARCH (1,1)	2.4172	.007903*	4.7246	.000001*	5.7665	5.39E− 09*
Skewed Student's *t* EGARCH (1,1)	2.4548	.007126*	4.7798	.000001*	5.9810	1.54E− 09*
Normal GJR-GARCH (1,1)	4.3344	.000008*	3.4908	.000251*	5.8684	2.99E− 09*
Student's *t* GJR-GARCH (1,1)	3.6091	.000161*	4.9291	4.81E− 07*	5.6071	1.33E− 08*
Skewed Student's *t* GJR-GARCH (1,1)	3.4175	.000328*	4.0377	.000029*	5.4033	4.09E− 08*
Model	**Period 4**		**Period 5**		**Period 6**	
	Test statistic	*P*-value	Test statistic	*P*-value	Test statistic	*P*-value
Normal GARCH (1,1)	2.6110	.004586*	5.3216	6.31E− 08*	5.8903	2.61E− 09*
Student's *t* GARCH (1,1)	2.7509	.003024*	5.2231	1.06E− 07*	4.9646	4.03E− 07*
Skewed Student's *t* GARCH (1,1)	2.4682	.006871*	6.3077	2.10E− 10*	5.4034	4.06E− 08*
Normal EGARCH (1,1)	1.9437	.026193	1.6794	.046682	3.8213	.000073*
Student's *t* EGARCH (1,1)	3.9026	.000051*	2.5272	.005817*	6.7386	1.33E− 11*
Skewed Student's *t* EGARCH (1,1)	3.8718	.000057*	2.5005	.006282*	6.4515	8.51E− 11*
Normal GJR-GARCH (1,1)	5.4734	2.71E− 08*	2.5198	.005956*	7.2867	3.16E− 13*
Student's *t* GJR-GARCH (1,1)	1.3995	.080991	1.8606	.031547	3.7447	.000095*
Skewed Student's *t* GJR-GARCH (1,1)	3.7595	.000089*	2.0243	.021617	1.9203	.027554

*Denotes statistical significance at 1% level.

Table 14
Diebold-Mariano test statistic and *p*-value for high frequency data: British Pound.

Model	Period 1		Period 2		Period 3	
	Test statistic	*P*-value	Test statistic	*P*-value	Test statistic	*P*-value
Normal GARCH (1,1)	4.3898	.000006*	3.2182	.000665*	4.7147	.000001*
Student's *t* GARCH (1,1)	6.0130	1.25E− 09*	3.6054	.000163*	5.3017	7.06E− 08*
Skewed Student's *t* GARCH (1,1)	5.4384	3.34E− 08*	3.6730	.000126*	5.7849	4.85E− 09*
Normal EGARCH (1,1)	3.6958	.000115*	3.8564	.000061*	2.5158	.006016*
Student's *t* EGARCH (1,1)	4.1089	.000021*	2.5209	.005935*	4.4156	.000005*
Skewed Student's *t* EGARCH (1,1)	4.0913	.000023*	2.5102	.006119*	4.3022	.000009*
Normal GJR-GARCH (1,1)	5.9356	1.97E− 09*	1.6499	.049632	4.8843	6.04E− 07*
Student's *t* GJR-GARCH (1,1)	3.4149	0.000331*	2.4856	.006546*	5.8732	2.91E− 09*
Skewed Student's *t* GJR-GARCH (1,1)	7.6605	2.06E− 14*	2.2931	.011028	5.6831	8.67E− 09*
Model	**Period 4**		**Period 5**		**Period 6**	
	Test statistic	*P*-value	Test statistic	*P*-value	Test statistic	*P*-value
Normal GARCH (1,1)	3.4275	.000316*	5.5846	1.50E− 08*	2.2302	.012970
Student's *t* GARCH (1,1)	4.0127	.000032*	5.2628	8.63E− 08*	2.2913	.011110
Skewed Student's *t* GARCH (1,1)	3.7121	.000108*	5.7619	5.49E− 09*	1.9615	.025043
Normal EGARCH (1,1)	2.7495	.002953*	4.7792	.000001*	2.3605	.009221*
Student's *t* EGARCH (1,1)	8.6976	2.21E− 16*	6.9767	2.71E− 12*	1.2961	.097612
Skewed Student's *t* EGARCH (1,1)	8.7316	2.21E− 16*	6.8923	4.79E− 12*	1.3113	.095034
Normal GJR-GARCH (1,1)	4.4363	.000005*	5.2254	1.05E− 07*	1.1832	.118517
Student's *t* GJR-GARCH (1,1)	8.3219	2.21E− 16*	4.9438	4.47E− 07*	2.1657	.012863
Skewed Student's *t* GJR-GARCH (1,1)	6.2295	3.45E− 10*	4.7118	.000001*	2.6445	.004153*

*Denotes statistical significance at 1% level.

and MAE (mean absolute error), whose expression is given by:

$$MAE = \frac{\sum_{t=1}^{T} |\tilde{h}_t - \widehat{h}_t|}{T} \tag{14}$$

Finally, in order to check the statistical significance of SVR-GARCH's superiority over GARCH models, we applied Diebold and Mariano (1995) predictive accuracy test for the nine GARCH models in both low and high frequencies, using SVR-GARCH as benchmark. Additionally, we applied the Model Confidence Test (Hansen et al., 2011) for each set of forecasts to further investigate whether the machine learning based approach yielded significantly better results. The description of both tests are displayed in appendices **A** and **B**.

5. Results and discussion

The results were widely favorable towards SVR-GARCH model. As shown in Tables 2–9, both RMSE and MAE of SVR-GARCH were lower than Normal, Student's *t* and Skewed Student's *t* distributions GARCHs, EGARCHs and GJR-GARCHs for all exchange rates and cryptocurrencies, in both low frequency and high frequency datasets. Similarly, the Diebold-Mariano test and Hansen's superior model sets also present strong evidences that the SVR models significantly outperforms the traditional GARCH models.

As shown in Tables 9–15, at the usual 95% confidence level only 19 out of 420 models failed to reject the null hypothesis, indicating that SVR models performed much better than the other models during all data range in both time frequencies for all assets –

Table 15
Diebold-Mariano test statistic and *p*-value for high frequency data: Japanese Yen.

Model	Period 1		Period 2		Period 3	
	Test statistic	*P*-value	Test statistic	*P*-value	Test statistic	*P*-value
Normal GARCH (1,1)	4.9330	4.69E− 07*	3.5532	.000199*	2.7268	.003254*
Student's *t* GARCH (1,1)	5.6116	1.28E− 08*	3.0526	.001163*	2.5908	.004857*
Skewed Student's *t* GARCH (1,1)	4.6434	1.93E− 06*	3.0732	.001087*	2.4158	.000794*
Normal EGARCH (1,1)	6.2383	3.17E− 10*	3.3271	.000454*	2.0245	.021597
Student's *t* EGARCH (1,1)	7.2093	5.29E− 13*	3.8657	.000059*	3.1253	.000914*
Skewed Student's *t* EGARCH (1,1)	7.2378	4.33E− 13*	3.8422	.000065*	3.2885	.000521*
Normal GJR-GARCH (1,1)	8.5345	2.21E− 16*	3.3737	.003853*	3.3127	.000479*
Student's *t* GJR-GARCH (1,1)	3.3764	.000098*	5.2041	1.18E− 07*	2.5059	006187*
Skewed Student's *t* GJR-GARCH (1,1)	6.0879	7.94E− 10*	1.0219	.153517	1.6333	.051369
Model	**Period 4**		**Period 5**		**Period 6**	
	Test statistic	*P*-value	Test statistic	*P*-value	Test statistic	*P*-value
Normal GARCH (1,1)	4.7986	9.17E− 07*	2.9258	.001756*	3.5971	.000168*
Student's *t* GARCH (1,1)	4.8939	5.73E− 07*	2.7229	.003290*	4.2148	.000014*
Skewed Student's *t* GARCH (1,1)	4.9584	4.15E− 07*	3.0272	.001265*	4.4181	.000006*
Normal EGARCH (1,1)	4.9784	3.76E− 07*	1.6405	.050607	3.9924	.000035*
Student's *t* EGARCH (1,1)	5.0017	3.34E− 07*	1.6904	.045638	4.3206	.000008*
Skewed Student's *t* EGARCH (1,1)	5.0091	3.22E− 07*	1.4656	.071532	4.2961	.000016*
Normal GJR-GARCH (1,1)	6.6291	2.72E− 11*	3.0451	.001193*	3.9831	.000036*
Student's *t* GJR-GARCH (1,1)	4.5513	.000003*	1.9769	.024166	3.3566	.000409*
Skewed Student's *t* GJR-GARCH (1,1)	3.2319	.000634*	1.1124	.133183	2.9392	.001685*

*Denotes statistical significance at 1% level.

even at the 99% confidence level, SVR-GARCH showed predictive superiority in 319 out of 420 models: 42 out of 60 models for low frequency and for 277 out of 360 models for high frequency data.

As for Hansen et al.'s (2011) model confidence set procedure, the SVR-GARCH model was in the set of superior models for all assets and all time periods – in fact, almost all SSMs (35 out of 42 sets) were composed only by the SVR-GARCH model, which provide further evidences the better performance of the machine learning approach. Overall, the SSMs generated by the MCS tests contained a small number of models due to the relatively small size of the initial set $\mathcal{M}^0$ (10 models). In addition, SVR-GARCH's RMSE and MAE were in general significantly lower than all other benchmark models, making it "superior" to the rest by a wide margin in many cases.

Overall, the forecasting errors of both GARCH and SVR models were higher for the cryptocurrencies than for the traditional currencies in both daily and hourly frequencies, an expected outcome given the big difference in the volatility levels for those kinds of asset.

For the low frequency dataset, the error metrics were generally higher than the high frequency one. This result is consistent with the findings in the literature: as seen in Xie and Li (2010), the RMSE for the volatility forecasting tend to decrease as the frequency increases, a behavior that was also identified for virtual currencies. As shown by the Diebold-Mariano test *p*-values and Hansen's superior model sets, the SVR models seemed to outperform GARCH models less emphatically for the cryptocurrencies than for the exchange rates for the daily volatility, whilst for the hourly volatility the superiority evidences were stronger for cryptocurrencies and milder for traditional currencies. This suggests a higher intraday volatility fluctuation in exchange rates than for cryptocurrencies, which can be associated to the huge liquidity of the foreign exchange market in comparison to the incipient acceptability of bitcoin, ethereum and dash.

The good predictive performance of the SVR based models can be linked to the nonlinearities that the Kernel function bring forth, inducing an infinite-dimensional feature space with a small number of parameters and incorporating nonlinear interactions that traditional linear models fail to capture. For assets with much higher volatility levels, like cryptocurrencies, the evidences of SVR's better predictive power are still very strong, suggesting the robustness of machine learning techniques in forecasting financial time series.

Analyzing the error metrics, the Diebold-Mariano test *p*-values and the composition of the set of superior models, the GJR-GARCH models seemed to perform slightly better than GARCH and EGARCH. Concerning the conditional distribution of ϵ_t, the Normal, Student's *t* and skewed Student's *t* distributions yielded overall similar results.

The results reveal that SVR models presented a significantly lower value for both RMSE and MAE error metrics in comparison to all nine GARCH models. The results are similar to the findings of Hsu et al. (2016), in which the authors conclude, based on various experiments, that machine learning techniques demonstrate superior predictive power than traditional econometric models. The results of this paper present evidences of such superiority not only for the exchange rates' volatility estimation, but also for the cryptocurrencies, a segment not explored by Hsu et al. (2016) and still not frequently studied in the finance literature.

Nonetheless, it was possible to see that cryptocurrencies have higher overall volatility than real world currencies. This result bring forth the discussion of whether cryptocurrencies can be treated as traditional currencies, specially concerning the debate regarding their fundamental or speculative nature, as discussed by Dowd (2014). The absence of a central monetary authority, while being the main feature of cryptocurrencies, is also one of the main sources of criticism, since a big share of their value is based on their notoriety and circulation on web, which creates a speculative profile for this kind of asset and a potential bubble. Authors like Baek and Elbeck (2015) expected that the increase in bitcoin's usage would make its volatility drop and exhibit a more investment-like behavior rather than a speculative tool. However, this did not happen, since bitcoin's market cap has been increasing all along, but its volatility is also going up, as seen in Fig. 1. Thus, the higher volatility levels showed by cryptocurrencies suggests a more cautious look over cryptocurrencies, and presents a possible evidence that they cannot yet be considered as "trusted currencies", mainly because their lack of maturity upon the store of value function.

6. Conclusion and remarks

This paper evaluated SVR-GARCH's predictive performance of daily and hourly volatiilty of three cryptocurrencies and three exchange rate pairs. The GARCH model was combined with machine learning approach, such that the mean and volatility equations were estimated using Support Vector Regression. Furthermore, we compared the models' predictive ability with Diebold-Mariano test and Hansen's Model Confidence Set. The results show that SVR-GARCH models managed to outperform all nine GARCH benchmarks – GARCHs, EGARCHs and GJR-GARCHs with Normal, Student's t and Skewed Student's t distributions – as seen by the value of error metrics RMSE and MAE, the Diebold-Mariano test p-values and the composition of the set of superior models.

The findings of this paper have the potential to aid scholars and market practitioners with an overview of the cryptocurrencies market features, discussing similarities and differences of their volatility patterns in comparison to real world currencies, presenting the extents in which the incorporation of a machine learning based technique yields better forecasting power for volatility over the GARCH benchmarks. The outcome of this research is a tool capable of estimating the risk for the cryptocurrencies in the future and can be used as a risk management for portfolios, as proposed by Dyhrberg (2016a). Furthermore, a more accurate model for volatility forecast in cryptocurrencies can be of interest for companies that accept them, as well as potential investors and traders of this market segment. More precise volatility predictions for cryptocurrencies can represent a measure of short-term risk to better evaluate the attractiveness of this kind of assets over alternative risky investments, potentially leading to better portfolio allocations, and guidance to investment decisions or corporate strategies.

Future researches are encouraged to replicate this study for other financial assets' volatility estimation, as well as to consider other distributions for the GARCH models' error term – such as Generalized Pareto Distribution (McNeil & Frey, 2000) – and other well known models for volatility estimation, such as TGARCH (Zakoian, 1994) and APARCH (Ding, Granger, & Engle, 1993). Also, the inclusion of SVR estimation to other volatility models apart from GARCH (1,1) can further contribute to better volatility predictions and may be an attractive and relevant issue in future developments in the finance literature.

Bearing in mind the huge popularity and prominence of machine learning methods many scientific fields, finance included, testing for different extensions of SVR – like Chang and Lin's (2002) ν-SVR – is also quite desirable. The use of different Kernel functions or mixture models, as seen in Bezerra and Albuquerque (2017), can also be incorporated to the SVR models. Finally, replications with different time periods and frequencies (*e.g.:* even higher frequency data, by minutes or even seconds) can contribute further for this research agenda.

Appendix A. Predictive accuracy test

Diebold and Mariano's (1995) predictive accuracy test compares the loss differential between the forecasting errors of two sets relative to the observed values. We used SVR-GARCH model as benchmark, so we defined d_t as the excess error of SVR-GARCH model over the other GARCH models:

$$d_t = [g(e_{SVR,t}) - g(e_{GARCH,t})] \tag{A.1}$$

where $e_{i,t} = \tilde{h}_t - \widehat{h}_{i,t}$ is the forecast error of the i-th model at time t and $g(.)$ is a loss function, which we defined as the squared error $g(e_{i,t}) = e_{i,t}^2$.

The Diebold-Mariano test evaluates the null hypothesis

$$H_0 : \mathbb{E}[d_t] \geq 0, \quad \forall\, t = 1, 2, \ldots, T \tag{A.2}$$

where T is the number of time periods in the test sets (thus, the number of forecasts generated). The null hypothesis states that SVR-GARCH models have equal or worse accuracy than GARCH models, while its rejection provides evidence of superiority over them.

Appendix B. Model confidence set

Hansen et al.'s (2011) Model Confidence Set (MCS) provides, at a given significance level α, a subset of "superior models" from an initial set $\mathcal{M}^0$ containing all m tested models. The superior set models (SSM) is obtained by recursively removing the worst model in $\mathcal{M}^0$ evaluating the null hypothesis of equal predictive ability for the i-th model in $\mathcal{M}^0, i = 1, 2, \ldots, m,$ given by:

$$H_0 : \mathbb{E}\left[\sum_{j=1}^{m-1}\sum_{t=1}^{T} g(e_{i,t}) - g(e_{j,t})\right] = 0, \quad i = 1, 2, \ldots, m \tag{B.1}$$

with T, $g(.)$ and $e_{i,t}$ as previously defined for the Diebold-Mariano test.

The MCS procedure basically tests whether the models in the initial set $\mathcal{M}^0$ have equal predictive power (null hypothesis); a block bootstrap procedure is used to compute the distribution under H_0. If H_0 is not rejected, then $\mathcal{M}^0$ is itself the superior set of models SSM. On the other hand, if H_0 is rejected, then at least one model "differs significantly" from the others in terms of predictive quality, so that one of the m models in $\mathcal{M}^0$ is chosen as the worst model and eliminated from $\mathcal{M}^0$, filtering it into a subset $\mathcal{M}^*$ containing "better models". The elimination rule removes the model with the worst relative performance in comparison to the average across all other models, measured by the test statistic:

$$t_i = \frac{\frac{1}{m-1}\sum_{j=1}^{m-1}\sum_{t=1}^{T} g(e_{i,t}) - g(e_{j,t})}{\sqrt{\widehat{\mathbb{V}}\left(\frac{1}{m-1}\sum_{j=1}^{m-1}\sum_{t=1}^{T} g(e_{i,t}) - g(e_{j,t})\right)}} \tag{B.2}$$

The routine is applied recursively, eliminating one model at a time. Each time the equal predictive ability null hypothesis is rejected, $\mathcal{M}^*$ is updated, eliminating the worst models. When H_0 ceases to be rejected, the current $\mathcal{M}^*$ is the SSM containing only the "superior models" at the significance level α.

References

Abu-Mostafa, Y. S., & Atiya, A. F. (1996). Introduction to financial forecasting. *Applied Intelligence, 6*(3), 205–213.
Andersen, T. G., & Bollerslev, T. (1998). Deutsche mark–dollar volatility: Intraday activity patterns, macroeconomic announcements, and longer run dependencies. *The Journal of Finance, 53*(1), 219–265.
Andersen, T. G., Bollerslev, T., Diebold, F. X., & Labys, P. (1999). Realized volatility and correlation.
Andrychowicz, M., Dziembowski, S., Malinowski, D., & Mazurek, Ł. (2014). Fair two–party computations via bitcoin deposits. In *Proceedings of international conference on financial cryptography and data security* (pp. 105–121). Springer.
Awartani, B. M., & Corradi, V. (2005). Predicting the volatility of the S&P-500 stock index via garch models: The role of asymmetries. *International Journal of Forecasting, 21*(1), 167–183.
Baek, C., & Elbeck, M. (2015). Bitcoins as an investment or speculative vehicle? A first look. *Applied Economics Letters, 22*(1), 30–34.
Balta, S., Creeze, A., Ezzra, A., & Sidokhine, F. (2016). "DinarDirham" whitepaper.
Baruník, J., & Křehlík, T. (2016). Combining high frequency data with non-linear models for forecasting energy market volatility. *Expert Systems with Applications, 55*, 222–242.
Belke, A., & Setzer, R. (2004). Contagion, herding and exchange-rate instabilitya survey. *Intereconomics, 39*(4), 222–228.
Bezerra, P. C. S., & Albuquerque, P. H. M. (2017). Volatility forecasting via svr–garch with mixture of gaussian kernels. *Computational Management Science, 14*(2), 179–196.
Blockchain (2017a). Bitcoin – confirmed transactions per day. https://blockchain.info/charts/n-transactions [Online; accessed 16-March-2017].
Blockchain (2017b). Bitcoin – market capitalization. https://blockchain.info/en/charts/market-cap [Online; accessed 16-March-2017].

Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics, 31*(3), 307–327.

Bouri, E., Azzi, G., & Haubo Dyhrberg, A. (2016). On the return-volatility relationship in the Bitcoin market around the price crash of 2013. *Economics Discussion Papers, No 2016-41*. Kiel Institute for the World Economy. https://ssrn.com/abstract=2869855.

Bouri, E., Gupta, R., Tiwari, A. K., & Roubaud, D. (2017). Does bitcoin hedge global uncertainty? Evidence from wavelet-based quantile-in-quantile regressions. *Finance Research Letters, 23*, 87–95.

Cao, L.-J., & Tay, F. E. H. (2003). Support vector machine with adaptive parameters in financial time series forecasting. *IEEE Transactions on Neural Networks, 14*(6), 1506–1518.

Çelik, S., & Ergin, H. (2014). Volatility forecasting using high frequency data: evidence from stock markets. *Economic Modelling, 36*, 176–190.

Chang, C.-C., & Lin, C.-J. (2002). Training ν -support vector regression: Theory and algorithms. *Neural Computation, 14*(8), 1959–1977.

Chaum, D. L. (1983). Blind signatures for untraceable payments. Online; access 07-November-2017.

Chen, S., Härdle, W. K., & Jeong, K. (2010). Forecasting volatility with support vector machine-based garch model. *Journal of Forecasting, 29*(4), 406–433.

Chen, S., Jeong, K., & Härdle, W. K. (2008). Support vector regression based garch model with application to forecasting volatility of financial returns.

Cohen, C. (1998). Beenz.com; the web's currency. Online; access 07-November-2017.

CoinDesk (2017a). Bitcoin – daily number of transactions. http://www.coindesk.com/data/bitcoin-daily-transactions/ [Online; accessed 16-March-2017].

CoinDesk (2017b). Bitcoin – market capitalization. http://www.coindesk.com/data/bitcoin-market-capitalization/ [Online; accessed 16-March-2017].

Community, B. (2017). "bitcoinwiki", Online; access 11-November-2017.

Cont, R. (2001). Empirical properties of asset returns: Stylized facts and statistical issues.

Darlington, J. (2014). The future of bitcoin: Mapping the global adoption of worlds largest cryptocurrency through benefit analysis.

Deboeck, G. (1994). *Trading on the edge: Neural, genetic, and fuzzy systems for chaotic financial markets*: 39. John Wiley & Sons.

Diebold, F. X., & Mariano, R. S. (1995). Comparing predictive accuracy. *Journal of Business & Economic Statistics, 13*(3), 253–263.

Ding, Z., Granger, C. W., & Engle, R. F. (1993). A long memory property of stock market returns and a new model. *Journal of Empirical Finance, 1*(1), 83–106.

Dowd, K. (2014). New private monies: A bit-part player?.

Drucker, H., Burges, C. J., Kaufman, L., Smola, A., & Vapnik, V. N. (1997). Support vector regression machines. In *Advances in neural information processing systems* (pp. 155–161). Morgan Kaufmann Publishers.

Dyhrberg, A. H. (2016a). Bitcoin, gold and the dollar–a garch volatility analysis. *Finance Research Letters, 16*, 85–92.

Dyhrberg, A. H. (2016b). Hedging capabilities of bitcoin. is it the virtual gold? *Finance Research Letters, 16*, 139–144.

Easley, D., López de Prado, M. M., & OHara, M. (2012). The volume clock: Insights into the high-frequency paradigm. *The Journal of Portfolio Management, 39*(1), 19–29.

Engle, R. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. *Econometrica, 50*(4), 987–1007.

Evans, C., Pappas, K., & Xhafa, F. (2013). Utilizing artificial neural networks and genetic algorithms to build an algo-trading model for intra-day foreign exchange speculation. *Mathematical and Computer Modelling, 58*(5), 1249–1266.

Evans, D. S. (2014). Economic aspects of bitcoin and other decentralized public-ledger currency platforms.

Ferguson, N. (2008). *The ascent of money: A financial history of the world*. Penguin.

Gandal, N., & Halaburda, H. (2014). Competition in the cryptocurrency market.

Gavrishchaka, V. V., & Banerjee, S. (2006). Support vector machine as an efficient framework for stock market volatility forecasting. *Computational Management Science, 3*(2), 147–160.

Gavrishchaka, V. V., & Ganguli, S. B. (2003). Volatility forecasting from multiscale and high-dimensional market data. *Neurocomputing, 55*(1), 285–305.

Glosten, L. R., Jagannathan, R., & Runkle, D. E. (1993). On the relation between the expected value and the volatility of the nominal excess return on stocks. *The Journal of Finance, 48*(5), 1779–1801.

G&SR (1998). E-gold; better money; since 1996. Online; access 07-November-2017.

G&SR (2006a). What is e-gold?. Online; access 07-November-2017.

G&SR (2006b). What is e-gold?. Online; access 07-November-2017.

Hansen, P. R., & Lunde, A. (2005). A forecast comparison of volatility models: Does anything beat a garch (1, 1)? *Journal of Applied Econometrics, 20*(7), 873–889.

Hansen, P. R., Lunde, A., & Nason, J. M. (2011). The model confidence set. *Econometrica, 79*(2), 453–497.

Hsu, M.-W., Lessmann, S., Sung, M.-C., Ma, T., & Johnson, J. E. (2016). Bridging the divide in financial market forecasting: machine learners vs. financial economists. *Expert Systems with Applications, 61*, 215–234.

Kim, T. H. (2016). A study of digital currency cryptography for business marketing and finance security. *Asia-pacific Journal of Multimedia Services Convergent with Art, Humanities, and Sociology, 6*(1), 365–376.

Kristoufek, L. (2015). What are the main drivers of the bitcoin price? Evidence from wavelet coherence analysis. *PloS One, 10*(4), e0123923.

Laurent, S., Rombouts, J. V., & Violante, F. (2012). On the forecasting accuracy of multivariate garch models. *Journal of Applied Econometrics, 27*(6), 934–955.

Levitan, R. (1999). "flooz.com". Online; posted 23-October-2017.

Li, M., & Suohai, F. (2013). Forex prediction based on SVR optimized by artificial fish swarm algorithm. Proceedings of the fourth global congress on intelligent systems, (pp. 47–52).

Li, X., & Wang, C. A. (2017). The technology and economic determinants of cryptocurrency exchange rates: The case of bitcoin. *Decision Support Systems, 95*, 49–60.

Lo, S., & Wang, J. C. (2014). Bitcoin as money?.

Marcucci, J., et al. (2005). Forecasting stock market volatility with regime-switching garch models. *Studies in Nonlinear dynamics and Econometrics, 9*(4), 1–53.

McNeil, A. J., & Frey, R. (2000). Estimation of tail-related risk measures for heteroscedastic financial time series: An extreme value approach. *Journal of Empirical Finance, 7*(3), 271–300.

Merton, R. C. (1980). On estimating the expected return on the market: An exploratory investigation. *Journal of Financial Economics, 8*(4), 323–361.

Nakamoto, S. (2008). Bitcoin: A peer-to-peer electronic cash system.

Nelson, D. B. (1991). Conditional heteroskedasticity in asset returns: A new approach. *Econometrica, 59*(2), 347–370.

Nelson, D. B. (1992). Filtering and forecasting with misspecified ARCH models i: Getting the right variance with the wrong model. *Journal of Econometrics, 52*(1–2), 61–90.

Polasik, M., Piotrowska, A. I., Wisniewski, T. P., Kotkowski, R., & Lightfoot, G. (2015). Price fluctuations and the use of bitcoin: An empirical inquiry. *International Journal of Electronic Commerce, 20*(1), 9–49.

Poon, S.-H., & Granger, C. (2003). Forecasting volatility in financial markets: A review. *Journal of Economic Literature, 41*(2), 478–539.

Premanode, B., & Toumazou, C. (2013). Improving prediction of exchange rates using differential emd. *Expert Systems with Applications, 40*(1), 377–384.

Reboredo, J. C., Matías, J. M., & Garcia-Rubio, R. (2012). Nonlinearity in forecasting of high-frequency stock returns. *Computational Economics, 40*(3), 245–264.

Rosenfeld, M. (2014). Analysis of hashrate-based double spending. arXiv preprint arXiv: 1402.2009.

Santamaría-Bonfil, G., Frausto-Solís, J., & Vázquez-Rodarte, I. (2015). Volatility forecasting using support vector regression and a hybrid genetic algorithm. *Computational Economics, 45*(1), 111–133.

Santos, A. A. P., da Costa, N. C. A., & dos Santos Coelho, L. (2007). Computational intelligence approaches and linear models in case studies of forecasting exchange rates. *Expert Systems with Applications, 33*(4), 816–823.

Segendorf, B. (2014). Have virtual currencies affected the retail payments market. *Sveriges Riksbank Economic Commentaries, 2*, 1–5.

Sermpinis, G., Stasinakis, C., Theofilatos, K., & Karathanasopoulos, A. (2015). Modeling, forecasting and trading the eur exchange rates with hybrid rolling genetic algorithms support vector regression forecast combinations. *European Journal of Operational Research, 247*(3), 831–846.

Shen, F., Chao, J., & Zhao, J. (2015). Forecasting exchange rate using deep belief networks and conjugate gradient method. *Neurocomputing, 167*, 243–253.

Sun, P., & Zhou, C. (2014). Diagnosing the distribution of garch innovations. *Journal of Empirical Finance, 29*, 287–303.

Urquhart, A. (2016). The inefficiency of bitcoin. *Economics Letters, 148*, 80–82.

Vapnik, V. N. (1995). *The nature of statical learning theory*. Springer Verlag.

Vasquez, A. (2017). Venezuelans use bitcoin 'mining' to escape inflation. Yahoo! News. Online; posted 23-October-2017.

Vigna, P., & Casey, M. J. (2016). *The age of cryptocurrency: How bitcoin and the blockchain are challenging the global economic order*. Macmillan.

Wang, Y.-H. (2009). Nonlinear neural network forecasting model for stock index option price: Hybrid gjr–garch approach. *Expert Systems with Applications, 36*(1), 564–570.

Xie, H., & Li, J. (2010). Intraday volatility analysis on s&p 500 stock index future. *International Journal of Economics and Finance, 2*(2), 26.

Xiong, T., Bao, Y., & Hu, Z. (2014). Multiple-output support vector regression with a firefly algorithm for interval-valued stock price index forecasting. *Knowledge-Based Systems, 55*, 87–100.

Yermack, D. (2013). Is Bitcoin a real currency? An economic appraisal. *Technical Report*. National Bureau of Economic Research.

Zakoian, J.-M. (1994). Threshold heteroskedastic models. *Journal of Economic Dynamics and Control, 18*(5), 931–955.