Survivable routing of logical topologies in WDM networks

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Abstract - Network restoration is often done at the electronic layer by rerouting traffic along a redundant path. With Wavelength Division Multiplexing (WDM) as the underlying physical layer, it is possible that both the primary and backup paths traverse the same physical links and would fail simultaneously in the event of a link failure. It is therefore critical that lightpaths are routed in such a way that a single link failure would not disconnect the network. We call such a routing *survivable* and develop algorithms for survivable routing of a logical topology. We prove necessary and sufficient conditions for a routing to be survivable and use this condition to formulate the problem as an Integer Linear Program. We use our new formulation to route various logical topologies over a number of different physical topologies and show that this new approach offers a much greater degree of protection than alternative routing schemes such as shortest path routing and a greedy routing algorithm.

I. Introduction

This paper deals with the problem of routing logical links (lightpaths) on a physical network topology in such a way that the logical topology remains connected in the event of single physical link failures (e.g., fiber cut). This is a relatively new view on the Routing and Wavelength Assignment (RWA) problem, that we believe to be critical to the design of WDM-based networks. We call this version of the RWA problem *survivable RWA*. In a WDM network the logical topology is defined by a set of nodes and lightpaths connecting the nodes while the physical topology is defined by the set of nodes and the fiber connecting them. Given the logical and physical topologies of the networks, one important question is how to embed the logical topology onto the physical topology. This leads to a static version of the routing and wavelength assignment (RWA) problem. In this version of the problem, the set of lightpaths, defined by the logical topology, are known in advance. In this context various researchers have developed RWA algorithms with the goal of minimizing network costs, including number of wavelengths required, number of wavelength converters, fiber use, etc. [1]. Since with WDM each physical fiber link can support many lightpaths (as many as there are wavelengths on the fiber), once the lightpaths are routed on the physical topology, it is possible (or in fact, likely) that two or more lightpaths would share the same physical link. Hence, the failure of a single physical link, can lead to the failure of multiple links in the logical topology. Since protected logical topologies are often designed to withstand only a single link failure, it is possible that a single physical link failure could leave the logical topology disconnected.

As a simple illustrative example, consider the logical and physical topologies shown in Figure 1. The logical topology is a ring with nodes ordered 1-3-4-5-2-1. Clearly, such a ring topology is 2-connected, and would remain connected if one of its links failed. The 5 logical links of this ring can be routed on the physical topology as shown in Figure 1a, where each physical link is labeled with the logical link that traverses it. For example logical link (1,3) traverses physical links (1,5) and (5,3). As can be seen from the figure, no physical link supports more than one logical link. Hence, the logical ring would remain protected even in the event of a physical link failure.

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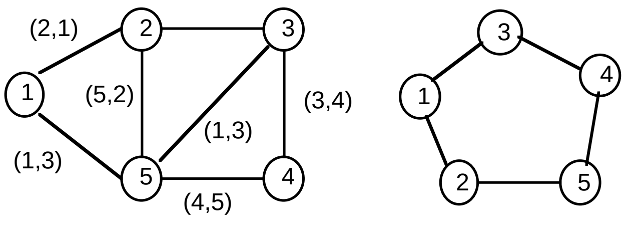

a) Physical topology b) Logical topology

Figure 1. Survivable routing of a logical topology on a physical topology.

Alternatively, had we routed logical link (1,3) on physical links (1,2) and (2,3) the routing would no longer be survivable because physical link (1,2) would have to support both logical links (1,3) and (2,1) hence its failure would leave the logical topology disconnected. Furthermore, for many logical topologies, no survivable routings can be found. For example, if the logical topology was a ring with nodes ordered 1-4-2-3-5-1 then it can be easily seen that no routing exists that can withstand a physical link failure. Hence, it is clear that although the logical topology of the network may be connected, once it is embedded on top of a WDM physical network, it may no longer withstand a physical link failure (e.g., fiber cut).

In the context of virtual private networks, the customer might request from the network provider that their lightpaths be routed in such a way that no single physical link failure would leave their VPN disconnected. One simple way to achieve this goal is to route the lightpaths so that no two lightpaths share a physical link. This seemingly simple solution by itself is difficult to obtain. In fact, it was shown in [15] that the related problem of finding disjoint paths for a collection of k source-destination pairs is NP-complete[1]. Furthermore, this simplified solution can be wasteful of resources. For many logical topologies, some of the lightpaths can be routed together while maintaining survivability.

Of course, there has been a significant body of work in the area of optical network protection [2-7,14,16]. Most previous work in WDM network protection is focused on restoration mechanisms that restore all lightpaths in the event of a physical link failure. Link based restoration recovers from a link failure by restoring the failed physical link, hence simultaneously restoring all of the associated lightpaths [2,3,6]. This is often done using optical loop-back protection [2,3,5]. In contrast, path based protection restores each of the lightpaths independently, by finding an alternative end-to-end path for each lightpath [2,3,14]. In many cases it is indeed necessary to restore all failed lightpaths. However, in other cases some level of protection is provided in the electronic layer and restoration at the physical layer may not be necessary. For example, when the electronic layer consists of SONET rings, single link failures can be recovered through loopback protection at the electronic layer. In this case, providing protection at both the optical and electronic layers is somewhat redundant. Another less obvious example is that of packet traffic in the internet where multiple electronic layer paths exist between the source and destination and the internet protocol (IP) automatically recovers from link failures by rerouting packets.

In such cases, a less stringent requirement may be imposed on the network – for example that the network remain connected in the event of a physical link failure. This approach, of course, is not suitable for all situations. For example, when a network is carrying high priority traffic with Quality of Service and protection guarantees, it may still be necessary to provide full restoration. However, when a network is used to support best effort internet traffic, guaranteeing connectivity may suffice. This approach is relatively new in the field of WDM network protection. A similar design goal was considered in [7], where heuristic algorithms were developed in order to minimize the number of source destination pairs that would become disconnected in the event of a physical link failure. The algorithm in [7] uses tabu search procedures to find disjoint alternate paths for all of the lightpaths.

In this paper we address the problem of routing the lightpaths of a logical topology on a given physical topology so that the logical topology can withstand a physical link failure. In section II we formulate the problem and give a necessary and sufficient condition for survivable routing. This condition, leads to some interesting insights into the survivable routing problem and allows us to formulate the problem as an Integer Linear Program (ILP). In section III we give an ILP formulation for the survivable routing problem. In order to obtain additional insight to the problem, in section IV we focus our attention on the problem of routing a bi-directional ring logical topology. In that case we are able to provide a simplified formulation that more easily renders a solution. We are also able to obtain necessary conditions for finding survivable routings for logical

[1] In [15] it was shown that the problem of finding node disjoint paths is NP-complete. This result can be easily extended to link disjoint paths in directed graphs.

rings. Finally, we use our ILP formulation to solve the survivable routing problem for some example networks and compare our results to alternative approaches.

II. Problem formulation

The physical topology of the network consists of a set of nodes N = {1..N} and a set of edges E where (i,j) is in E if a link exists between nodes i and j. We assume a bi-directional physical topology, where if (i,j) is in E so is (j,i). Furthermore, we assume that a failure (cut) of (i,j) will also result in a failure in (j,i). This assumption stems from the fact that the physical fiber carrying the link from i to j is typically bundled together with that from j to i. Furthermore, in some systems the same fiber is used for communicating in both directions. Lastly, we assume that WDM is employed and each physical link (fiber) is capable of supporting W wavelengths in each direction.

The logical topology of the network can be described by a set of logical nodes N_L and logical edges E_L, where N_L is a subset of N and an edge (i,j) is in E_L if both i and j are in N_L and there exists a logical link between them. Given a logical topology, we wish to find a way to route the logical topology on the physical topology such that the logical topology remains connected even in the event of a physical link failure.

In order to route a logical link (s,t) on the physical topology one must find a corresponding lightpath on the physical topology between nodes s and t. Such a lightpath consists of a set of physical links connecting nodes s and t as well as wavelengths along those links. If wavelength changers are available then any wavelength can be used on any link. However, without wavelength changers, the same wavelength must be used along the route. In this paper we assume that either wavelength changers are available or that the number of wavelengths exceeds the number of lightpaths. This assumption allows us, for now, to ignore the wavelength continuity constraints and focus only on survivable design.

Let f_{ij}^{st} =1 if logical link (s,t) is routed on physical link (i,j) and 0 otherwise. Now in order to find a routing for the logical topology, we must find a route for every logical link (s,t) in E_L. For much of this paper we consider bi-directional logical topologies where if (s,t) $\in E_L$ so is (t,s). Therefore, implicit in finding a route from s to t is also the route from t to s.

In this work we are concerned with finding routings that are survivable. We call a routing *survivable* if the failure of any physical link leaves the (logical) network connected. Of course, a routing cannot possibly be survivable if the underlying logical topology is not redundant. The logical topology is redundant (i.e., 2-connected) if the removal of any logical link does not cause the topology to be disconnected. The following theorems, give some simple yet useful necessary and sufficient conditions for survivability in a network. First we must define the following notions:

A *cut* is a partition of the set of nodes N into two parts S and $\overline{S}$ = N- S. Each cut defines a set of edges consisting of those edges in E with one endpoint in S and the other in N-S. We refer to this set of edges as the cut-set associated with the cut <S,N-S>, or simply the *CS(S,N-S)*. Let $|CS(S, N-S)|$ equal the size of the cut-set <S,N-S>; that is, the number of edges in the cut-set. The following Lemma, also known as Menger's Theorem [12], relates the connectivity of a network to the size of its cut-sets.

Lemma 1: A logical topology with set of nodes N_L and set of edges E_L is redundant (two-connected) if and only if every non-trivial cut <S,N_L-S> has a corresponding cut-set of size greater than or equal to 2.

Proof: (see [12]) Necessity is due to the fact that if any cut-set consists of only a single link, removal of that link would leave the topology disconnected. Sufficiency is a direct result of the max-flow min-cut theorem. ■

Consider a routing for a logical topology given by the assignment of values to the variables f_{ij}^{st} (for all physical links (i,j) and logical links (s,t)) which correspond to the physical links used to route the various logical links. The following Theorem gives a necessary and sufficient condition for a routing of a logical topology to be survivable.

Theorem 1: A routing is survivable *if and only if* for every cut-set CS(S,N_L-S) of the logical topology the following holds. Let E(s,t) be the set of physical links used by logical link (s,t), i.e., E(s,t) = {(i,j) $\in$ E for which f_{ij}^{st} =1}. Then, for every cut-set CS(S,N_L-S),

$$\bigcap_{(s,t)\in CS(S,N_L-S)} E(s,t) = \emptyset.$$

The above condition requires that no single physical link is shared by all logical links belonging to a cut-set of the logical topology. In other words, not all of the logical links belonging to a cut-set can be routed on the same physical link. This condition must hold for all cut-sets of the logical topology. To prove the theorem we must show that the above condition is both necessary and sufficient. Necessity is obvious because if there exists a physical link that carries all of the logical links belonging to a cut-set, failure of that link would leave the network disconnected. To see that the condition is also sufficient, notice that the removal of any physical link leaves at least one logical link in each cut-set of the logical topology connected. Hence the network must still be connected. ■ Notice that it is a direct result of the above theorem that if the logical topology was not redundant then no routing could be survivable. This is because if the logical topology was not redundant then at least one cut-set must exist with size equal to 1. The failure of the corresponding link would leave the topology disconnected.

III. Integer Linear Programming formulation

Using Theorem 1, we are able to formulate the problem of survivable routing of a logical topology on a given physical topology as an Integer Linear Program (ILP). Given a physical topology and a corresponding logical topology, we wish to find a way to route the logical topology on the physical topology such that the logical topology remains connected even in the event of a physical link failure.

In order to route a logical link (s,t) on the physical topology one must find a corresponding path on the physical topology between nodes s and t. Such a lightpath consists of a set of physical links connecting nodes s and t as well as wavelengths along those links. Let f_{ij}^{st}=1 if logical link (s,t) is routed on physical link (i,j) and 0 otherwise. Clearly f_{ij}^{st}>0 implies that there exists a physical link between nodes i and j. When the logical links are bi-directional, implicit in finding a route from s to t is also the route from t to s. Using standard network flow formulation finding a route from s to t amounts to routing a unit of flow from node s to node t [10]. This can be expressed by the following set of constraints on the value of the flow variables associated with the logical link (s,t).

$$\sum_{j\,s.t.(i,j)\in E} f_{ij}^{st} - \sum_{j\,s.t.(j,i)\in E} f_{ji}^{st} = \begin{cases} 1 & if \quad s=i \\ -1 & if \quad t=i \\ 0 & otherwise \end{cases} \qquad (1)$$

$$\forall i \in N.$$

The set of constraints above are flow conservation constraints for routing one unit of traffic from node s to node t. Equation (1) requires that equal amounts of flow due to lightpath (s,t) enter and leave each node that is not the source or destination of (s,t). Furthermore, node s has an exogenous input of 1 unit of traffic that has to find its way to node t. There are many possible combinations of flow variable values that can satisfy the constraint of eq.(1). Any feasible solution to eq.(1) has a route from s to t embedded in it. It is easy to see that if in addition we required that the path length be minimized (i.e., min $\sum_{(i,j)\in E} f_{ij}^{st}$ subject to (1)) , the solution would also be the unique shortest path [11, p.73].

Now in order to find a survivable routing for the logical topology, we must find a route for every logical link (s,t) in E_L. Using theorem 1, the connectivity requirement can be expressed using the following constraint,

$$\begin{matrix}\forall (i,j)\in E \\ \forall S \subset N_L\end{matrix}, \qquad \sum_{(s,t)\in CS(S,N_L-S)} f_{ij}^{st} + f_{ji}^{st} < \left|CS(S,N_L-S)\right|.$$

The above constraint simply states that for all proper cuts of the logical topology, the number of cut-set links flowing on any given physical link (in either direction) is less than the size of the cut-set. This implies that not all logical links belonging to a cut-set can be carried on a single physical link and immediately satisfies Theorem 1.

If the number of wavelengths on a fiber is limited to W, a capacity constraint can be imposed as follows,

$$\forall (i,j)\in E, \qquad \sum_{(s,t)\in E_L} f_{ij}^{st} \le W.$$

There are a number of objective functions that one can consider. Perhaps the simplest is to find a survivable routing that uses the least capacity. That is, minimize the total number of wavelengths used on all links (i.e., if one link uses 2 wavelengths and another uses three that amounts to a total cost of 5). An alternative formulation goal may be to minimize the total number of physical links used. Such an approach would lend itself to solutions that maximize physical link sharing by the lightpaths (subject to survivability constraints). Here we focus on the first objective of minimizing total number of wavelengths used and the optimal survivable routing problem can be expressed as the following integer linear program.

$$\text{Minimize} \quad \sum_{\substack{(i,j)\in E \\ (s,t)\in E_L}} f_{ij}^{st}$$

Subject to:

a) Connectivity constraints: for each pair (s,t) in E_L:

$$\sum_{j\, s.t.(i,j)\in E} f_{ij}^{st} - \sum_{j\, s.t.(j,i)\in E} f_{ji}^{st} = \begin{cases} 1 & if \quad s=i \\ -1 & if \quad t=i \\ 0 & otherwise \end{cases}$$

$$\forall i \in N.$$

b) Survivability constraints:

$$\begin{matrix}\forall (i,j)\in E \\ \forall S \subset N_L\end{matrix}, \qquad \sum_{(s,t)\in CS(S,N_L-S)} f_{ij}^{st} + f_{ji}^{st} < \left|CS(S,N_L-S)\right|$$

c) Capacity constraints

$$\forall (i,j)\in E, \qquad \sum_{(s,t)\in E_L} f_{ij}^{st} \le W$$

d) Integer flow constraints: $f_{ij}^{st} \in \{0,1\}$

The above ILP can now be solved using a variety of techniques. We implemented this ILP using the CPLEX software package. CPLEX uses branch and bound techniques for solving ILPs and is capable of solving ILPs consisting of up to one million variables and constraints [13]. To illustrate the utility of this approach, we implemented the ILP for the NSFNET physical topology shown in figure 2. We attempted to embed random logical topologies of degree 3, 4, and 5, where we define a logical topology of degree k to be logical topology where every node has degree k.

For each, we generated 100 random logical topologies and used the ILP to find optimal survivable routing on the NSFNET. Since we are mainly concerned with the survivable routing, in our implementation we ignored the capacity constraint (i.e., we assume no wavelength restriction). Obviously, if needed, the capacity constraints can be easily incorporated into the solution. We also compare our approach to the survivability provided by shortest path routing for the same random logical topologies. In each case we checked to see if the shortest path solution yields a survivable routing. This can be accomplished by individually removing each physical link and checking to see if the remaining topology is connected.

Our results are summarized in Table 1. Shown in the table are results for both the Shortest Paths solution (labeled SP) and the ILP solution (labeled ILP). As can be seen from the table, the ILP was able to find a protected solution for every one of the random logical topologies. In contrast, the shortest path approach resulted in 86 out of 100 of the degree 3 topologies being unprotected. With higher degree logical topologies, shortest path was able to protect more of the topologies, still 38 and 27 of the random degree 4 and 5 topologies, respectively, remained unprotected. However, as expected, the ILP solution on average required both more physical links and more total wavelengths (wavelength*links). This difference in link requirements appears to be small and well justified by the added protection that it provides.

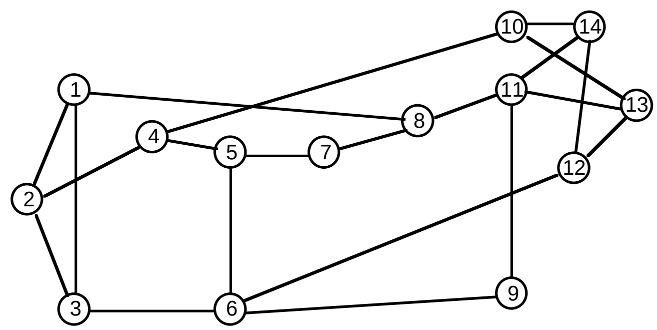

Figure 2. The 14-node, 21 link NSFNET.

	Logical Top's	Unprotected solution	Ave. links	Ave. λ*links
Degree 3 -ILP	100	0	19.55	46.07
Degree 3 - SP	100	86	19.31	45.25
Degree 4 -ILP	100	0	20.30	60.64
Degree 4 - SP	100	38	20.17	60.47
Degree 5 - ILP	100	0	20.50	75.40
Degree 5 - SP	100	27	20.48	75.31

Table 1. Embedding random logical topologies on the NSFNET of figure 2.

IV. Ring Logical topologies

We can gain some additional insight into the survivable routing problem by considering special forms of the logical topology. For example, the ring logical topology, which is the most widely used protected logical topology has a special structure that leads to a simpler problem formulation. In this section we discuss the special case of embedding ring logical topologies on arbitrary physical topologies

A unidirectional ring logical topology is an ordered set of nodes ($n_1..n_L$) where (n_i, n_{i+1}) is in E_L for 0<i<L and (n_L, n_1) is also in E_L. In a bi-directional ring, the reverse connections (n_{i+1}, n_i) and (n_1,n_L) are also in E_L. Since we focus on protected topologies, here we only consider bi-directional rings. Hence, for simplicity, we assume that all links are bi-directional and refer to the pair of links connecting nodes n_i and n_{i+1} as (n_i, n_{i+1}). Implied in this notation is that the pair of links between two nodes are treated as a single bi-directional link. It is also possible to treat the links (n_i, n_{i+1}) and (n_{i+1},n_i) as two separate links. That approach is subsumed in the general logical topology discussion of the previous section.

Recall that a routing of the logical topology is *survivable* if the failure of any physical link leaves the (logical) network connected. The following corollary to Theorem 1 gives a necessary and sufficient condition for a routing of a bi-directional logical ring to be survivable.

Corollary 1: A bi-directional logical ring is survivable if and only if no two logical links share the same physical link.

Proof: It can be easily seen that every cut-set of the ring logical topology contains exactly two links and every pair of logical links share a cut-set, hence by Theorem 1 no two logical links can share a physical link. ■

Corollary 1 leads to a significant simplification of the survivability constraints. While in the general logical topology case the survivability constraints were expressed in terms of constraints on all of the cut-sets (notice that there can be as many as 2^{N-1} such cut-sets). For the ring topology the survivability constraint can be simply replaced by a capacity constraint on the physical links. Specifically we require,

$$\sum_{(s,t)\in E_L} f_{ij}^{st} + \sum_{(s,t)\in E_L} f_{ji}^{st} \le 1 \quad .$$
$$\forall (i,j)\in L$$

That is, there can be at most one logical link routed on any given physical link. Note that since the logical links are bi-directional, when route (s,t) uses physical link (i,j), implicitly it uses the link in both directions. Also note that since no two lightpaths can share a physical link, the objective of minimizing the total number of physical links and that of minimizing the total number of wavelength*links used are in fact the same (in contrast to the general case where a physical link may be used by multiple logical links). The optimal survivable routing problem for logical rings can be expressed as the following integer linear program:

$$\text{Minimize} \sum_{\substack{(i,j)\in E \\ (s,t)\in E_L}} f_{ij}^{st}$$

Subject to:

a) Connectivity constraints: for each pair (s,t) in E_L:

$$\sum_{j\ s.t.(i,j)\in E} f_{ij}^{st} - \sum_{j\ s.t.(j,i)\in E} f_{ji}^{st} = \begin{cases} 1 & if \quad s=i \\ -1 & if \quad t=i \\ 0 & otherwise \end{cases}$$

$$\forall i \in N.$$

b) Survivability constraints:

$$\sum_{(s,t)\in E_L} f_{ij}^{st} + \sum_{(s,t)\in E_L} f_{ji}^{st} \le 1$$
$$\forall (i,j)\in L$$

c) Integer flow constraints: $f_{ij}^{st} \in \{0,1\}$

Again, the above ILP can now be solved using a variety of search techniques. While general ILP's can be rather difficult to solve, this particular ILP is relatively simple. First notice that without the survivability constraints the ILP amounts to solving a shortest path problem. The addition of the survivability constraints make the solution more difficult to obtain. However, the total number of constraints used is small, relative to the exponential number of constraints used in the general case, hence the above ILP can be solved very quickly. We were able to solve this ILP using the CPLEX software package running on a SUN SPARC Ultra 10 machine for 10 node rings in less than a second.

A. Necessary conditions for survivable routing

In this section we develop some necessary conditions on the physical topology to ensure survivable routing of ring logical topologies. Clearly, it is not always possible to route a logical topology on a given physical topology in a manner that preserves the survivability of the logical topology. For example, in the case of a ring, there may be instances where we cannot find disjoint paths for all of the links. In such cases some of the lightpaths will have to share a physical link and the ring would not be survivable. It is interesting to determine under what circumstances it will be possible (or not possible) to find survivable routings. Consider any random ring logical topology. For any cut <S, N-S> of the physical topology, let $|CS_P(S,N\text{-}S)|$ be the number of physical links along this cut and $|CS_L(S,N\text{-}S)|$ be the number of logical links traversing the same cut. Clearly, in order to be able to route the logical links along disjoint physical paths, $|CS_P(S,N\text{-}S)|$ must be greater than or equal to $|CS_L(S,N\text{-}S)|$. Hence, for a given logical topology one requirement is that for all possible cuts of the physical topology <S,N-S>, the following must hold,

$$|CS_P(S,N\text{-}S)| \geq |CS_L(S,N\text{-}S)|.$$

The above condition is necessary, but not sufficient to insure that a survivable routing exists for a particular ring logical topology.

There are situations where one may want to design a physical topology that can support all possible ring logical topologies. One such example may be a service provider that regularly receives requests for ring topologies. Such a service provider may want to design the physical topology of his network so that it can support all possible rings in a survivable manner. Another possible situation is when the logical topology can be dynamically reconfigured [8,9] for the purpose of load balancing. Here, again, one may want to ensure that the resulting topology can be routed in a survivable manner. The following theorem provides a necessary condition on the physical topology for supporting all possible ring logical topologies in a survivable manner.

Theorem 2: In order for a physical topology to support any possible ring logical topology in a survivable manner the following must hold. For any cut of the physical topology <S, N-S>,

$$|CS_P(S, N\_S)| \geq 2\min(|S|,|N-S|).$$

Theorem 2 says that for all cuts of the physical topology, the number of physical links in the cut set must be greater than or equal to twice the number of nodes on the smaller side of the cut. The condition of theorem 2 is only a necessary condition. To prove its necessity we must show that there exists a ring logical topology that requires 2 min (|S|,|N-S|) physical links along the given cut. To show the existence of such a topology we construct the following ring. Suppose without loss of generality that S achieves the minimum of (S,N-S) and let S contain nodes $n_1..n_s$. Now, construct a logical ring consisting of the following links: $\{(n_1', n_1), (n_1, n_2'), (n_2', n_2) \ldots (n_s, n_s'), (n_s', n_1')\}$, where $n_i \in S$ and $n_i' \in (N\text{-}S)$. Since $|N\text{-}S| \geq |S|$, such a construction always exists. Figure 3 shows an example where S contains 2 nodes and |N-S| = 3. A ring with 4 links traversing the cut-set is constructed using the above procedure.

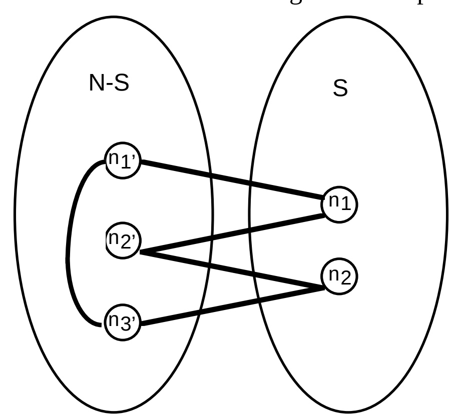

Figure 3. A logical ring that requires the maximum number of cut-set links.

Shortest path bound: Another useful yet simple lower bound on the number of links that the physical topology must contain is obtained by observing that each link in the logical topology will use at least as many physical links as would be required if it were routed on the shortest path. Since no logical link can share a physical link, the number of physical links in the physical topology must obey the following inequality,

$$|E| \geq \sum_{(s,t)\in E_L} |SP(s,t)|,$$

Where, $|SP(s,t)|$ is the length (in physical links) of the shortest path from s to t.

B. Logical ring results

We implemented the ILP for the ring logical topology using the CPLEX software package. We know from the previous section that in order to embed randomly ordered logical rings on a physical topology the physical topology must be densely connected. Hence, for the analysis in this section we consider the 6 and 10 node physical topologies of figures 4 and 5. Both of these topologies obey the conditions of Theorem 2 and every node is of degree four. Furthermore, it can be shown that both topologies are 4-connected. We therefore believe that we should be able to find survivable routings for most logical rings.

We attempted to embed all possible 6 and 10 node logical rings on the 6 and 10 nodes physical rings. Notice that there are 120 (5!) 6-node logical ring orders and 362880 (9!) 10-node logical ring topologies. We used the ring ILP to determine survivable routings for all of these topologies. In addition, we also considered two simple heuristic algorithms for finding a routing for the lightpaths. The *shortest path* solution where each lightpath of the logical topology is routed along the shortest path. Of course, in the case of shortest path, some lightpaths may be routed along the same physical link. In such cases, the shortest path approach would result in an unprotected routing. A somewhat more sophisticated approach is a greedy algorithm that routes lightpaths sequentially using the shortest available path. In order to prevent two lightpaths from sharing a physical link, whenever a physical link is used for routing a lightpath, it is removed from the physical topology so that no other lightpaths can be routed through it. Note that this greedy algorithm is useful for embedding ring logical topologies since rings require that no two logical links share a physical link. Unfortunately a similar approach cannot be used to embed arbitrary logical topologies since the connectivity of the logical topology cannot be easily determined by inspecting the routing of individual lightpaths.

Our results are summarized in Table 2. For the 6-node physical topology, our ILP was able to find a survivable routing for all 120 logical ring orders. The average number of physical links used to route a logical topology was 7.4. Also, since each physical link supports at most one lightpath, the average number of wavelength*links used was also 7.4. For the 10-node physical topology, our ILP was not able to find a survivable routing for 9.3% of the 362880 logical topologies. When a routing was found, the average number of links used to route a logical topology was 17.8. The greedy algorithm also found a survivable routing for all 6 node logical topologies, but it could not find a survivable routing for 61% of the 10 node rings. With shortest path routing, 53% of the 6-node ring logical topologies were left unprotected and 99% of the 10-node rings were left unprotected. As expected, the ILP was able to protect many more of the logical topologies. Of course, this added protection comes at a price. Shortest path routing used an average of 7.2 wavelengths*links for the 6-node rings and 15.5 wavelengths*links for the 10 node rings, only slightly less than the number of links used by the ILP solution. However, shortest path routing used significantly fewer physical links than the ILP solution. This is, of course, because shortest path routing allows lightpaths to share a physical link, while the ILP does not. Also shown in the table are the number of links used by the greedy algorithm. By definition, the greedy algorithm does not yield a routing when a protected solution is not found, thus the number of links used can only be calculated when a protected solution is obtained. As expected, the greedy solution used more links than both the ILP and the shortest-path solutions.

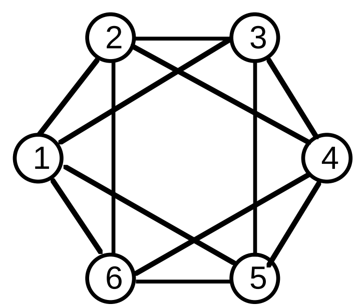

Figure 4. 6 node degree 4 physical topology.

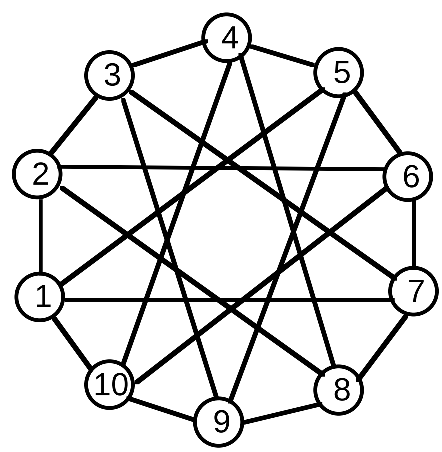

Figure 5. 10 node degree 4 physical topology.

	Logical Top's	No protected solution	Ave. links	Ave. λ*links
6 node-ILP	120	0	7.4	7.4
6 node - SP	120	64 (53%)	6.4	7.2
6 node - GR	120	0	8.1	8.1
10 node-ILP	362880	33760 (9%)	17.8	17.8
10 node - SP	362880	358952 (99%)	11.8	15.5
10 node - GR	362880	221312 (61%)	18.4	N/A

Table 2. Embedding ring logical topologies on 6 and 10 node 3-connected physical topologies.

Next we consider the 10-node physical topology of figure 5 and attempt to embed random logical ring topologies of various sizes. We attempted to embed 10,000 random logical rings of each size between 5 and 10 nodes. For each ring the set of nodes and their order was chosen at random. Again, we compare the results of our ILP to those obtained using the shortest path routing algorithm and the greedy algorithm. In figure 6 we plot the percent of logical topologies for which we failed to obtain a protected routing. As can be seen from the figure, when we used the ILP we were able to find a protected routing for 100% of the logical rings of size 5 to 9, and fewer than 10% of the 10 node rings were left unprotected. Notice that this latter number is consistent with the results in Table 2. However, when shortest path routing was used, the majority of the logical topologies were left unprotected. The greedy approach was able to protect more of the topologies, but not nearly as many as the ILP. In Figure 7 we plot the average number of physical links used per logical topology. As can be seen from the figure, the shortest path approach indeed uses fewer physical links. However, at a relatively small cost in number of physical links, the ILP solution is able to offer a much greater level of protection. Also notice that the number of wavelengths*links used with the ILP solution is the same as the number of physical links used. In contrast the shortest path solution uses more wavelength*links than physical links because some physical links support multiple wavelengths. As expected, the greedy approach used the most links. Also notice that in the case of the greedy approach, the average number of links represents only those topologies for which a protected routing was found. Hence for those cases the number of physical links is the same as the number of wavelengths*links. As mentioned previously, in the case of the greedy algorithm, the number of links used is only calcuated in cases where a protected solution is found.

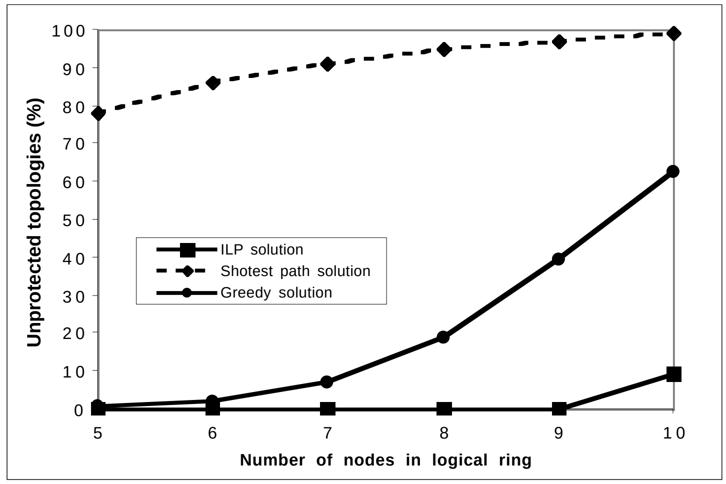

Figure 6. Fraction of logical ring topologies that cannot be protected on the 10 node physical topology of figure 5.

For extremely large topologies, solving the Integer Linear Program may become difficult. Thus it would be nice to determine what information can be obtained from the Linear Programming (LP) relaxation of the problem. The linear programming relaxation will either find (1) no solution exists, (2) determine a solution with integer flows, or (3) determine a solution with non-integer flows. If the LP relaxation results in no solution, this is a simple way to determine that there is no solution to the ILP either. If the LP relaxation finds an integer solution, then this solution will also be the solution for the ILP. In the third case where the LP relaxation finds a non-integer solution, one must solve the ILP to determine a survivable routing. We solved the LP relaxation for the 6-node and 10-node cases described above to determine the effectiveness of the LP relaxation in solving the integer problem. In the 6-node case, 11.6% of the logical topologies resulted in a non-integer solution. The remaining logical topologies produced integer solutions. In the 10-node case, 97% of the logical topologies that were determined to be unprotectable by the ILP were also found to be infeasible by the LP relaxation. Unfortunately, 57% of the ring logical topologies produced non-integer solutions to the LP relaxation. As mentioned above, to determine a survivable routing for these logical topologies requires solving the ILP.

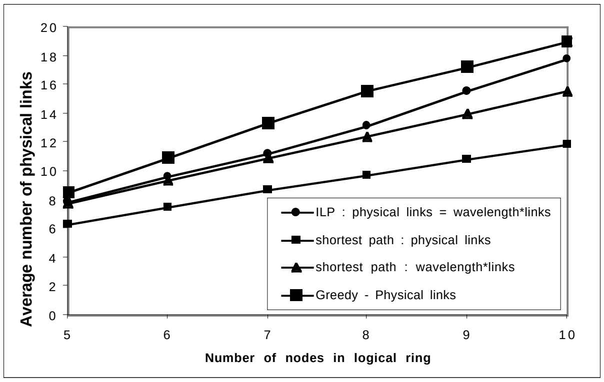

Figure 7. Average number of physical links used to embed ring logical topologies on the 10 node physical topology of figure 5.

V. Conclusion

This paper considers the problem of embedding protected logical topologies on a WDM physical topology so that the resulting network remains connected in the event of a physical link failure. We proved necessary and sufficient conditions for the survivable routing of the logical topology and used these conditions to develop an ILP formulation for the problem. We used the new ILP formulation to find survivable routings for a variety of network topologies. Our results show that this new formulation is able to offer a much greater degree of protection when compared to shortest path routing. This added protection, of course, comes at the expense of additional network resources. However, it appears from our examples that the additional number of links and wavelengths needed is rather small.

Since this problem is relatively new, the work in this paper is rather preliminary and many extensions are possible. For example, this approach can be used to design a network to various degrees of protection. While here we focused on single link failures, multiple failures can be captured in a similar manner. Also, while here we focused on minimizing the total number of wavelength*links used, other objective functions, such as total number of physical links used, can also be minimized. Lastly, while here we focused on the survivability constraints only, future work could also consider wavelength limitation and wavelength continuity constraint.

Perhaps the most important area for future work is in the search for an efficient solution to the ILP problem. Since large ILPs are generally difficult to solve, it would be useful to find efficient relaxations and alternative formulations that yield feasible solutions.

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