
List of Symbols

The bold number(s) at the end of each line refer(s) to the page where the symbol is introduced or used in an alternative way.

$\mathbb{N} = \{0, 1, 2, \dots\}$ the set of natural numbers, **14**

\mathbb{R} the set of real numbers, **13**

$[A]_{ij}$ element (i, j) of matrix A ; also denoted by a_{ij} , **17**

A^\top the transpose of matrix A ; that is, $[A^\top]_{ij} = a_{ji}$, **19**

\oplus the operation max, or maximization, **13**

\oplus' the operation min, or minimization, **16**

\otimes plus, or addition, **13**

ε the zero element in max-plus algebra; numerical value is $\varepsilon = -\infty$, **13, 39**

ε' the zero element in min-plus algebra; numerical value is $\varepsilon' = +\infty$, **16**

$\mathcal{E}(n, m)$ the $n \times m$ matrix with all elements equal to ε , **18**

$\mathcal{E}'(n, m)$ the $n \times m$ matrix with all elements equal to ε' , **180**

$E(n, m)$ the $n \times m$ matrix with element e on the diagonal and ε elsewhere, **18**

\mathbf{u} the unit vector; numerical value is $\mathbf{u} = (0, \dots, 0)^\top$, **19**

$\mathbf{u}[\mu] = \mu \otimes \mathbf{u}$ the vector with elements equal to $\mu \in \mathbb{R}_{\max}$, **59**

e the unit in max-plus and min-plus algebra; numerical value is $e = 0$, **13, 16**

e_j the j th base vector of \mathbb{R}_ε^n with j th element zero and all other elements equal to ε , **19**

$\lceil a \rceil$ the smallest integer greater than or equal to $a \in \mathbb{R}$, **131**

A_τ the matrix A with τ subtracted from every element: $[A_\tau]_{ij} = a_{ij} - \tau$, **39, 61**

A^* the formal power series $A^* = \bigoplus_{k \geq 0} A^{\otimes k}$, **42**

A^+ the formal power series $A^+ = \bigoplus_{k \geq 1} A^{\otimes k}$, **31**

$[B]_{\cdot k}$ the k th column of matrix B , **39, 74**

$V(A, \mu)$ the eigenspace of matrix A for the eigenvalue μ , **36**

$V(A)$ the eigenspace of matrix A in the case where the eigenvalue is known and unique, **36**

$\lambda = \lambda(A)$ the eigenvalue of matrix A , and in the stochastic setup, the Lyapunov exponent of $\{A(k) : k \in \mathbb{N}\}$, **36, 173**

λ^{top} the top Lyapunov exponent of $\{A(k) : k \in \mathbb{N}\}$, **170**

λ^{bot} the bottom Lyapunov exponent of $\{A(k) : k \in \mathbb{N}\}$, **170**

- $\sigma = \sigma(A)$ the cyclicity of matrix A , **50**
 $\sigma_{\mathcal{G}} = \sigma_{\mathcal{G}(A)}$ the cyclicity of $\mathcal{G}(A)$, **33**
 $t(A)$ the transient time of matrix A , **55**
 \mathbb{R}_{\max} the set $\mathbb{R} \cup \{-\infty\}$, **13**
 \mathbb{R}_{\min} the set $\mathbb{R} \cup \{+\infty\}$, **16**
 \mathcal{R}_{\max} the structure $(\mathbb{R}_{\max}, \oplus, \otimes, \varepsilon, e)$, **13**
 \mathcal{R}_{\min} the structure $(\mathbb{R}_{\min}, \oplus', \otimes, \varepsilon', e)$, **16**
 \underline{n} the set $\{1, \dots, n\}$ for $n \in \mathbb{N} \setminus \{0\}$, **17**
 $\mathcal{G}(A)$ the communication graph of matrix A , **28**
 $\mathcal{N}(A)$ the set of nodes of $\mathcal{G}(A)$, **28**
 $\mathcal{D}(A)$ the set of arcs of $\mathcal{G}(A)$, **28**
 $\mathcal{G}^c(A)$ the critical graph of matrix A , **38**
 $\mathcal{N}^c(A)$ the set of nodes of $\mathcal{G}^c(A)$, **38**
 $\mathcal{D}^c(A)$ the set of arcs of $\mathcal{G}^c(A)$, **38**
 $\pi(i)$ the set of direct predecessors of node i , **33**
 $\pi^+(i)$ the set of all predecessors of node i , **33**
 $\pi^*(i)$ the set $\pi^+(i) \cup \{i\}$, **33**
 $\sigma(i)$ the set of direct successors of node i , **33**
 $\sigma^+(i)$ the set of all successors of node i , **33**
 $\sigma^*(i)$ the set $\sigma^+(i) \cup \{i\}$, **34**
 $\mathcal{C}(A)$ the set of all elementary circuits in $\mathcal{G}(A)$, **38**
 $|p|_l$ the length of path p , **28**
 $|p|_w$ the weight of path p , **29**
 $i\mathcal{R}j$ node j is reachable from node i , **31**
 $i\mathcal{C}j$ node j communicates with node i : $i\mathcal{R}j$ as well as $j\mathcal{R}i$, with $i\mathcal{C}i$ always true, **31**
 \bar{x} the equivalence class of vectors that are colinear to x , **24**
 $\|\cdot\|_{\mathbb{P}}$ the projective norm, **80**
 (η, v) a generalized eigenmode, **58**
 γ^{top} the maximal entry of vector γ , **60**
 $\|v\|_{\infty}$ the supremum norm of vector v , **56**
 $\|A\|_{\max}$ the maximal finite element of matrix A , **168**
 $\|A\|_{\min}$ the minimal finite element of matrix A , **168**

