Multi-objective optimization of workload to study the locus of control in a novel airspace

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Abstract

The current transformation of the national airspace system in the United States through the NextGen initiative requires an increased capacity in air traffic management. In order to safely and efficiently achieve this increase, we must better understand the optimal divisions of control between autonomous systems and human operators within an airspace. Using a simulation tool developed at Georgia Tech, we are able to study the optimal divisions of control between agents in a novel airspace. We examine the total workload within the airspace and the balance of workload between the agents in the airspace in a multi-objective optimization setting. In doing so, we find clear trends among the simulator inputs that minimize our objectives.

Keywords: NextGen, fractional factorial designs, linear models, Pareto set, Pareto frontier, multi-objective optimization, gradient-free optimization

1 Introduction

The fast-paced growth of civil aviation in the United States requires the national airspace system (NAS) to be transformed in order to meet future demands and avoid gridlock in the sky and in airports. The Next Generation Air Transportation System (NextGen) is the Federal Aviation Administration's plan to modernize and advance the NAS. NextGen is a wide-ranging transformation of the entire national air transportation system currently being implemented across the US (Federal Aviation Administration, 2007).

Among other goals of this initiative, NextGen requires an increased capacity in air traffic management. A higher level of autonomy in both air and ground systems will play an important role in achieving this increased capacity. Yet the safest division of control between autonomous systems and human operators remains an open and complex question. Human-computer interaction mistakes are an increasing source of mishaps in aviation (Billings (1997); Sarter et al. (1995); Sarter et al. (1997)). Thus along with the implementation of the NextGen initiative, questions relating to the *locus of control* within air traffic systems arise as important areas of research.

1.1 Problem Statement

We define the locus of control to be the distribution of air traffic control decisions to resolve conflicts between agents in an airspace. There are two commonly studied systems of control within an airspace: centralized and decentralized. A completely centralized control paradigm requires the ability for a centralized, typically automated, decision maker to carry out actions taken by all aircraft within the immediate airspace. We will refer to this centralized decision maker as air traffic control (ATC). A fully-centralized system is commonly asserted to provide an airspace-wide optimal traffic management solution due to a more complete picture of the state of the airspace. At the other extreme, a completely decentralized control paradigm requires each aircraft's flight crew (ie. pilots) to execute all actions taken by the aircraft. A decentralized system is commonly asserted to provide a more user-specific optimal traffic management solution, to distribute the traffic management workload, and potentially to be more robust. Since both system extremes have significant benefits, we need to establish a shared framework to allow comparisons between very different control paradigms and study the benefits of less-extreme distributions of control.

A series of actions must take place in order to safely and efficiently land an aircraft. These actions can be carried out by either the flight crew on the aircraft or by ATC and the division of these actions between these agents will significantly vary the locus of control within an airspace. We are interested in finding the optimal ways for the flight crew and ATC to carry out these actions.

We propose to study this problem using a simulation tool recently developed at Georgia Tech. Bigelow (2011) provided an initial exploration of this simulator. In this project, we implement multi-agent analysis and simulation to examine the potential sets of optimal locus of control points within a novel airspace.

1.2 Objectives

We use the aforementioned simulator to model an airspace with multiple agents: three flight crews on three distinct aircraft and an ATC agent (referred to as FC1, FC2, FC3, and ATC, respectively). We want to study the actions of these four agents over a period of approximately 13 minutes. During this time period, the three aircraft begin in the air and must successively land on a chosen runway. We assume the conditions in the airspace remain constant throughout the flight period and from trial to trial. We also ensure the agents obey certain regulations in regards to safety and reliability. In moving to a new aviation system, greater efficiency is important, but maintaining safety and reliability is absolutely critical.

The model inputs a set of 2-level factor variables that define which agents have the authority and which agents have the responsibility to carry out a set of pre-defined actions (see Section 2.2 for more details) . Upon simulation, the model outputs the time required to carry out each of these actions. We use these measures of time to create two relevant metrics. First, we find the total amount of time each agent spends completing the actions they are responsible for carrying out. We use these metrics to create a total workload measure to describe the total amount of work within the system. Second, we find the positive difference between the total workload for the flight crews and ATC. This measure is used to capture the balance of work between all agents in the system. We use these two measures (total workload and balance of workload) to conduct a multi-objective, gradient-free optimization routine in order to find the sets of inputs that best minimize the total work within the system and the balance of work between agents in the system.

2 Methods

2.1 Simulation Platform

WMC (Work Models that Compute) is a simulation engine under development at the Cognitive Engineering Center at Georgia Tech. WMC allows for the modeling of activity in complex work environments (Pritchett, 2014). In this context, *work* refers specifically to "the pattern(s) of activity required to meet some goal(s) within a complex environment." The WMC platform allows for the modeling of agents, the actions these agents perform, and the resources these actions work on.

This project uses the WMC engine to model the interaction between multiple agents in an airspace. During the final stages of flight, an aircraft's flight crew must interact with flight crews of other nearby aircraft, as well as with ATC. Using WMC, we are able to examine how multiple aircraft interact with the airspace, subject to common safety restrictions. The platform allows us to deterministically model the interactions between the various agents (flight crews and ATC) as they carry out all the actions required to land each of the planes.

2.2 Airspace actions

In a novel airspace, we can consider a number of actions that need to be completed in order to successfully land an aircraft. For a single aircraft, these actions may be completed by the aircraft's flight crew or by ATC. In this project, we consider a set of 19 such actions, which are listed in the Action column of Table 1.

Responsibility and *authority* for each of these actions can be assigned to either the aircraft's flight crew or ATC. In this context, an agent (flight crew or ATC) has *authority* of an action if they are supposed to execute the action. With this authority, the agent may execute the action, or they may decide to assign responsibility of the action to the other agent. In this case, the agent who is assigned responsibility of the action then has *responsibility* to carry out the action and they must be the one to execute the action.

If an agent who is authorized to execute an action chooses to delegate the action to the other agent in the system, the authorized agent is then responsible for monitoring the other agent to ensure the action gets completed. Thus assigning responsibility to another agent will spawn the creation of a new action, referred to as a *monitoring action*, which the authorized agent must carry out. As such, delegating actions to other agents introduces more total workload into the system. Yet depending on how many actions each agent is authorized to carry out, it may be preferred to introduce more total workload in order to balance the workload between the agents.

2.3 Model inputs

Our model considers 38 binary inputs. From each of the 19 distinct actions described in the Action column of Table 1, we create an input to describe the agent who has authority over the action and an input for the agent who is responsible for the action, defined in the Authorized Agent and Responsible Agent columns of Table 1, respectively. For a single aircraft, the authorized agent for action x_i can be either the aircraft's flight crew ($x_i = 1$) or ATC ($x_i = 0$). Similarly, the responsible agent for action x_j can be either the flight crew ($x_j = 1$) or ATC ($x_j = 0$). In our model, responsibility and authority over actions is identical for the three distinct aircraft.

Action	Authorized Agent	Responsible Agent
calculate distance to runway	x_1	x_2
start descent	x_3	x_4
intercept glideslope	x_5	x_6
land aircraft	x_7	x_8
set flaps speedbrakes	x_9	x_{10}
deploy gear	x_{11}	x_{12}
direct to waypoint	x_{13}	x_{14}
calculate distance to waypoint	x_{15}	x_{16}
set airspeed	x_{17}	x_{18}
manage waypoint progress	x_{19}	x_{20}
execute maneuver	x_{21}	x_{22}
clear for descent	x_{23}	x_{24}
clear for final approach	x_{25}	x_{26}
command OPD airspeed	x_{27}	x_{28}
calculate IM airspeed	x_{29}	x_{30}
command maneuver	x_{31}	x_{32}
calculate distance to MP	x_{33}	x_{34}
determine sequence	x_{35}	x_{36}
set lead aircraft	x_{37}	x_{38}

Table 1: List of actions to be completed in order to land each aircraft, as defined in the WMC simulator.

With this definition of inputs, if the authorized agent is the same as the responsible agent for a given action, this implies the authorized agent carries out the action and no monitoring action is created. Yet if the authorized agent and the responsible agent differ, then a monitoring action will be created in order for the action to be carried out. For instance, consider the *set airspeed* action. Suppose ATC has authority over this action, yet the flight crew is responsible for this action. In this scenario, $x_{17} = 0$, $x_{18} = 1$, and ATC will monitor FC1, FC2, and FC3 independently as they execute the set airspeed action for their respective planes.

Our goal is to find the optimal ways to assign responsibility and authority to agents in the airspace, while minimizing the total workload in the system and balancing the distribution of work between agents in the system.

2.4 Model Mapping

Recall our two objectives for optimization are total workload in the system and balance of workload between agents in the system. The WMC simulator inputs 38 2-level factor inputs ($\mathbf{x} \in S$), where S is the input space, made up of the 2^{38} possible combinations of the 2-level factors, and then outputs the total time an agent spends on each action during the simulation. Using these simulator outputs, we can define y_i to be the sum of time spent by agent i on any action listed in the Action column of Table 1 and z_i to be the sum of time spent by agent i on any monitoring action, where the relevant agents in our model are $i \in \{FC1, FC2, FC3, ATC\}$, as defined previously. Then, we can define our objective functions for optimization as $t(\mathbf{x})$ (total workload in the system) and $b(\mathbf{x})$ (balance of workload in the system):

$$t(\mathbf{x}) = y_{ATC} + z_{ATC} + \frac{1}{3} \sum_{\text{all } j} (y_j + z_j) \tag{1}$$

$$b(\mathbf{x}) = | y_{ATC} - \frac{1}{3} \sum_{\text{all } j} y_j | \tag{2}$$

where $j \in \{FC1, FC2, FC3\}$. These two objectives define our output space.

2.5 Experimental Design

In general, a 2^n full factorial experiment is an experiment whose design consists of n 2-level, discrete factors and whose experimental units take on all possible combinations of these levels across all such factors. Thus the design for an experiment with n 2-level factors will have 2^n experimental units. Full factorial experiments allow investigators to study the effect of each factor on the response variable, as well as the effects of interactions between the factors on the response variable. The *Yates matrix* for a 2^n full factorial design is a matrix in which each column represents a distinct factor or interaction and each row describes a single experimental unit.

Notice the number of experimental units required to test a 2^n full factorial design increases geometrically with n. As such, with 38 input factors, a full factorial design would contain 2^{38} experimental units. With the WMC simulator, it would take over 174,000 years to run a full factorial design for our experiment. Hence we need to consider more efficient ways to explore the output space for our system.

Fractional factorial designs (FFDs) use only a fraction of the 2^n experimental units and are used in experimentation as a means of effectively obtaining information on factors in the early stages of implementation or when a full factorial design is inconceiveable, among various other reasons.

Constructing the Yates matrix from n factors to include only 2^k experimental units, one starts with the Yates matrix of a full factorial design for k 2-level factors (the base factors) and subsequently assigns $n-k$ additional factors (the generated factors) to interaction columns among the k base factors.

The more base factors there are in a design, the more effects can be estimated without risk of bias. FFDs are grouped into classes according to their *resolution*, which quantifies the amount of confounding among factors in the design. With a design of *Resolution VI*, investigators are able to estimate the main factor effects unconfounded by four-factor (or less) interactions. Additionally, two-factor interaction effects will be unconfounded by three-factor (or less) interactions. And three-factor interaction effects can be estimated, but may be confounded with other three-factor interactions. Please see Lawson and Erjavec (2000) and Kuehl (2000) for more on factorial designs and FFDs.

In the initial stage of our experiment, we create a 2^{12} FFD of Resolution VI in order to get a partial view of the output space in our system. To efficiently create a 2^{12} FFD, we utilize the `R` package `FrF2`. This package ensures blocking and randomization during creation of the FFD. See Grömping (2014) for more information about how this package creates FFDs.

We proceed by running the WMC simulator with the distinct 2^{12} sets of inputs. This gives us a respective set of outputs, which gives us a partial glimpse of the output space, shown in Figure 1.

2.6 Linear Model Creation

The standard method for fitting a response from a factorial design is a linear model (Box et al., 2005). Using the results from our initial experimentation, we build two linear models. The response for the first model is total workload (t), while the response for the second is balance of workload (b). Each linear model initially contains first- and second-order interactions for the 38 predictors (x_1, ..., x_{38}). These full linear models can be described as follows:

$$t = \beta_0 + \sum_{i=1} \beta_i x_i + \sum_{i \neq j} \beta_{i,j} x_i x_j \,, \tag{3}$$

$$b = \alpha_0 + \sum_{i=1} \alpha_i x_i + \sum_{i \neq j} \alpha_{i,j} x_i x_j \,. \tag{4}$$

Using these initial linear models, we perform a stepwise regression variable selection procedure in order to explore explanatory variables that are most significant. We find 21 significant first-order variables for the t model and 18 significant first-order variables for the b model, as well as many significant second-order interactions in both models. Examining the residuals for both models, we see no worrisome trends. Detailed information on the coefficient estimates and goodness of fit for these initial linear models can be found in Appendix A.

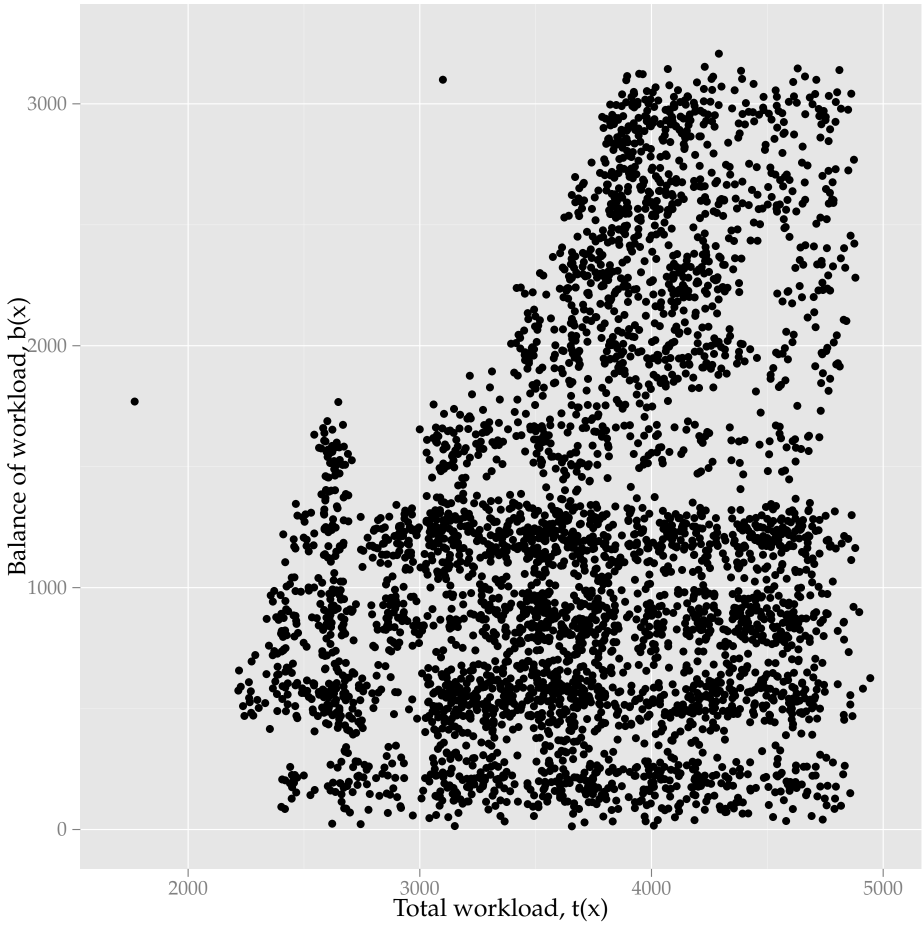

Figure 1: Initial view of the output space.

2.7 Pareto Front Exploration

The goal of our analysis is to find the sets of inputs ($\mathbf{x}$) that minimizes the total workload, $t(\mathbf{x})$, and the balance of workload, $b(\mathbf{x})$, in our system. We consider a multi-objective optimization:

$$\min_{\mathbf{x}} t(\mathbf{x}), b(\mathbf{x})$$
$$\mathbf{x} \in S.$$

In a multi-objective problem, $\mathbf{x}^* \in S$ is said to be *Pareto optimal* if all other vectors $\mathbf{x} \in S$ have a higher value for at least one of the objective functions. In most multi-objective problems, there exists a set of Pareto optimal outputs. When there are two objective functions, these outputs can be represented as points and the image of this set of points is called the Pareto front, or Pareto curve. The shape of the Pareto curve indicates the nature of the trade-off between the different objective functions.

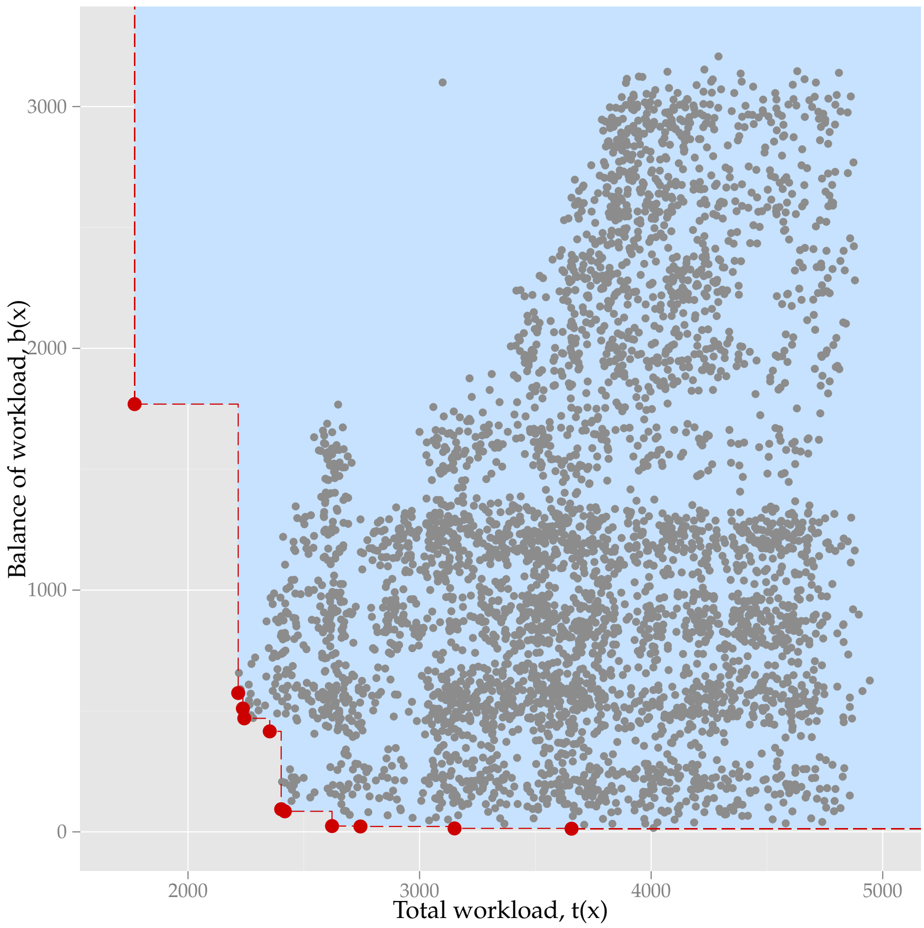

Figure 2: The initial Pareto curve, C_0, is represented by the red dotted line. Pareto optimal points (ie. points on the curve) are shown in red and points that are not Pareto optimal (ie. points not on the curve) are shown in gray. Recall C_0 was created using the output from running the input sets in the initial FFD.

After running the WMC simulator with the initial set of inputs from the FFD, we are able to define an initial Pareto curve $C_0 = C_0(f, b)$, shown in Figure 2. From here, we want to find points that may exist beyond this curve. To do so, we use an algorithm known as *maximum expected area* (MEA), which selects a new candidate input set $\mathbf{x}^*_{MEA}$ by maximizing the area between the current curve C_i and the curve produced by the output from accepting and running $\mathbf{x}^*_{MEA}$, denoted C^*_{MEA}. Possible candidate input sets, $\{\mathbf{x}^*\}$, are selected by randomly drawing a large number of times from a Bernoulli distribution, such that:

$$x_i^* \sim Bernoulli(0.5), \qquad \text{for } i = 1, ..., 38.$$

We disregard any sets of inputs that have already been run, and then evaluate the regression functions from our linear models in order to predicted values for $t(\mathbf{x}^*)$ and $b(\mathbf{x}^*)$ for all possible candidate input sets, $\{\mathbf{x}^*\}$. We then select the candidate input set $\mathbf{x}^*_{MEA}$ that is predicted to maximize the area between the current curve C_i and the curve produced by the output from accepting and running $\mathbf{x}^*_{MEA}$, denoted C^*_{MEA}. We proceed by running the simulator for $\mathbf{x}^*_{MEA}$ and obtaining the true values of $t(\mathbf{x}^*_{MEA})$ and $b(\mathbf{x}^*_{MEA})$. From here, we update our linear model to include information from $\mathbf{x}^*_{MEA}$, recalculate the Pareto curve C_{i+1}, and repeat the algorithm until the set of Pareto optimal points reasonably converges, or no points can be found beyond the current Pareto curve. A simplified version of the MEA algorithm is depicted in Figure 4.

3 Results

After implementing the MEA algorithm and continuously updating C and the linear models, we find a few clear trends in the Pareto optimal points. These trends develop after approximately 200 repetitions of the MEA algorithm and remain fairly constant for repetitions beyond 200. Figure 4 shows how the front develops after 100, 200, and 500 repetitions of the MEA algorithm.

After continuously updating the linear models for t and b throughout the 200 repetitions of the MEA algorithm, we find the estimates for the coefficients in the model change slightly, yet the significance of the coefficients remain mostly constant. We don't see significant changes in goodness of fit statistics for either model, which is not concerning. We suspect slightly better fit could be achieved by conducting a variable selection procedure during each run of the MEA algorithm. Detailed information on the updated estimates for the coefficients and goodness of fit for these models can be found in Appendix B.

The trends can be summarized into three groups of output points, which are highlighted in Figure 5. These three trends are discussed further in Sections 3.1, 3.2, 3.3

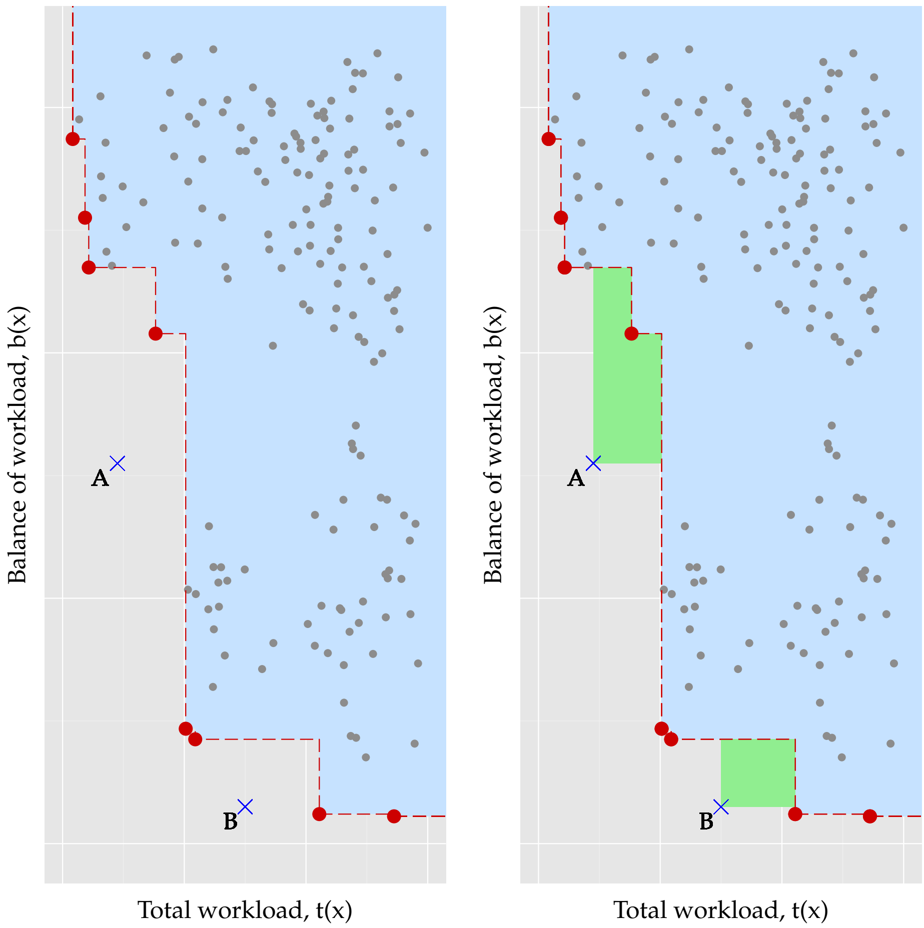

Figure 3: (*Left*) **A** and **B** are the predicted output points from two candidate input sets, $\mathbf{x}_A^*$ and $\mathbf{x}_B^*$, respectively. Both predicted output points lie beyond the current Pareto curve C, represented by the red dashed line. (*Right*) Comparing the expected area gained by including **A** in C and the expected area gained by including **B** in C, both shown in green, we see the expected area is greater with the inclusion of **A**. Hence, if we are only considering these two points in our MEA algorithm, we would set $\mathbf{x}_{MEA}^* = \mathbf{x}_A^*$ and simulate the true values for $t(\mathbf{x}_A^*)$ and $b(\mathbf{x}_A^*)$. We would then update C after considering the new point, $(t(\mathbf{x}_A^*), b(\mathbf{x}_A^*))$.

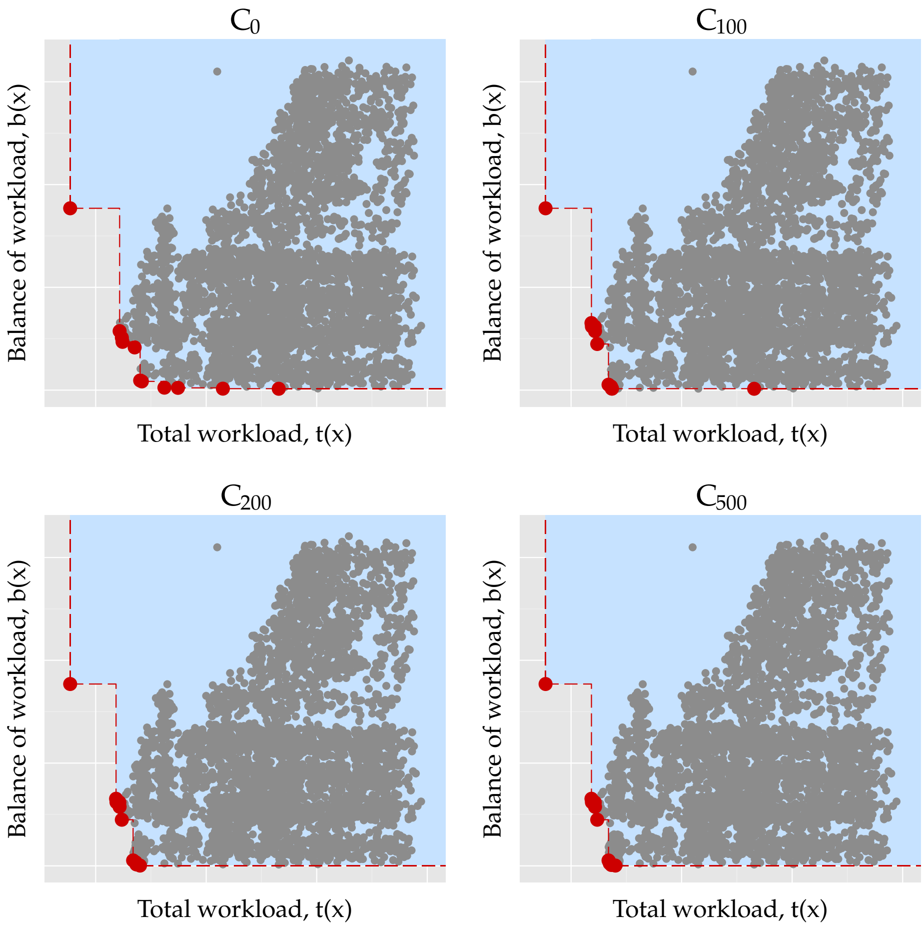

Figure 4: The initial Pareto curve, labelled C_0, as well as the Pareto curves created after 100, 200, and 500 repetitions of the MEA algorithm, labeled C_{100}, C_{200}, and C_{500}, respectively, are all shown by the red dashed lines. Notice how the curve develops from its initial state to C_{100} and C_{200}, yet remains fairly constant after more repetitions, as can be seen by comparing C_{200} and C_{500}.

3.1 Output Trend 1

The set of Pareto optimal points highlighted in blue in Figure 5 have a more equal balance of workload within the system than other Pareto optimal points. These points have certain trends among the actions, which are highlighted in Table 2 by their respective variables.

Action	Variables
calculate distance to runway	$x_1 = x_2 = 1$
intercept glideslope	$x_5 = 1$
set flaps speedbrakes	$x_{10} = 1$
direct to waypoint	$x_{13} = x_{14} = 1$
calculate distance to waypoint	$x_{15} = x_{16} = 0$
set airspeed	$x_{17} = 1$
manage waypoint progress	$x_{19} = x_{20} = 1$
clear for descent	$x_{23} = 1$
clear for final approach	$x_{25} = 1$
calculate IM airspeed	$x_{29} = x_{30} = 1$

Table 2: First set of observed trends in the Pareto optimal input sets.

3.2 Output Trend 2

The majority of other Pareto optimal points are highlighted in green in Figure 5. These points have a less equal balance of workload and require ATC to carry out more actions than the flight crews. Yet the trade-off between the higher levels of $b(\mathbf{x})$ results in a lower total workload in the system, since there are less monitoring actions created. The trends among the actions for these output points are highlighted in Table 3.

Action	Variables
calculate distance to runway	$x_1 = x_2 = 1$
set flaps speedbrakes	$x_9 \neq x_{10}$
deploy gear	$x_{11} \neq x_{12}$
direct to waypoint	$x_{13} = x_{14} = 0$
calculate distance to waypoint	$x_{15} = x_{16} = 0$
manage waypoint progress	$x_{19} = x_{20} = 0$
calculate IM airspeed	$x_{29} = x_{30} = 0$

Table 3: Second set of observed trends in the Pareto optimal input sets.

3.3 Output Trend 3

The final trend is represented in Figure 5 by the orange point, $(t(\mathbf{x}), b(\mathbf{x})) = (1769.25, 1769.25)$. For this output point, the flight crews are authorized and responsible to carry out all actions for their respective aircraft. Hence, $\mathbf{x} = (1, 1, ..., 1)$. Under this scenario, total workload is minimized, yet the balance of workload is much higher than any other Pareto optimal point.

This is an example of a completely decentralized system. Although this point is Pareto optimal by definition, we consider it to be somewhat less optimal than other points along the front.

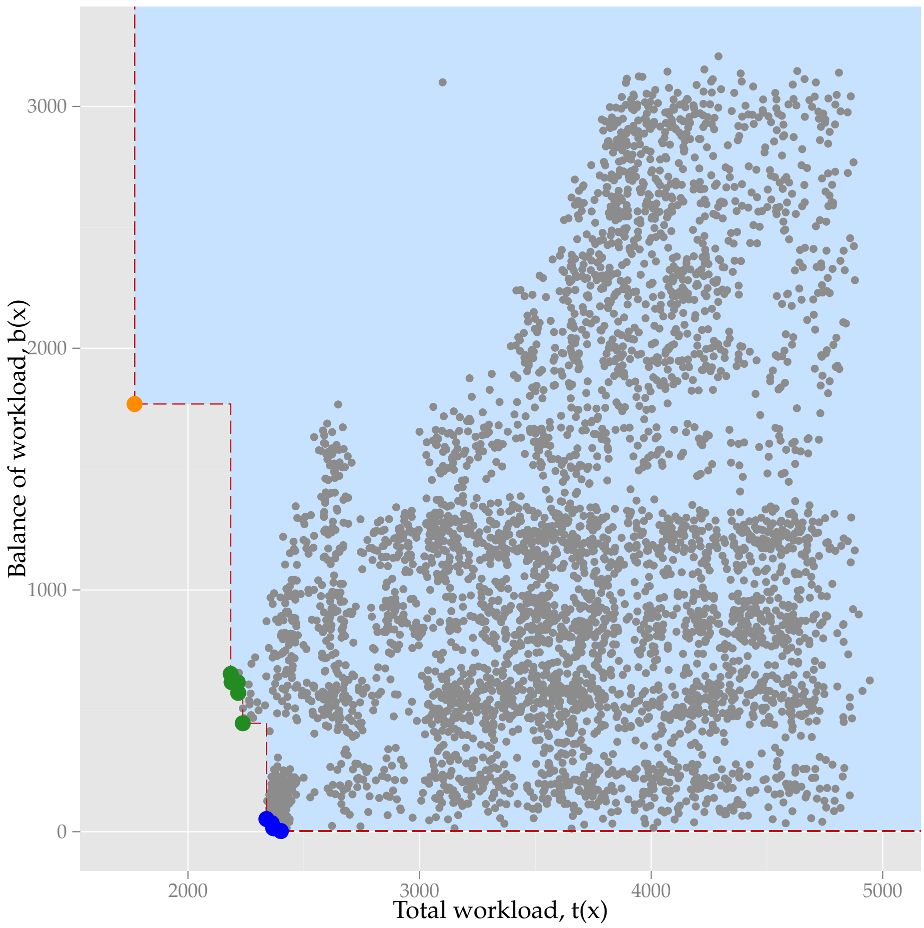

Figure 5: The Pareto curve after 200 repetitions of the MEA algorithm is shown in red. Pareto optimal points are highlighted in orange, green, and blue, where each color represents a different trend in these optimal points.

4 Discussion and future work

Overall, we find some clear trends among all the Pareto optimal points. We see evidence the authorized agent should also be the responsible agent for the direct to waypoint, the calculate distance to waypoint, the manage waypoint progress, and the calculate IM airspeed actions. We also find evidence that the flight crews should be authorized and responsible for carrying out the calculate distance to runway action. Additionally, we find some interesting interactions between the variables, suggesting certain agents should carry out specific sets of actions in order to minimize the objectives.

We hope results of our optimization will be of use to researchers interested in further studying the trade-offs of control within an airspace with the actions we have defined here. After further study, we hope these methods can be applied to a more general airspace.

With the WMC simulator, future researchers could consider fitting a joint multivariate model on the total workload and balance of workload responses. Additionally, we suggest modifying the MEA algorithm to include a variable selection procedure. We suspect this modification would yield slightly better model fit. Future researchers should consider including the economic trade-offs of the objectives in their model and incorporating appropriate weights on the objective functions. And lastly, we suggest future researchers also consider redefining the objective functions by placing different weights on the required actions and the monitoring actions.

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Appendix A Coefficient estimates for the initial linear models

Table 4: Estimated regression coefficients from the initial linear model for the response variable t after applying a stepwise regression procedure.

Coefficient	Estimate	Std. Error	t-value	Pr(> | t |)
(Intercept)	3656.6955	11.9208	306.75	0.0000
β_1	227.6552	10.1089	22.52	0.0000
β_2	320.3712	8.7545	36.59	0.0000
β_{16}	279.0589	9.7879	28.51	0.0000
β_{15}	271.9138	9.4560	28.76	0.0000
β_{30}	157.0692	7.9918	19.65	0.0000
β_{29}	150.3771	7.5816	19.83	0.0000
β_9	9.7773	7.5816	1.29	0.1973
β_{10}	5.3037	7.1480	0.74	0.4581
β_7	-2.4656	5.0544	-0.49	0.6257
β_8	-9.5967	5.6510	-1.70	0.0895
β_{13}	-5.6801	6.6864	-0.85	0.3957
β_{14}	2.6299	7.1480	0.37	0.7130
β_{11}	3.1000	6.1904	0.50	0.6166
β_{12}	-7.4166	6.1904	-1.20	0.2310
β_{25}	22.8031	5.6510	4.04	0.0001
β_{26}	17.7088	5.0544	3.50	0.0005
β_{18}	36.5655	3.5740	10.23	0.0000
β_{17}	36.7009	3.5740	10.27	0.0000
β_{20}	-10.7812	5.0544	-2.13	0.0330
β_{19}	-1.2118	6.1904	-0.20	0.8448
β_5	7.7086	4.3773	1.76	0.0783
$\beta_{1,2}$	-1315.4644	5.0544	-260.26	0.0000
$\beta_{16,15}$	-1513.1083	5.0544	-299.36	0.0000
$\beta_{30,29}$	-626.2076	5.0544	-123.89	0.0000
$\beta_{2,16}$	98.7182	5.0544	19.53	0.0000
$\beta_{2,15}$	98.7182	5.0544	19.53	0.0000
$\beta_{1,15}$	87.0922	5.0544	17.23	0.0000
$\beta_{1,16}$	87.0922	5.0544	17.23	0.0000
$\beta_{2,29}$	41.7897	5.0544	8.27	0.0000
$\beta_{1,29}$	40.9110	5.0544	8.09	0.0000
$\beta_{1,30}$	39.4045	5.0544	7.80	0.0000
$\beta_{2,30}$	39.1364	5.0544	7.74	0.0000
$\beta_{9,10}$	106.6541	5.0544	21.10	0.0000

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Table 4 – *Continued from previous page*

Coefficient	Estimate	Std. Error	t-value	Pr(> | t |)
$\beta_{15,29}$	33.5945	5.0544	6.65	0.0000
$\beta_{16,29}$	33.0434	5.0544	6.54	0.0000
$\beta_{15,30}$	31.5185	5.0544	6.24	0.0000
$\beta_{16,30}$	31.4922	5.0544	6.23	0.0000
$\beta_{7,8}$	34.6223	5.0544	6.85	0.0000
$\beta_{1,9}$	-21.0661	5.0544	-4.17	0.0000
$\beta_{1,10}$	-21.0188	5.0544	-4.16	0.0000
$\beta_{15,13}$	124.2796	5.0544	24.59	0.0000
$\beta_{16,13}$	123.7388	5.0544	24.48	0.0000
$\beta_{1,13}$	-22.7388	5.0544	-4.50	0.0000
$\beta_{13,14}$	-215.4025	5.0544	-42.62	0.0000
$\beta_{15,14}$	123.7311	5.0544	24.48	0.0000
$\beta_{16,14}$	123.5573	5.0544	24.45	0.0000
$\beta_{1,14}$	-22.8645	5.0544	-4.52	0.0000
$\beta_{11,12}$	35.9118	5.0544	7.11	0.0000
$\beta_{2,10}$	-15.4540	5.0544	-3.06	0.0022
$\beta_{2,9}$	-15.4489	5.0544	-3.06	0.0023
$\beta_{16,9}$	-14.9058	5.0544	-2.95	0.0032
$\beta_{15,9}$	-14.8795	5.0544	-2.94	0.0033
$\beta_{16,10}$	-14.8735	5.0544	-2.94	0.0033
$\beta_{15,10}$	-14.8473	5.0544	-2.94	0.0033
$\beta_{18,17}$	-86.5388	5.0544	-17.12	0.0000
$\beta_{16,7}$	-12.9618	5.0544	-2.56	0.0104
$\beta_{15,20}$	23.3970	5.0544	4.63	0.0000
$\beta_{16,20}$	23.3970	5.0544	4.63	0.0000
$\beta_{2,14}$	-11.8963	5.0544	-2.35	0.0186
$\beta_{2,13}$	-11.6727	5.0544	-2.31	0.0210
$\beta_{1,11}$	-11.5357	5.0544	-2.28	0.0225
$\beta_{20,19}$	-37.3389	5.0544	-7.39	0.0000
$\beta_{15,19}$	23.6777	5.0544	4.68	0.0000
$\beta_{16,19}$	23.6777	5.0544	4.68	0.0000
$\beta_{1,12}$	-10.9564	5.0544	-2.17	0.0302
$\beta_{11,19}$	-10.7936	5.0544	-2.14	0.0328
$\beta_{7,12}$	9.0136	5.0544	1.78	0.0746
$\beta_{8,12}$	8.9084	5.0544	1.76	0.0781
$\beta_{8,11}$	8.8815	5.0544	1.76	0.0790
$\beta_{2,11}$	-8.7488	5.0544	-1.73	0.0835
$\beta_{1,25}$	-8.5583	5.0544	-1.69	0.0905
$\beta_{29,9}$	-8.4415	5.0544	-1.67	0.0950

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Table 4 – *Continued from previous page*

Coefficient	Estimate	Std. Error	t-value	Pr(> | t |)
$\beta_{9,25}$	-8.3768	5.0544	-1.66	0.0975
$\beta_{14,5}$	-15.2816	5.0544	-3.02	0.0025
$\beta_{29,5}$	-8.4899	5.0544	-1.68	0.0931
$\beta_{1,19}$	-8.3059	5.0544	-1.64	0.1004
$\beta_{2,12}$	-8.2458	5.0544	-1.63	0.1029
$\beta_{30,10}$	-8.1032	5.0544	-1.60	0.1090
$\beta_{30,9}$	-8.0515	5.0544	-1.59	0.1112
$\beta_{29,10}$	-7.9595	5.0544	-1.57	0.1154
$\beta_{30,14}$	-7.6882	5.0544	-1.52	0.1283
$\beta_{1,8}$	-7.6570	5.0544	-1.51	0.1299
$\beta_{30,13}$	-7.5352	5.0544	-1.49	0.1361
$\beta_{1,26}$	-7.4057	5.0544	-1.47	0.1429
$\beta_{16,26}$	-7.1741	5.0544	-1.42	0.1559
$\beta_{15,25}$	-7.1739	5.0544	-1.42	0.1559
$\beta_{16,25}$	-7.1682	5.0544	-1.42	0.1562
$\beta_{15,26}$	-7.1582	5.0544	-1.42	0.1568
Residuals:				
Min	1Q	Median	3Q	Max
-655.50	-49.77	-2.83	47.57	541.83

Residual standard error: 80.87 on 4007 degrees of freedom
Multiple R-squared: 0.9829, **Adjusted R-squared:** 0.9826
F-statistic: 2624 on 88 and 4007 DF, p-value: 0.0000

Table 5: Estimated regression coefficients from the initial linear model for the response variable b after applying a stepwise regression procedure.

Coefficient	Estimate	Std. Error	t-value	Pr(> | t |)
(Intercept)	3071.0825	22.3743	137.26	0.0000
α_1	-2157.4734	25.8356	-83.51	0.0000
α_{15}	-2186.7504	24.2683	-90.11	0.0000
α_{29}	-979.4597	12.5321	-78.16	0.0000
α_{13}	-482.6014	10.8531	-44.47	0.0000
α_{16}	196.5729	12.5321	15.69	0.0000
α_{17}	-143.1959	10.8531	-13.19	0.0000
α_{10}	105.3316	10.8531	9.71	0.0000
α_{14}	61.5831	8.8615	6.95	0.0000
α_{19}	-93.4432	10.8531	-8.61	0.0000
α_8	39.0772	12.5321	3.12	0.0018
α_2	150.5123	14.0113	10.74	0.0000
α_{23}	-50.3686	10.8531	-4.64	0.0000
α_{12}	26.1262	8.8615	2.95	0.0032
α_5	-35.5739	10.8531	-3.28	0.0011
α_7	-63.2101	12.5321	-5.04	0.0000
α_{25}	-41.0691	10.8531	-3.78	0.0002
α_{26}	10.9825	6.2660	1.75	0.0797
α_{27}	-31.5250	10.8531	-2.90	0.0037
$\alpha_{1,15}$	2031.5973	12.5321	162.11	0.0000
$\alpha_{1,29}$	654.5122	12.5321	52.23	0.0000
$\alpha_{15,29}$	643.7682	12.5321	51.37	0.0000
$\alpha_{15,13}$	356.1167	12.5321	28.42	0.0000
$\alpha_{1,13}$	313.5367	12.5321	25.02	0.0000
$\alpha_{1,16}$	-135.0695	12.5321	-10.78	0.0000
$\alpha_{15,17}$	98.1717	12.5321	7.83	0.0000
$\alpha_{1,17}$	97.0253	12.5321	7.74	0.0000
$\alpha_{15,16}$	-75.0196	12.5321	-5.99	0.0000
$\alpha_{1,10}$	-67.4254	12.5321	-5.38	0.0000
$\alpha_{15,10}$	-62.8238	12.5321	-5.01	0.0000
$\alpha_{1,14}$	-58.6596	12.5321	-4.68	0.0000
$\alpha_{15,19}$	73.8754	12.5321	5.89	0.0000
$\alpha_{1,19}$	56.7062	12.5321	4.52	0.0000
$\alpha_{1,2}$	-81.5924	12.5321	-6.51	0.0000
$\alpha_{16,2}$	-78.5870	12.5321	-6.27	0.0000
$\alpha_{15,2}$	-77.1548	12.5321	-6.16	0.0000
$\alpha_{15,8}$	-36.9713	12.5321	-2.95	0.0032

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Table 5 – *Continued from previous page*

Coefficient	Estimate	Std. Error	t-value	Pr(> | t |)
$\alpha_{1,23}$	34.7365	12.5321	2.77	0.0056
$\alpha_{15,23}$	33.7307	12.5321	2.69	0.0071
$\alpha_{1,8}$	-29.3022	12.5321	-2.34	0.0194
$\alpha_{15,5}$	23.6691	12.5321	1.89	0.0590
$\alpha_{1,5}$	23.4761	12.5321	1.87	0.0611
$\alpha_{1,12}$	-23.2831	12.5321	-1.86	0.0633
$\alpha_{1,7}$	41.5189	12.5321	3.31	0.0009
$\alpha_{8,7}$	34.0430	12.5321	2.72	0.0066
$\alpha_{15,7}$	28.3300	12.5321	2.26	0.0238
$\alpha_{15,25}$	30.9320	12.5321	2.47	0.0136
$\alpha_{1,25}$	28.9456	12.5321	2.31	0.0210
$\alpha_{29,2}$	-19.9651	12.5321	-1.59	0.1112
$\alpha_{15,27}$	22.3777	12.5321	1.79	0.0742
$\alpha_{1,27}$	21.8807	12.5321	1.75	0.0809
Residuals:				
Min	1Q	Median	3Q	Max
-685.03	-171.19	-7.69	164.29	820.99

Residual standard error: 200.5 on 4045 degrees of freedom
Multiple R-squared: 0.9374, **Adjusted R-squared:** 0.9366
F-statistic: 1211 on 50 and 4045 DF, p-value: 0.0000

Appendix B Coefficient estimates for the updated linear models

Table 6: Estimated regression coefficients from the updated linear model after 200 MEA repetitions for the response variable t.

Coefficient	Estimate	Std. Error	t-value	Pr(>|t|)
(Intercept)	3652.1758	11.5966	314.94	0.0000
β_{1}	218.7477	9.8212	22.27	0.0000
β_{2}	308.2098	8.5652	35.98	0.0000
β_{16}	288.5748	9.3986	30.70	0.0000
β_{15}	282.1723	9.0768	31.09	0.0000
β_{30}	145.1201	7.8528	18.48	0.0000
β_{29}	135.1741	7.4025	18.26	0.0000
β_{9}	15.7133	7.3512	2.14	0.0326
β_{10}	12.1367	6.9571	1.74	0.0811
β_{7}	-2.1761	4.6162	-0.47	0.6374
β_{8}	-5.6280	5.4581	-1.03	0.3025
β_{13}	1.2683	6.5797	0.19	0.8472
β_{14}	8.8081	6.9919	1.26	0.2078
β_{11}	6.1985	5.8929	1.05	0.2929
β_{12}	-2.6464	5.9938	-0.44	0.6589
β_{25}	20.9453	5.3489	3.92	0.0001
β_{26}	16.3436	4.7932	3.41	0.0007
β_{18}	33.9620	3.4467	9.85	0.0000
β_{17}	34.6422	3.4196	10.13	0.0000
β_{20}	-8.6208	4.7996	-1.80	0.0725
β_{19}	1.7614	6.0024	0.29	0.7692
β_{5}	6.8468	4.2620	1.61	0.1082
$\beta_{1,2}$	-1303.5311	4.9354	-264.12	0.0000
$\beta_{16,15}$	-1501.1750	4.9354	-304.17	0.0000
$\beta_{30,29}$	-614.2744	4.9354	-124.46	0.0000
$\beta_{2,16}$	99.6062	4.9324	20.19	0.0000
$\beta_{2,15}$	99.6062	4.9324	20.19	0.0000
$\beta_{1,15}$	87.9802	4.9324	17.84	0.0000
$\beta_{1,16}$	87.9802	4.9324	17.84	0.0000
$\beta_{2,29}$	53.9104	4.9339	10.93	0.0000
$\beta_{1,29}$	53.0317	4.9339	10.75	0.0000
$\beta_{1,30}$	51.5252	4.9339	10.44	0.0000
$\beta_{2,30}$	51.2571	4.9339	10.39	0.0000
$\beta_{9,10}$	94.2750	4.8562	19.41	0.0000

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Table 6 – *Continued from previous page*

Coefficient	Estimate	Std. Error	t-value	Pr(>|t|)
$\beta_{15,29}$	34.6699	4.9317	7.03	0.0000
$\beta_{16,29}$	34.1188	4.9317	6.92	0.0000
$\beta_{15,30}$	32.5939	4.9317	6.61	0.0000
$\beta_{16,30}$	32.5677	4.9317	6.60	0.0000
$\beta_{7,8}$	29.1621	4.7577	6.13	0.0000
$\beta_{1,9}$	-20.7544	4.8590	-4.27	0.0000
$\beta_{1,10}$	-21.4862	4.8602	-4.42	0.0000
$\beta_{15,13}$	112.3463	4.9354	22.76	0.0000
$\beta_{16,13}$	111.8056	4.9354	22.65	0.0000
$\beta_{1,13}$	-23.6268	4.9324	-4.79	0.0000
$\beta_{13,14}$	-203.4692	4.9354	-41.23	0.0000
$\beta_{15,14}$	111.7978	4.9354	22.65	0.0000
$\beta_{16,14}$	111.6240	4.9354	22.62	0.0000
$\beta_{1,14}$	-23.7525	4.9324	-4.82	0.0000
$\beta_{11,12}$	27.9949	4.7494	5.89	0.0000
$\beta_{2,10}$	-15.9213	4.8602	-3.28	0.0011
$\beta_{2,9}$	-15.1372	4.8590	-3.12	0.0018
$\beta_{16,9}$	-14.9951	4.8424	-3.10	0.0020
$\beta_{15,9}$	-14.9688	4.8424	-3.09	0.0020
$\beta_{16,10}$	-14.6286	4.8436	-3.02	0.0025
$\beta_{15,10}$	-14.6023	4.8436	-3.01	0.0026
$\beta_{18,17}$	-77.3127	4.8434	-15.96	0.0000
$\beta_{16,7}$	-11.4766	4.7634	-2.41	0.0160
$\beta_{15,20}$	16.7284	4.8427	3.45	0.0006
$\beta_{16,20}$	16.7284	4.8427	3.45	0.0006
$\beta_{2,14}$	-12.7843	4.9324	-2.59	0.0096
$\beta_{2,13}$	-12.5608	4.9324	-2.55	0.0109
$\beta_{1,11}$	-12.3558	4.7948	-2.58	0.0100
$\beta_{20,19}$	-30.0713	4.8293	-6.23	0.0000
$\beta_{15,19}$	15.0331	4.8854	3.08	0.0021
$\beta_{16,19}$	15.0331	4.8854	3.08	0.0021
$\beta_{1,12}$	-9.8991	4.7861	-2.07	0.0387
$\beta_{11,19}$	-6.7183	4.7688	-1.41	0.1590
$\beta_{7,12}$	7.2751	4.7403	1.53	0.1249
$\beta_{8,12}$	7.9663	4.7432	1.68	0.0931
$\beta_{8,11}$	7.3463	4.7380	1.55	0.1211
$\beta_{2,11}$	-9.5690	4.7948	-2.00	0.0460
$\beta_{1,25}$	-9.5599	4.7882	-2.00	0.0459
$\beta_{29,9}$	-7.9424	4.8593	-1.63	0.1022

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Table 6 – *Continued from previous page*

Coefficient	Estimate	Std. Error	t-value	Pr(> | t |)
$\beta_{9,25}$	-9.0009	4.7376	-1.90	0.0575
$\beta_{14,5}$	-13.7413	4.7791	-2.88	0.0041
$\beta_{29,5}$	-4.1331	4.7841	-0.86	0.3877
$\beta_{1,19}$	-8.9388	4.8837	-1.83	0.0673
$\beta_{2,12}$	-7.1886	4.7861	-1.50	0.1332
$\beta_{30,10}$	-8.7580	4.8606	-1.80	0.0716
$\beta_{30,9}$	-7.5524	4.8593	-1.55	0.1202
$\beta_{29,10}$	-8.6143	4.8606	-1.77	0.0764
$\beta_{30,14}$	-8.7636	4.9317	-1.78	0.0756
$\beta_{1,8}$	-9.6876	4.7708	-2.03	0.0424
$\beta_{30,13}$	-8.6107	4.9317	-1.75	0.0809
$\beta_{1,26}$	-10.2483	4.7935	-2.14	0.0326
$\beta_{16,26}$	-5.8090	4.7932	-1.21	0.2256
$\beta_{15,25}$	-5.0040	4.7884	-1.05	0.2961
$\beta_{16,25}$	-4.9983	4.7884	-1.04	0.2966
$\beta_{15,26}$	-5.7931	4.7932	-1.21	0.2269
Residuals:				
Min	1Q	Median	3Q	Max
-675.03	-46.40	-0.40	42.27	550.76

Residual standard error: 80 on 4485 degrees of freedom

Multiple R-squared: 0.9875, **Adjusted R-squared:** 0.9873

F-statistic: 4031 on 88 and 4485 DF, p-value: 0.0000

Table 7: Estimated regression coefficients from the updated linear model after 200 MEA repetitions for the response variable b.

Coefficient	Estimate	Std. Error	t-value	Pr(> | t |)
(Intercept)	3095.9540	21.5331	143.78	0.0000
α_1	-2118.9906	24.5182	-86.43	0.0000
α_{15}	-2237.5844	22.8702	-97.84	0.0000
α_{29}	-976.2707	12.1652	-80.25	0.0000
α_{13}	-501.2413	10.5060	-47.71	0.0000
α_{16}	201.1125	12.1675	16.53	0.0000
α_{17}	-150.6946	10.4317	-14.45	0.0000
α_{10}	110.2360	10.4153	10.58	0.0000
α_{14}	61.5831	8.6348	7.13	0.0000
α_{19}	-109.0087	10.4755	-10.41	0.0000
α_8	36.3656	11.8727	3.06	0.0022
α_2	140.2762	13.5342	10.36	0.0000
α_{23}	-52.2684	10.4136	-5.02	0.0000
α_{12}	26.1262	8.6348	3.03	0.0025
α_5	-39.6005	10.4201	-3.80	0.0001
α_7	-69.8220	11.8728	-5.88	0.0000
α_{25}	-40.9045	10.4140	-3.93	0.0001
α_{26}	14.6705	5.7895	2.53	0.0113
α_{27}	-33.1604	10.4149	-3.18	0.0015
$\alpha_{1,15}$	2059.7980	11.9833	171.89	0.0000
$\alpha_{1,29}$	618.5829	11.9715	51.67	0.0000
$\alpha_{15,29}$	673.3195	11.9765	56.22	0.0000
$\alpha_{15,13}$	393.3966	11.9695	32.87	0.0000
$\alpha_{1,13}$	285.3359	11.9833	23.81	0.0000
$\alpha_{1,16}$	-106.8688	11.9833	-8.92	0.0000
$\alpha_{15,17}$	113.1691	11.7065	9.67	0.0000
$\alpha_{1,17}$	78.1215	11.7179	6.67	0.0000
$\alpha_{15,16}$	-112.2996	11.9695	-9.38	0.0000
$\alpha_{1,10}$	-61.3529	11.6645	-5.26	0.0000
$\alpha_{15,10}$	-72.6327	11.6478	-6.24	0.0000
$\alpha_{1,14}$	-86.8603	11.9833	-7.25	0.0000
$\alpha_{15,19}$	105.0066	11.8621	8.85	0.0000
$\alpha_{1,19}$	34.8325	11.8790	2.93	0.0034
$\alpha_{1,2}$	-118.8723	11.9695	-9.93	0.0000
$\alpha_{16,2}$	-50.3862	11.9833	-4.20	0.0000
$\alpha_{15,2}$	-48.9541	11.9833	-4.09	0.0000
$\alpha_{15,8}$	-44.4706	11.6671	-3.81	0.0001

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Table 7 – *Continued from previous page*

Coefficient	Estimate	Std. Error	t-value	Pr(> | t |)
$\alpha_{1,23}$	31.8379	11.6595	2.73	0.0063
$\alpha_{15,23}$	37.5305	11.6420	3.22	0.0013
$\alpha_{1,8}$	-22.6858	11.6875	-1.94	0.0523
$\alpha_{15,5}$	31.7222	11.6651	2.72	0.0066
$\alpha_{1,5}$	14.8796	11.6798	1.27	0.2027
$\alpha_{1,12}$	-24.3034	11.6238	-2.09	0.0366
$\alpha_{1,7}$	46.5456	11.6872	3.98	0.0001
$\alpha_{8,7}$	46.9656	11.6160	4.04	0.0001
$\alpha_{15,7}$	28.6311	11.6688	2.45	0.0142
$\alpha_{15,25}$	30.6028	11.6434	2.63	0.0086
$\alpha_{1,25}$	27.9695	11.6605	2.40	0.0165
$\alpha_{29,2}$	-55.8944	11.9715	-4.67	0.0000
$\alpha_{15,27}$	25.6485	11.6464	2.20	0.0277
$\alpha_{1,27}$	17.3981	11.6635	1.49	0.1359
Residuals:				
Min	1Q	Median	3Q	Max
-696.32	-135.38	-35.09	157.73	806.54

Residual standard error: 195.4 on 4523 degrees of freedom
Multiple R-squared: 0.9437, **Adjusted R-squared:** 0.943
F-statistic: 1515 on 50 and 4523 DF, p-value: 0.0000