Quasi-Convex Programming

Quasi-convex programming is a powerful optimization technique with diverse applications in electronic design automation. It enables solving complex problems by leveraging the properties of quasi-convex functions - a class of functions that generalize the well-known convex functions.

by Wai-Shing Luk

Definitions and Properties

Quasi-Convex Functions

A function is quasi-convex if its sublevel sets are convex. This allows for more flexibility compared to traditional convex optimization.

Key Properties

Quasi-convex functions preserve many desirable properties of convex functions, such as local optimality implying global optimality.

Practical Significance

Quasi-convex formulations enable solving a wide range of non-convex problems in electronic design automation efficiently.

Examples of Quasi-Convex Functions

Square Root of Absolute Value

$\sqrt{|y|}$ is quasi-convex on $\mathbb{R}$.

Logarithm

$\log(y)$ is quasi-linear on $\mathbb{R}_{++}$.

Product Function

$f(x, y) = x y$ is quasi-concave on $\mathbb{R}_{++}^2$.

Linear-Fractional Function

$f(x) = (a^{\mathsf{T}} x + b)/(c^{\mathsf{T}} x + d)$ with domain ${x \,|\, c^{\mathsf{T}} x + d > 0 }$.

Properties of Quasi-Convex Functions

Convex Sublevel Sets

If **f** is quasi-convex, there exists a family of convex functions **ϕt** such that the **t**-sublevel set of **f** is the **0**-sublevel set of **ϕt**.

Monotonicity

ϕt is non-increasing with respect to **t** for fixed **β**, meaning the sublevel sets of **f** are nested.

Fractional Functions

An example of a quasi-convex function is **$f(\beta) = p(\beta)/q(\beta)$**, where **$p$** is convex, **$q$** is concave, **$p(\beta) \geq 0$**, and **$q(\beta) > 0$** on the domain of **$f$**.

Optimization Problems

1. **Constraint Formulations 🤔**

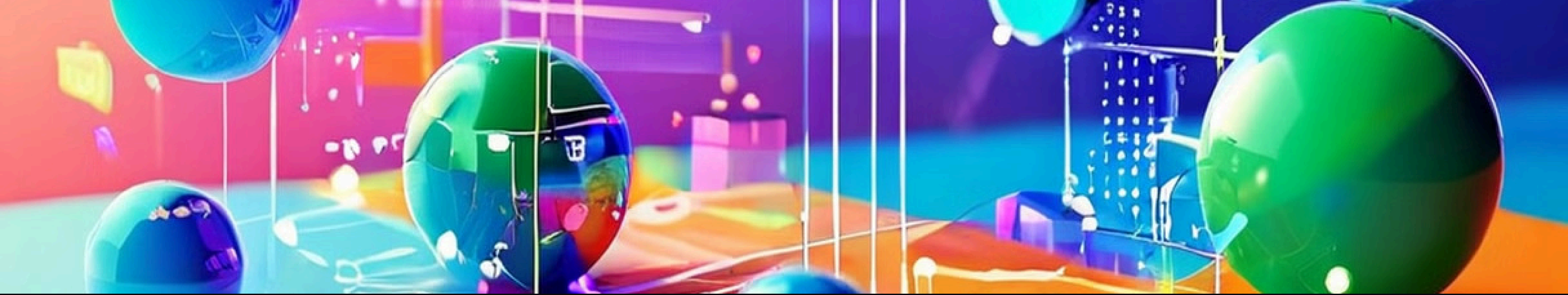
   Quasi-convex constraints can model a variety of non-convex requirements in electronic design.

2. **Global Optimality**

   Quasi-convex optimization can often guarantee global optimality, unlike general non-convex programming.

3. **Efficient Algorithms**

   Specialized algorithms have been developed to solve quasi-convex programs effectively.



Solving Quasi-Convex Programs

1

Bisection

Iterative algorithms that leverage the quasi-convexity of the objective function.

2

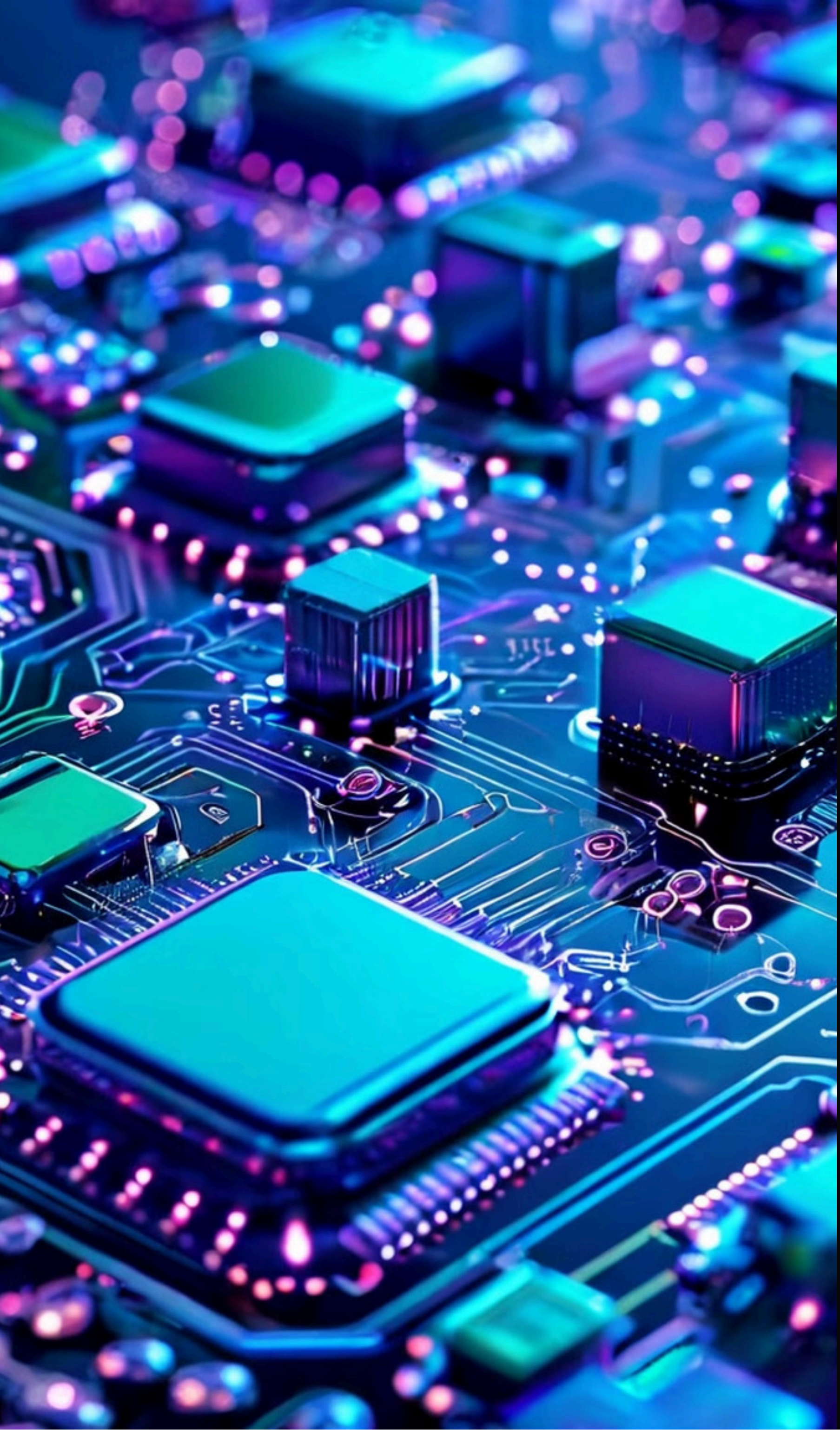
Outer Approximation

Constructing a sequence of convex relaxations to solve the original non-convex problem.

3

Geometric Programming

Transforming certain quasi-convex problems into an equivalent convex formulation.



Applications in EDA

Circuit Design

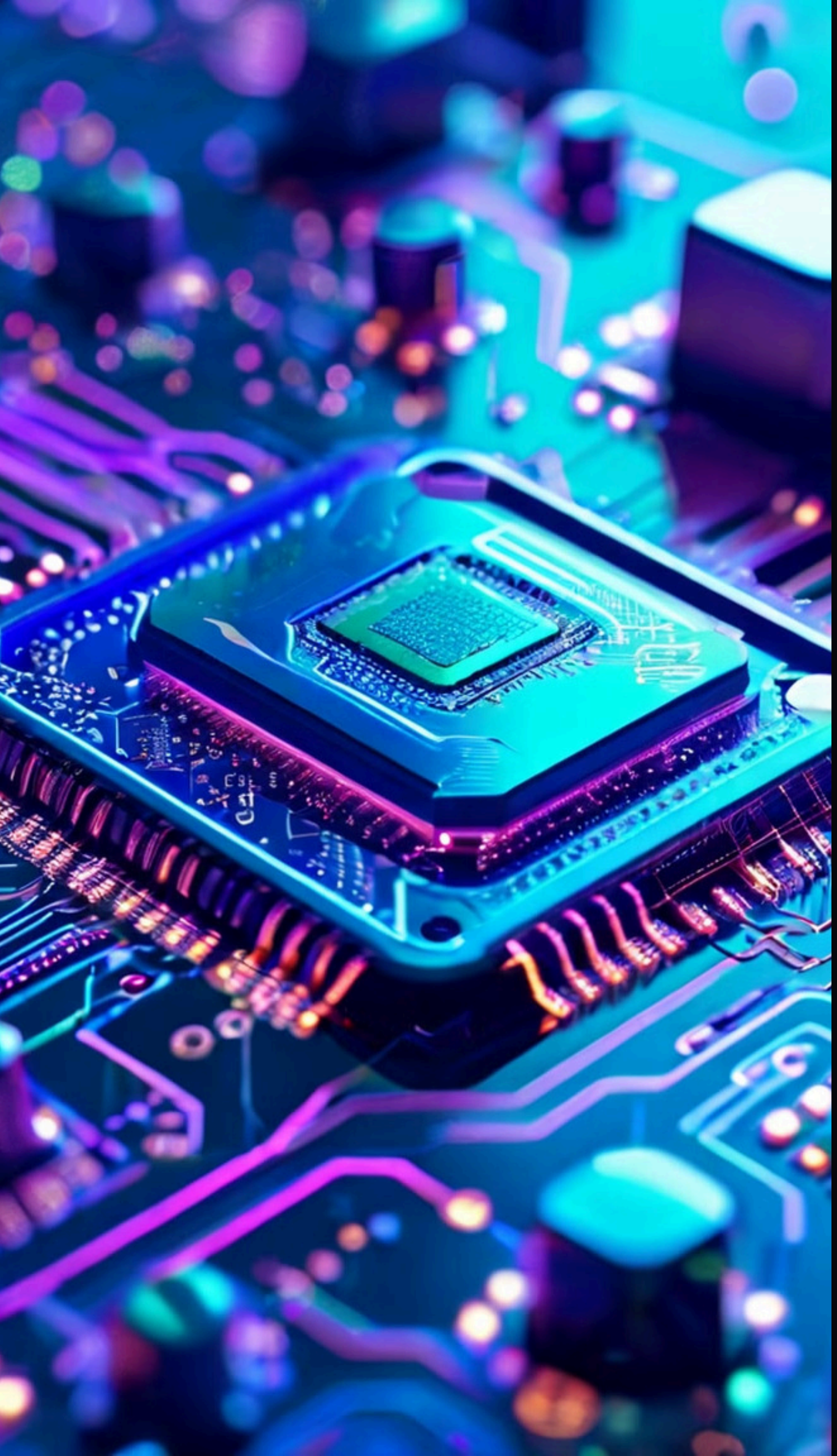
Quasi-convex optimization can be applied to analog circuit design, device sizing, and other circuit-level problems.

VLSI Design

Power optimization, yield maximization, and other VLSI design challenges can be formulated as quasi-convex programs.

EDA Tools

Quasi-convex programming is being increasingly integrated into modern electronic design automation (EDA) software tools.



Placement and Routing Optimization

1

Component Placement 🤔

Quasi-convex formulations can optimize the positioning of circuit components on a chip or PCB to minimize wiring length and congestion.

2

Wire Routing 🤔

Quasi-convex programming can be used to find optimal routes for interconnections between components, balancing constraints such as shortest path and minimum crosstalk.

3

Design Convergence

The iterative nature of quasi-convex optimization algorithms can help achieve convergence to high-quality, manufacturable design solutions.

Analog Circuit Design 🤔

Device Sizing

Quasi-convex programming can be used to optimally size transistors and other analog components to meet performance targets.

Biasing Circuits

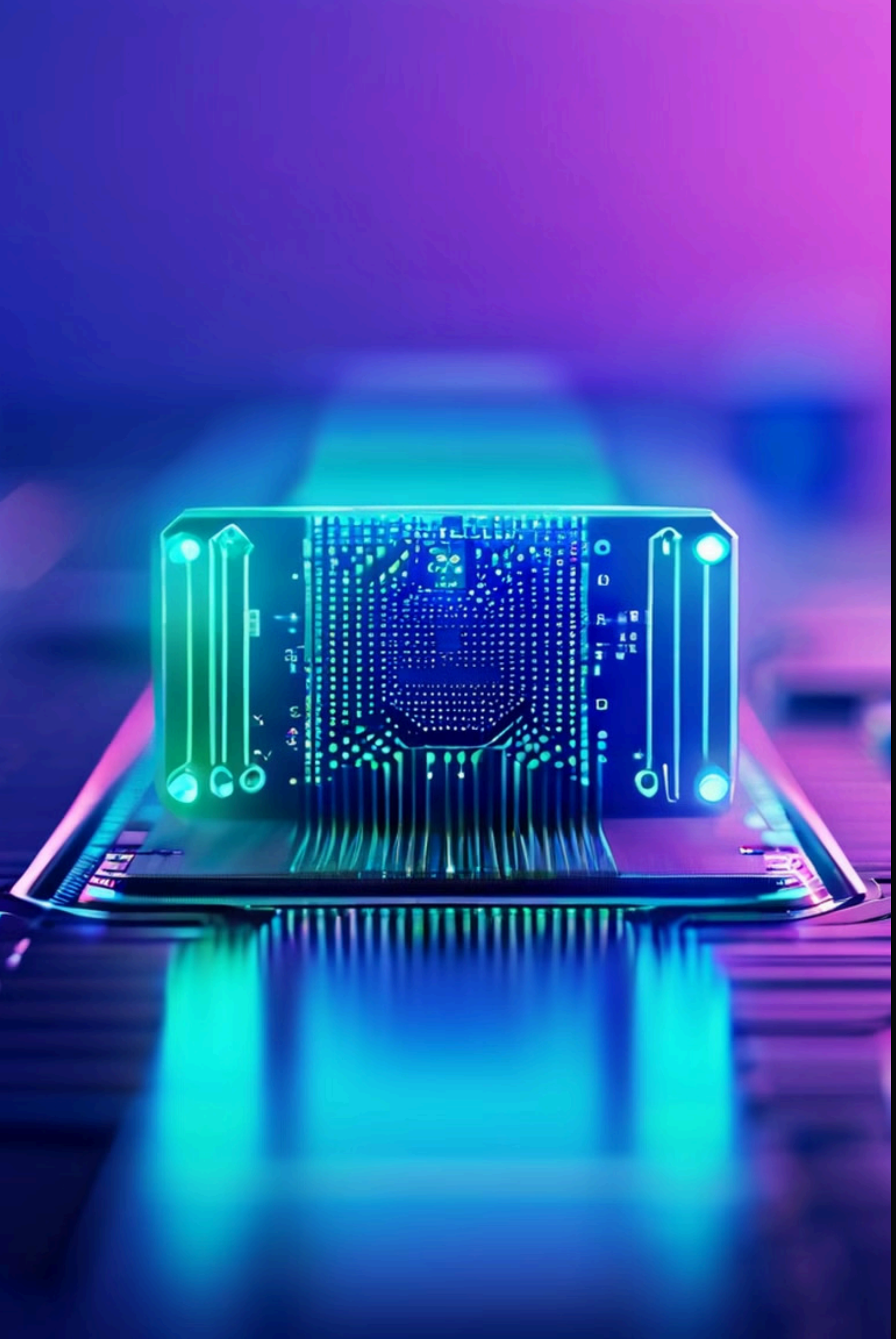
Quasi-convex techniques can optimize the design of biasing networks to ensure stable and robust analog circuit operation.

Layout Synthesis

Quasi-convex formulations can guide the physical layout of analog circuits to enhance parameters like matching and parasitics.

Yield Optimization

Quasi-convex programming can be employed to maximize the manufacturing yield of analog integrated circuits.



Power Optimization in VLSI

Objective	Minimize power consumption
Constraints	Performance, area, and reliability requirements
Quasi-Convex Approach	Model power as a quasi-convex function of design parameters
Benefits	Global optimality, efficient algorithms, scalable solutions

Yield Optimization

Manufacturing Variability

Quasi-convex programming can model the impact of process variations on circuit performance and yield.

Robust Design

Quasi-convex optimization can generate designs that are less sensitive to manufacturing uncertainties, improving overall yield.

Statistical Modeling

Quasi-convex formulations can leverage statistical techniques to optimize for manufacturability and yield targets.

Future Directions

1. **Emerging Applications**

   Expanding the use of quasi-convex programming to new domains in electronic design automation, such as RF circuit design and machine learning-based EDA.

2. **Algorithmic Advances**

   Developing more efficient and scalable quasi-convex optimization algorithms to handle the increasing complexity of modern electronic designs.

3. **Integration with EDA Tools**

   Seamlessly integrating quasi-convex programming techniques into mainstream electronic design automation software for broader industry adoption.