Preface

This textbook is concerned with sequences of events, where events are viewed as sudden changes in a process to be studied. Some examples of events are a message arrives, a train leaves the station, and a door opens. This book deals with the modeling, analysis, and timing of such events, all subject to synchronization constraints. These constraints are relations that exist between the events, such as a message must have been sent before it can arrive and a certain train should not depart before another train has arrived (in order to allow the changeover of passengers).

Apart from the introductory Chapter 0, the book consists of three parts. Part I (Chapters 1–6) deals with max-plus algebra, i.e., an algebra in which the evolution of events can be described in a convenient way. Part II (Chapters 7–10) covers two specific applications, both related to timetable design for railway networks. Part III (Chapters 11–13) deals with various extensions. Later on in this Preface, we give a brief description of each chapter.

The level of the book is last-year undergraduate student mathematics. The book will be of interest for applied mathematicians, operations researchers, econometricians, and civil, electrical, and mechanical engineers with quantitative backgrounds. Most important is a basic knowledge of conventional algebra and matrix calculus, with the addition of some knowledge of system theory (or recurrence relations). Some knowledge of stochastic processes is required for Chapter 11 only. No prior knowledge of graph theory and the modeling tool of Petri nets is required.

Each chapter can be taught conveniently in two hours. The only exceptions to this rule are Chapter 3 (which requires at least three hours) and Chapter 10 (for which one hour is enough). With two hours per week, this would mean that fourteen weeks would suffice to teach the whole book. Of course, depending on interest, some chapters can be studied in more depth, requiring more than two hours, while others can be skipped or can be treated superficially in one hour. A minimum course would cover the material of Chapters 0, 1, 2, 7 and 8. From a mathematical point of view, Chapter 3 is probably the most challenging in presenting key results. All chapters contain an exercise section as well as a notes section with suggestions for further reading and sometimes other remarks.

On the timescale of mathematical evolution, max-plus algebra and its applications are a recent phenomenon. Originally, some results were published in journals (e.g., [30], [43]), and even earlier traces exist. The first major leap forward in this algebra was the appearance of the book [31] in 1979. The next book, written from a system-theoretical perspective, was [5]. It can be viewed as a product of the "French school." Neither book can be called a textbook. The current book is believed to be the first textbook in the area of max-plus algebra and its applications. Other textbooks with introductions to the much wider area of discrete event systems do exist (e.g., [20]). Though the book at hand concentrates on applications related to timetables, other realistic applications do exist, for instance, in the areas of production lines and network calculus. For references the reader is referred to the notes sections of the chapters of this book.

A brief description of the book now follows. Chapter 0 gives an overview of some concepts to be dealt with and some problems to be solved. These concepts, problems, and solutions are elucidated by means of a simple academic example.

Part I, consisting of Chapters 1–6, contains the core of the theory and forms the basis for Parts II and III. Chapter 1 introduces max-plus algebra, which can be viewed as a mutation of conventional algebra. In max-plus algebra, the operations max (being maximization) and plus (being addition) play a fundamental role. Vectors, matrices, and the notion of linearity are introduced within this new algebra. The "heaps of pieces" point of view provides a first application. Chapter 2 deals with eigenvalues and eigenvectors of matrices in max-plus algebra and their graph-theoretic interpretations. Sets of linear equations are studied also. In Chapters 3 and 4, we explore linear systems in max-plus algebra and study their behavior in terms of throughput, growth rate, and periodicity. Various concepts are introduced, such as (ir)reducibility of the system matrix or its graph having a sunflower shape. Chapters 5 and 6 deal with numerical procedures to calculate characteristic quantities of a matrix, such as the eigenvalue and its extension, the so-called generalized eigenmode. The three procedures treated are named Karp's algorithm, the power algorithm, and Howard's algorithm. The last one especially is very efficient for large-scale matrices.

In Part II we examine Petri nets and real-life applications, mainly drawn from everyday railway issues in the Netherlands. The subject of Chapter 8 is a study of the timetable for the whole of the Dutch railway system. Since the detailed description of the whole network would obscure the methods used, we decided to describe a subnetwork of dimension 24. This subnetwork contains all the details of how to arrive at a max-plus model starting from line and synchronization data as provided by the railway company. Petri nets form a very convenient intermediate tool to connect this data to max-plus models. Therefore, the preceding chapter, Chapter 7, is devoted to the introduction of Petri nets. Chapter 9 deals with delay propagation and various stability measures for railway networks. We also discuss issues such as an optimal allocation of trains and their ordering. The application of Chapter 10 concerns a series of railway tunnels for which capacity issues are discussed. Having come to the end of Part II, the reader should be able to conclude that max plus is at work indeed!

In Part III we explore some extensions of the theory treated so far, and this section can be read independently of Part II. Chapter 11 deals with various stochastic extensions. The subject of Chapter 12 is min-max-plus systems, which are max-plus systems, described by the max and plus operation, to which the min operation, being minimization, is added. Thus, a larger class of problems can be modeled. The relationship to the theory of nonnegative matrices and nonexpansive mappings is indicated. Lastly, Chapter 13 deals with continuous flows on networks, which, theoretically speaking, can be viewed as the continuous counterpart of discrete events on networks. Though we had been thinking about including a chapter on the control of input/output systems, we decided not to do so. The subject concerned requires a background in residuation theory that is beyond the scope of this book. Those interested are referred to [29] and the references therein.

The book ends with a bibliography, a list of frequently used symbols, and an index.

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Dear reader: We hope that you will enjoy reading this book as much as we enjoyed writing it. Have a good max-plus trip. Bon voyage!

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