Multilayer Survivable Optical Network Design

Sylvie Borne[1], Virginie Gabrel[2], Ridha Mahjoub[2], and Raouia Taktak[2]

[1] Institut Galilée, Avenue J.B. Clément 93430 Villetaneuse, France
sylvie.borne@lipn.univ-paris13.fr

[2] LAMSADE, Université Paris Dauphine, Place du Maréchal de Lattre de Tassigny 75775 Paris Cedex 16, France
{gabrel,mahjoub,taktak}@lamsade.dauphine.fr

Abstract. With the explosive growth of traffic data, telecommunication networks have evolved toward a model of high-speed IP routers interconnected by intelligent optical core networks. This IP-over-optical architecture is particularly considered as an important opportunity for telecommunication carriers who want to vary services and add more multimedia applications.

In our work, we are interested in the problem of survivability in multilayer IP-over-optical networks. Given a set of traffic demands for which we know a survivable logical routing in the IP layer, our purpose is to determine the corresponding survivable topology in the optical layer. We show that the problem is NP-hard even for one demand. We formulate the problem in terms of $0-1$ linear program based on path variables. We discuss the pricing problem and prove that it reduces to a shortest path problem. Using this, we propose a Branch-and-Price algorithm. Some preliminary computational results are also discussed.

1 Introduction

Telecommunication networks have witnessed within the past years an explosive growth of traffic data. This rapid evolution has induced a need to a new promising architecture that enable an efficient management of huge amount of data. Telecommunication networks have hence evolved toward a multilayer architecture consisting of high-speed routers interconnected by intelligent optical core networks. The IP-over-WDM networks are composed of a virtual (IP/MPLS) layer over a physical (WDM) layer. Multilayer Network Design problems has recently interested many researchers [3,5]. Moreover, survivability of this networks has become unavoidable in order to ensure a continuously routing of data in case of failures [2].

The problem that we are studying belongs actually to the multilayer survivability context. Consider an IP-over-WDM network consisted of an IP/MPLS layer over a WDM layer. The logical layer is composed of IP routers which are interconnected by virtual links and the optical layer consists of a number of Optical Cross Connects OXC interconnected by physical links. To each IP router corresponds an OXC. Consider also a set of demands and for each demand two node-disjoint paths routing it in the virtual layer. Finally, for each physical link we associate a cost corresponding to its cost of installation. The Multilayer Survivable Optical Network Design (MSOND) problem is to find, for each demand, two elementary node-disjoint physical paths routing it in

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the optical layer going in order through the OXCs corresponding to the routers in the logical paths and such that the total cost of installation is minimum. Apart from the importance of MSOND in the telecommunication context, our problem is very interesting and raised from challenging classical problems. In fact, for a single demand (Single Commodity MSOND or SC-MSOND), the problem can be seen as a Steiner cycle that should visits specific nodes with some precedence constraints between them. This is in a close relationship with classical problems such as the shortest path with specified nodes [4], the Steiner cycle [6] and the travelling salesman problem with precedence constraints [1].

The paper is organized as follows. In the following section we give some definitions and notations that are necessary for the sequel. In Section 3, we prove that MSOND is NP-hard even for a single commodity. Section 4 will be devoted to present the path formulation, discuss the corresponding pricing problem and give some preliminary results. We conclude in Section 5 by some future works and perspectives.

2 Notations

We associate to the logical layer an undirected graph $G_1 = (V_1, E_1)$ where nodes correspond to routers and edges to possible links between these routers. We associate to the optical layer an undirected graph $G_2 = (V_2, E_2)$ where nodes correspond to the OXCs and edges to the physical links between these OXC. To every router $v_i \in V_1$ we associate an OXC $w_i \in V_2$. We assume that between nodes of G_1 there exist traffic demands. Let us denote by K the set of these demands. Denote by (O_k, D_k) the pair of routers origin-destination for $k \in K$ and by O'_k and D'_k the corresponding OXCs in the optical layer. Let $L_k^1 = (v_k^{1,1}, \ldots, v_k^{1,j}, \ldots, v_k^{1,l_{1,k}})$ and $L_k^2 = (v_k^{2,1}, \ldots, v_k^{2,j}, \ldots, v_k^{2,l_{2,k}})$ be the two paths routing demand $k \in K$ in G_1. These paths pass through terminal routers $v_k^{i,j}$ for which are associated terminal optical end-nodes OXCs $w_k^{i,j} (k \in K, i \in \{1,2\}, j = 1, \ldots, l_{1,k} + l_{2,k})$. Denote by T_k the set of these *terminals*. The other nodes in V_2 are called *Steiner nodes* for the demand $k \in K$ and are denoted $S_k = V_2 \setminus T_k$. Denote by $\mathscr{T}_k = \{T_k^q, q = 1, \ldots, n_k, n_k = l_{1,k} + l_{2,k} - 2, k \in K\}$ the set of sections between the different pairs of terminals OXC. A graph $G^{q,k}$ is the induced graph obtained from G_2 by deleting all terminals T_k of the demand k but extremities of section q or T_k^q. Graph G_2 is assumed to be complete with infinite capacities on the edges. Let $c(e) > 0$ be the cost of an edge $e \in E_2$.

3 Complexity

In this section, we study the complexity of the problem MSOND. We are in particular interested in the problem SC-MSOND (case of $|K| = 1$). The SC-MSOND can be defined as follows:

Input: an undirected graph $G' = (V', E')$, a cost $w'_e \geq 0$ associated to each $e' \in E'$ and $T' = (v_1, \ldots, v_l)$ terminals.

Output: An elementary cycle going in order through the terminals T' such that the total cost is minimum.

The corresponding decision problem is to find if there exists an elementary cycle going in order through the terminals T' such that the total cost is at most equal to a positive integer U'. Recall that T' constitutes the terminals corresponding actually to the source, the destination and intermediary nodes used in the two paths between the source and the destination. Since the two given paths are vertex disjoint, we have always $|T'| \geq 3$.

Theorem 1. *SC-MSOND Problem is NP-hard.*

Proof. We prove that the decision problem associated to SC-MSOND is NP-hard by proposing a polynomial reduction from the decision problem associated to Weighted Min-Sum Vertex Disjoint Paths WMSVDP proved to be NP-hard in [7,8]. This problem can be defined as follows:

Input: an undirected graph $G = (V,E)$, a cost $w_e \geq 0$ associated to each $e \in E$ and $T = \{(s_i,t_i) \in V, i = 1,...,k\}$ pairs of origin-destination, we assume that k is fixed and greater than or equal to 3.

Output: Does it exist k vertex disjoint paths $P_1,...,P_k$, P_i is a path from s_i to t_i, $i = 1,...,k$ such that the total cost is at most equal to a positive integer U.

Consider an instance (G,W,T) of the WMSVDP. We construct from (G,W,T) an instance (G',W',T') of MSOND as follows. We add to a copy of the graph G, k vertices $u_1,...,u_k$ and $2k$ edges $\{t_i,u_i\},\{u_i,s_{i+1}\}, i = 1,...,k$ ($s_1 = s_{k+1}$). Denote E_u the added edges. Let $w'_e = w_e$ if $e \in E$ and 0 otherwise (see Figure 1). Finally we set $T' = (s_1,t_1,u_1,s_2,...,s_j,t_j,u_j...,s_k,t_k,u_k,s_1)$ the terminals.

In the following we show that there exist k vertex disjoint paths between the pairs of T in G such that the total cost is at most equal to U if and only if there exists in G' an elementary cycle going in order through the terminals T' such that the total cost is at most equal to U.

Consider first a solution of WMSVDP in G with a total cost $C \leq U$. The solution consists of k vertex disjoint paths between the pairs $(s_i,t_i), i = 1,...,k$. These paths plus the set of edges E_u constitute by construction an elementary cycle in G' going in order through the terminals of T'. And since, the weights of all edges in E_u is equal to 0, the cost of the cycle is equal to C which is at most equal to U. Consider now an elementary cycle in G' going in order through the terminals T' with a total cost $C' \leq U$. Consider the sections between the terminals $(s_i,t_i), i = 1,...,k$. Since the cycle is elementary, these sections are

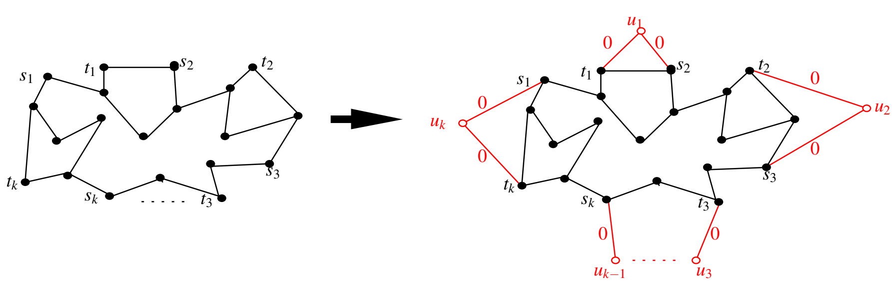

Fig. 1. WMSVDP reduction to SC-MSOND

vertex disjoint. Moreover, as the weights of all edges in E_u are 0, the total weight of the sub-paths between $(s_i,t_i), i = 1,...,k$ is exactly equal to C' which is at most equal to U.

Corollary 1. *Since SC-MSOND is a particular case of MSOND, MSOND is NP-hard.*

4 Path Formulation

We denote by P_k^q the set of paths routing the section q of demand k calculated in the reduced graph $G^{q,k}$ previously defined. We associate for each path $p \in P_k^q$ a binary variable $x_p^{q,k}$ which takes 1 if $p \in P_k^q$ is selected to rout section q of demand k and 0 otherwise. Let $y_e = 1$ if the edge $e \in E_2$ is installed and 0 if not. We define coefficients $a = (a_p^{q,k}, k \in K, q \in T_k, p \in P_k^q)$ and $b = (b_p^{q,k}, k \in K, q \in T_k, p \in P_k^q)$ as follows. $a_p^{q,k}(w)$ characterize the degree of a vertex w in a path p routing section q of demand k: it is equal to 1 if w is one of the extremities of section q, 2 if w belongs to p and 0 otherwise. $b_p^{q,k}(e)$ designs the belonging of an edge e to the path p routing section q of demand k: it is equal to 1 if e belongs p and 0 otherwise. The MSOND problem is equivalent to the following $0-1$ linear program.

$$min \sum_{e \in E_2} c(e) y_e$$

$$\sum_{p \in P_k^q} x_p^{q,k} = 1 \qquad \forall k \in K, \forall q \in \mathscr{T}_k \tag{1}$$

$$\sum_{q \in \mathscr{T}_k} \sum_{p \in P_k^q} a_p^{q,k}(w) x_p^{q,k} \leq 2 \quad \forall w \in V_2, \forall k \in K \tag{2}$$

$$\sum_{p \in P_k} b_p^{q,k}(e) x_p^{q,k} \leq y_e \qquad \forall e \in E_2, \forall k \in K, \forall q \in \mathscr{T}_k \tag{3}$$

$$0 \leq x_p^{q,k} \leq 1 \qquad \forall k \in K, \forall q \in \mathscr{T}_k, \forall p \in P_k^q \tag{4}$$

$$x_e^{q,k} \in \{0,1\} \qquad \forall k \in K, \forall q \in \mathscr{T}_k, \forall p \in P_k^q \tag{5}$$

$$0 \leq y_e \leq 1 \qquad \forall e \in E_2 \tag{6}$$

$$y_e \in \{0,1\} \qquad \forall e \in E_2 \tag{7}$$

Constraints (1) ensure routing of the demands through terminals with respect to the order constraints since paths are calculated in reduced graphs. Constraints (2) ensure the elementarity and disjunction of the two paths. Constraints (3) force routing variables to be equal to 0 if design variables are equal to 0 as well. Finally, constraints (4), (6) and (5), (7) represent, respectively, the trivial and integrity constraints.

4.1 Pricing Problem

Let us denote by $\pi^{q,k}$, λ_w^k and $\beta_e^{q,k}$ the dual variables associated respectively with constraints (1), (2) and (3), with respect to primal variable $x_p^{q,k}$. The reduced cost of the variable $x_p^{q,k}$ is given by $R_p^{q,k} = -(\pi^{q,k} + \sum_{w \in V_2} \lambda_w^k a_p^{q,k}(w) + \sum_{e \in E_2} b_p^{q,k}(e) \beta_e^{q,k})$. Here, the pricing problem is to find, for each section q of a demand k, a path of P_k^q such as

$R_p^{q,k} = min_{p\prime \in P_k^q} R_{p\prime}^{q,k}$ and $R_p^{q,k} < 0$. This can be seen as a shortest path problem in a reduced graph $G^{k,q}$ with weights λ_w^k on vertices and $\beta_e^{q,k}$ on edges. λ_w^k can be after split and hence weights are then only on edges. As dual variables λ_w^k and $\beta_e^{q,k}$ are negative, edge weights are non negative and the shortest path pricing problem can be solved in polynomial time.

4.2 Preliminary Results

We compute two relaxations of the previous program. The first relaxing both integer constraints and the second relaxing only x variables integrality. Results are reported in Table 1. The columns represent the numbers respectively of nodes in G_2, nodes in G_1, demands, generated paths for the first and second relaxations and finally the gaps of these relaxations comparing to the optimal value obtained by a Branch-and-Cut algorithm based on a cut formulation of the problem. The results show that branching only on y variables is interesting for small instances but is inefficient for larger ones. In addition, both relaxations are weak with a mean gap of near to 25% and have to be strengthened mainly by identifying and adding new valid inequalities.

Table 1. Preliminary computational results

V_1	V_2	K	paths1	paths2	gap1	gap2
6	4	4	55	65	21.26	0.00
7	4	3	99	202	21.26	0.00
7	4	5	147	227	21.26	0.00
8	5	4	340	1533	21.44	0.00
8	5	6	214	908	27.16	0.00
10	7	8	2260	18947	12.57	0.00
12	10	8	11163	348697	24.71	0.00
12	10	12	2619	195999	22.63	100.00

V_1	V_2	K	paths1	paths2	gap1	gap2
13	8	12	2474	228436	27.29	100.00
14	12	18	3620	2071	20.16	17.89
16	13	16	4890	4324	25.68	24.88
16	13	18	8071	5179	26.17	24.73
17	15	18	10403	7773	24.72	23.81
17	15	20	11474	6377	30.68	29.61
18	15	25	18478	8044	25.08	23.03
20	17	25	23168	14506	25.43	25.04

5 Conclusion

In this paper, we study the problem of Multilayer Survivable Optical Networks Design. We prove that this problem is NP-hard and we propose a path-based formulation to it. We discuss the corresponding pricing problem and give some preliminary computational results for two relaxations of the formulation. Current experimentations concern the test of different branching rules to achieve the Branch-and-Price algorithm. These results will be shown later.

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