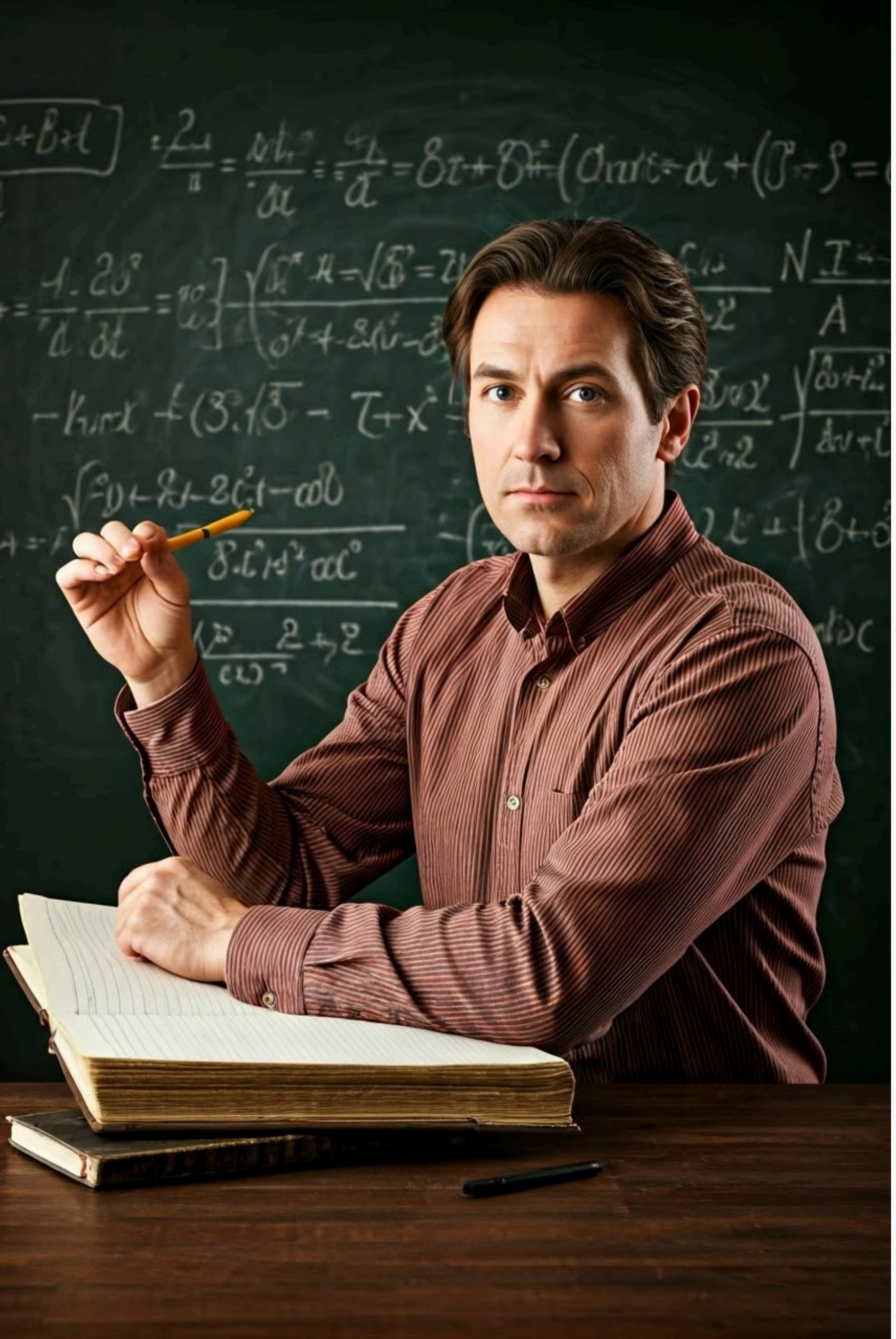
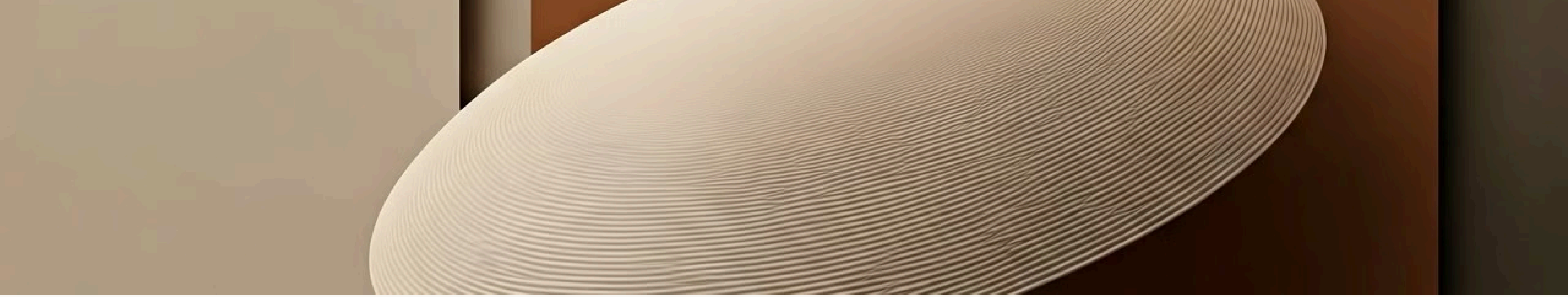


The Ellipsoid Method and Amazing Oracles

The ellipsoid method is a powerful optimization technique that offers distinct advantages over other methods, particularly in problems with a large or infinite number of constraints. Unlike interior-point methods, the ellipsoid method does not require the evaluation of all constraint functions, but instead relies on a separation oracle to provide cutting planes. This makes the method ideal for exploiting certain problem structures. In this presentation, we will explore three key applications of the ellipsoid method and discuss the importance of the separation oracle in each case.

by Wai-Shing Luk



Robust Optimization with the Ellipsoid Method

Handling Uncertainty

The ellipsoid method is particularly well-suited for robust optimization problems, where the goal is to find a solution that performs well even in the presence of uncertain parameters. By using a separation oracle to identify the worst-case scenario, the method can efficiently navigate the search space and find an optimal solution.

Exploiting Problem Structure

The separation oracle can take advantage of the problem's structure, such as the use of affine arithmetic to handle interval uncertainties. This allows the method to quickly identify cutting planes and update the search space, leading to faster convergence.

Versatility

The ellipsoid method's ability to handle a large number of constraints makes it a versatile tool for a wide range of robust optimization problems, from portfolio management to engineering design.

Parametric Network Optimization with the Ellipsoid Method

1

Negative Cycle Detection

The key to solving parametric network optimization problems with the ellipsoid method is the ability to efficiently detect negative cycles in the network. Algorithms like Tarjan's can exploit the network's structure to quickly identify cutting planes.

2

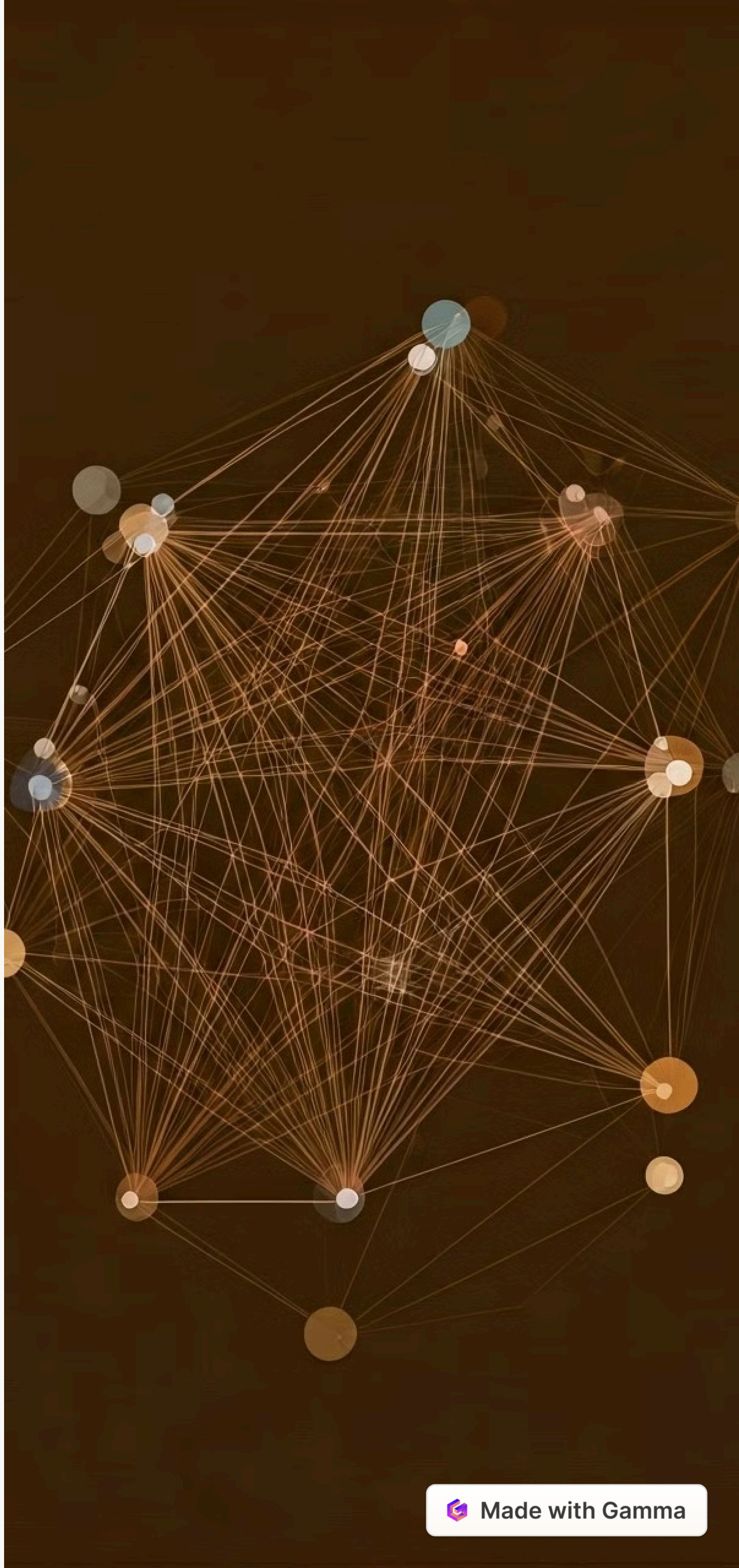
Exploiting Locality

By taking advantage of the network's locality and other properties, the separation oracle can construct cutting planes in a highly efficient manner, further enhancing the performance of the ellipsoid method.

3

Diverse Applications

Parametric network optimization problems arise in a wide range of applications, from transportation planning to VLSI design. The ellipsoid method's ability to handle these problems makes it a valuable tool in many engineering and operations research domains.



Semidefinite Programming and the Ellipsoid Method

Cholesky Factorization

The ellipsoid method's separation oracle can leverage the Cholesky factorization to efficiently check the positive definiteness of symmetric matrices, a key step in solving semidefinite programming problems.

Lazy Evaluation

By using lazy evaluation techniques, the separation oracle can construct cutting planes in O(p^3) time, where p is the row at which the Cholesky factorization fails, rather than the full O(m^3) time, making the method highly efficient.

Matrix Norm Minimization

The ellipsoid method can be applied to matrix norm minimization problems, where the goal is to find the matrix with the smallest norm that satisfies certain constraints.

Improving the Ellipsoid Method with Parallel Cuts

1

Reducing Computation Time

The use of parallel cuts, where two constraints are used simultaneously to update the ellipsoid, can significantly improve the computation time of the ellipsoid method, especially in problems where certain constraints have tight upper and lower bounds.

2

Efficient Implementation

By carefully implementing the parallel cut updates, the method can ensure that every update, whether it uses a deep cut or a parallel cut, results in at most one square root operation, further reducing the computational burden.

3

Practical Applications

The parallel cut technique has been shown to be particularly effective in the design of FIR filters, where the tight constraints on the frequency response can be exploited to achieve significant speedups.

Applying the Ellipsoid Method to Discrete Optimization

Handling Discrete Variables

The cutting-plane method, of which the ellipsoid method is a part, can also be applied to discrete optimization problems where some design variables are restricted to discrete forms. The key is to use a separation oracle that can locate the nearest discrete solutions.

Overcoming Limitations

Unlike traditional methods that rely on relaxation and branch-and-bound, the ellipsoid method can exploit the convexity of the problem, even in the presence of discrete constraints, leading to more efficient solutions.

Applications in Engineering

Discrete optimization problems arise in many engineering domains, such as the design of multiplierless FIR filters, where the coefficients must be represented using a limited set of discrete values. The ellipsoid method offers a promising approach for tackling these challenges.

The Importance of Separation Oracles

1. **Exploiting Problem Structure**

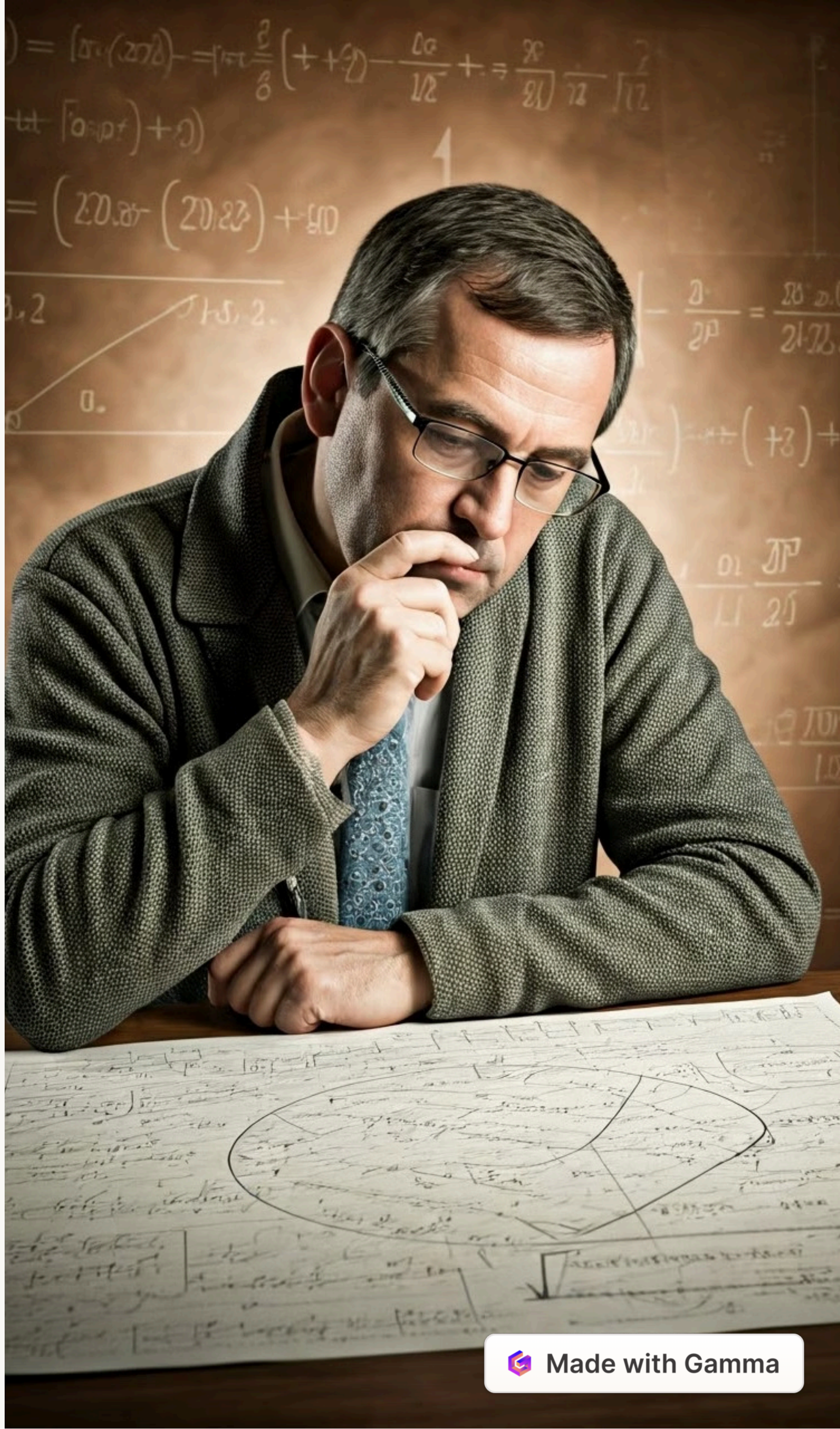
   The separation oracle is the key to the ellipsoid method's ability to take advantage of certain problem structures, such as network locality or the Cholesky factorization of symmetric matrices.

2. **Reducing Computational Burden**

   By providing efficient cutting planes, the separation oracle can significantly reduce the computational cost of the ellipsoid method, making it a viable alternative to interior-point methods in many practical applications.

3. **Versatility and Adaptability**

   The flexibility of the separation oracle allows the ellipsoid method to be applied to a wide range of optimization problems, from robust optimization to discrete programming, making it a powerful and versatile tool in the optimization toolbox.



The Ellipsoid Method: A Companion, Not a Competitor

Complementary Strengths

While the ellipsoid method may be perceived as slower than interior-point methods for solving convex problems, it offers distinct advantages, such as the ability to handle problems with a large or infinite number of constraints.

Ongoing Improvements

Techniques like parallel cuts and efficient implementations have helped to improve the performance of the ellipsoid method, making it a valuable tool in the optimization landscape.

A Collaborative Approach

Rather than viewing the ellipsoid method as a competitor to other optimization techniques, it should be seen as a companion, with each method offering unique strengths that can be leveraged to solve a wide range of optimization problems effectively.