Syllabus for the First Year Exam (2015)

203: Introduction to Probability Theory

Textbook:

- DeGroot, M.H. and Schervish M.J. Probability and Statistics. Fourth Edition, Addison Wesley.

Course Topics:

- Definition of probability and finite sample spaces. Counting methods. Combinatorial methods. Multinomial coefficients.
- Probability of a union of events. Conditional probability and independent events. Bayes' theorem.
- Discrete random variables. Examples: Bernoulli, Binomial, Hypergeometric, Poisson and Negative Binomial random variables.
- Continuous random variables. Normal, Gamma and Beta distributions.
- Density function. Quantile function.
- Bivariate distributions and marginal distributions.
- Conditional distributions and multivariate distributions.
- Functions of a single random variable. Functions of two or more random variables.
- Expectation and variance. Conditional expectation.
- The normal distribution and properties of normal random variables.
- Markov and Chebyshev's inequalities. The law of large numbers. The central limit theorem.
- The multinomial distribution.
- Markov chains.

205B: Intermediate Classical Inference

Textbook:

- Casella G, Berger R. (2002). *Statistical Inference*, second edition. Pacific Grove CA: Duxbury.

Course Topics:

- Sufficient and minimal sufficient statistics. Finding a sufficient statistic and showing that it is minimal sufficient. The factorization theorem. Examples from common distributions. Sufficiency and the exponential family. Ancillary statistics. Complete statistics. Techniques to find complete sufficient statistics. Examples mainly from exponential family. Basu's theorem connecting sufficiency, completeness and ancillary properties.
- Principles of statistical inference: Sufficiency, conditionality and likelihood.
- Methods of finding estimators: method of moments; maximum-likelihood estimation; unbiased estimators. Definitions and properties; examples with common distributions. Possible logical inconsistency of frequentist estimators (e.g., a negative estimate of a positive parameter). Introduction to point estimation using Bayesian techniques. Bayes estimator, minimax estimators. How to find each of them with examples.
- Methods for evaluating estimators: mean squared error, minimum variance, the Rao-Blackwell theorem, the Lehman-Scheffe theorem, the Cramer-Rao inequality.
- Methods for finding tests: likelihood-ratio tests, Union-Intersection test, Intersection-Union test. Introduction to Bayesian testing.
- Methods for evaluating tests: error probabilities. Neyman-Pearson lemma for point null vs. point alternative. Extend it to composite null vs. composite alternative by using monotone likelihood ratio (MLR) property. UMP tests, Unbiased tests and UMPU tests. How to evaluate them, p-values.
- Methods for finding interval estimators: Pivotal quantities, inverting a test statistic.
- Methods for evaluating interval estimators: coverage probability and size, illustrate in location and scale families how to find intervals with shortest length. Idea of false coverage, connecting UMP test with intervals having minimum false coverage, UMA intervals.
- Touch upon the concept of Bayesian credible intervals and make a brief comparative study with frequentist confidence intervals.

206B: Intermediate Bayesian Inference

Textbook:

- Christian Robert, *The Bayesian Choice: From Decision-Theoretic Foundations to Computational Implementation.* Springer, 2nd edition, 2001.
- Berger, J.O. (1984) *Statistical Decision Theory and Bayesian Analysis.* Springer.

Course Topics:

- Interpretations of probability: Subjective, Classical, Frequentist; Elicitation and representation of beliefs and uncertainty through probability distributions; Likelihoods

- Priors and Posteriors: Bayes Theorem; Choice of priors; Calculation of posteriors; Conjugate analysis; Predictive distributions; Jeffreys priors and improper priors
- Decision theoretic approaches to statistical inference; Expected losses; Frequentist and Bayesian risk; Optimality of Bayesian procedures.
- Random number generation; Inversion, transformation and rejection methods; Monte Carlo integration; Importance sampling; Laplace approximations.
- Markov chain Monte Carlo: Gibbs and Metropolis-Hasting sampling; Estimation of posterior distributions via MCMC; Drawing inferences, making predictions; Simple hierarchical modeling.
- Bayesian Inference: Point estimation; Estimation Error; Interval estimation; Hypothesis testing; Bayes factors.
- Interpretation of results: Understanding posterior uncertainty.

207: Intermediate Bayesian Statistical Modeling

Textbook: Bayesian Data Analysis, Third Edition. A. Gelman, J.B. Carlin, H.S. Stern, D.B. Dunson, A. Vehtari and D.B. Rubin. Chapman and Hall/CRC.
Course Topics:

- Single and multi-parameter models (Chapters 2 and 3)
- Normal approximations to the posterior distribution (Chapter 4)
- Bayesian inference for hierarchical models (Chapters 5 and 6)
- Model comparison and model assessment (Chapter 6)
- Modeling accounting for data collection (Chapter 7)
- Posterior simulation (Chapter 11-12)
- Approximations to the posterior distribution based on posterior modes (Chapter 13)
- Bayesian inference for regression models and hierarchical linear models (Chapters 14 and 15)
- Models for robust inference (Chapter 17)
- Mixture models (Chapter 22)

211: Foundations of Applied Mathematics

Textbook: Riley, K.F., Hobson, M.P. & Bence, S.J. *Mathematical Methods for Physics and Engineering*, Cambridge University Press.
Course Topics:

- Multivariate calculus (Partial differentiation, multiple integrals. ...)
- Linear algebra (vectors, matrices, vector spaces, eigenvectors, ...)
- Vector calculus
- Line, surface, volume integrals
- Fourier series
- Integral transforms (Fourier, Laplace)
- Complex variable analysis and applications (complex differentiation, complex integration)
- Ordinary differential equations (1st order all degrees, 2nd order linear)

212A: Applied Mathematical Methods I

Textbook: Applied Partial Differential Equations (with Fourier Series and Boundary Value Problems) by Richard Haberman, 4th edition. Chapters 1-5, 7-12.
Course Topics:

- Pass/Fail questions: Recognizing Elliptic/Parabolic/Hyperbolic PDEs and knowing their basic properties
- 1st order PDES: Method of Characteristics for quasilinear equations.
- 2nd order PDEs: Transformation to canonical form
- 2nd order PDEs: Method of separation of variables using Fourier series, for linear PDEs with constant coefficients. Homogeneous and non-homogeneous equations.
- 2nd order PDEs: Sturm Liouville Theory
- 2nd order PDEs: Applications of Sturm Liouville Theory to linear PDEs with non-constant coefficients. Homogeneous and non-homogeneous equations
- Green's functions

212B: Applied Mathematical Methods II

Textbook: *Perturbation Methods for Engineers and Scientists* by Alan W. Bush, CRC Press.
Course Topics:

- Iterative method for asymptotic expansion
- Singular perturbation and re-scaling
- Boundary layer problems, matched asymptotic, matching condition and the composite expansion
- Multiple scale expansions
- Method of strained variable, renormalization technique for strained variable expansions
- Method of dominant balance, the WKB method, eigenvalue problems (excluding the case of turning point)
- Region of dominant contribution, Watson's lemma, Laplace method

213: Numerical Solutions of Differential Equations

Textbook: Lecture notes
Course Topics:

- Numerical linear algebra: Direct methods for linear system (LU decomposition, Gaussian elimination, Choleski decomposition). Least square problems (QR decomposition, Gram-Schmitz, Householder transformation). Eigenvalue problems (Power iteration, Rayleigh quotient iteration, and QR iteration). Error analysis, matrix norm and condition number. Iterative methods (steepest descent, conjugate gradient).
- Numerical methods for ODEs: Initial value problems using Single-step methods (Euler method, Runge-Kutta methods) and Multi-step methods. Error analysis (Local truncation error, discrete error, roundoff error, global error, the order of a numerical method). Stiff ODE systems. Boundary value problems (Finite difference, shooting methods and Galerkin method).
- Numerical methods for PDEs: Finite difference for hyperbolic PDEs (upwind, Lax-Friedrich, Lax-Wendroff methods.) and parabolic PDEs (explicit and Crank-Nicolson methods). Consistency; stability and convergence. Lax Equivalence Theorem. CFL condition.

214: Applied Dynamical Systems

Textbook: *Nonlinear Dynamics and Chaos: With Applications to Physics, Biology, Chemistry and Engineering* by Steven H. Strogatz, Westview Press.
Course Topics:

- 1D Flows
- Bifurcations
- 2D Flows/Linear Systems/Linearization
- Limit cycles and their stability
- Poincaré-Bendixson theorem
- Chaos

Note: The material for the in-class exam corresponds to the topics covered in the course from textbook chapters 1-3 and 5-10.

256: Linear Statistical Models

Textbooks:

- Monahan J.F. (2008) A primer on linear regression models. Chapman & Hall/CRC Texts in Statistical Science. (M).
- Rencher, A.C. and Schaalje B.G. (2008) *Linear Models in Statistics.* Wiley-Interscience (Second edition). (RS)
- Faraway, J.J. (2000). Linear Models with R. Chapman & Hall/CRC Texts in Statistical Science. (F)
- Ravishanker N. and Dey. D (2001) *A First Course in Linear Model Theory.* Chapman & Hal. (RD)

Course Topics:

- Basic notions of linear algebra, e.g., vector spaces, column and null spaces of a matrix, inverse and generalized inverse, solutions to systems of linear equations, bases, orthogonal matrices, idempotent matrices, eigenvalues and eigenvectors. (M, RS and RD)
- Definition and examples of the general linear model, including simple and multiple linear regression, analysis of variance, and analysis of covariance models. (M, RS and RD)
- Ordinary and generalized Least Squares Estimation. Estimable functions. Best linear unbiased estimators and the Gauss-Markov Theorem. (M, RS and RD)
- Distribution Theory. Class notes but available in many books. Covariances. Properties of covariances. Quadratic forms. Expectations of quadratic forms. Multivariate Normal distribution and its properties. Orthogonal transformations of MVN vectors. Partitions and conditional distributions. Quadratic forms in Multivariate Normal Variables and its distributions. Cochran's theorem. Non-central F distribution. (M, RS and RD)
- Maximum likelihood estimation, interval estimation and hypothesis testing under the Gaussian Gauss-Markov model. (M, RS and RD)
- You should also be familiar with fitting linear models using `R`. (F)