When “Convex Optimization” Meets “Network Flow”

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Introduction

Overview

- Network flow problems can be solved efficiently and have a wide range of applications.
- Unfortunately, some problems may have other additional constraints that make them impossible to solve with current network flow techniques.
- In addition, in some problems, the objective function is quasi-convex rather than convex.
- In this lecture, we will investigate some problems that can still be solved by network flow techniques with the help of convex optimization.

Parametric Potential Problems

Parametric potential problems

Consider:

$$\begin{array}{ll} \text{maximize} & g(\beta), \\ \text{subject to} & y \leq d(\beta), \\ & Au = y, \end{array}$$

where $g(\beta)$ and $d(\beta)$ are concave.

Note: the parametric flow problems can be defined in a similar way.

Network flow says:

- For fixed β, the problem is feasible precisely when there exists no negative cycle
- Negative cycle detection can be done efficiently using the Bellman-Ford-like methods
- If a negative cycle C is found, then $\sum_{(i,j)\in C} d_{ij}(\beta) < 0$

Convex Optimization says:

- If both sub-gradients of $g(\beta)$ and $d(\beta)$ are known, then the *bisection method* can be used for solving the problem efficiently.
- Also, for multi-parameter problems, the *ellipsoid method* can be used.

Quasi-convex Minimization

Consider:

$$\begin{array}{ll} \text{maximize} & f(\beta), \\ \text{subject to} & y \leq d(\beta), \\ & Au = y, \end{array}$$

where $f(\beta)$ is *quasi-convex* and $d(\beta)$ are concave.

Example of Quasi-Convex Functions

- $\sqrt{|y|}$ is quasi-convex on $\mathbb{R}$
- $\log(y)$ is quasi-linear on $\mathbb{R}_{++}$
- $f(x, y) = xy$ is quasi-concave on $\mathbb{R}^2_{++}$
- Linear-fractional function:
  - $f(x) = (a^\mathsf{T} x + b)/(c^\mathsf{T} x + d)$
  - $\mathrm{dom}\, f = \{x \mid c^\mathsf{T} x + d > 0\}$
- Distance ratio function:
  - $f(x) = \|x - a\|_2 / \|x - b\|_2$

- dom $f = \{x \mid \|x - a\|_2 \leq \|x - b\|_2\}$

Convex Optimization says:

If f is quasi-convex, there exists a family of functions ϕ_t such that:

- $\phi_t(\beta)$ is convex w.r.t. β for fixed t
- $\phi_t(\beta)$ is non-increasing w.r.t. t for fixed β
- t-sublevel set of f is 0-sublevel set of ϕ_t, i.e., $f(\beta) \leq t$ iff $\phi_t(\beta) \leq 0$

For example:

- $f(\beta) = p(\beta)/q(\beta)$ with p convex, q concave $p(\beta) \geq 0$, $q(\beta) > 0$ on dom f,
- can take $\phi_t(\beta) = p(\beta) - t \cdot q(\beta)$

Convex Optimization says:

Consider a convex feasibility problem:

$$\begin{array}{ll} \text{find} & f(\beta), \\ \text{s. t.} & \phi_t(\beta) \leq 0, \\ & y \leq d(\beta), Au = y, \end{array}$$

- If feasible, we conclude that $t \geq p^*$;
- If infeasible, $t < p^*$.

Binary search on t can be used for obtaining p^*.

Quasi-convex Network Problem

- Again, the feasibility problem ([eq:quasi]) can be solved efficiently by the bisection method or the ellipsoid method, together with the negatie cycle detection technique.
- Any EDA's applications ???

Monotonic Minimization

- Consider the following problem:

$$\begin{array}{ll} \text{minimize} & \max_{ij} f_{ij}(y_{ij}), \\ \text{subject to} & Au = y, \end{array}$$

where $f_{ij}(y_{ij})$ is non-decreasing.

- The problem can be recast as:

$$\begin{array}{ll} \text{maximize} & \beta, \\ \text{subject to} & y \leq f^{-1}(\beta), \\ & Au = y, \end{array}$$

where $f^{-1}(\beta)$ is non-deceasing w.r.t. β.

E.g. Yield-driven Optimization

- Consider the following problem:

$$\begin{array}{ll} \text{maximize} & \min_{ij} \Pr(y_{ij} \leq \tilde{d}_{ij}) \\ \text{subject to} & Au = y, \end{array}$$

where $\tilde{d}_{ij}$ is a random variables.

- Equivalent to the problem:

$$\begin{array}{ll} \text{maximize} & \beta, \\ \text{subject to} & \beta \leq \Pr(y_{ij} \leq \tilde{d}_{ij}), \\ & Au = y, \end{array}$$

where $f_{ij}^{-1}(\beta)$ is non-deceasing w.r.t. β.

E.g. Yield-driven Optimization (II)

- Let $F(x)$ is the cdf of $\tilde{d}$.

- Then:

$$
\begin{aligned}
& \beta \leq \Pr(y_{ij} \leq \tilde{d}_{ij}) \leq t \\
\Rightarrow \quad & \beta \leq 1 - F_{ij}(y_{ij}) \\
\Rightarrow \quad & y_{ij} \leq F_{ij}^{-1}(1-\beta)
\end{aligned}
$$

- The problem becomes:

$$
\begin{array}{ll}
\text{maximize} & \beta, \\
\text{subject to} & y_{ij} \leq F_{ij}^{-1}(1-\beta), \\
& Au = y,
\end{array}
$$

Network flow says

- Monotonic problem can be solved efficiently using cycle-cancelling methods such as Howard's algorithm.

Min-cost flow problems

Min-Cost Flow Problem (linear)

Consider:

$$
\begin{array}{ll}
\min & d^{\mathsf{T}} x + p \\
\text{s. t.} & c^- \leq x \leq c^+, \\
& A^{\mathsf{T}} x = b, \; b(V) = 0
\end{array}
$$

- some c^+ could be $+\infty$ some c^- could be $-\infty$.
- A^{T} is the incidence matrix of a network G.

Conventional Algorithms

- Augmented-path based:
  - Start with an infeasible solution
  - Inject minimal flow into the augmented path while maintaining infeasibility in each iteration
  - Stop when there is no flow to inject into the path.
- Cycle cancelling based:

- Start with a feasible solution x_0
- find a better sol'n $x_1 = x_0 + \alpha \triangle x$, where α is positive and $\triangle x$ is a negative cycle indicator.

General Descent Method

1. **Input**: a starting $x \in \text{dom } f$
2. **Output**: x^*
3. **repeat**
    1. Determine a descent direction p.
    2. Line search. Choose a step size $\alpha > 0$.
    3. Update. $x := x + \alpha p$
4. **until** a stopping criterion is satisfied.

Some Common Descent Directions

- For convex problems, the search direction must satisfy $\nabla f(x)^\mathsf{T} p < 0$.
- Gradient descent:
    - $p = -\nabla f(x)^\mathsf{T}$
- Steepest descent:
    - $\triangle x^{nsd} = \text{argmin}\{\nabla f(x)^\mathsf{T} v \mid \|v\| = 1\}$.
    - $\triangle x^{sd} = \|\nabla f(x)\| \triangle x^{nsd}$ (un-normalized)
- Newton's method:
    - $p = -\nabla^2 f(x)^{-1} \nabla f(x)$

Network flow says (II)

- Here, there is a better way to choose p!
- Let $x := x + \alpha p$, then we have:

$$\begin{array}{lll} \min & d^\mathsf{T} x_0 + \alpha d^\mathsf{T} p & \Rightarrow d^\mathsf{T} < 0 \\ \text{s. t.} & -x_0 \leq \alpha p \leq c - x_0 & \Rightarrow \text{residual graph} \\ & A^\mathsf{T} p = 0 & \Rightarrow p \text{ is a cycle!} \end{array}$$

- In other words, choose p to be a negative cycle with cost d!
    - Simple negative cycle, or
    - Minimum mean cycle

Network flow says (III)

- Step size is limited by the capacity constraints:
  - $\alpha_1 = \min_{ij}\{c^+ - x_0\}$, for $\triangle x_{ij} > 0$
  - $\alpha_2 = \min_{ij}\{x_0 - c^-\}$, for $\triangle x_{ij} < 0$
  - $\alpha_{\text{lin}} = \min\{\alpha_1, \alpha_2\}$
- If $\alpha_{\text{lin}} = +\infty$, the problem is unbounded.

Network flow says (IV)

- An initial feasible solution can be obtained by a similar construction of the residual graph and cost vector.
- The LEMON package implements this cycle cancelling algorithm.

Min-Cost Flow Convex Problem

- Problem Formulation:

$$\begin{array}{ll} \min & f(x) \\ \text{s. t.} & 0 \le x \le c, \\ & A^{\mathsf{T}} x = b,\ b(V) = 0 \end{array}$$

Common Types of Line Search

- Exact line search: $t = \operatorname{argmin}_{t>0} f(x + t\triangle x)$
- Backtracking line search (with parameters $\alpha \in (0, 1/2), \beta \in (0, 1)$)
  - starting from $t = 1$, repeat $t := \beta t$ until

$$f(x + t\triangle x) < f(x) + \alpha t \nabla f(x)^{\mathsf{T}} \triangle x$$

  - graphical interpretation: backtrack until $t \le t_0$

Network flow says (V)

- The step size is further limited by the following:
  - $\alpha_{\text{cvx}} = \min\{\alpha_{\text{lin}}, t\}$
- In each iteration, choose $\triangle x$ as a negative cycle of G_x, with cost $\nabla f(x)$ such that $\nabla f(x)^{\mathsf{T}} \triangle x < 0$

Quasi-convex Minimization (new)

- Problem Formulation:

$$\begin{array}{ll} \min & f(x) \\ \text{s. t.} & 0 \leq x \leq c, \\ & A^\mathsf{T} x = b,\ b(V) = 0 \end{array}$$

- The problem can be recast as:

$$\begin{array}{ll} \min & t \\ \text{s. t.} & f(x) \leq t, \\ & 0 \leq x \leq c, \\ & A^\mathsf{T} x = b,\ b(V) = 0 \end{array}$$

Convex Optimization says (II)

- Consider a convex feasibility problem:

$$\begin{array}{ll} \text{find} & x \\ \text{s. t.} & \phi_t(x) \leq 0, \\ & 0 \leq x \leq c, \\ & A^\mathsf{T} x = b,\ b(V) = 0 \end{array}$$

  - If feasible, we conclude that $t \geq p^*$;
  - If infeasible, $t < p^*$.
- Binary search on t can be used for obtaining p^*.

Network flow says (VI)

- Choose $\triangle x$ as a negative cycle of G_x with cost $\nabla\phi_t(x)$
- If no negative cycle is found, and $\phi_t(x) > 0$, we conclude that the problem is infeasible.
- Iterate until x becomes feasible, i.e. $\phi_t(x) \leq 0$.

E.g. Linear-Fractional Cost

- Problem Formulation:

$$
\begin{array}{ll}
\min & (e^{\mathsf{T}} x + f)/(g^{\mathsf{T}} x + h) \\
\text{s. t.} & 0 \leq x \leq c, \\
& A^{\mathsf{T}} x = b,\ b(V) = 0
\end{array}
$$

- The problem can be recast as:

$$
\begin{array}{ll}
\min & t \\
\text{s. t.} & (e^{\mathsf{T}} x + f) - t(g^{\mathsf{T}} x + h) \leq 0 \\
& 0 \leq x \leq c, \\
& A^{\mathsf{T}} x = b,\ b(V) = 0
\end{array}
$$

Convex Optimization says (III)

- Consider a convex feasibility problem:

$$
\begin{array}{ll}
\text{find} & x \\
\text{s. t.} & (e - t \cdot g)^{\mathsf{T}} x + (f - t \cdot h) \leq 0, \\
& 0 \leq x \leq c, \\
& A^{\mathsf{T}} x = b,\ b(V) = 0
\end{array}
$$

  - If feasible, we conclude that $t \geq p^*$;
  - If infeasible, $t < p^*$.
- Binary search on t can be used for obtaining p^*.

Network flow says (VII)

- Choose $\triangle x$ to be a negative cycle of G_x with cost $(e - t \cdot g)$, i.e. $(e - t \cdot g)^{\mathsf{T}} \triangle x < 0$
- If no negative cycle is found, and $(e - t \cdot g)^{\mathsf{T}} x_0 + (f - t \cdot h) > 0$, we conclude that the problem is infeasible.
- Iterate until $(e - t \cdot g)^{\mathsf{T}} x_0 + (f - t \cdot h) \leq 0$.

E.g. Statistical Optimization

- Consider the quasi-convex problem:

$$\begin{array}{ll} \min & \Pr(\mathbf{d}^\mathsf{T} x > \alpha) \\ \text{s. t.} & 0 \le x \le c, \\ & A^\mathsf{T} x = b,\ b(V) = 0 \end{array}$$

- $\mathbf{d}$ is random vector with mean d and covariance Σ.
- Hence, $\mathbf{d}^\mathsf{T} x$ is a random variable with mean $d^\mathsf{T} x$ and variance $x^\mathsf{T} \Sigma x$.

Statistical Optimization

- The problem can be recast as:

$$\begin{array}{ll} \min & t \\ \text{s. t.} & \Pr(\mathbf{d}^\mathsf{T} x > \alpha) \le t \\ & 0 \le x \le c, \\ & A^\mathsf{T} x = b,\ b(V) = 0 \end{array}$$

Note:

$$\begin{array}{ll} & \Pr(\mathbf{d}^\mathsf{T} x > \alpha) \le t \\ \Rightarrow & d^\mathsf{T} x + F^{-1}(1-t) \|\Sigma^{1/2} x\|_2 \le \alpha \end{array}$$

(convex quadratic constraint w.r.t x)

Recall...

Recall that the gradient of $d^\mathsf{T} x + F^{-1}(1-t)\|\Sigma^{1/2} x\|_2$ is $d + F^{-1}(1-t)(\|\Sigma^{1/2} x\|_2)^{-1} \Sigma x$.

Problem w/ additional Constraints (new)

- Problem Formulation:

$$\begin{array}{ll} \min & f(x) \\ \text{s. t.} & 0 \le x \le c, \\ & A^\mathsf{T} x = b,\ b(V) = 0 \\ & \color{green}{s^\mathsf{T} x \le \gamma} \end{array}$$

E.g. Yield-driven Delay Padding

- Consider the following problem:

$$\begin{aligned} \text{maximize} \quad & \gamma\,\beta - c^{\mathsf{T}} p, \\ \text{subject to} \quad & \beta \leq \Pr(y_{ij} \leq \mathbf{d}_{ij} + p_{ij}), \\ & Au = y,\ p \geq 0 \end{aligned}$$

  - p: delay padding
  - γ: weight (determined by a trade-off curve of yield and buffer cost)
  - $\mathbf{d}_{ij}$: Gaussian random variable with mean d_{ij} and variance s_{ij}.

E.g. Yield-driven Delay Padding (II)

.pull-left[

- The problem is equivalent to:

$$\begin{aligned} \max \quad & \gamma\,\beta - c^{\mathsf{T}} p, \\ \text{s.t.} \quad & y \leq d - \beta s + p, \\ & Au = y, p \geq 0 \end{aligned}$$

]

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- or its dual:

$$\begin{aligned} \min \quad & d^{\mathsf{T}} x \\ \text{s.t.} \quad & 0 \leq x \leq c, \\ & A^{\mathsf{T}} x = b,\ b(V) = 0 \\ & s^{\mathsf{T}} x \leq \gamma \end{aligned}$$

]

Recall ...

- Yield drive CSS:

$$\begin{aligned} \max \quad & \beta, \\ \text{s.t.} \quad & y \leq d - \beta s, \\ & Au = y, \end{aligned}$$

- Delay padding

$$\begin{aligned} \max \quad & -c^\intercal p, \\ \text{s.t.} \quad & y \le d + p, \\ & Au = y,\ p \ge 0 \end{aligned}$$

Considering Barrier Method

- Approximation via logarithmic barrier:

$$\begin{aligned} \min \quad & f(x) + (1/t)\phi(x) \\ \text{s.t.} \quad & 0 \le x \le c, \\ & A^\intercal x = b,\ b(V) = 0 \end{aligned}$$

  - where $\phi(x) = -\log(\gamma - s^\intercal x)$
  - Approximation improves as $t \to \infty$
  - Here, $\nabla\phi(x) = s/(\gamma - s^\intercal x)$

Barrier Method

- **Input**: a feasible x, $t := t^{(0)}$, $\mu > 1$, tolerance $\varepsilon > 0$
- **Output**: x^*
- **repeat**
  1. Centering step. Compute $x^*(t)$ by minimizing $t\,f + \phi$
  2. Update $x := x^*(t)$.
  3. Increase t. $t := \mu t$
- **until** $1/t < \varepsilon$.

Note: Centering is usually done by Newton's method in general.

Network flow says (VIII)

In the centering step, instead of using the Newton descent direction, we can replace it with a negative cycle on the residual graph.

Useful Skew Design Flow

Useful Skew Design: Why vs. Why Not

Why not

Some common challenges when implementing useful skew design include:

- need more engineer training
- difficulty in building a balanced clock-tree
- uncertainty in how to handle process variation and multi-corner multi-mode issues …, etc.

Why

If these challenges are overcome and useful skew design is implemented correctly,

- it can lead to less time spent on timing issues
- get better chip performance or yield

Clock Arrival Time vs. Clock Skew

- Clock signal runs periodically.
- Thus, absolute clock arrival time u_i is not so important.
- Instead, the skew $y_{ij} = u_i - u_j$ is more important in this scenario.

Useful Skew Design vs. Zero-Skew Design

- "Critical cycle" instead of "critical path".
- "Negative cycle" instead of "negative slack".
- If there is a negative cycle, it means that there is no positive slack solution no matter how to schedule.
- Others are pretty much the same.
- Same design principle:
  - Always tackle the most critical one first!

Linear Programming vs. Network Flow Formulation

- Linear programming formulation
  - can handle more complex constraints
- Network flow formulation
  - usually more efficient
  - return the most critical cycle as a bonus
  - can handle quantized buffer delay (???)
- Anyway, timing analysis is much more time-consuming than the optimization solving.

Target Skew vs. Actual Skew

Don't mess up these two concepts:

- Target skew:
  - the skew we want to achieve in the scheduling stage.
  - Usually deterministic (we schedule a meeting at 10:00, rather than 10:00 $\pm$ 34 minutes, right?)
- Actual skew
  - the skew that the clock tree actually generates.
  - Can be formulated as a random variable.

A Simple Case

To warm up, let us start with a simple case:

- Assume equal path delay variations.
- Single-corner.
- Before a clock tree is built.
- No adjustable delay buffer (ADB).

Network

Definition (Network)

A *network* is a collection of finite-dimensional vector spaces of *nodes* and *edges/arcs*:

- $V = \{v_1, v_2, \cdots, v_N\}$, where $|V| = N$
- $E = \{e_1, e_2, e_3, \cdots, e_M\}$ where $|E| = M$

which satisfies 2 requirements:

1. The boundary of each edge is comprised of the union of nodes
2. The intersection of any edges is either empty or a boundary node of both edges.

Example

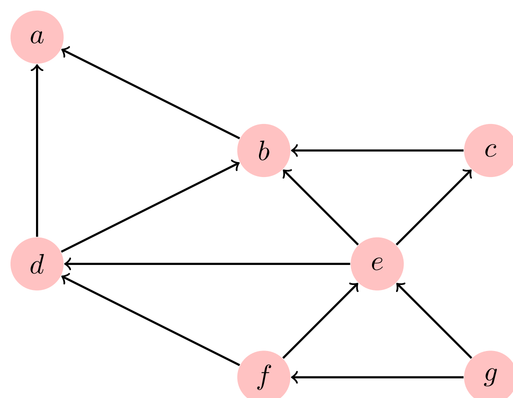


图 1: A network

Orientation

Definition (Orientation)

An *orientation* of an edge is an ordering of its boundary node (s, t), where

- s is called a source/initial node
- t is called a target/terminal node

Definition (Coherent)

Two orientations to be the same is called *coherent*

Node-edge Incidence Matrix

Definition (Incidence Matrix)

A $N \times M$ matrix A^{T} is a node-edge incidence matrix with entries:

$$A(i,j) = \begin{cases} +1 & \text{if } e_i \text{ is coherent with } v_j, \\ -1 & \text{if } e_i \text{ is not coherent with } v_j, \\ 0 & \text{otherwise.} \end{cases}$$

Example (II)

$$A^{\mathsf{T}} = \begin{bmatrix} 0 & -1 & 1 & 1 & 0 \\ 1 & 1 & 0 & -1 & -1 \\ -1 & 0 & -1 & 0 & 1 \end{bmatrix}$$

Timing Constraint

- Setup time constraint

$$y_{\text{skew}}(i,f) \leq T_{\text{CP}} - D_{if} - T_{\text{setup}} = u_{if}$$

  While this constraint destroyed, cycle time violation (zero clocking) occurs.
- Hold time constraint

$$y_{\text{skew}}(i,f) \geq T_{\text{hold}} - d_{if} = l_{if}$$

  While this constraint destroyed, race condition (double clocking) occurs.

Timing Constraint Graph

- Create a graph (network) by
  - replacing the hold time constraint with an *h-edge* with cost $-(T_{\text{hold}} - d_{ij})$ from FF_i to FF_j, and
  - replacing the setup time constraint with an s-edge with cost $T_{\text{CP}} - D_{ij} - T_{\text{setup}}$ from FF_j to FF_i.
- Two sets of constraints stemming from clock skew definition:

- The sum of skews for paths having the same starting and ending flip-flop to be the same;
- The sum of clock skews of all cycles to be zero

Timing Constraint Graph (TCG)

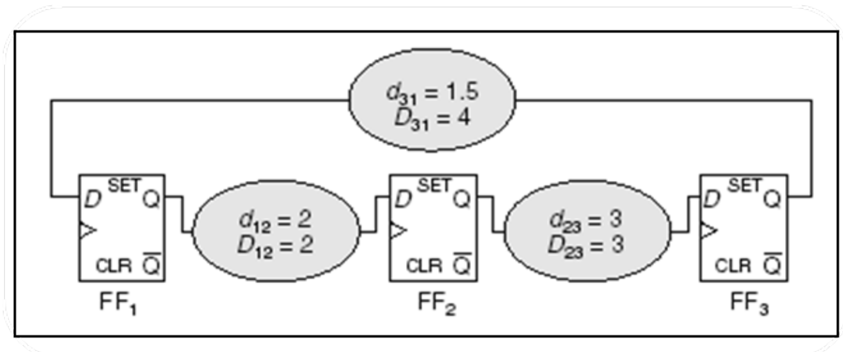
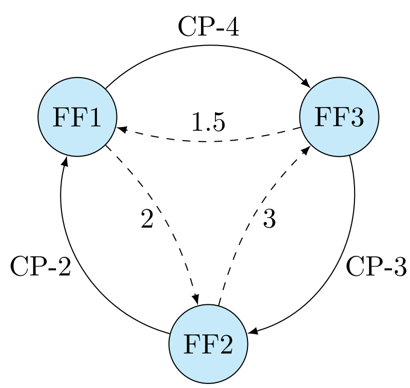


图 2: Example circuit



First Thing First

Meet all timing constraints

- Find y in $\{y \in \mathbb{R}^n \mid y \leq d, A\,u = y\}$
- How to solve:
  1. Find a negative cycle, fix it.

2. Iterate until no negative cycle is found.

- Bellman-Ford-like algorithm (and its variants are publicly available):
  - Strongly suggest “Lazy Evaluation”:
    - Don’t do full timing analysis on the whole timing graph at the beginning!
    - Instead, perform timing analysis only when the algorithm needs.
  - Stop immediately whenever a negative cycle is detected.

Delay Padding (DP)

- Delay padding is a technique that fixes the timing issue by intentionally **solely** “increasing” delays.
- Usually formulated as:
  - Find p, y in $\{p, y \in \mathbb{R}^n \mid y \leq d + p, A\, u = y, p \geq 0\}$
- If the objective is to minimize the sum of p, then the problem is the dual of the standard *min-cost flow* problem, which can be solved efficiently by the *network simplex* algorithm (publicly available).
- Beautiful right?

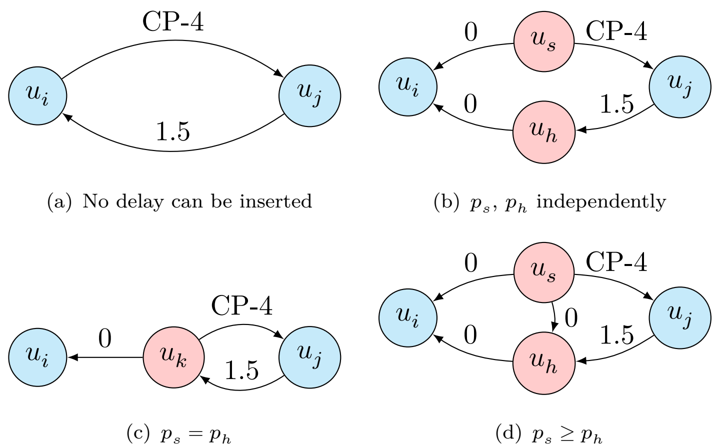
Delay Padding (II)

- No, the above formulation is impractical.
- In modern design, “inserting” a delay may mean swapping a faster cell with a slower cell from the cell library. Thus, no need to minimize the sum of p.
- More importantly, it may not be possible to find a position to insert delay for some delay paths.
- Some papers consider only allowing insert delays to the max-delay path only. Some papers consider only allowing insert delays to both the max- and min-delay paths together only. None of them are perfect.

Delay Padding (III)

- My suggestion. Instead of calculating the necessary $p's$ and then look for the suitable position to insert, it is easier (and more flexible) to determine the position first and then calculate the suitable values.
- It can be achieved by modifying the timing graph and solve a feasibility problem. Easy enough!
- Quantized delay can be handled too (???).

Four possible ways to insert delay



(a) No delay can be inserted

(b) p_s, p_h independently

(c) $p_s = p_h$

(d) $p_s \geq p_h$

图 3:

Delay Padding (cont'd)

- If there exists a negative cycle in the modified timing graph, it implies that the timing problem cannot be fixed by simply the delay padding technique.
  - Then, try decrease D_{ij}, or increase T_{CP}
- Be aware of the min-delay path is still the min-delay path after a certain amount of delay is inserted (how???).

Variation Issue

Yield-driven Clock Skew Scheduling

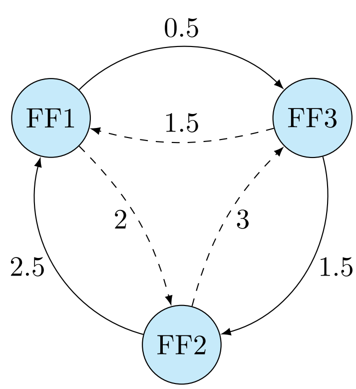
- Assume all timing issues are fixed.
- Now, how to schedule the arrival times to maximize yield?
- According to the critical-first principle, we seek for the most critical cycle first.
- The problem can be formulated as:
  - $\max\{\beta \in \mathbb{R} \mid y \leq d - \beta, A\,u = y\}$.
- It is equivalent to the *minimum mean cycle* problem, which can be solved efficiently by for example *Howard's algorithm* (publicly available).

Minimum Balancing Algorithm

- Then we evenly distribute the slack on this cycle.
- To continue the next most critical cycle, we contract the first one into a "super vertex" and repeat the process.
- The process stops when the timing graph remains only a single vertex.
- The overall method is known as *minimum balancing* (MB) algorithm in the literature.

Example: Most timing-critical cycle

The most vulnerable timing constraint



Example: Distribute the slack

- Distribute the slack evenly along the most timing-critical cycle.

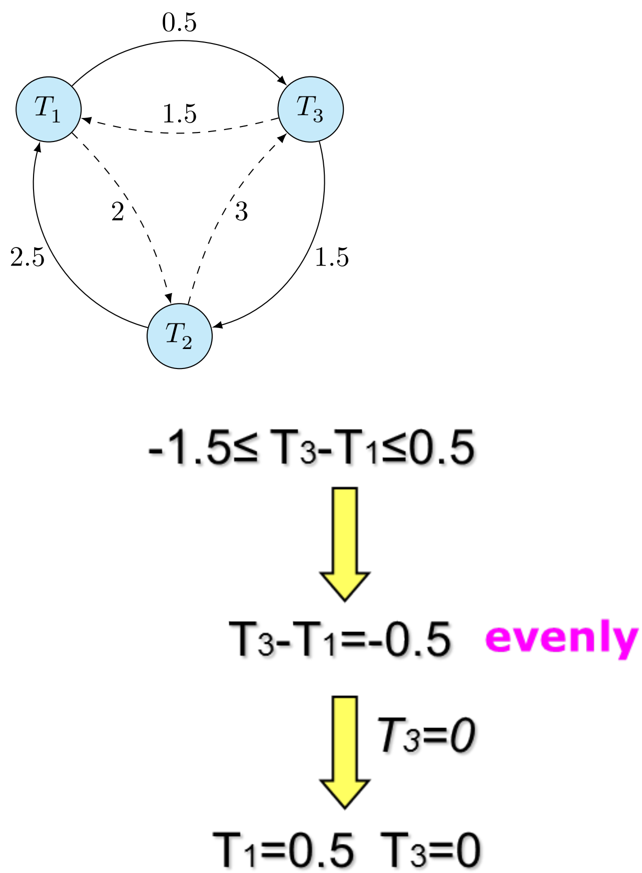


图 4: img

Example: Distribute the slack (cont'd)

- To determine the optimal slacks and skews for the rest of the graph, we replace the critical cycle with a super vertex.

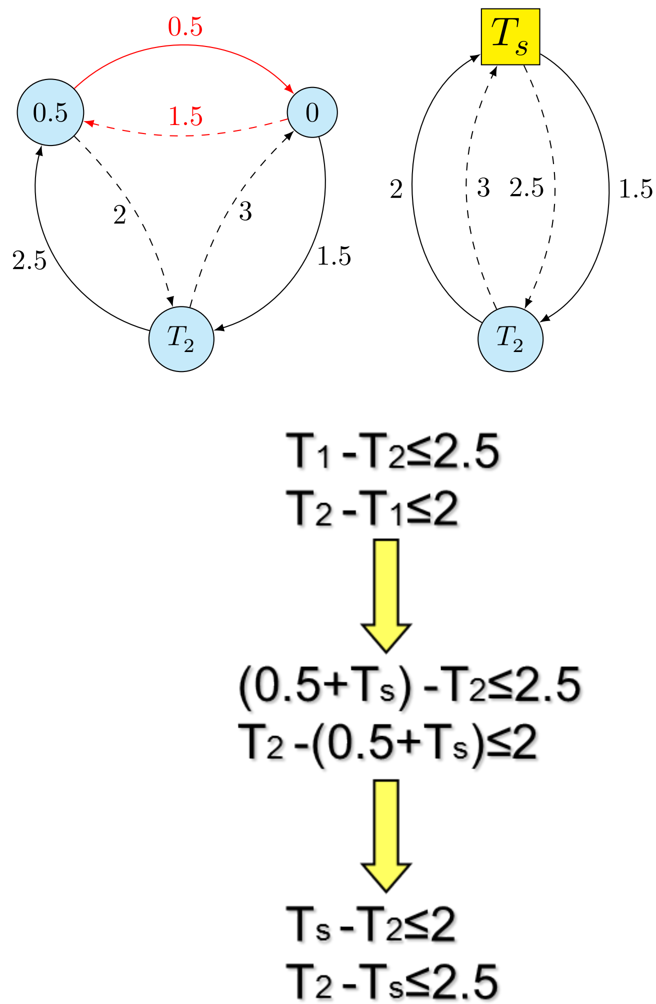
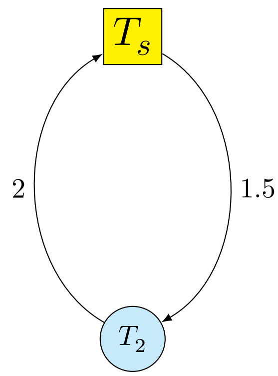

0.5
0.5
1.5
0
2
3
2.5
1.5
T_2
T_s
2
3
2.5
1.5
T_2
$T_1 - T_2 \le 2.5$
$T_2 - T_1 \le 2$
$(0.5+T_s) - T_2 \le 2.5$
$T_2 - (0.5+T_s) \le 2$
$T_s - T_2 \le 2$
$T_2 - T_s \le 2.5$

图 5: img

Repeat the process iteratively



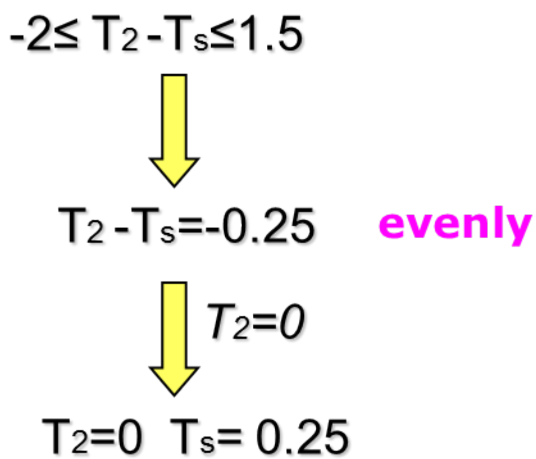


图 6: img

Repeat the process iteratively (II)

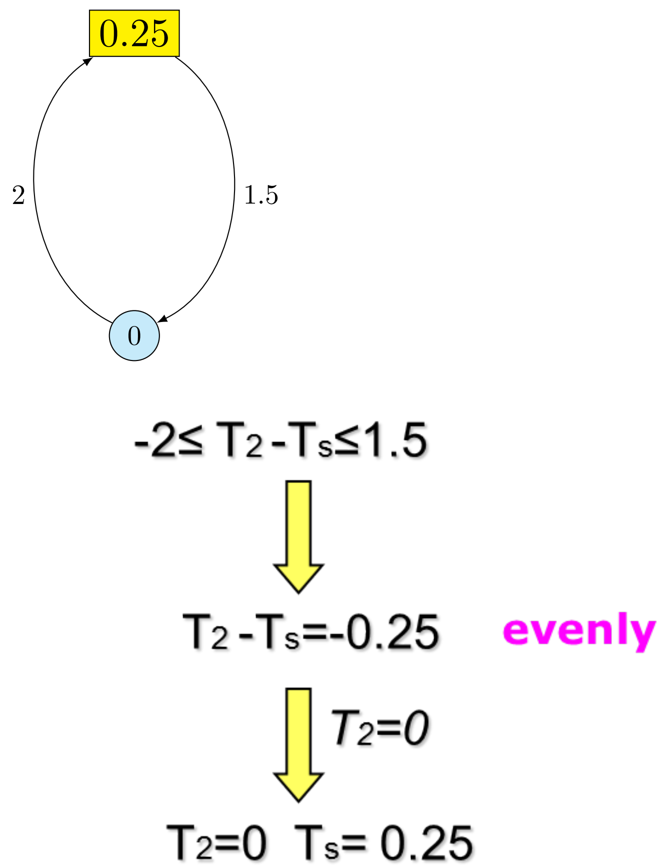


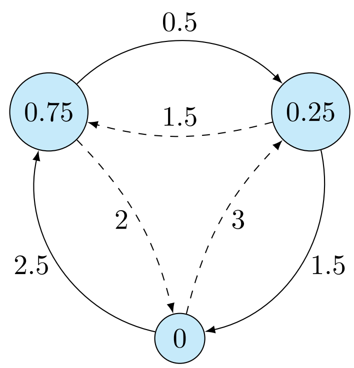
图 7: img

Final result

- Skew_{12} = 0.75
- Skew_{23} = -0.25
- Skew_{31} = -0.5
- Slack_{12} = 1.75
- Slack_{23} = 1.75

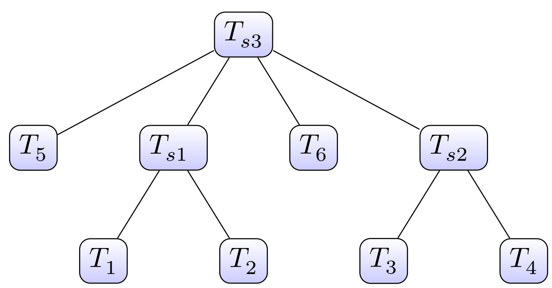
- $\text{Slack}_{31} = 1$

  where $\text{Slack}_{ij} = \text{CP} - \text{D}_{ij} - \text{T}_{\text{setup}} - \text{Skew}_{ij}$



What the MB algorithm really give us?

- The MB algorithm not only give us the scheduling solution, but also a tree-topology that represents the order of "criticality"!



Clock-tree Synthesis and Placement

- I strongly suggest that the topology of the clock-tree precisely follows the order of "criticality"!
  - since the lower branch of clock-tree has smaller skew variation.
- I also suggest that the placer should follow the topology of the clock-tree:
  - Physically place the registers of the same branch together.
  - The locality implies stronger correlation of variations and implies even smaller skew variation due to the cancellation effect.

- Note that the current SSTA does not provide the correlation information, so this is the best you can do!

Second Example: Yield-driven Clock Skew Scheduling

- Now assume that SSTA (or STA+OCV, POCV, AOCV) is performed.
- Let ($\bar{d}$, s) be the (mean, variance) of $\mathbf{d}$
- The most critical cycle can be obtained by solving:
  - $\max\{\beta \in \mathbb{R} \mid y \leq \bar{d} - \beta s, A\,u = y\}$
- It is equivalent to the minimum cost-to-time ratio cycle problem, which can be solved efficiently by for example Howard's algorithm (publicly available).
- Gaussian distribution is assumed. For arbitrary distribution, see my DAC'08 paper.

What About the Correlation?

- In the above formulation, we minimum the maximum possibility of timing violation of each *individual* timing constraint. So only individual delay distribution is needed.
- Yes, the objective function is not the true timing-yield. But it is reasonable, easy to solve, and is the best you can do so far.

Multi-Corner Issue

Meet all timing constraints in Multi-Corner

- Assume no Adjustable Delay Buffer (ADB)
- Find y in $\{y \in \mathbb{R}^n \mid y \leq d^{(k)}, A\,u = y, \forall k \in [1..K]\}$
- Equivalent to finding y in $\{y \in \mathbb{R}^n \mid y \leq \min_k\{d^{(k)}\}, A\,u = y\}$
- Feasibility problem
- How to solve:
  1. Find a negative cycle, fix it.
  2. Iterate until no negative cycle is found.

- Better avoid fixing the timing issue corner-by-corner. Inducing ping-pong effect.

Delay padding (DP) in Multi-Corner

- The problem CANNOT be formulated as a network flow problem. But still you can solve it by a linear programming formulation.
- Or, decompose the problem into sub-problems for each corner.
- Again use the modified timing graph technique.
- Then, y's are shared variables of sub-problems.
- If we solve each sub-problem individually, the solution will not agree with each other. Induce *ping-pong effect*.
- Need something to drive the agreement.

Delay Padding (DP) in Multi-Corner (cont'd)

- Follow the idea of *dual decomposition*: If a solution is above the average. then introduce a punishment cost. If a solution is below the average, then introduce a rewarding cost.
- Then, each subproblem is a min-cost potential problem, which can be solved efficiently.
- If some subproblems do not have feasible solutions, it implies that the problem cannot be fixed by simply delay padding.
- The process repeats until all solutions converge. If not, it implies that the problem cannot be fixed by simply delay padding.

Yield-driven Clock Skew Scheduling

- $\max\{\beta \in \mathbb{R} \mid y \leq d^{(k)} - \beta s, A\, u = y, \forall k \in [1..K]\}$
- More or less the same as in Single Corner.

Clock-Tree Issue

Clock Tree Synthesis (CTS)

- Construct merging location

- DME algorithm, Elmore delay, buffer insertion

- Some research on *bounded-skew DME algorithm*. But the algorithm is too complicated in my opinion.
- If the previous stage is over-optimized, the clock tree is hard to implement. If it happens, some budgeting techniques should be invoked (engineering issue)
- After a clock tree is constructed, more detailed timing (rather than Elmore delay) can be obtained via timing analysis.

Co-optimization Issue

- After a clock tree is built, we have a clearer picture.
- Should I perform the re-scheduling? And how?
- Some papers suggest adding a factor to the timing constraint, say:

$$1.2u_i - 0.8u_j \leq w_{ij}$$

.

- Then the formulation is not a kind of network-flow, but may still be solvable by linear programming.
- Need to investigate more deeply.

Adjustable Delay Buffer Issue

Adjustable delay buffers in Multi-Mode

- Assume adjustable delay buffers are added solely to the clock tree
- Hence, each mode can have a different set of arrival times.
- Easier for clock skew scheduling, harder for clock-tree synthesis.

Meet timing constraint in Multi-Mode:

- find $y^{(m)}$ in $\{y^{(m)} \in \mathbb{R}^n \mid y^{(m)} \leq d^{(m)}, A\,u^{(m)} = y^{(m)}, \forall m \in [1..M]\}$
- Can be done in parallel.
- find a negative cycle, fix it (do not need to know all $d_i^{(m)}$ at the beginning) for every mode in parallel.

Delay Padding (DP) in Multi-mode

- Again use a modified timing graph technique.
- NOT a network flow problem. Use LP, or
- Dual decomposition -> min-cost potential problem for each mode
  - Only p's are shared variables.
  - Initial feasible solution obtained by the single-mode method
    - A negative cycle => problem cannot be fixed by DP
- Not converge => problem cannot be fixed by DP
  - Try decrease D_{ij}, or increase T_{CP}

Yield-driven Clock Skew Scheduling

- $\max\{\beta \in \mathbb{R} \mid y^{(m)} \leq d^{(m)} - \beta s, A\, u^{(m)} = y^{(m)}, \forall m \in [1..M]\}$
- Pretty much the same as Single-Mode.

Difficulty in ADB Multi-Mode Design

- How to design the clock-tree?
- What is the order of criticality?
- How to determine the minimum range of ADB?