BASKIN SCHOOL OF ENGINEERING
Department of Applied Mathematics
and Statistics

Student number: ____________________

First Year Exam: June 11th 2010

TAKE-HOME EXAM: INSTRUCTIONS

You are to work individually on this problem. Do not share with anyone any information or comments about your findings or the models and methods you use. You must turn in your solution by **9am on Monday 14th June**. You can either print it and take it to Nic Brummell's office (BE125), or send a PDF file to `brummell@ams.ucsc.edu` . Please take care to organize and present the material in the best possible way; be informative but concise. Your paper should consist of no more than 12 letter-size pages (with 11pt or larger type and margins on all four sides of at least 1 inch), including tables and figures. The grading will based on both technical correctness and the presentation of the report. All questions should be answered in an accurate, complete and logically-argued manner.

Problem 1

a) Describe the qualitative behavior of the solutions of

$$\frac{dx}{dt} = -x^3 + \alpha x + \beta \tag{1}$$

where x, α and β are all real and α and β vary.

b) Append to Eqn 1

$$\frac{d\beta}{dt} = -\epsilon x \tag{2}$$

where $\epsilon > 0$ and much less than 1 and describe the qualitative properties of the solution of the system of equations.

c) Compute the numerical solution–software of your choice – of Eqns 1 and 2 for $\alpha = 1, \epsilon = 0.005, x(0) = 2, \beta(0) = x(0)^3 - \alpha x(0)$. Compare your numerical and analytical solutions.

Problem 2

Consider the following two equations. First VB:

$$\frac{dx}{dt} = r(K - x)$$

and second LOG

$$\frac{dx}{dt} = rx(1 - \frac{x}{K})$$

where r and K are positive parameters.

a) Describe the qualitative behavior of VB and LOG in the $t - x$ plane.

b) Solve the equations to determine the exact solutions for VB and LOG for $x(0) = x_0$ and plot solutions in the $t - x$ plane for $r = 0.2$, $K = 100$, to confirm the analysis in part a)

c) Show that the numerical scheme

$$x(t + \Delta t) = K(1 - exp(-r\Delta t)) + x(t) \cdot exp(-r\Delta t) \tag{3}$$

is an exact solution of VB. Explain why

$$x(t + \Delta t) = x(t) + r \cdot x(t) \cdot (1 - \frac{x(t)}{K}) \cdot \Delta t \tag{4}$$

is only an approximate numerical solution of LOG.

d) Compute the solutions of Eqns 3 and 4 for $\Delta t = 1$, $K = 100$, $x(0) = 0.15K$ and for $r = 0.4, 1.0, 2.0, 2.3, 2.6$ and 3.0. Interpret your numerical results.