Lecture 05b - Clock Skew Scheduling Under Process Variations (2)

Overview

- A Review of CSS Issues
- General Formulation
- Yield-driven Clock Skew Scheduling
- Numerical Results

Minimum Clock Period Problem

- Linear programming (LP) formulation

$$\begin{array}{ll} \text{minimize} & T_{\text{CP}} \\ \text{subject to} & l_{ij} \leq T_i - T_j \leq u_{ij} \end{array}$$

  where FF_i and FF_j are sequentially adjacent to each other.

- The above constraints are called *system of difference constraints* (see Introduction to Algorithms, MIT):
  - Key: it is easy to check if a feasible solution exists by detecting negative cycles using the Bellman-Ford algorithm.

System of Difference Constraints

- In some cases, you may need to do some transformations, e.g.
  - $T_i \leq \min_k\{T_k + a_{ik}\} \rightarrow T_i - T_k \leq a_{ik},\ \forall k$
  - $T_i \geq \max_k\{T_k + b_{ik}\} \rightarrow b_{ik} \leq T_i - T_k,\ \forall k$

Slack Maximization (EVEN)

- Slack Maximization Scheduling

$$\begin{array}{ll} \text{maximum} & t \\ \text{subject to} & T_j - T_i \leq \mu_{ij} - t \end{array}$$

  (Note: μ_{ij} is not equal to μ_{ji})

- is equivalent to the so-called *minimum mean cycle problem* (MMC), where:
  - $t^* = \sum_{(i,j)\in C} \mu_{ij}/|C|$,
  - C: critical cycle (first negative cycle)
- Can be efficiently solved by the parametric shortest path methods.

Slack Maximization (C-PROP)

- Slack Maximization Scheduling

$$\begin{array}{ll} \text{maximum} & t \\ \text{subject to} & T_j - T_i \leq \mu_{ij} - \sigma_{ij} t \end{array}$$

  (we show the correctness later)

- is equivalent to the *minimum cost-to-time ratio problem* (MCR), where:
  - $t^* = \sum_{(i,j)\in C} \mu_{ij} / \sum_{(i,j)\in C} \sigma_{ij}$,
  - C: critical cycle

General Formulation

- General form:

$$\begin{array}{ll} \text{maximum} & g(t) \\ \text{subject to} & T_i - T_j \leq f_{ij}(t),\ \forall (i,j) \in E \end{array}$$

  where $f_{ij}(t)$ a linear function that represents various problems defined above.

Problem	$g(t)$	$f_{ij}(t)$ (setup)	$f_{ji}(t)$ (hold)
Min. CP	$-t$	$t - D_{ij} - T_{\text{setup}}$	$-T_{\text{hold}} + d_{ij}$
EVEN	t	$T_{\text{CP}} - D_{ij} - T_{\text{setup}} - t$	$-T_{\text{hold}} + d_{ij} - t$
C-PROP	t	$T_{\text{CP}} - D_{ij} - T_{\text{setup}} - \sigma_{ij} t$	$-T_{\text{hold}} + d_{ij} - \sigma_{ij} t$

General Formulation (cont'd)

- In fact, $g(t)$ and $f_{ij}(t)$ are not necessarily linear functions. Any monotonic decreasing function will do.
- Theorem: if $g(t)$ and $f_{ij}(t)$ are *monotonic decreasing* functions for all i and j, then there is a unique solution to the problem. (prove later).
- Question 1: Does this generalization have any application?
- Question 2: What if $g(t)$ and $f_{ij}(t)$ are convex but not monotone?

Non-Gaussian Distribution

- 65nm and below, the path delay is likely to have a non-Gaussian distribution:

  Note: central limit theorem does not apply because

  - random variables are correlated (why?)
  - delays are non-negative

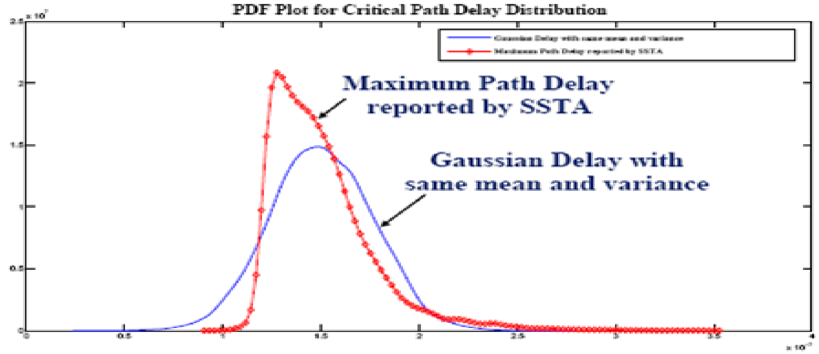


Figure 1: image

Timing Yield Maximization

- Formulation:
  - $\max\{\min\{\Pr\{T_j - T_i \leq \tilde{W}_{ij}\}\}\}$
  - is not exactly timing yield but reasonable.
- It is equivalent to:

$$\begin{array}{ll} \text{maximum} & t \\ \text{subject to} & T_i - T_j \leq T_{\text{CP}} - F_{ji}^{-1}(t) \\ & T_j - T_i \leq F_{ij}^{-1}(1-t) \end{array}$$

  where $F_{ij}(\cdot)$ is CDF of $\tilde{W}_{ij}$

- Luckily, any CDF must be a monotonic increasing function.

Statistical Interpretations of C-PROP

- Reduce to C-PROP when $\tilde{W}_{ij}$ is Gaussian, or precisely

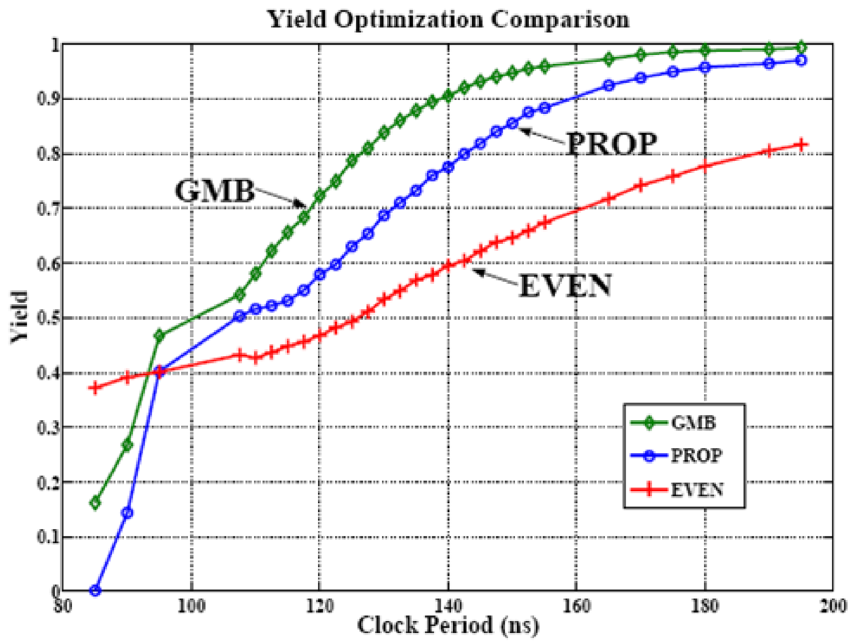
$$F_{ij}(x) = K((x - \mu_{ij})/\sigma_{ij})$$

- EVEN: identical distribution up to shifting

$$F_{ij}(x) = H(x - \mu_{ij})$$

  Not necessarily worse than C-PROP

Comparison



Three Solving Methods in General

- Binary search based
  - Local convergence is slow.
- Cycle based
  - Idea: if a solution is infeasible, there exists a negative cycle which can always be "zero-out" with minimum effort (proof of optimality)
- Path based
  - Idea: if a solution is feasible, there exists a (shortest) path from where we can always improve the solution.

Parametric Shortest Path Algorithms

- Lawler's algorithm (binary search)
- Howard's algorithm (based on cycle cancellation)
- Hybrid method
- Improved Howard's algorithm
- Input:

  - Interval [tmin, tmax] that includes t*
  - Tol: tolerance
  - G(V, E): timing graph
- Output:
  - Optimal t* and its corresponding critical cycle C

Lawler's Algorithm

```
@startuml
 while ((tmax - tmin) > tol)
    : t := (tmin + tmax) / 2;
    if (a neg. cycle C under t exists) then
       : tmax := t;
    else
       : tmin := t;
    endif
 endwhile
 : t* := t;
@enduml
```

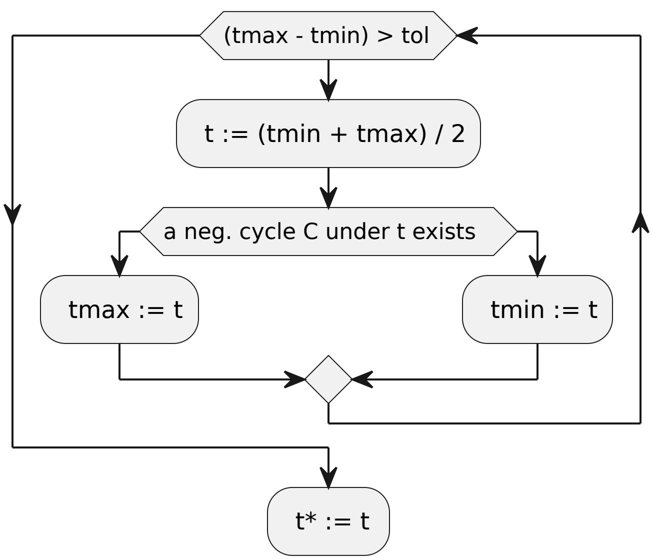


Figure 2: image

Howard's Algorithm

```
@startuml
```

```
 : t := tmax;
 while (a neg. cycle C under t exists)
    : find t' such that
      sum{(i,j) in C | fij(t')} = 0;
    : t := t';
 endwhile
 : t* := t;
@enduml
```

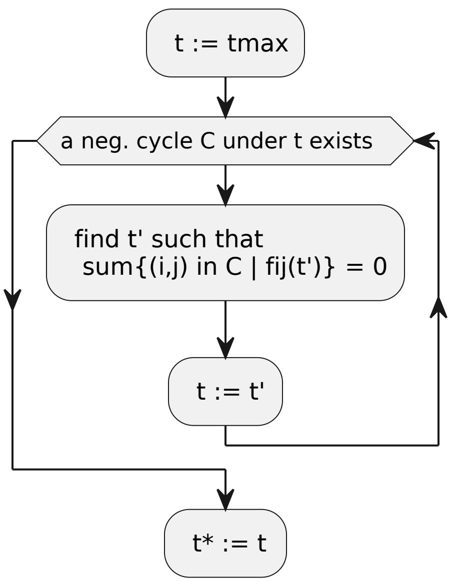


Figure 3: image

Hybrid Method

```
@startuml
 while ((tmax - tmin) > tol)
    : t := (tmin + tmax) / 2;
    if (a neg. cycle C under t exists) then
       : find t' such that
         sum{(i,j) in C | fij(t')} = 0;
       : t := t';
       : tmax := t;
    else
       : tmin := t;
    endif
 endwhile
```

```
 : t* := t;
@enduml
```

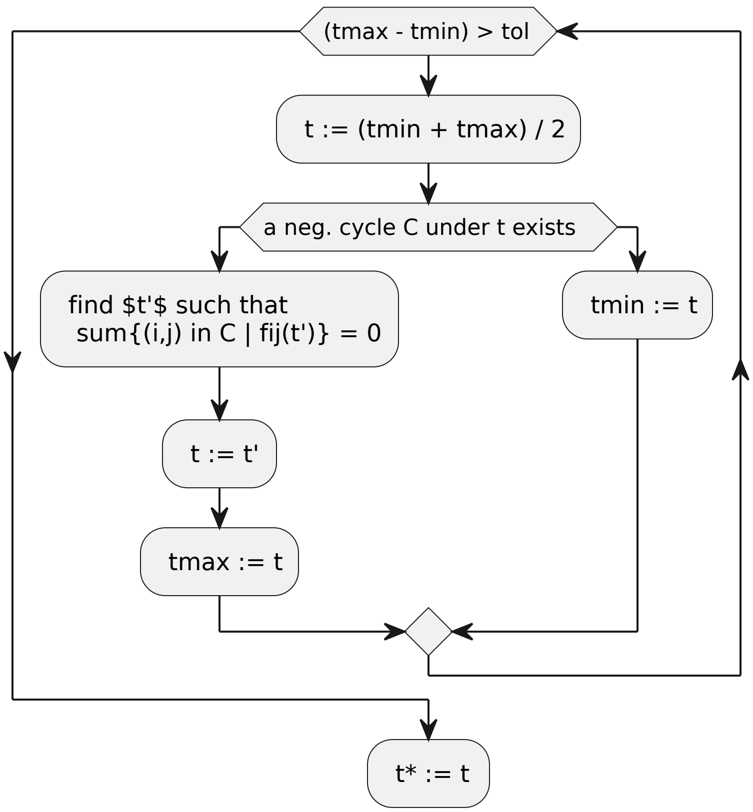
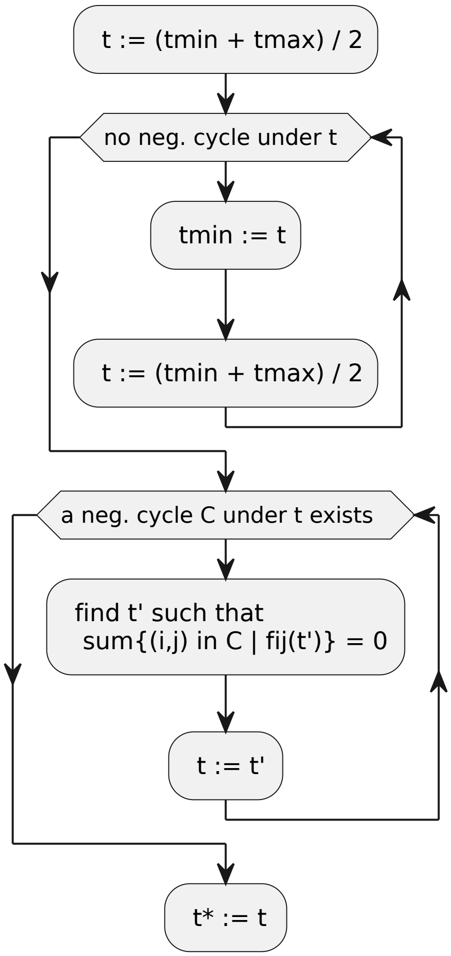


Figure 4: image

Improved Howard's Algorithm

```
@startuml
 : t := (tmin + tmax) / 2;
 while (no neg. cycle under t)
    : tmin := t;
    : t := (tmin + tmax) / 2;
 endwhile
 while (a neg. cycle C under t exists)
    : find t' such that
      sum{(i,j) in C | fij(t')} = 0;
    : t := t';
 endwhile
```

```
 : t* := t;
@enduml
```



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Clock Skew Scheduling for Unimodal Distributed Delay Models

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Useful Skew Design: Why and Why not?

Bad :

- Needs more engineer training.
- Balanced clock-trees are harder to build.
- Don't know how to handle process variation, multi-corner multi-mode, ..., etc.

Good :

If you do it right,

- spend less time struggling about timing, or
- get better chip performance or yield.

What can modern STA tools do today?

- Manually assign clock arrival times to registers (all zeros by default)
- Grouping: Non-critical parts can be grouped as a single unit. In other words, there is no need for full-chip optimization.
- Takes care of multi-cycle paths, slew rate, clock-gating, false paths etc. All we need are the reported **slacks**.
- Provide 3-sigma statistics for slacks/path delays (POCV).
- However, the full probability density function and correlation information are not available.

Unimodality

- In statistics, a unimodal probability distribution or unimodal distribution is a probability distribution with a single peak.
- In continuous distributions, unimodality can be defined through the behavior of the cumulative distribution function (cdf). If the cdf is *convex* for $x < m$ and *concave* for $x > m$, then the distribution is unimodal, m being the *mode*.
- Examples
  - Normal distribution
  - Log-normal distribution
  - Log-logistic distribution
  - Weibull distribution

Quantile function

- The quantile function z_p of a distribution is the inverse of the cumulative distribution function $\Phi^{-1}(p)$.
- Close-form expression for some unimodal distributions:
  - Normal: $\mu + \sigma\sqrt{2}\mathrm{erf}^{-1}(2p-1)$
  - Log-normal: $\exp\left(\mu + \sigma\sqrt{2}\mathrm{erf}^{-1}(2p-1)\right)$

- Log-logistic: $\alpha\left(\frac{p}{1-p}\right)^{1/\beta}$
- Weibull: $\lambda(-\ln(1-p))^{1/k}$

- For log-normal distribution:
  - mode: $\exp(\mu-\sigma^2)$
  - CDF at mode: $1/2(1+\mathrm{erf}(-\sigma/\sqrt{2}))$

Normal vs. Log-normal Delay Model

Normal/Gaussian:

- Convertible to a linear network optimization problem.
- Supported over the whole real line. Negative delays are possible.
- Symmetric, obviously not adaptable to the 3-sigma results.

Log-normal:

- Non-linear, but still can be solved efficiently with network optimization.
- Supported only on the positive side.
- Non-symmetric, may be able to fit into the 3-sigma results. (???)

Setup- and Hold-time Constraints

- Let $T_{\text{skew}}(i,f)=t_i-t_f$, where
  - t_i: clock signal delay at the initial register
  - t_f: clock signal delay at the final register
  - Assume in zero-skew, i.e. $T_{\text{skew}}(i,f)=0$, the reported setup- and hold-time slacks are S_{if} *and* H_{if} respectively.
- Then, in useful skew design:

$$T_{\text{skew}}(i,f)\le S_{if} \implies t_i-t_f\le S_{if}$$

$$T_{\text{skew}}(i,f)\ge -H_{if} \implies t_f-t_i\le H_{if}$$

- In principle, H_{if} and $T_{\text{CP}}-S_{if}$ represent the minimum- and maximum-path delay, and should be always greater than zero.
- Let $D_{if}=T_{\text{CP}}-S_{if}$

Yield-driven Optimization

- Max-Min Formulation:
  - $\max\{\min\{\Pr\{t_j-t_i\le \tilde{W}_{ij}\}\}\}$,
  - No need for correlation information between paths.
  - Not exactly the timing yield objective but reasonable.
- Equivalent to:

$$\begin{array}{ll} \text{maximum} & \beta \\ \text{subject to} & \Pr\{t_i - t_j \leq T_{\text{CP}} - \tilde{D}_{ij}\} \geq \beta \\ & \Pr\{t_j - t_i \leq \tilde{H}_{ij}\} \geq \beta \end{array}$$

- or:

$$\begin{array}{ll} \text{maximum} & \beta \\ \text{subject to} & t_i - t_j \leq T_{\text{CP}} - \Phi_{D_{ij}}^{-1}(\beta) \\ & t_j - t_i \leq \Phi_{H_{ij}}^{-1}(1-\beta) \end{array}$$

Yield-driven Optimization (cont'd)

- In general, Lawler's algorithm (binary search) can be used.
- Depending on the distribution, there are several other ways to solve problem.

Gaussian Delay Model

- Reduce to:

$$\begin{array}{ll} \text{maximum} & \beta \\ \text{subject to} & t_i - t_j \leq T_{\text{CP}} - (\mu_{ij}^D + \sigma_{ij}^D \sqrt{2}\text{erf}^{-1}(2\beta - 1)) \\ & t_j - t_i \leq \mu_{ij}^H + \sigma_{ij}^H \sqrt{2}\text{erf}^{-1}(2(1-\beta) - 1) \end{array}$$

- Linearization. Since $\text{erf}^{-1}(\cdot)$ is anti-symmetric and monotonic, we have:

$$\begin{array}{ll} \text{maximum} & \beta' \\ \text{subject to} & t_i - t_j \leq T_{\text{CP}} - \mu_{ij}^D - \sigma_{ij}^D \beta' \\ & t_j - t_i \leq \mu_{ij}^H - \sigma_{ij}^H \beta' \end{array}$$

- is equivalent to the minimum cost-to-time ratio (linear).
- However, actual path delay distributions are non-Gaussian.

Log-normal Delay Model

- Reduce to:

$$\begin{array}{ll} \text{maximum} & \beta \\ \text{subject to} & t_i - t_j \leq T_{\text{CP}} - \exp(\mu_{ij}^D + \sigma_{ij}^D \sqrt{2}\text{erf}^{-1}(2\beta - 1)) \\ & t_j - t_i \leq \exp(\mu_{ij}^H + \sigma_{ij}^H \sqrt{2}\text{erf}^{-1}(2(1-\beta) - 1)) \end{array}$$

- Since $\text{erf}^{-1}(\cdot)$ is anti-symmetric and monotonic, we have:

$$
\begin{array}{ll}
\text{maximum} & \beta' \\
\text{subject to} & t_i - t_j \leq T_{\text{CP}} - \exp(\mu_{ij}^D + \sigma_{ij}^D \beta') \\
& t_j - t_i \leq \exp(\mu_{ij}^H - \sigma_{ij}^H \beta')
\end{array}
$$

- Bypass evaluating error function. Non-linear and non-convex, but still can be solved efficiently by for example binary search on β'.

Weibull Delay Model

- Reduce to:

$$
\begin{array}{ll}
\text{maximum} & \beta \\
\text{subject to} & t_i - t_j \leq T_{\text{CP}} - \lambda_{ij}^D(-\ln(1-\beta))^{1/k_{ij}^D} \\
& t_j - t_i \leq \lambda_{ij}^H(-\ln(\beta))^{1/k_{ij}^H}
\end{array}
$$