Syllabus for the First Year Exam (2013)

203: Introduction to Probability Theory

Textbook:

- DeGroot, M.H. and Schervish M.J. Probability and Statistics. Fourth Edition, Addison Wesley.

Course Topics:

- Definition of probability and finite sample spaces. Counting methods. Combinatorial methods. Multinomial coefficients.
- Union of events. Conditional probability and independent events. Bayes' theorem.
- Discrete random variables. Examples: Bernoulli, Binomial, Hypergeometric, Poisson and Negative Binomial random variables.
- Continuous random variables. Normal, Gamma and Beta distributions.
- Conditional distributions and multivariate distributions.
- Functions of a single random variable. Functions of two or more random variables.
- Expectation and variance. Conditional expectation and conditional variance.
- The normal distribution and properties of normal random variables.
- Markov and Chebyshev's inequalities. The law of large numbers. The central limit theorem.
- The Poisson process.
- Markov chains.

205B: Intermediate Classical Inference

Textbook:

- Casella G, Berger R. (2002). *Statistical Inference*, second edition. Pacific Grove CA: Duxbury.

Course Topics:

- Sufficient and minimal sufficient statistics. Finding a sufficient statistic and showing that it's minimal sufficient. The factorization theorem. Examples from common distributions. Sufficiency and the exponential family.
- Principles of statistical inference: Sufficiency, conditionality and likelihood. Birbaum's Theorem.

- Methods of finding estimators: method of moments; maximum-likelihood estimation; unbiased estimators. Definitions and properties; examples with common distributions. Possible logical inconsistency of frequentist estimators (e.g., a negative estimate of a positive parameter).
- Methods for evaluating estimators: mean squared error, minimum variance, the Rao-Blackwell theorem, asymptotic relative efficiency.
- Methods for finding tests: likelihood-ratio tests. Large sample hypothesis testing.
- Methods for evaluating tests: error probabilities. Most powerful tests. p-values.
- Methods for finding interval estimators: Pivotal quantities. Large-sample interval estimation.
- Methods for evaluating interval estimators: coverage probability and size.
- Univariate and multivariate Δ–method; double-expectation theorem.
- The bootstrap as a method for improving the performance of likelihood-based procedures in small samples.

206B: Intermediate Bayesian Inference

Textbook:

- Christian Robert, *The Bayesian Choice: From Decision-Theoretic Foundations to Computational Implementation.* Springer, 2nd edition, 2001.

Course Topics:

- Interpretations of probability: Subjective, Classical, Frequentist; Elicitation and representation of beliefs and uncertainty through probability distributions; Likelihoods
- Priors and Posteriors: Bayes Theorem; Choice of priors; Calculation of posteriors; Conjugate analysis; Predictive distributions; Jeffreys priors and improper priors
- Decision theoretic approaches to statistical inference; Expected losses; Frequentist and Bayesian risk; Optimality of Bayesian procedures.
- Random number generation; Inversion, transformation and rejection methods; Monte Carlo integration; Importance sampling; Laplace approximations.
- Markov chain Monte Carlo: Gibbs and Metropolis-Hasting sampling; Estimation of posterior distributions via MCMC; Drawing inferences, making predictions; Basic MCMC diagnostics; Simple hierarchical modeling
- Model selection and Bayes factors; Model Averaging; relationship with the BIC and other model selection approaches.
- Interpretation of results: Drawing conclusions in context; Understanding posterior uncertainty; Comparison to Frequentist results and interpretations

207: Intermediate Bayesian Statistical Modeling

Textbook: Bayesian Data Analysis, Second Edition. A. Gelman, J.B. Carlin, H.S. Stern and D.B. Rubin. Chapman and Hall/CRC.
Course Topics:

- Single and multi-parameter models (Chapters 2 and 3).
- Normal approximations to the posterior distribution (Section 4.1 + class notes).
- Bayesian inference for hierarchical models (Chapters 5, 6)
- Modeling accounting for data collection (Sections 7.1-7.3 and 7.8).
- Posterior simulation (Chapter 11).
- Approximations to the posterior distribution based on posterior modes (Chapter 12). Importance sampling (Section 13).
- Model comparison and model assessment (Chapter 6 plus class notes).
- Bayesian inference for regression models and hierarchical linear models (Chapters 14 and 15).
- Models for robust inference (Chapter 17).
- Mixture models (Chapter 18).

211: Foundations of Applied Mathematics

Textbook: Riley, K.F., Hobson, M.P. & Bence, S.J. *Mathematical Methods for Physics and Engineering*, Cambridge University Press.
Course Topics:

- Multivariate calculus (Partial differentiation, multiple integrals. ...)
- Linear algebra (vectors, matrices, vector spaces, eigenvectors, ...)
- Vector calculus
- Line, surface, volume integrals
- Fourier series
- Integral transforms (Fourier, Laplace)
- Complex variable analysis and applications (complex differentiation, complex integration)
- Ordinary differential equations (1st order all degrees, 2nd order linear)

212A: Applied Mathematical Methods I

Textbook: Applied Partial Differential Equations (with Fourier Series and Boundary Value Problems) by Richard Haberman, 4th edition. Chapters 1-5,7-12.
Course Topics:

- Pass/Fail questions: Recognizing Elliptic/Parabolic/Hyperbolic PDEs and knowing their basic properties
- 1st order PDES: Method of Characteristics for quasilinear equations.
- 2nd order PDEs: Transformation to canonical form
- 2nd order PDEs: Method of separation of variables using Fourier series, for linear PDEs with constant coefficients. Homogeneous and non-homogeneous equations.
- 2nd order PDEs: Sturm Liouville Theory
- 2nd order PDEs: Applications of Sturm Liouville Theory to linear PDEs with non-constant coefficients. Homogeneous and non-homogeneous equations
- Green's functions

212B: Applied Mathematical Methods II

Textbook: Lecture notes and the book
Perturbation Methods by E. J. Hinch, Cambridge University Press [Paperback], 1991
Course Topics:

- Iterative method for asymptotic expansion
- Singular perturbation and re-scaling
- Boundary layer problems, matched asymptotic, matching condition and the composite expansion
- Multiple scale expansions
- Method of strained variable, renormalization technique for strained variable expansions
- Method of dominant balance, the WKB method, eigenvalue problems (excluding the case of turning point)
- Region of dominant contribution, Watson's lemma, Laplace method
- Riemann-Lebesgue Lemma, Method of stationary phase

213: Numerical Solutions of Differential Equations

Textbook: Lecture notes
Course Topics:

- Numerical linear algebra. LU decomposition; Choleski decomposition; Householder reflection; QR decomposition. Iterative methods; steepest descent; conjugate gradient. Power iteration and QR algorithm for eigenvalue problems. Error analysis, matrix norm and condition number; least square approximation.
- Numerical methods for ODEs. Euler method; Runge-Kutta methods; multistep methods. Local error; global error; the order of a numerical method; stiff ODEs. Finite difference and shooting methods for boundary value problems.
- Numerical methods for PDEs. Finite difference for Poisson equation, heat equation and wave equation. FTCS, BTCS; Crank-Nicolson and Lax-Friedrich methods. Consistency; stability and convergence. Lax Richtmyer Theorem; CFL condition.

214: Introduction to Dynamical Systems and Chaos

Literature: Lecture notes and "Nonlinear Dynamics and Chaos: With Applications to Physics, Biology, Chemistry and Engineering" by Steven H. Strogatz, Westview Press (required).

Additional readings: H. K. Khalil, "Nonlinear Systems", Prentice Hall; S. Sastry, "Nonlinear Systems: Analysis, Stability and Control", Springer; and M. Vidyasagar, "Nonlinear Systems Analysis", SIAM.

Course Topics:

- 1D Flows
- Bifurcations
- 2D Flows/Liner Systems/Linearization
- Limit Cycles and Bifurcations
- Describing Functions
- Chaos and Fractals

The course relies on a working knowledge of matrix algebra, calculus, differential equations and physics.

256: Linear Statistical Models

Textbooks:

- Casella G. and Berger R.L. (2001) Statistical Inference (Second Edition). Duxbury Advanced Series. (CB)
- A First Course in Linear Model Theory by Nalini Ravishanker and Dipak K. Dey (RD)
- Linear Models with R by Julian Faraway (F)
- Weisberg S. (2005) Applied Linear Regression (Third Edition). Wiley. (W)
- Linear Regression Analysis by Seber (S)
- Monahan J.F. (2008) A primer on linear regression models.
- Rencher and Schaalje (2008) Linear models in statistics.

Course Topics:

- Introduction to Simple Linear Regression (CB, Chapter 11)
- Distribution Theory. Class notes but available in many books (e.g. RD or S). Covariances. Properties of covariances. Quadratic forms. Expectations of quadratic forms. Multivariate Normal distribution and its properties. Orthogonal transformations of MVN vectors. Partitions and conditional distributions. Quadratic forms in Multivariate Normal Variables and its distributions. Cochran's theorem. Non-central F distribution.
- Estimation. Ordinary Least Squares Estimation. Generalized Least Squares Estimation. BLUE. MLE. Estimable functions. Gauss-Markov Theorem. Hypothesis Testing and interval estimation.
- Multiple Regression: Model building and residual analysis. Balanced One-way Classification. Two-way Classification.
- You should also be familiarized with fitting linear models. W includes several data analyses. Another good reference is: Linear Models with R by Julian Faraway (Chapters: 1,2,3,4,5,6,7,14 and 15)