Lecture 8: Phase Shifting Mask

@luk036

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Overview

- Background
- What is Phase Shifting Mask?
- Phase Conflict Graph
- Phase Assignment Problem
  - Greedy Approach
  - Planar Graph Approach

class: middle, center

Background

Background

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- In the past, chips have continued to get smaller and smaller, and therefore consume less and less power.
- However, we are rapidly approaching the end of the road and optical lithography cannot take us to the next place we need to go.

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Process of Lithography

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1. Photo-resist coating ( )
2. Illumination ( )
3. Exposure ( )
4. Etching ( )
5. Impurities doping ( )
6. Metal connection

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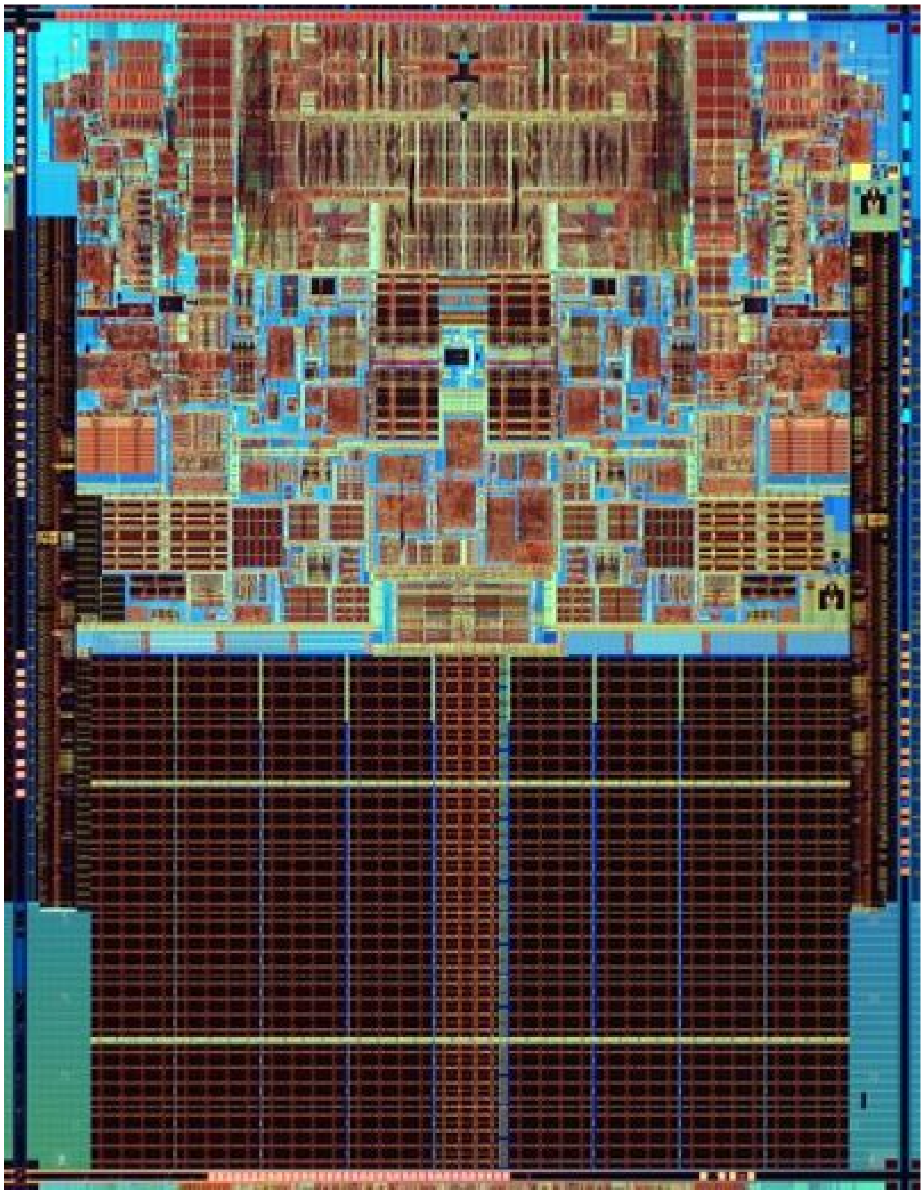

Figure 1: image

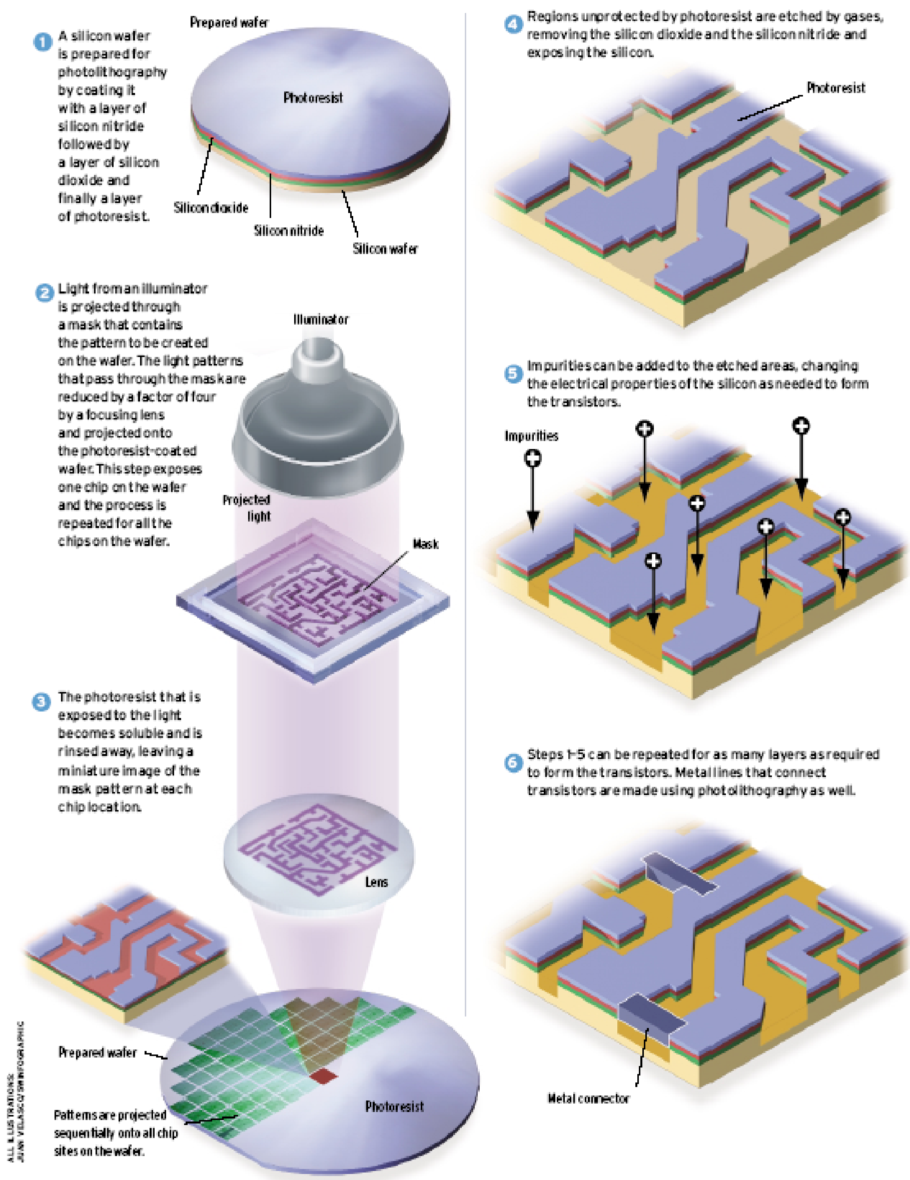

Figure 2: image

Sub-wavelength Lithography

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- Feature size is much smaller than the lithography wavelength
  - 45nm vs. 193nm

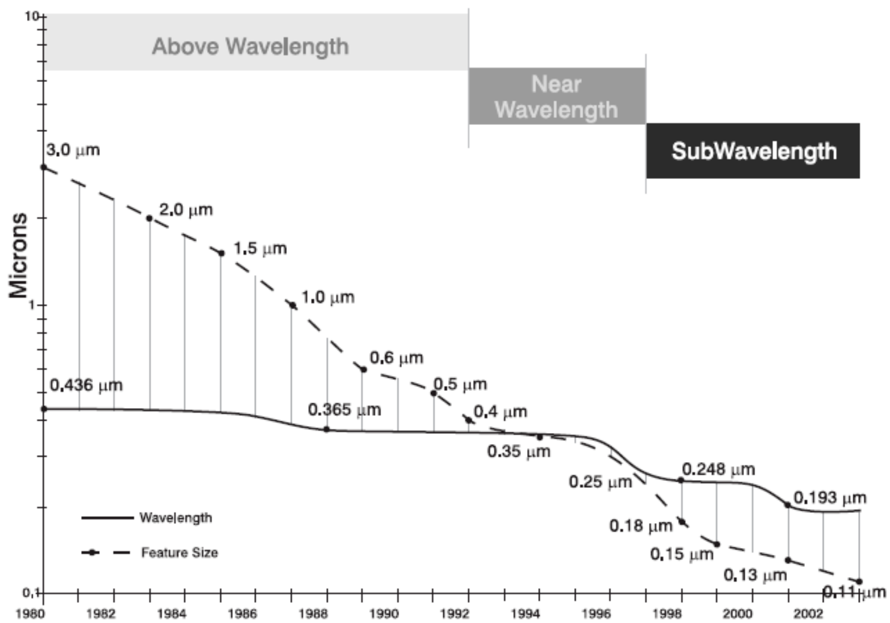

Figure 1: Shift to subwavelength optical lithography since the 0.35-micron process generation.

Figure 3: image

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- What you see in the mask/layout is **not** what you get on the chip:
  - Features are distored
  - Yields are declined

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DFM Tool (Mentor Graphics)

OPC and PSM

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- Results of OPC on PSM:

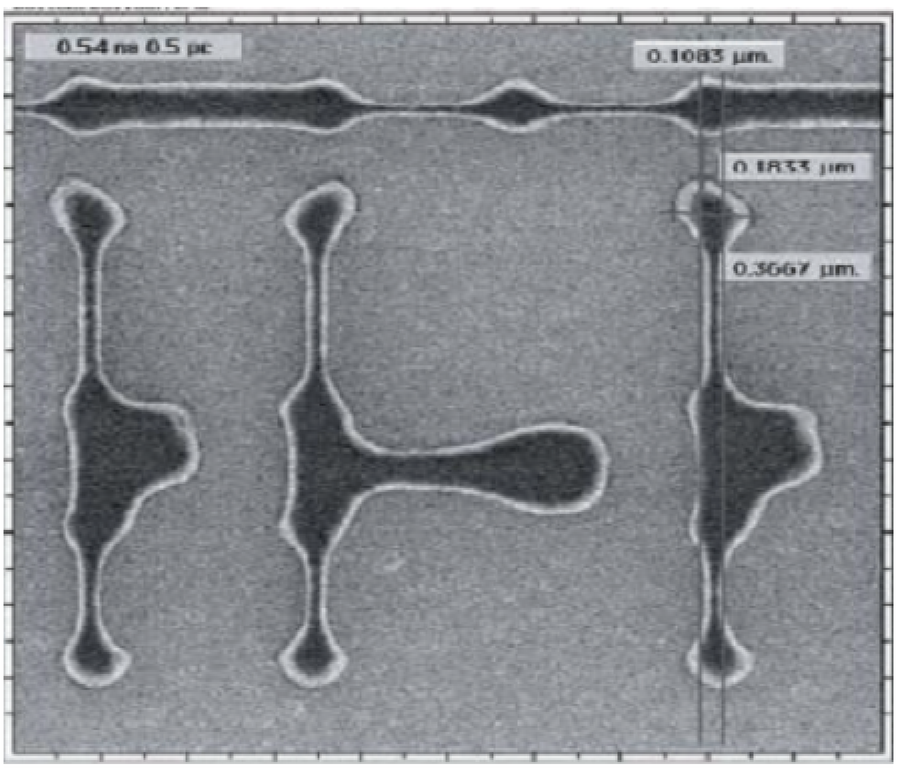
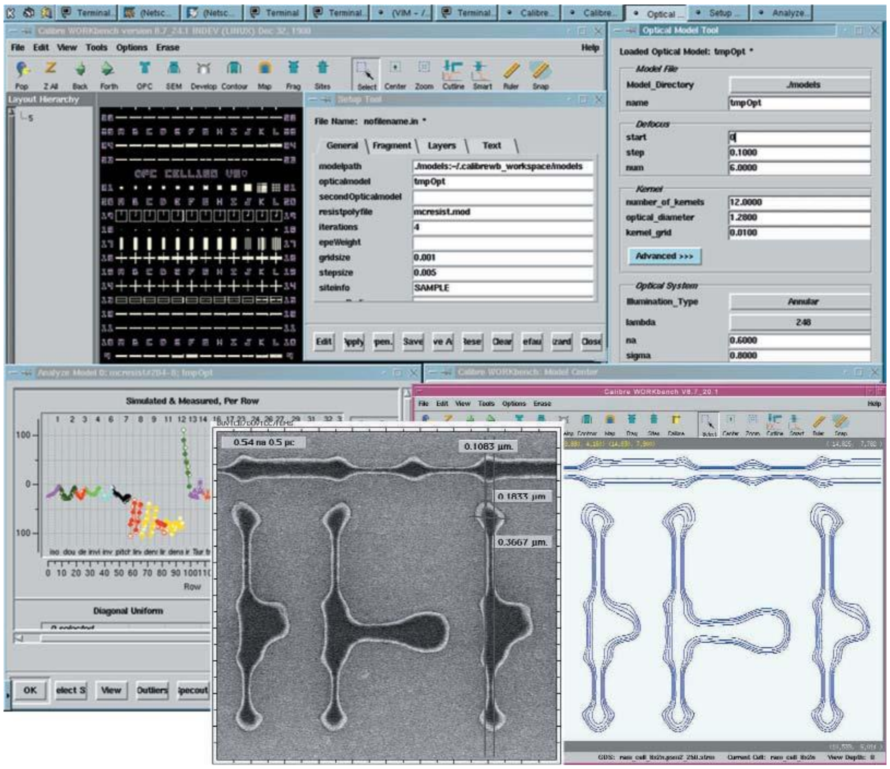


Figure 4: image



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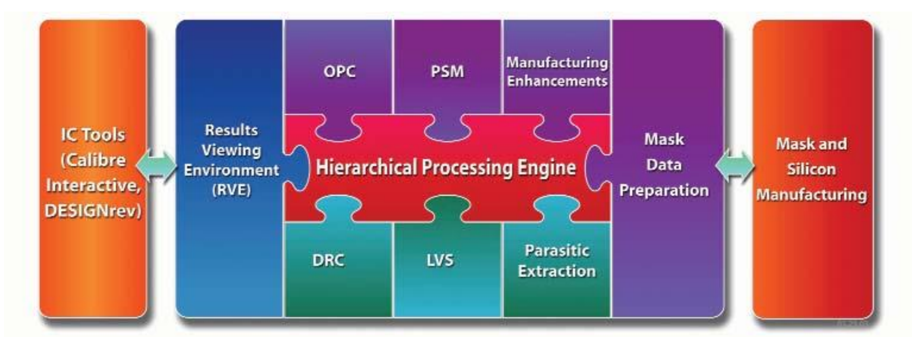

Figure 6: image

- A = original layout
- B = uncorrected layout
- C = after PSM and OPC

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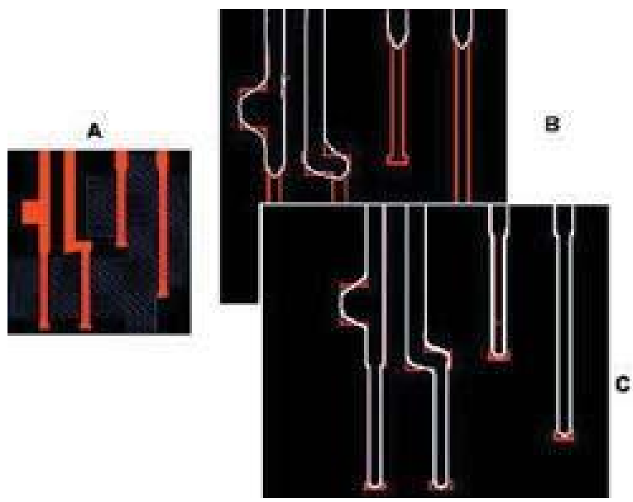

Figure 7: image

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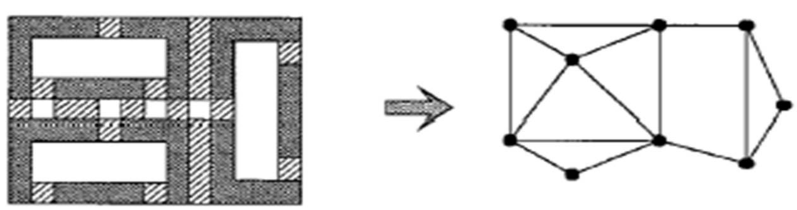
Phase Shifting Mask

Phase Conflict Graph

- Edge between two features with separation of $\leq b$ (dark field)
- Similar conflict graph for "bright field".
- Construction method: plane sweeping method + dynamic priority search



tree

Phase Assignment Problem

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- Instance: Graph $G = (V, E)$
- Solution: A color assignment $c : V \to [1..k]$ (here $k = 2$)

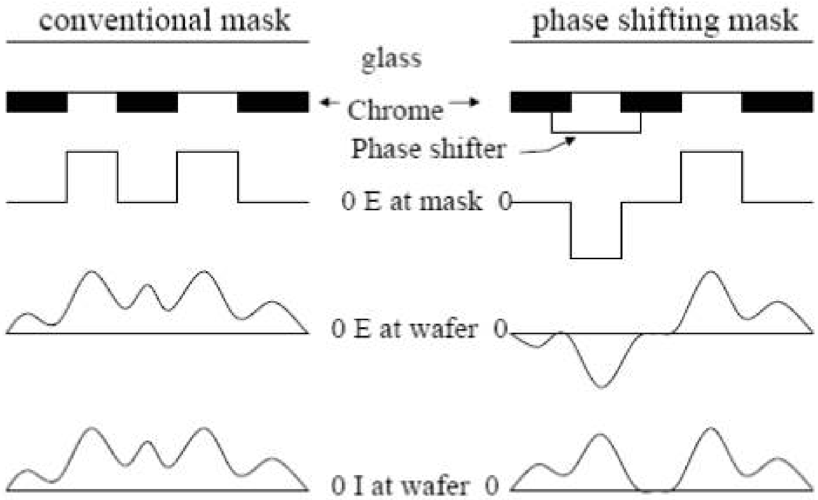

Figure 8: image

- Goal: Minimize the weights of the monochromatic edges. (Question: How can we model the weights?)

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Phase Assignment Problem

- In general, the problem is NP-hard.
- It is solvable in polynomial time for planar graphs with $k = 2$, since the problem is equivalent to the T-join problem in the dual graph [Hadlock75].
- For planar graphs with $k = 2$, the problem can be solved approximately in the ratio of two using the primal-dual method.

Overview of Greedy Algorithm

- Create a maximum weighted spanning tree (MST) of G (can be found in LEDA package)
- Assign colors to the nodes of the MST.
- Reinsert edges that do not conflict.
- Time complexity: $O(N \log N)$
- Can be applied to non-planar graphs.

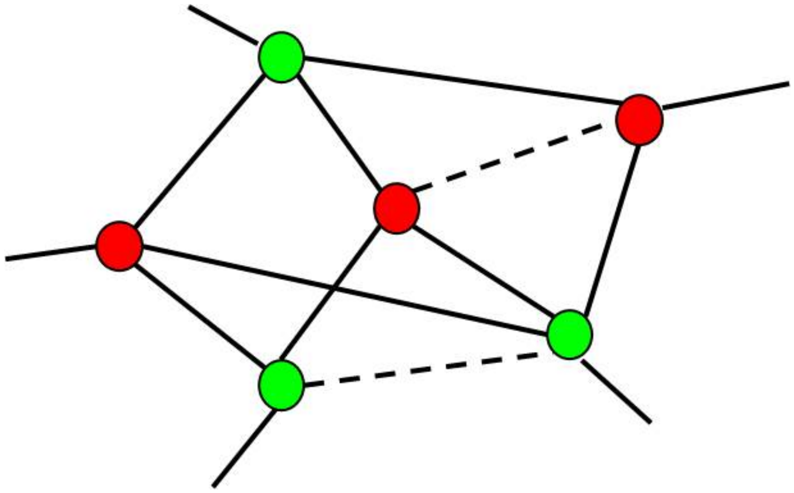

Figure 9: image

Greedy Algorithm

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- Step 1: Construct a maximum spanning tree T of G (using e.g. Kruskal's algorithm, which is available in the LEDA package).

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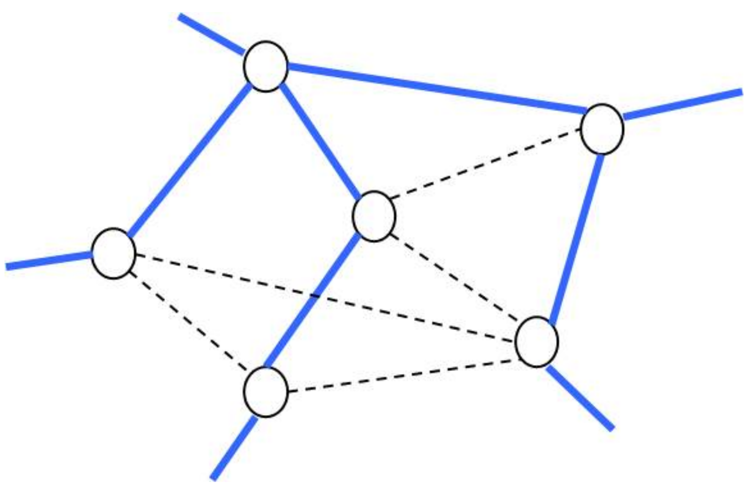

Figure 10: image

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Greedy Algorithm (Cont'd)

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- Step 2: Assign colors to the nodes of T.

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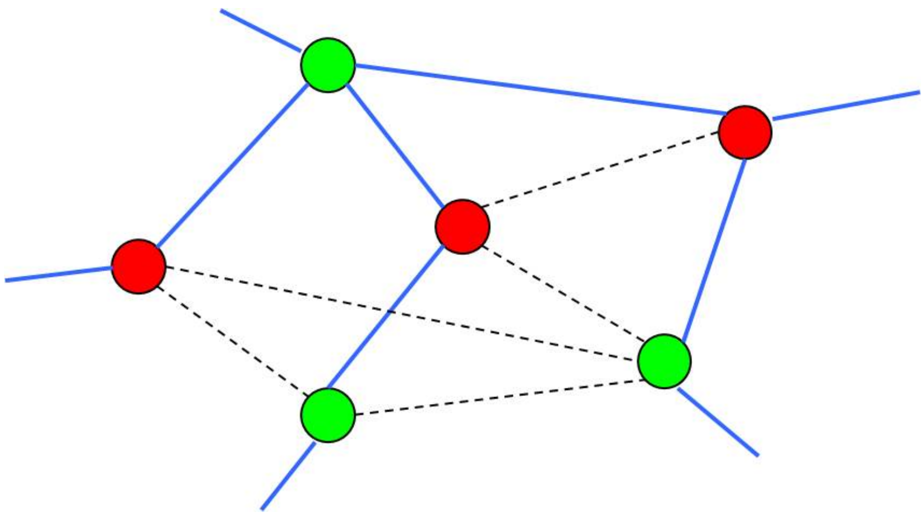

Figure 11: image

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Greedy Algorithm (Cont'd)

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- Step 3: Reinsert edges that do not conflict.

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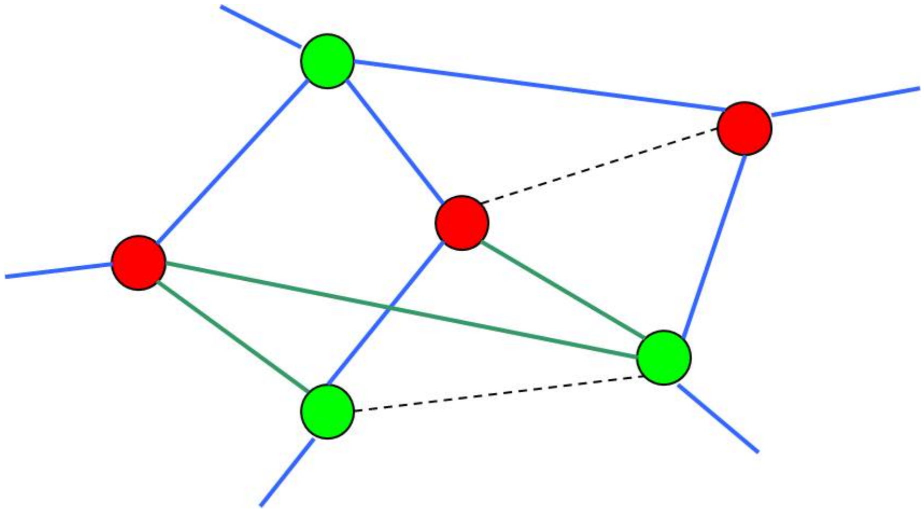
Other Approaches

- Reformulate the problem as a MAX-CUT problem. Note that the MAX-CUT problem is approximatable within a factor of 1.1383 using the "semi-definite programming" relaxation technique [Goemans and Williamson 93].
- Planar graph approach: Convert G to a planar graph by removing the minimal edges, and then apply the methods to the resulting planar graph.

  Note: the optimal "planar sub-graph" problem is NP-hard.



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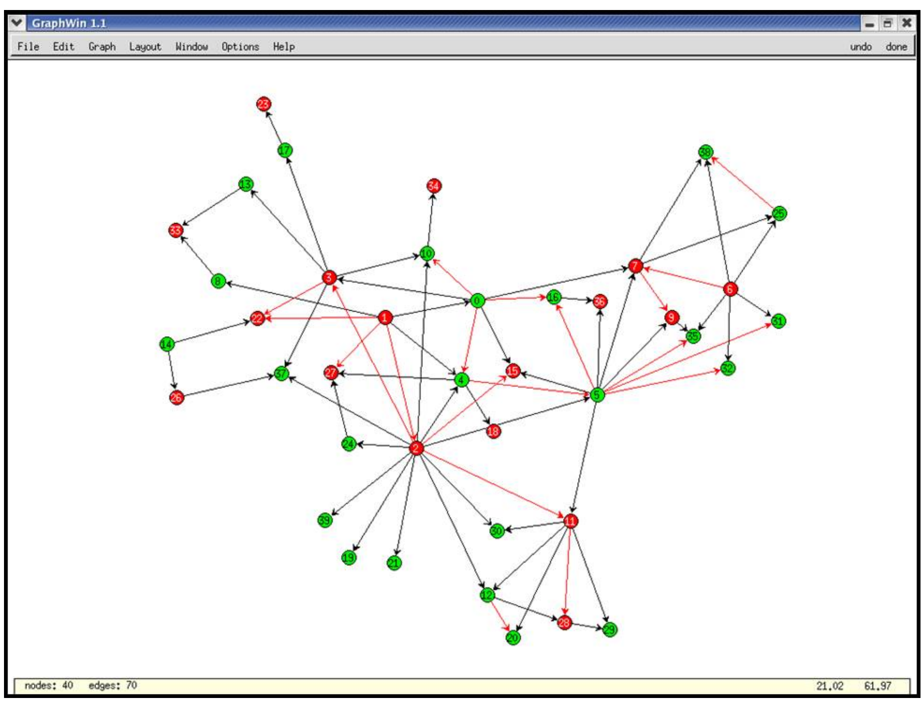

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Overview of Planar Graph Approach (Hadlock's algorithm)

1. Approximate G by a planar graph G'
2. Decompose G' into its bi-connected components.
3. For each bi-connected component in G',
    1. construct a planar embedding
    2. construct a dual graph G^*
    3. construct a complete graph $C(V, E)$, where
        - V is a set of odd-degree vertices in G^*
        - the weight of each edge is the shortest path of two vertices
    4. find the minimum perfect matching solution. The matching edges are the conflict edges that have to be deleted.
4. Reinsert the non-conflicting edges from G.

Planar Graph Approach

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- Step 1: Approximate G with a planar graph G'
    - It is NP-hard.
    - The naive greedy algorithm takes $O(n^2)$ time.
    - Any good suggestion?

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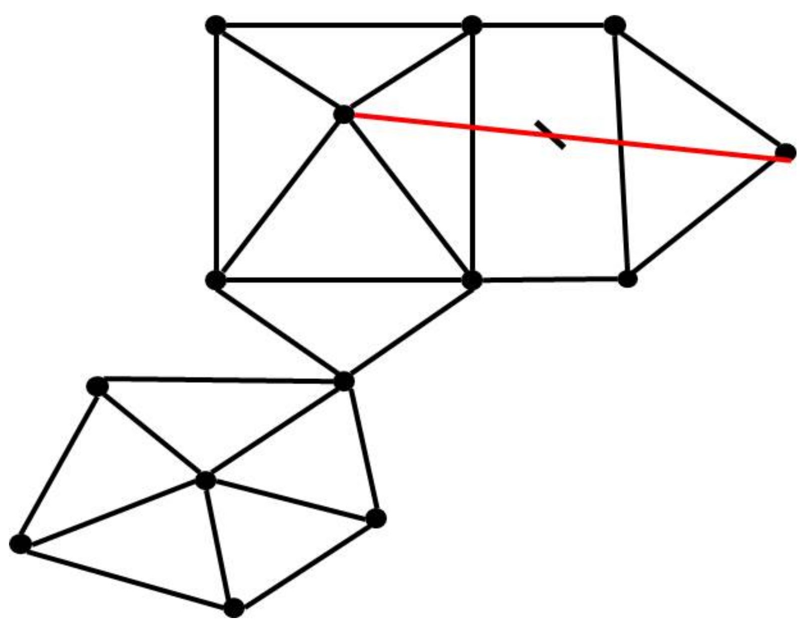

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Planar Graph Approach

- Step 2: Decompose G' into its bi-connected components in linear time (available in the LEDA package).

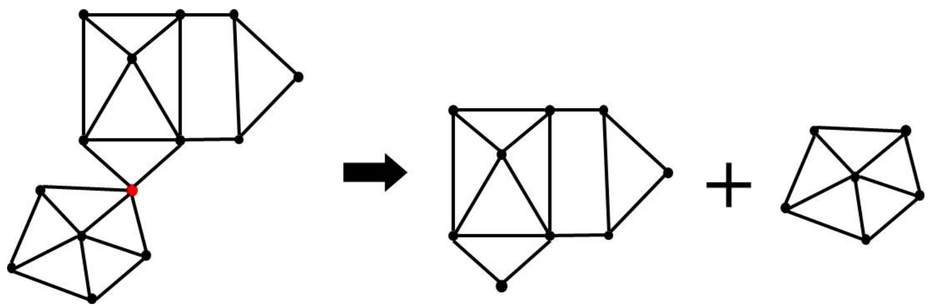

Figure 15: image

Planar Graph Approach

- Step 3: For each bi-connected component in G', construct a planar embedding in linear time (available in the LEDA package)

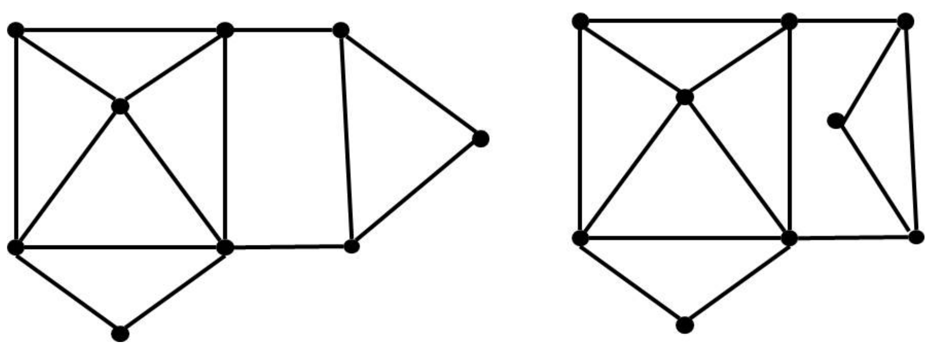

Figure 16: image

Note: planar embedding may not be unique unless G is tri-connected.

Planar Graph Approach

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- Step 4: For each bi-connected component, construct its dual graph G^* in linear time.

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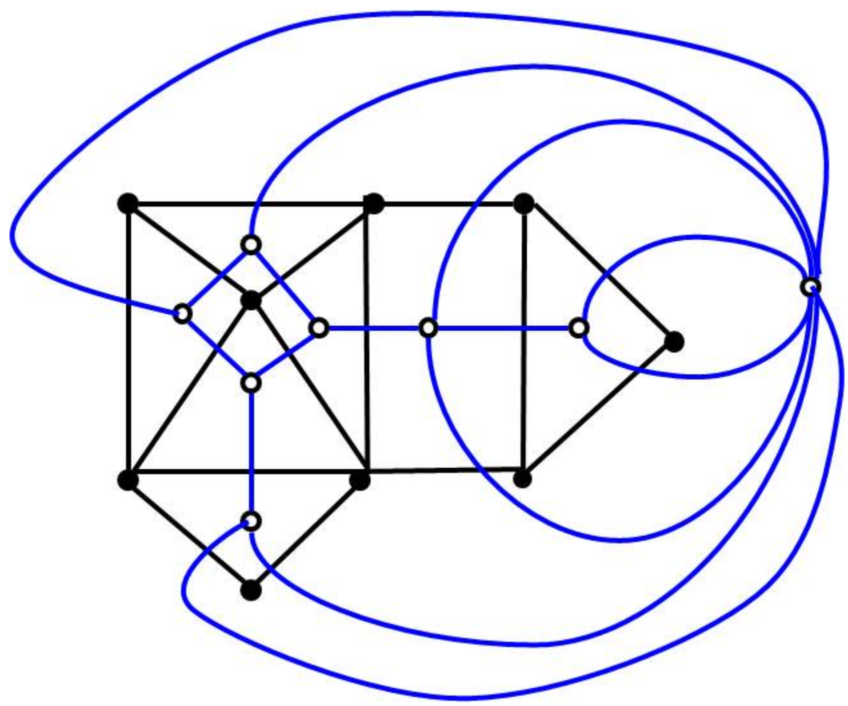

Figure 17: image

Planar Graph Approach

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- Step 5: Find the minimum weight perfect matching of G^*.
  - Polynomial time solvable.
  - Can be formulated as a network flow problem.

  Note: complete graph vs. Voronoi graph

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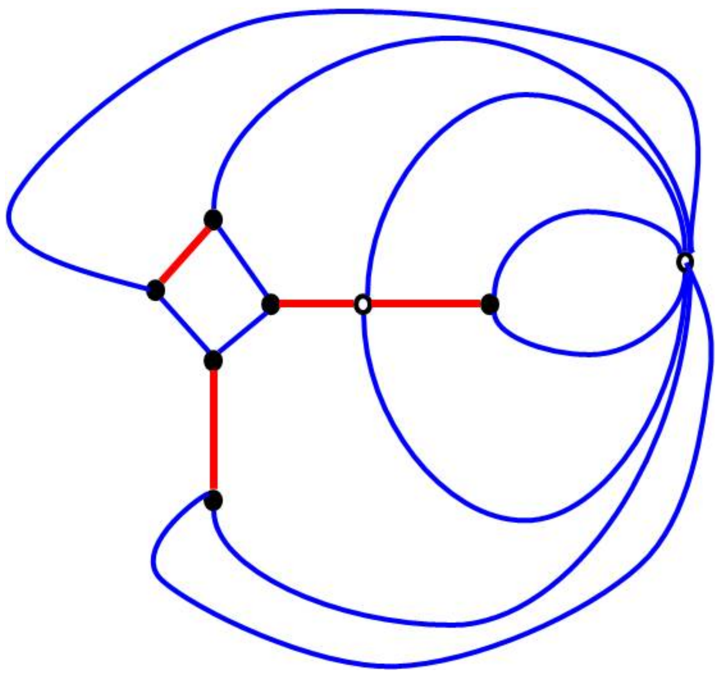

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Planar Graph Approach

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- Step 6: reinsert the non-conflicting edges in G.

Note: practically we keep track of conflicting edges.

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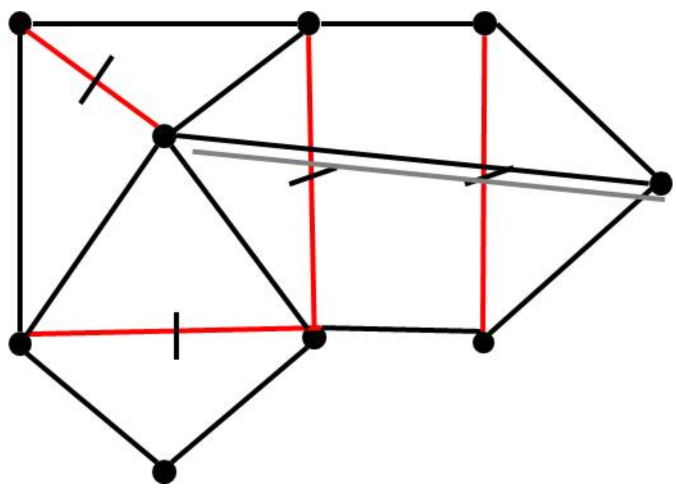

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