Lecture 9: Double Patterning

@luk036

2022-11-23

] .pull-right[

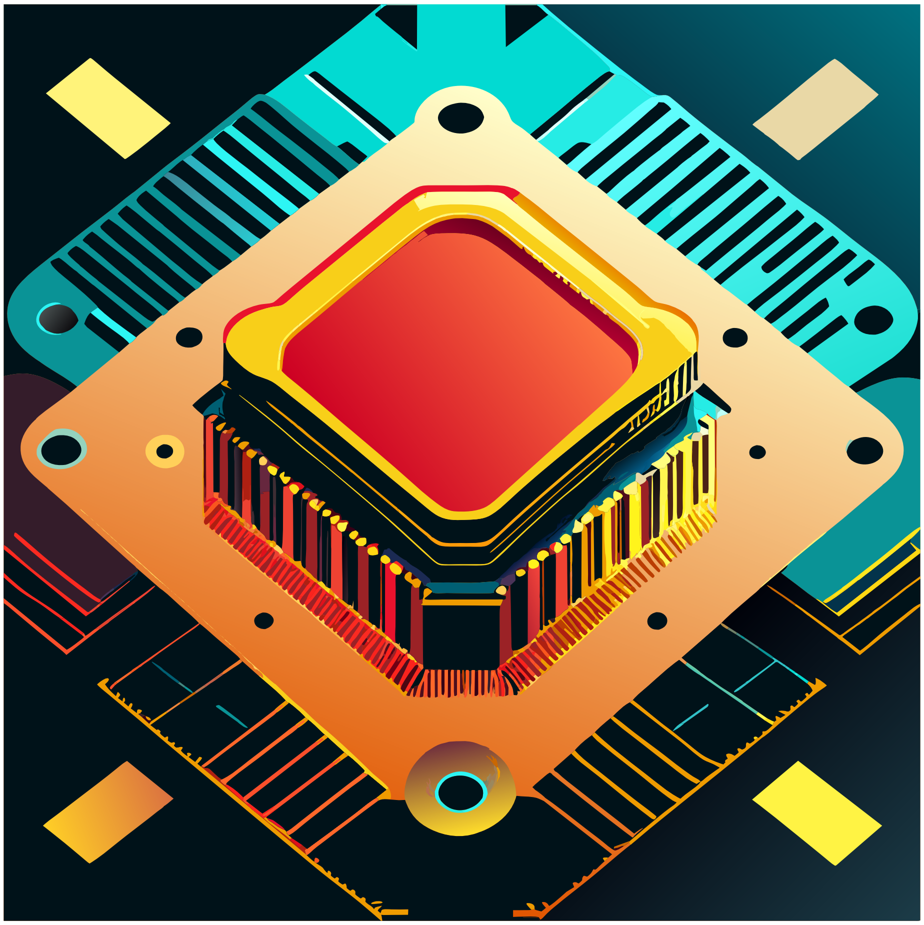


Figure 1: image

]

Background

.pull-left[

- In the past, chips have continued to get smaller and smaller, and therefore consume less and less power.
- However, we are rapidly approaching the end of the road and optical lithography cannot take us to the next place we need to go.

] .pull-right[

]

Process of Lithography

.pull-left[

1. Photo-resist coating ( )
2. Illumination ( )
3. Exposure ( )
4. Etching ( )
5. Impurities doping ( )
6. Metal connection

] .pull-right[

]

Sub-wavelength Lithography

.pull-left[

- Feature size « lithography wavelength
  - 45 nm vs. 193 nm

] .pull-right[

- What you see in the mask/layout is **not** what you get on the chip:
  - Features are distorted
  - Yields are declined

]

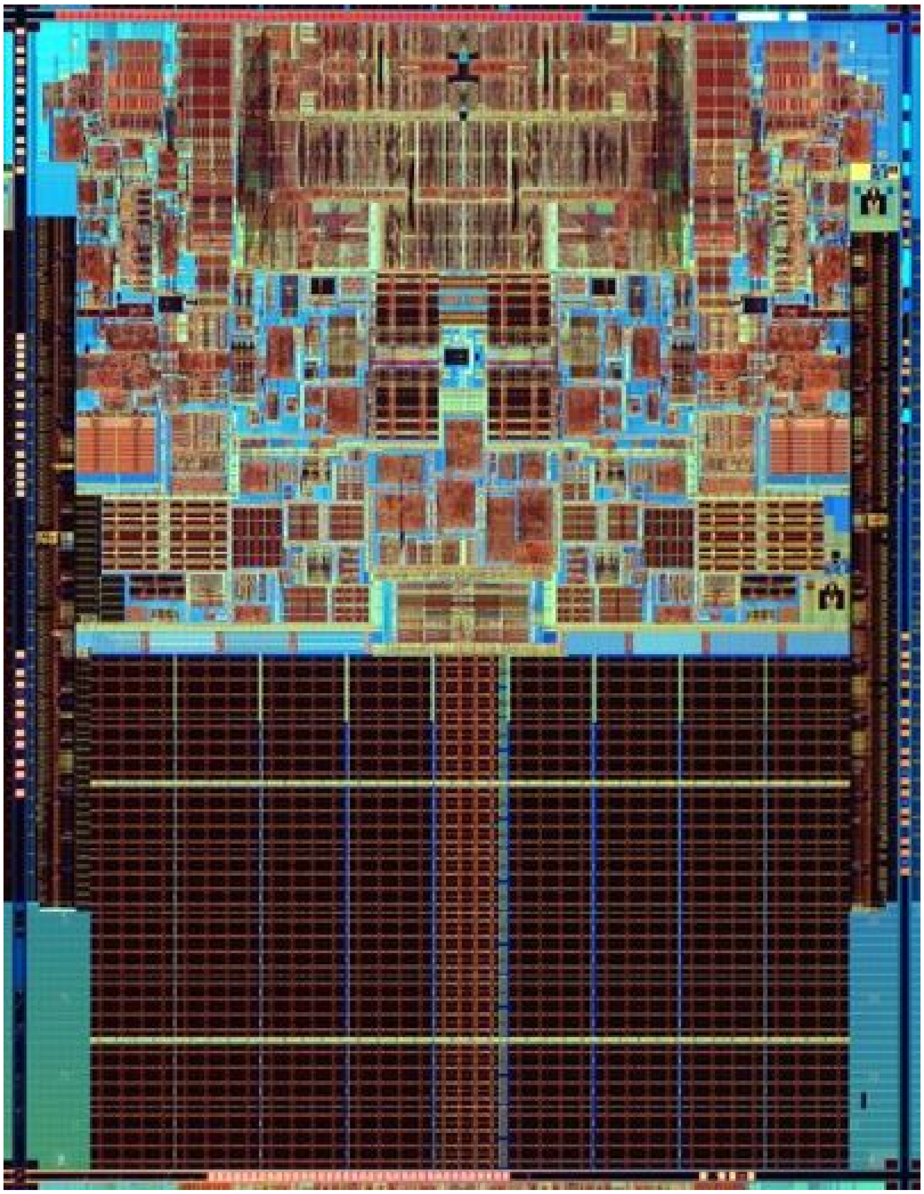


Figure 2: image

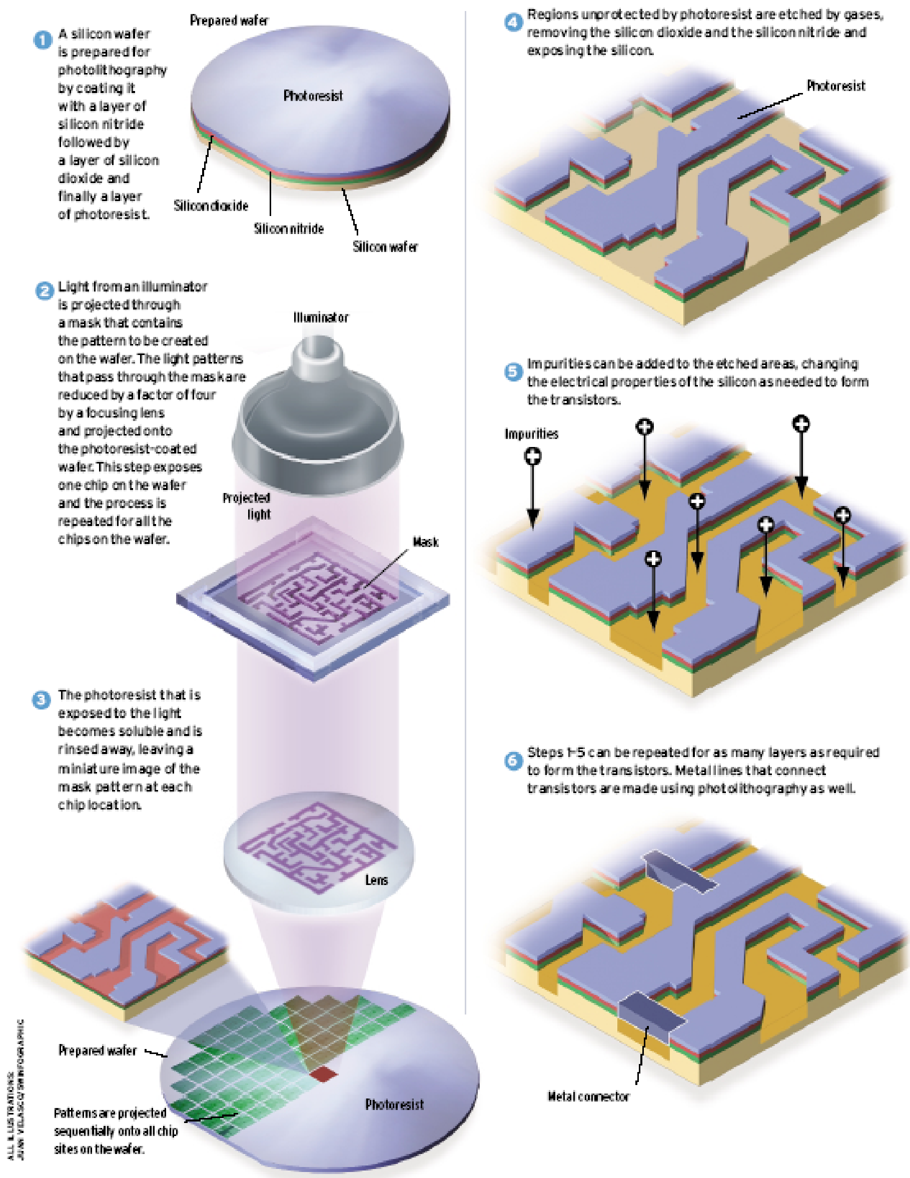


Figure 3: image

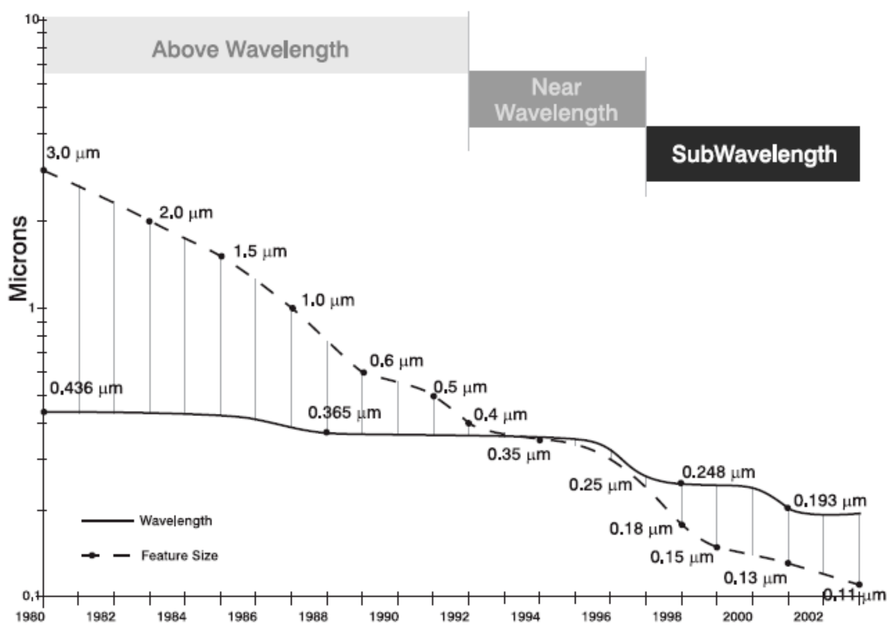


Figure 1: Shift to subwavelength optical lithography since the 0.35-micron process generation.

Figure 4: image

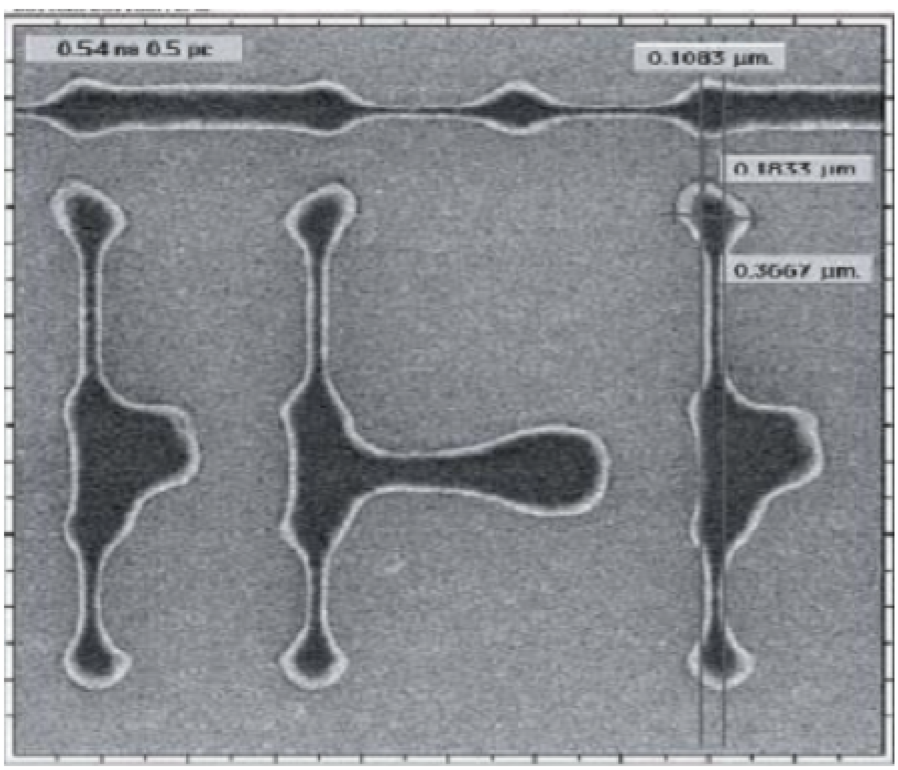


Figure 5: image

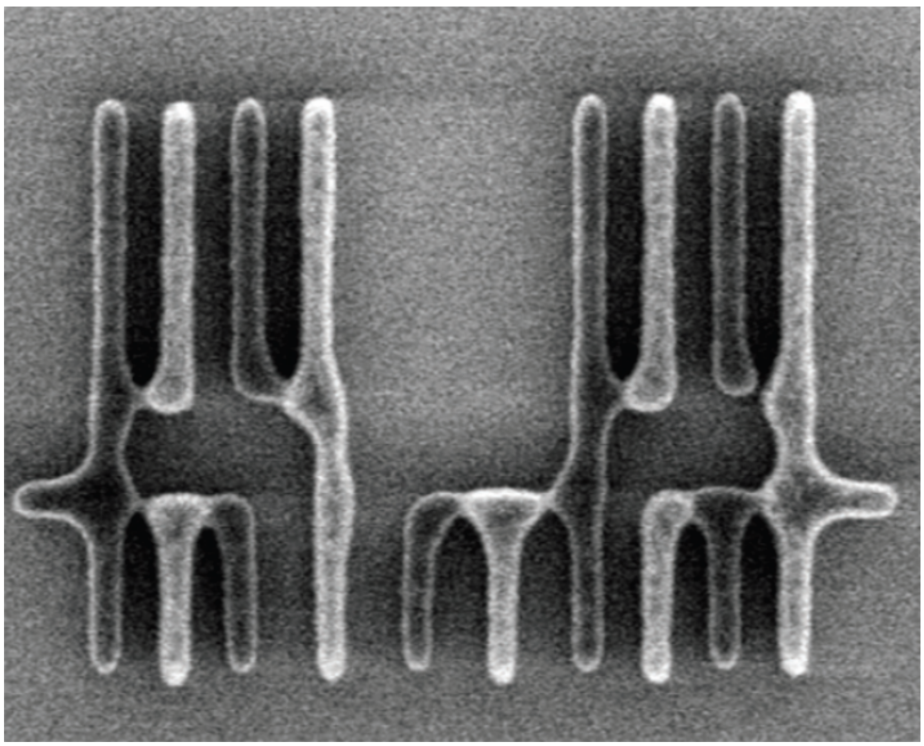


Figure 6: image

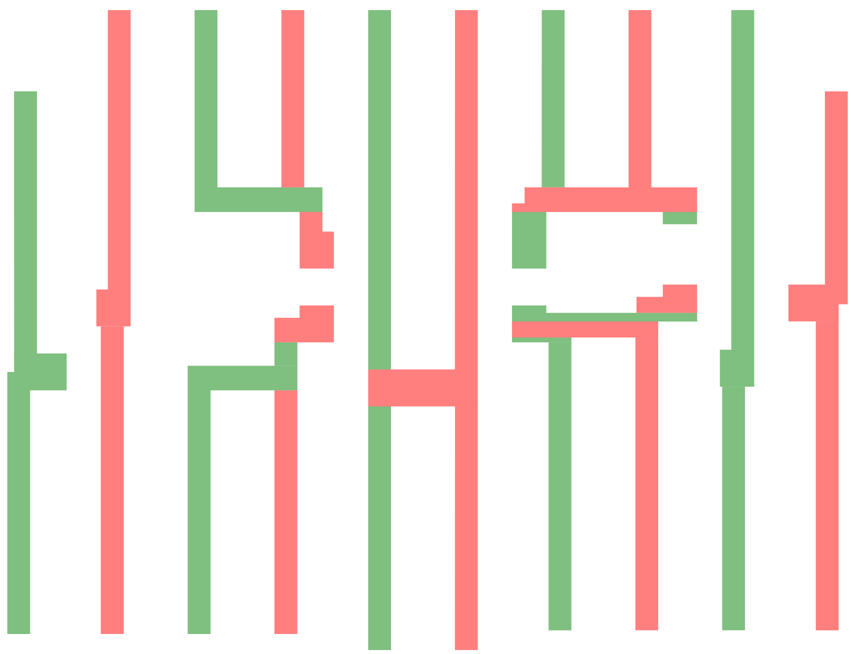


Figure 7: image

What is Double Patterning?

.pull-left[

] .pull-right[

]

Unlike conventional optical lithography, which exposes the photoresist once under one mask, masks is exposed twice by splitting them into two, each with half its feature density.

Key technologies

- *Layout fracturing* algorithm to reduce the number of rectangles and the total cut length.
- *Dynamic priority search tree* for plane sweeping.
- Graph-theoretic approach:
  - Convert the coloring problem to a T-join problem and then solve it with Hadlock's algorithm.
- Decompose the underlying conflict graph into its tri-connected components using a data structure named *SPQR-tree*.

Polygon Fracturing Algorithm

- Allow minimal overlap to reduce the number of rectangles, and thus the number of conflicts.

Conflict Detection

.pull-left70[

- Rule 1: If the distance between two rectangles is $\geq b$, then the two rectangles are *not* in conflict.
- Rule 2: Two overlapping/contacting rectangles are **NOT** conflict.
- Rule 3:
  - Definition: A polygon is said to be *rectilinearly convex* if it is both *x-monotone* and *y-monotone*.
  - Two rectangles X and Y are in conflict if they are $\leq b$ apart and there is a path from X to Y that reconstructs a "concave" polygon.

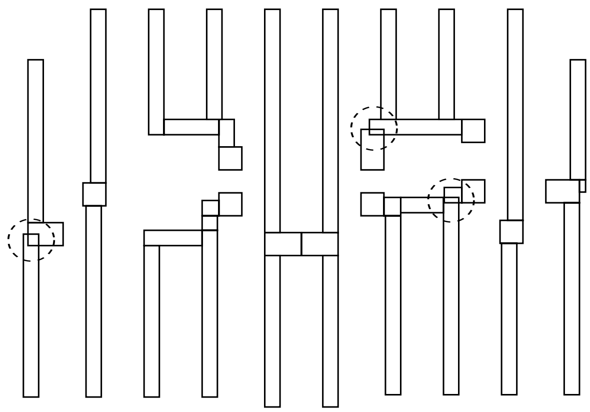


Figure 8: image

– Conflicting: (A, C), (B, D), but not (A, B), (A, D) and (B, C).

] .pull-right30[

]

Conflict Graph

- Blue edge: positive weight (opposite color preferred)
- Green edge: negative weight (same color preferred)

Formulation of the Layout Decomposition Problem

- INSTANCE: Graph $G = (V, E)$ and weight function $w : E \to Z$
- SOLUTION: Disjoint subsets of vertices V_0 and V_1 so that $V_0 \cup V_1 = V$ and $V_0 \cap V_1 = \emptyset$.
- MINIMIZE: total cost $\sum_{e \in E_c} w(e)$ where $E_c = \{(u, v) : u, v \in V_0$ or $u, v \in V_1, (u, v) \in E\}$

Note: the problem is - Linear time solvable for bipartite graphs. - Polynomial time solvable for planar graphs. - But in general, NP-hard (even for tripartite graphs)

Graph-Theoretic Approach

- Q: How can we produce a high-quality result when the problem is NP-hard?

- A: Observe that G is a nearly planar graph: we can use Hadlock's algorithm.

- However, the time complexity of this method is still very high.

- Solution: Graph division methods.

---

class: middle, center

SPQR-Tree

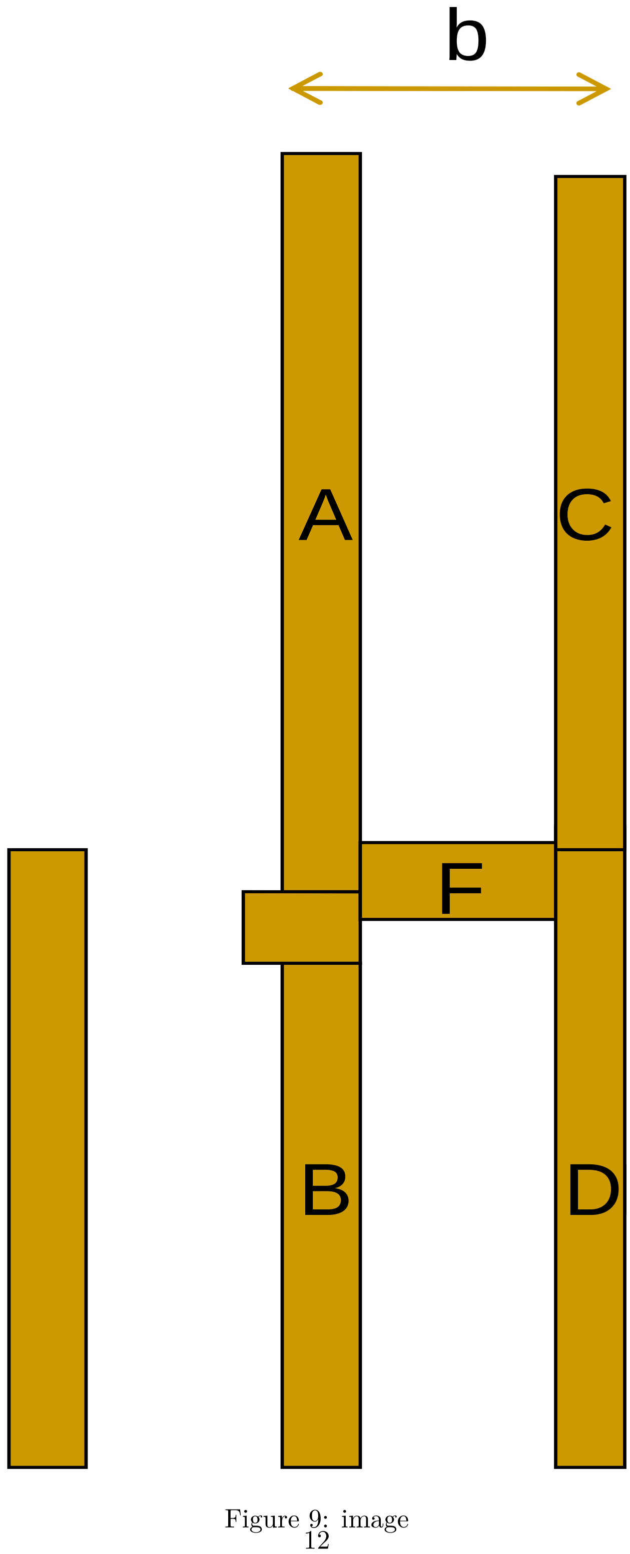


Figure 9: image

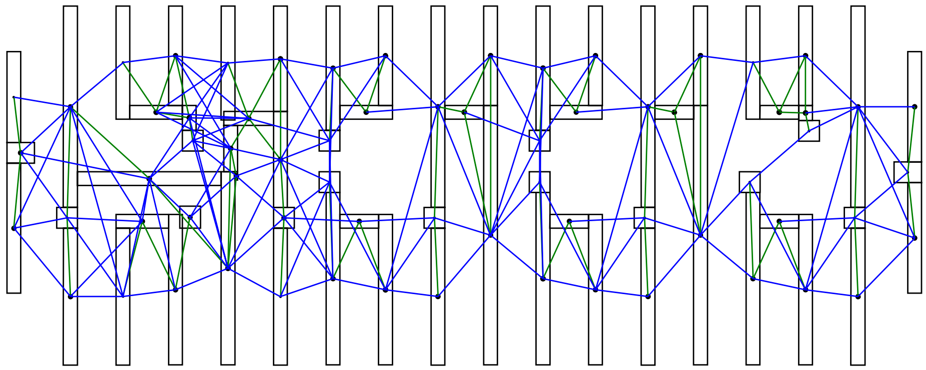


Figure 10: image

Connected Graph

- Recall that a graph $G = (V, E)$ is a *connected* if every pair of vertices u, v in G is connected by a path.
- A graph can be divided into its connected components in linear time.
- Clearly, the color assignment problem can be solved independently for each connected component without affecting any QoR.

Bi-connected Graph

- A vertex is called a *cut-vertex* of a connected graph G if removing it disconnects G.
- If no cut-vertex is found in G, then the graph is called a bi-connected graph.
- In the following example, a, b and c are cut-vertices.

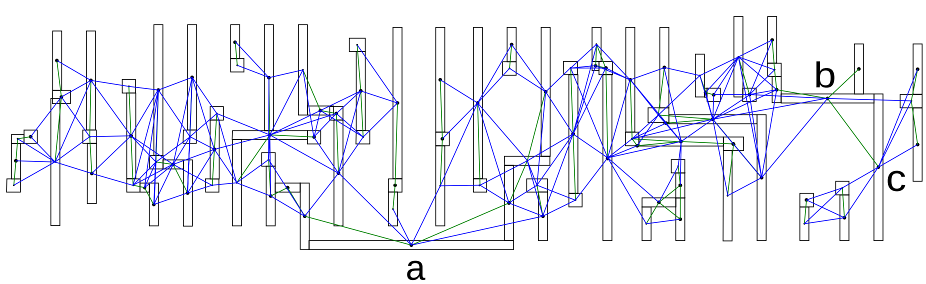


Figure 11: An example of a conflict graph with its bi-connected components. Vertices a, b, and c are cut-vertices.

Bi-connected Components G'

- A division of G into its bi-connected components can be performed in linear time by using a simple depth-first search to identify cut-vertices.
- It can be easily shown that the color assignment problem can be solved for each bi-connected component separately without affecting any QoR [@chiang_fast_2005]
- Question: Is it possible to further decompose the graph?

Tri-connected Graph

- If removing a pair of vertices will disconnect G', the pair is called a *separation pair* of G'.
- If no separation pair can be found in G', then it is called a *tri-connected graph*.
- In the following example, (a, b), (g, h), (c, d), (c, e) and (c, f) are separation pairs.

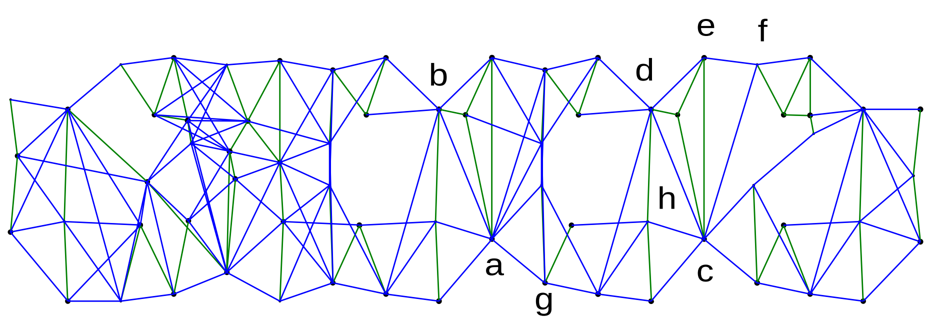


Figure 12: An example of a conflict graph and its tri-connected components. $\{a, b\}$, $\{c, d\}$, $\{c, e\}$, $\{c, f\}$ and $\{g, h\}$ are separation pairs.

Tri-connected Graph Division

SPQR-tree

- A division of G' into its tri-connected components can be performed by identifying the separation pairs in linear time with the help of SPQR-

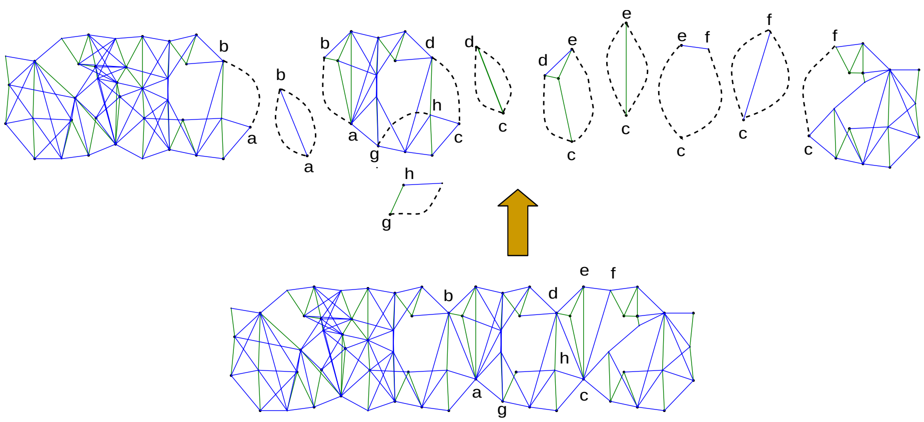


Figure 13: image

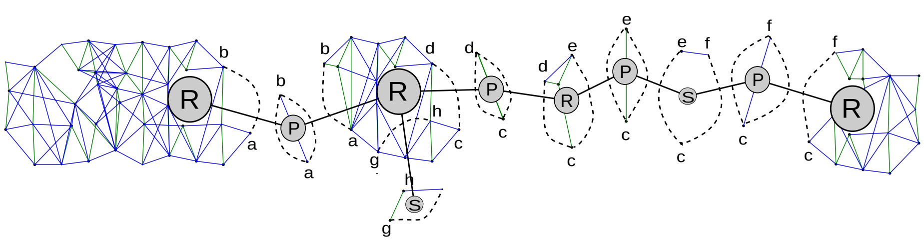


Figure 14: image

tree [@gutwenger_linear_2001].

Skeleton

- Each tree node of SPQR-tree is associated with a tri-connected component of G' called *skeleton*
- A skeleton represents a contraction of G' based on a set of *virtual edges*.
- A skeleton was classified into four types:
  - Series (S): the skeleton is a cycle graph.
  - Parallel (P): the skeleton contains only two vertices s and t, and k parallel edges between s and t where $k \geq 3$.
  - Trivial (Q): the skeleton contains only two vertices s and t, and two parallel edges between s and t, one of which is virtual and the other is real.
  - Rigid (R): the skeleton is a tri-connected graph of a type other than the above.

---

class: middle, center

Divide-and-Conquer Method

Divide-and-conquer method

Consists of three basic steps:

1. Divide a conflict graph into its tri-connected components.
2. Conquer each tri-connected component in a bottom-up manner.
3. Combine the solutions into a complete solution in a top-down manner.

We calculate two possible solutions for each component, namely (s, t) of the same color and (s, t) of the opposite color.

Bottom-up Conquering: S Type

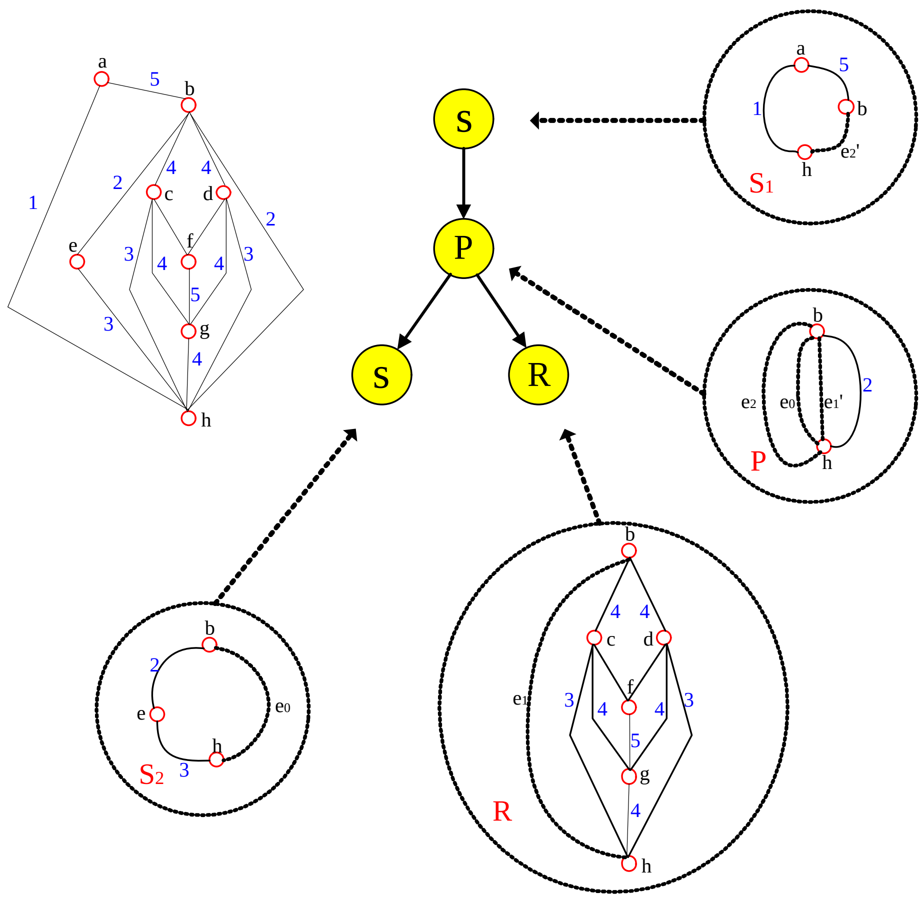


Figure 15: image

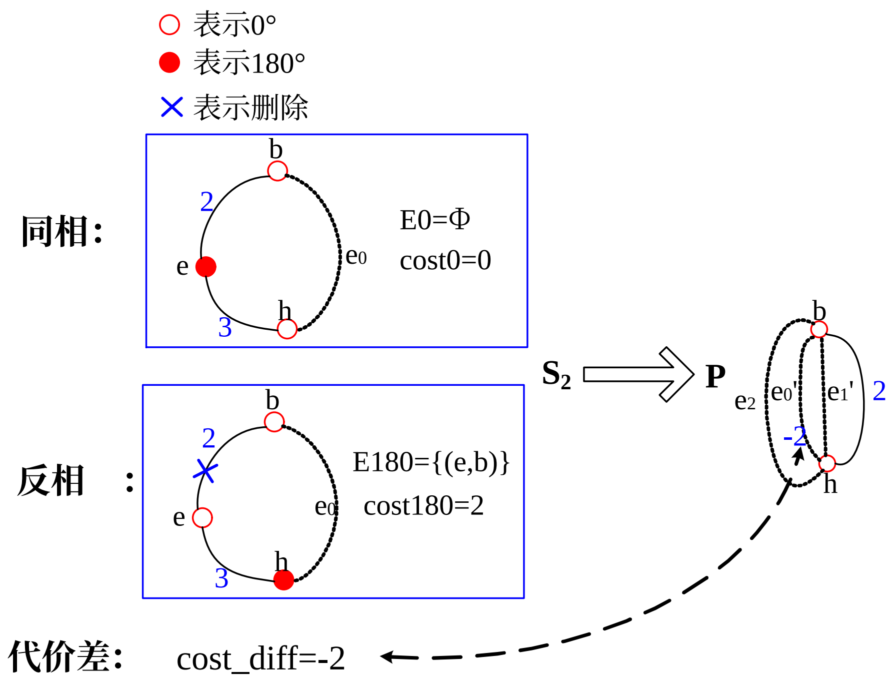


Figure 16: image

P Type

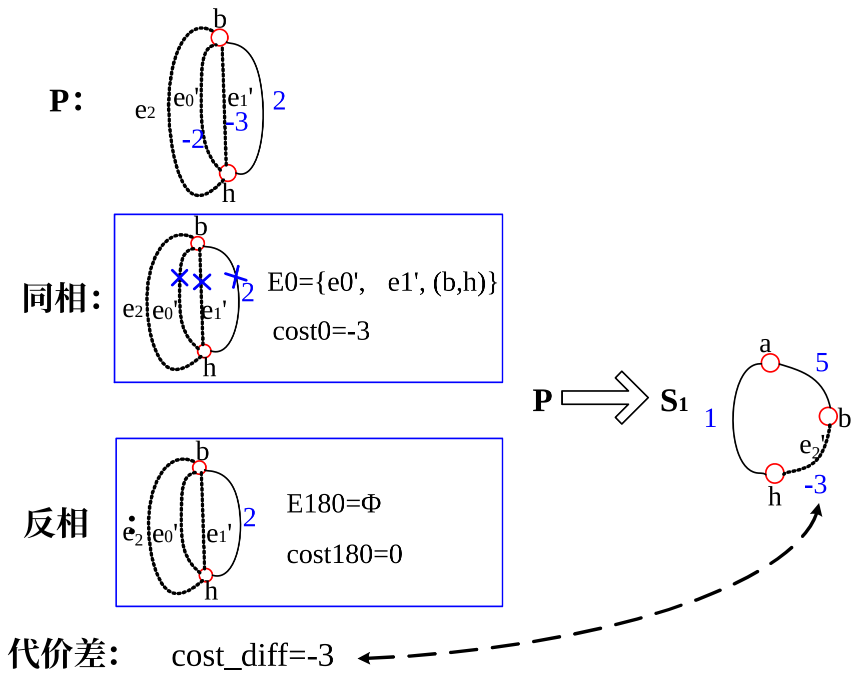


Figure 17: image

R Type

Top-down Merging

Node Splitting

- Node splitting (additional rectangle splitting) for resolving conflicts.
- To reduce the number of “cuts”, we apply node splitting after one color assignment and then recolor.

.pull-left[

Before:

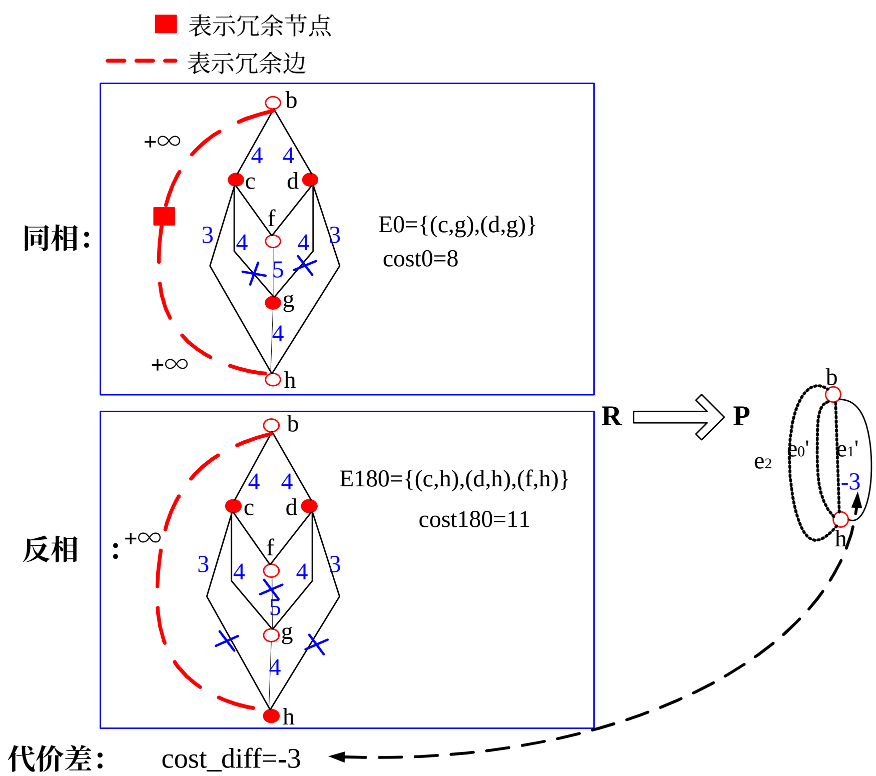


Figure 18: image

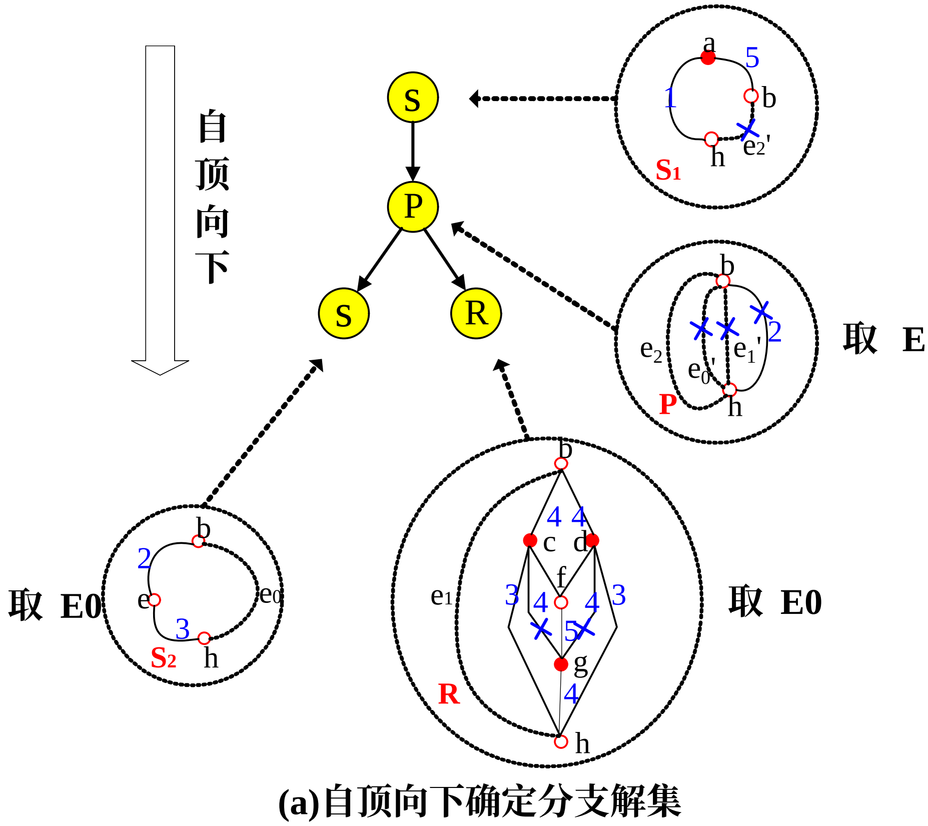


Figure 19: image

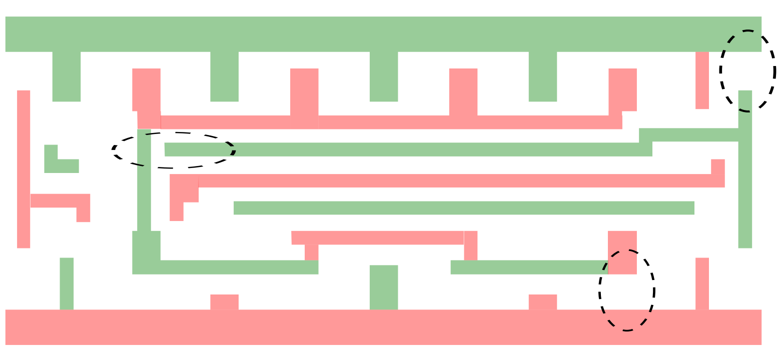


Figure 20: image

] .pull-right[

After:

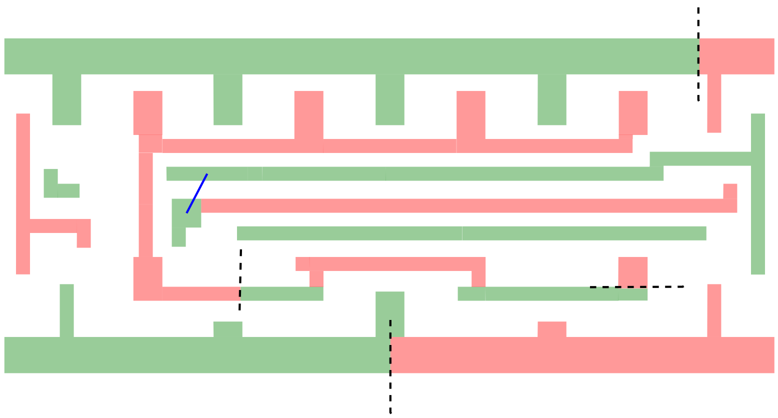


Figure 21: image

]

---

class: middle, center

Experimental Results

---

45 nm SDFFRS_X2 Layer 11, 9

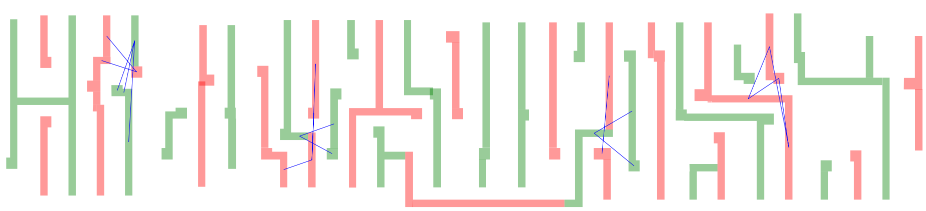


Figure 22: image

---

45 nm Example

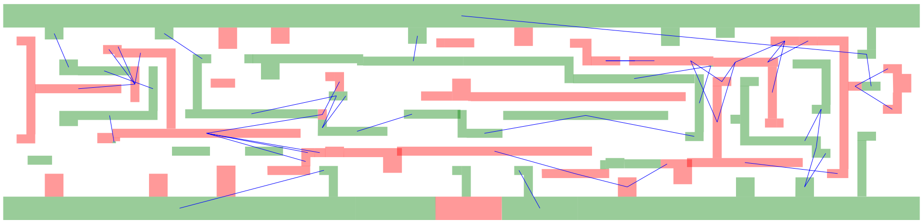


Figure 23: image

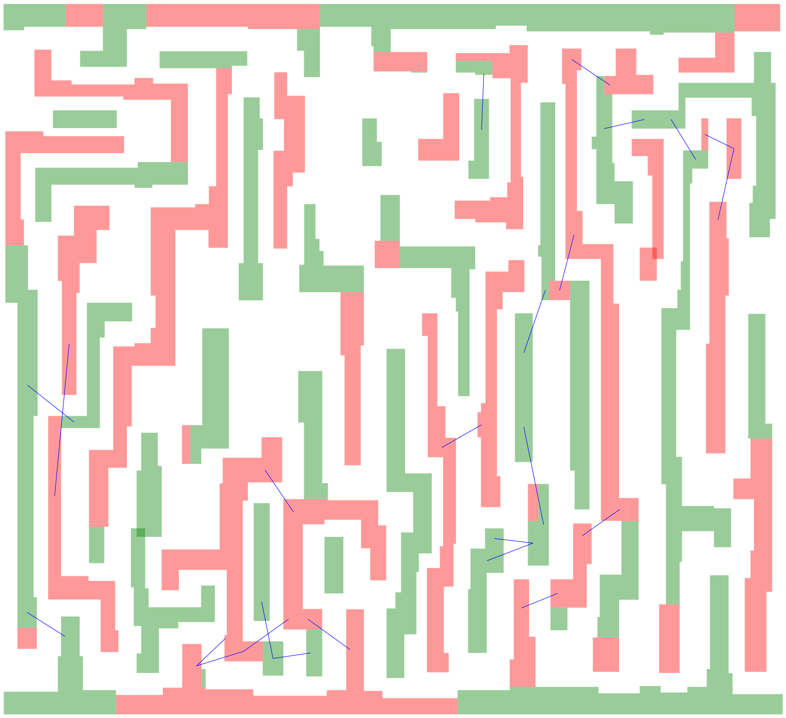


Figure 24: image

Random, 4K rectangles

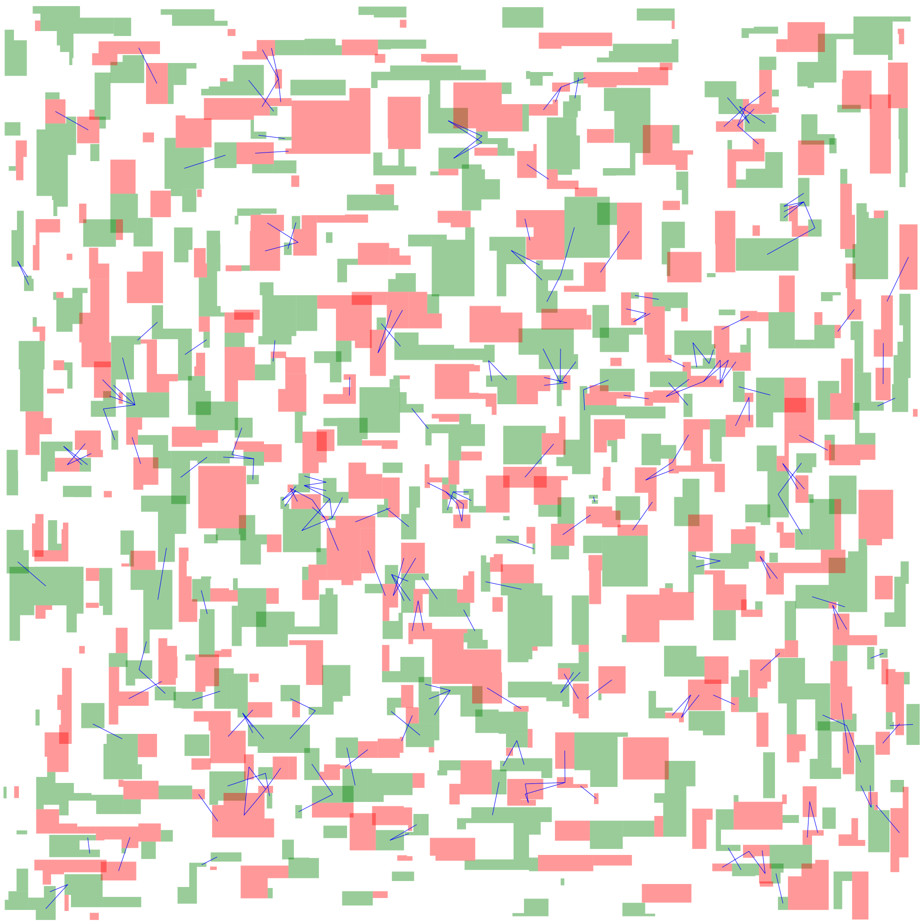
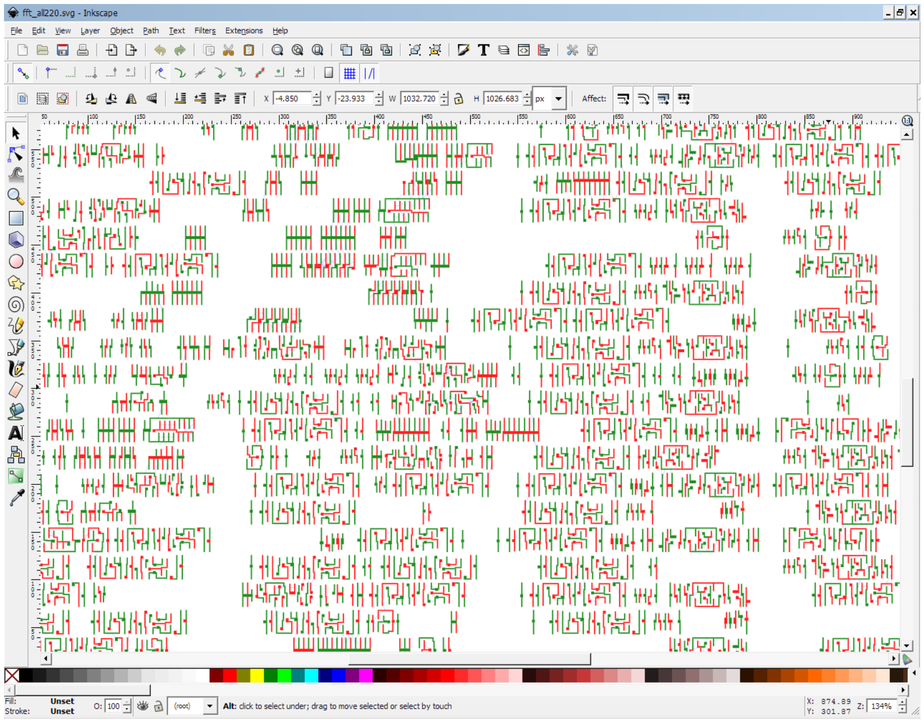


Figure 25: image



fft_all, 320K polygons

Experimental Results

Table 1: Experimental results of the runtime and cost reduction (with minimizing the number of stitches)

#poly	#nodes/#edges	w/ spqr	w/o spqr	time	cost
3631	31371/52060	13.29	38.25	65.3%	4.58%
9628	83733/138738	199.94	2706.12	92.6%	2.19%
18360	159691/265370	400.43	4635.14	91.4%	1.18%
31261	284957/477273	1914.54	9964.18	80.7%	1.61%
49833	438868/738759	3397.26	15300.9	77.8%	1.76%
75620	627423/1057794	3686.07	17643.9	79.1%	2.50%