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## Max Plus at Work

Modeling and Analysis of Synchronized Systems: A Course on Max-Plus Algebra and Its Applications

Bernd Heidergott Geert Jan Olsder Jacob van der Woude

PRINCETON UNIVERSITY PRESS
PRINCETON AND OXFORD

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Published by Princeton University Press, 41 William Street, Princeton, New Jersey 08540 In the United Kingdom: Princeton University Press, 3 Market Place, Woodstock, Oxfordshire OX20 1SY

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 ${\it Library of Congress Cataloging-in-Publication Data}$ 

Heidergott, Bernd.

Max Plus at work: modeling and analysis of synchronized systems: a course on Max-Plus algebra and its applications / Bernd Heidergott, Geert Jan Olsder, Jacob van der Woude.

p. cm.—(Princeton series in applied mathematics) Includes bibliographical references and index. ISBN-13: 978-0-691-11763-8 (acid-free paper) ISBN-10: 0-691-11763-2 (acid-free paper)

 ${\bf 1. Matrices-Textbooks. \, 2. \, System \, theory--Textbooks. \, I. \, Olsder, Geert \, Jan. \, II.}$ 

Woude, J.W. van der. III. Title.

QA188.H445 2006

512-dc22 2005048690

British Library Cataloging-in-Publication Data is available

The publisher would like to acknowledge the authors of this volume for providing the camera-ready copy from which this book was printed.

Printed on acid-free paper.

pup.princeton.edu

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1