EPFL, LABORATORY OF COMPUTATIONAL NEUROSCIENCE

BIOLOGICAL MODELING OF NEURAL NETWORKS

Miniproject: The response of a neuronal network to a transient current

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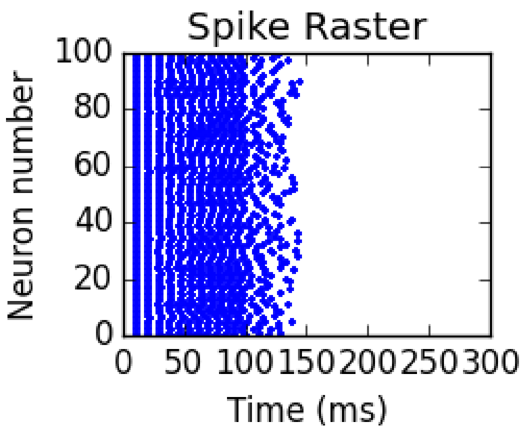
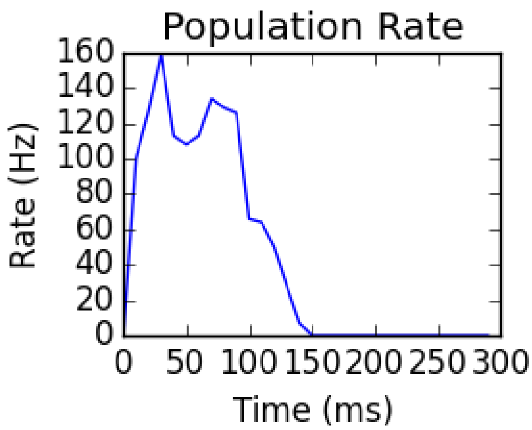
This mini project examines the phenomenon of late response of a neural network to a transient current and how this behaviour varies with different properties of the network.

1 SIMULATION OF A POPULATION

Our reference population contains 100 leaky integrate-and-fire excitatory neurons that are randomly connected with a 30% connection probability and that is simulated during 300ms. The population is injected with a current of 5nA for the first 100ms.

We define the late response as the mean of population rate registered during the last 50ms of the simulation. For the simulation with the default settings, the late response is 0.0, meaning that the population can't hold the activity after the stimulus disappears as is also visible in the plots.





To calculate the shape of the post synaptic current (PSC) we need to solve the differential equation:

$$\tau_{syn}\frac{dI_{syn}}{dt} = -I_{syn} \tag{1.1}$$

We use the integrating factor:

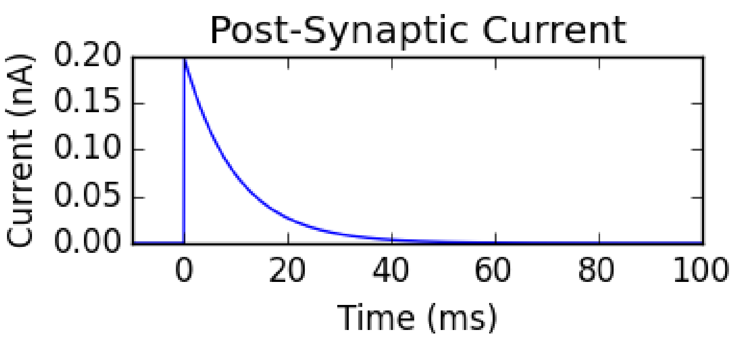
$$\mu = c_1 e^{\frac{t}{\tau_{syn}}} \qquad \frac{d\mu}{dt} = \frac{c_1}{\tau_{syn}} e^{\frac{t}{\tau_{syn}}} \tag{1.2}$$

Where c_1 is an arbitrary constant. We reorganize, multiply both sides by μ and solve:

$$\begin{aligned}
\frac{dI_{syn}}{dt} + \frac{1}{\tau_{syn}} I_{syn} &= 0 \\
ce^{\frac{t}{\tau_{syn}}}\frac{dI_{syn}}{dt} + ce^{\frac{t}{\tau_{syn}}}\frac{1}{\tau_{syn}} I_{syn} &= 0 \\
\mu\frac{dI_{syn}}{dt} + \frac{d\mu}{dt} I_{syn} &= 0 \\
\frac{d(\mu I_{syn})}{dt} &= 0 \\
\mu I_{syn} &= c_2 \\
I_{syn} &= ke^{-\frac{t}{\tau_{syn}}} \qquad k = \frac{c_2}{c_1}
\end{aligned} \tag{1.3}$$

The shape of the PSC is an exponential decline with a maximum at $t = 0$. In terms of the equation, k is just an arbitrary constant. In the context of the PSC, it defines the value of the current at $t = 0$, i.e. the value of the current right after a spike has occurred. Because in the simulation we scale this current jump using the synaptic weights (default: $w = 0.2nA$), we will simply pick $k = 1$ from now on.

The shape of the current after multiplying by the synaptic weight looks like:

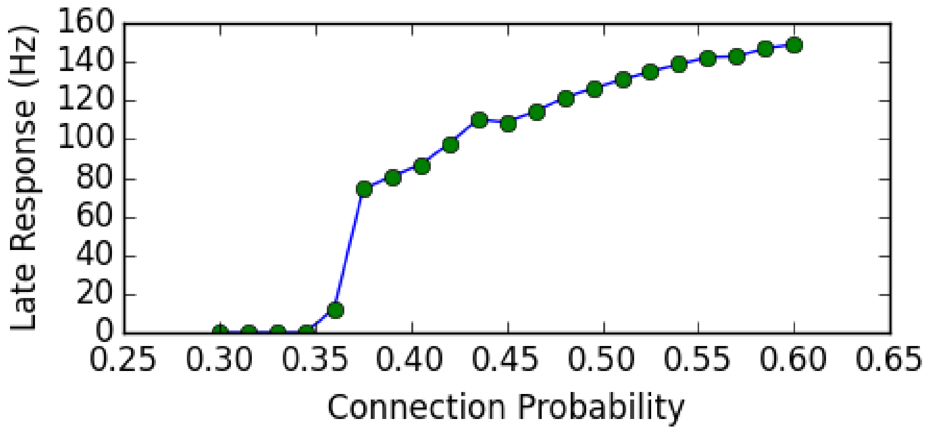


Because it only makes sense to talk about a PSC *after* a spike has arrived, we can define $I_{syn} = \alpha(t)$ as:

$$\alpha(t) = \begin{cases} 0 & \text{if } x < 0 \\ e^{-\frac{t}{\tau_{syn}}} & \text{if } x \geq 0 \end{cases} \tag{1.4}$$

2 Changing the connection probability

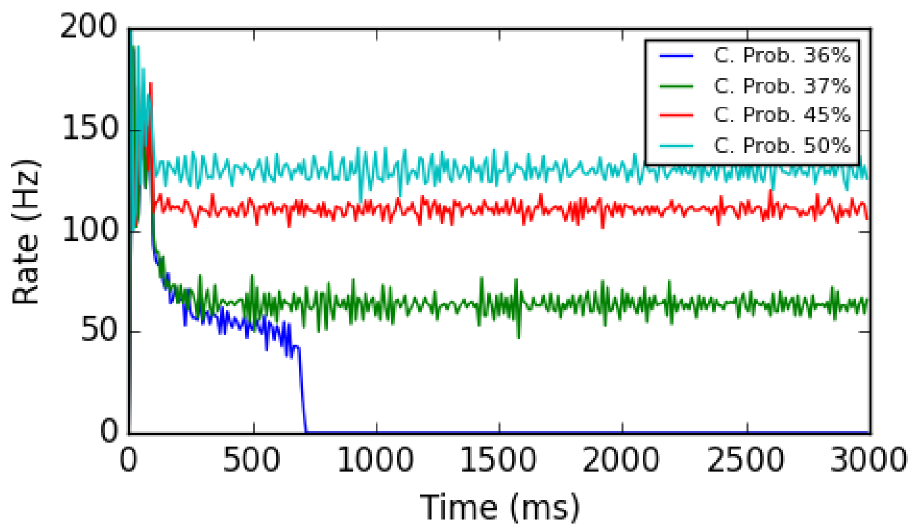
The late response jumps from 0Hz to around 55Hz when the probability is increased over a value around 36% and from that point on it increases with a slow decay.



2.1 Later responses

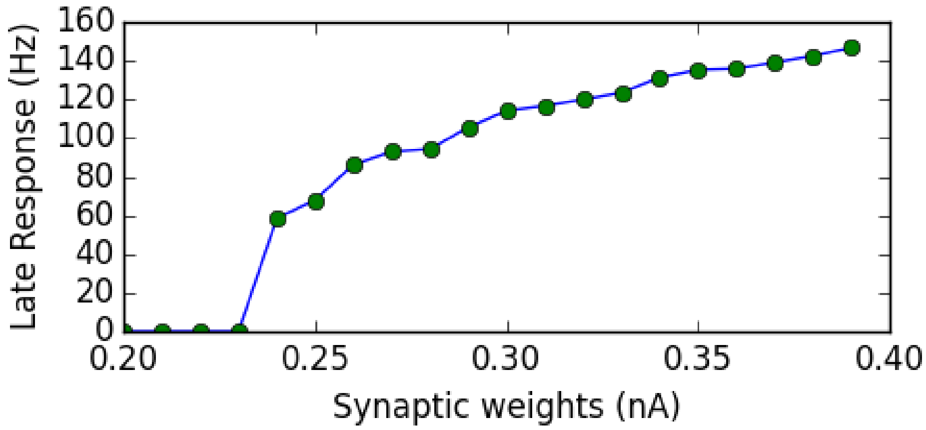
This initial discontinuity is interesting because it seems to indicate that there is a point in which the response of the population switches from just decaying to being hold indefinitely. If on the contrary the population was simply responding more strongly but its response still decayed, we would expect our late response metric to continuously raise from zero instead of jumping.

This can be illustrated by looking at the population rate on two values that are very close on both sides of the jump boundary over a longer timescale.



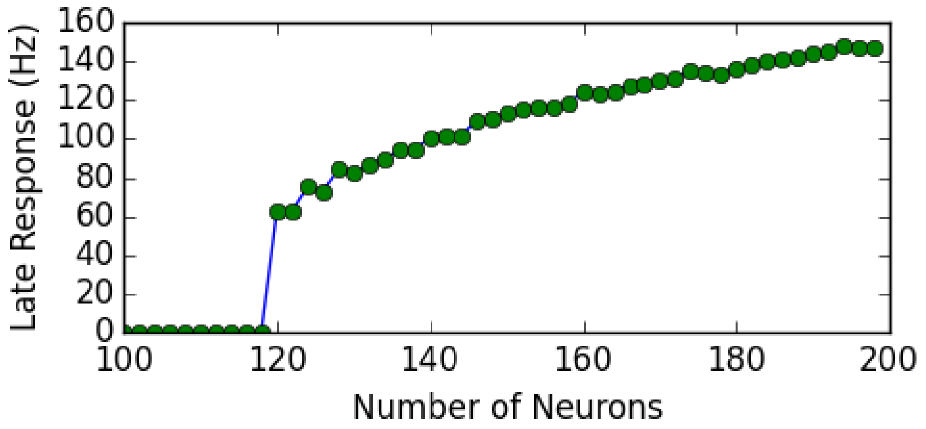
3 Changing the synaptic weights

The late response jumps from $0Hz$ to around $55Hz$ when the synaptic weights are increased over a value around $25nA$ and from that point on it increases with a slow decay. An effect very similar to that of changing the connection probabilities.



4 CHANGING THE NUMBER OF NEURONS

The late response jumps from $0Hz$ to around $55Hz$ when the number of neurons is increased over a value around 120 and from that point on it increases with a slow decay. Again, an effect very similar to that of changing the connection probabilities.



5 EFFECTIVE COEFFICIENT OF THE POPULATION

From the equation of the synaptic input of each neuron, we can derive a relationship between the population activity and the synaptic current received by each neuron.

Note that this is a randomly connected network, so not all of the spikes are relevant: n here will represent the number of neurons that a given one is connected to. On average, we expect $n = pN$ where p is the connection probability and N the total size of the population.

Note also that all our neurons are equivalent and that we can approximate the average of incoming spike trains to a single neuron $\frac{1}{n}\sum_j S_j(t)$ by the the activity of the whole population $A(t)$.

After making both substitutions we can take the average over time in both sides:

$$
\begin{aligned}
I &= \sum_j w \int_{-\infty}^{+\infty} S_j(t-s)\alpha(s)ds \\
I &= wn \sum_j \int_{-\infty}^{+\infty} \frac{1}{n} S_j(t-s)\alpha(s)ds \\
I &= wpN \int_{-\infty}^{+\infty} A(t-s)\alpha(s)ds \\
\langle I \rangle &= \langle wpN \int_{-\infty}^{+\infty} A(t-s)\alpha(s)ds \rangle \\
\langle I \rangle &= wpNA \int_{-\infty}^{+\infty} \alpha(s)ds
\end{aligned} \tag{5.1}
$$

where A is the average population activity and $\alpha(t)$ is the post-synaptic current calculated in Exercise 1. The integration of $\alpha(t)$ can thus be limited to start from 0 and it could be interpreted as the post-synaptic charge with unit in coulombs.

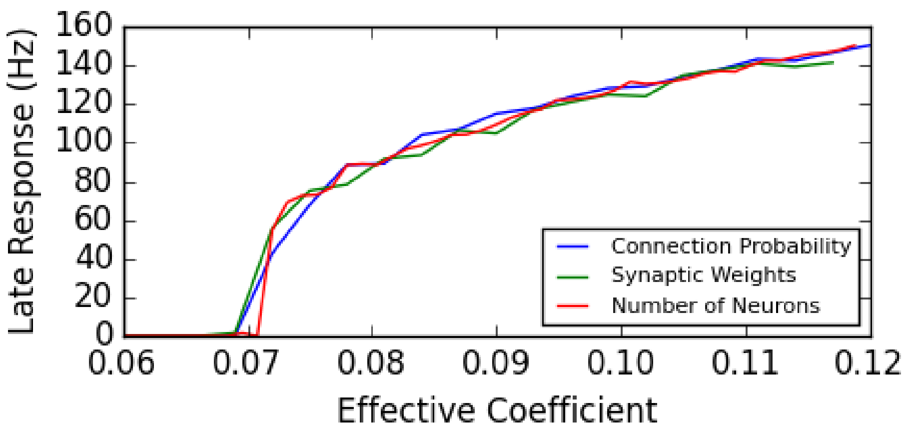
$$
\int_{-\infty}^{+\infty} \alpha(s)ds = \int_0^{+\infty} e^{-\frac{t}{\tau_{syn}}} ds = -\tau_{syn} e^{-\frac{t}{\tau_{syn}}} \Big|_0^{+\infty} = 0 + \tau_{syn} e^{-\frac{0}{\tau_{syn}}} = \tau_{syn} \tag{5.2}
$$

Substituting in the previous equation we arrive to our definition of the Effective Coefficient.

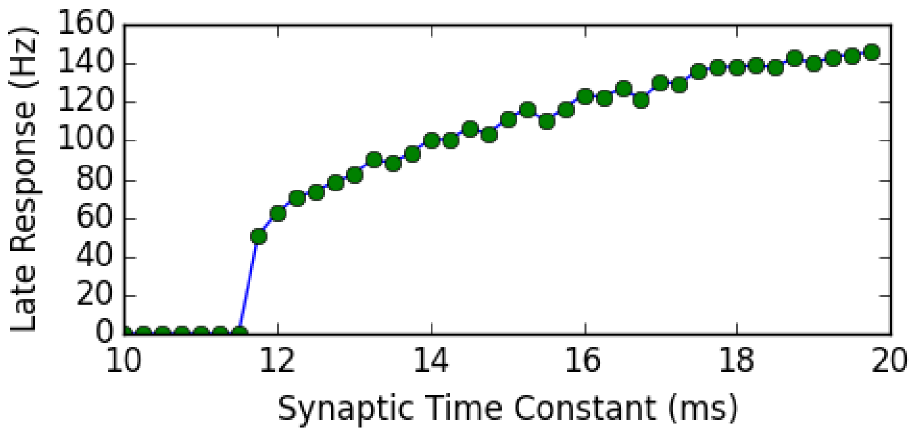
$$
\langle I \rangle = CA \qquad C = wpN\tau_{syn} \tag{5.3}
$$

From that formula and the plots below we can confirm the suspicion from exercises 2, 3 and 4 where the late response seemed to react equally to proportional changes to any of the three parameters. It does in fact, react to changes of the Effective Coefficient which is composed of all of them.

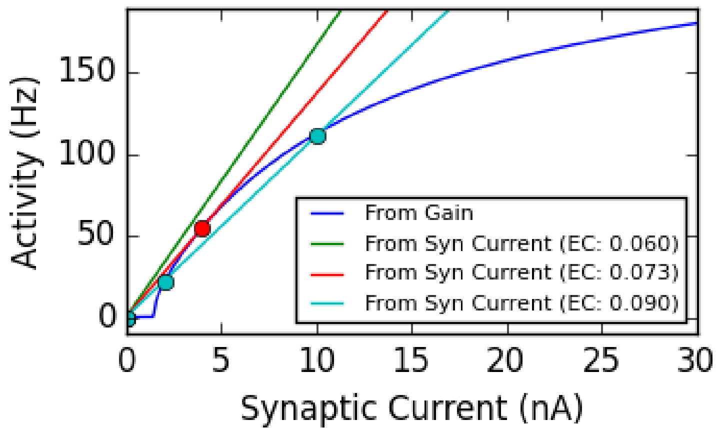
Taking the experiments from the previous exercises we can plot the results against the value of the Effective Coefficient and see how the curves, ignoring some noise, are actually aligned:



In fact, it's possible to repeat the parameter sweep experiment once more for the remaining component (τ_{syn}) and see how the curve follows the same pattern:



We can now compare this relationship with that derived from the gain function.



Using the initial values of the simulation we see that the only intersection point is on the origin, when both the activity and the synaptic current are zero. This stable point attracts the dynamics of the population to a resting state of no-activity.

However, if we increase any of the components of Effective Coefficient enough, the two curves will meet in a second point around $A = 55Hz$ and, as we continue to increase the Effective Coefficient (Exercises 2,3,4), this new tangent point splits in two intersections.

The third and highest intersection point becomes a stable one and will compete with the original one by attracting the dynamics to a state of continued activity. The existence of these two stable points, and of an unstable one between them is what creates the need for an external stimulus in order to be able to transition from one to the other.

In other words, networks with a high enough Effective Coefficient present a bistability that allows the network to be able to "remember" the transient by being able to switch from a state of no activity to one of continued activity and stay in it. Networks with lower Effective Coefficient values on the other hand, are unable to hold the high activity response and will eventually come back to rest.