

Research 1

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Tools

- Python
- Numpy, matplotlib

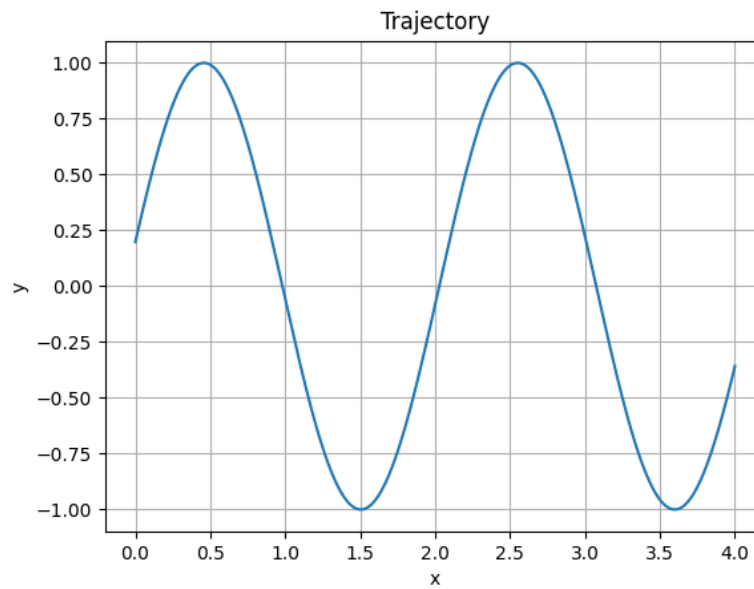
Solution

Source code

1. Introduce basic equations:

$$y = A \sin(O_m x + \theta_0) \quad (1)$$

$$y'_x = A O_m \cos(O_m x + \theta_0) \quad (2)$$



2. Find natural form of motion:

$$\sigma(x) = \int_{X_{min}}^x \sqrt{1 + y'_x(x)^2} dx \quad (3)$$

$$\dot{\sigma}(x) = \sqrt{1 + y'_x(x)^2} \quad (4)$$

3. As we prepared initial formulas, we can proceed with explanations.

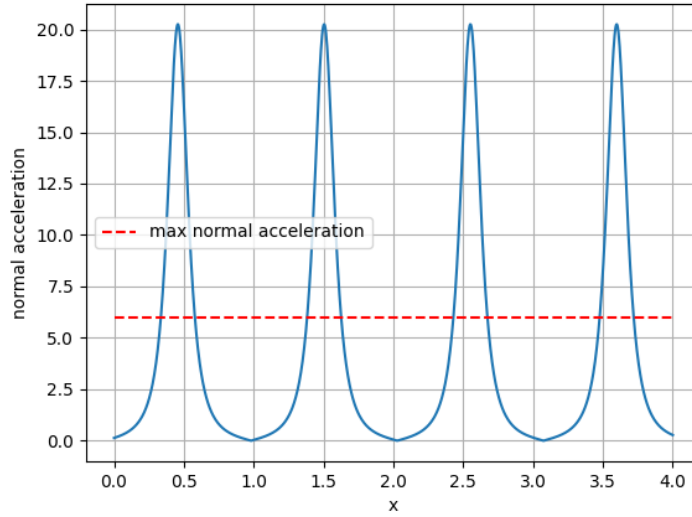
4. Find how limitation on normal acceleration affects the motion.

(a) Curvature formula:

$$\kappa(x) = \frac{|y''_x(x)|}{\sqrt{1 + y'_x(x)^2}} \quad (5)$$

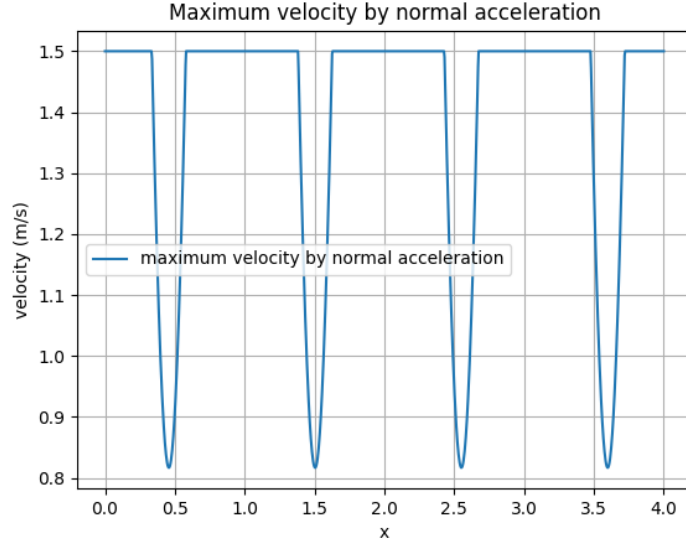
(b) Let us imagine we are moving with v_{max} , what our normal acceleration would be throughout the x

$$a_n(x) = v_{max}^2 \kappa(x) \quad (6)$$



(c) As we see that normal acceleration exceeds the limitation, we could reverse calculations and obtain the maximum velocity we could take through the curve.

$$v_{max}(x) = \sqrt{\frac{a_n(x)}{\kappa(x)}} \quad (7)$$



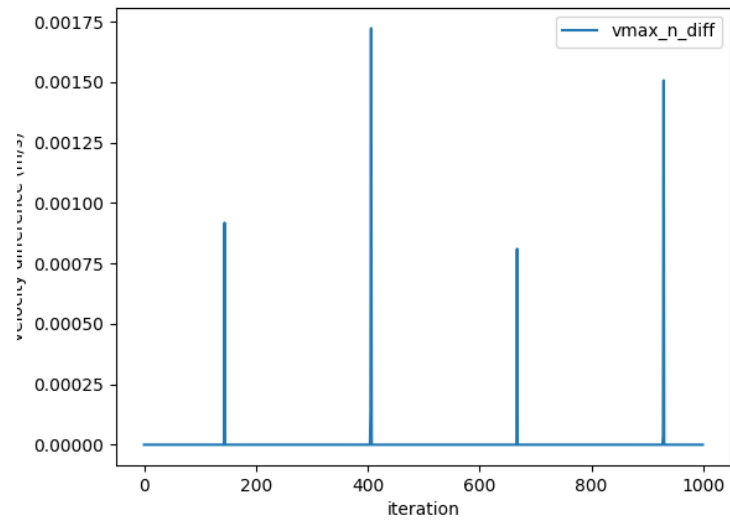
5. Find how limitation on tangential acceleration affects the motion.

- (a) We can observe that at some point we cannot accelerate and decelerate on the given dx interval. I would thank Ilya Miloshin for explanation of this case.

$$a_\tau(t) = \frac{v'_{max}(x(\sigma(t)))}{dt} = \frac{dv_{max}}{dx} \frac{dx}{d\sigma} \frac{d\sigma}{dt} \quad (8)$$

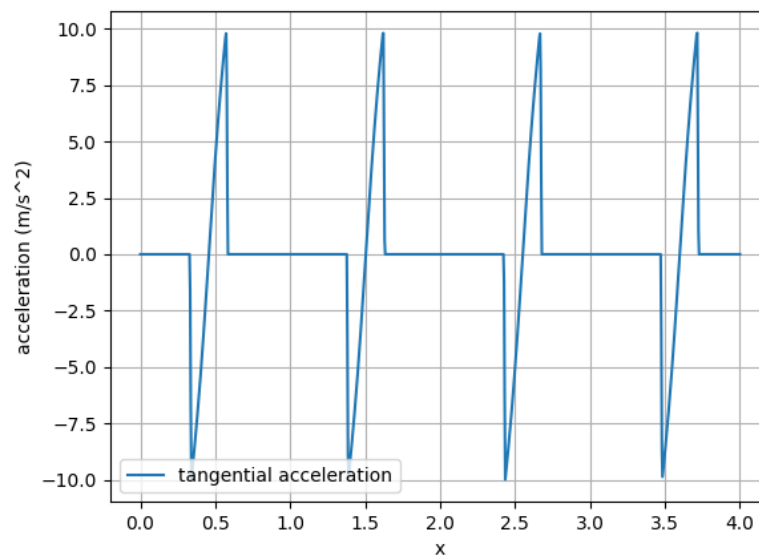
$$\frac{dx}{d\sigma} = \frac{1}{\sqrt{1 + y_x'^2}} \quad (9)$$

- (b) Actually, the difference between velocity limited by normal acceleration and further by tangential component was not big.



6. Tangential acceleration

By differentiating the velocity by x , we can find the tangential acceleration for each x .



7. Acceleration and deceleration

So far there was no talk about start and finish of trajectory.

- (a) We have to start from 0 velocity and end with 0 velocity.
- (b) We will use trapezoidal profile during simulation to accelerate and decelerate.
- (c) Position to start deceleration:

$$t_{decel} = \frac{v_{max}}{a_{\tau}} \approx 0.15 \quad (10)$$

$$s_{decel} = a_{\tau max} * t_{decel}^2 / 2 \quad (11)$$

- (d) So, when we are left with s_{decel} distance (we will convert to x_{decel} for simplicity in code), we start deceleration.

8. Simulation

- (a) We will use trapezoidal profile for acceleration and deceleration
- (b) General approach:
 - i. Choose appropriate acceleration
 - ii. Store current parameters to use in next iteration
 - iii. Velocity simulation:

$$v(t) = v(t - dt) + a(t) \cdot dt \quad (12)$$

- iv. Position simulation:

$$x(t) = x(t - dt) + v(t) \cdot dt \cdot \left(\sqrt{1 + y_x'^2} \right)^{-1} \quad (13)$$

Multiplication by $\dot{\sigma}(x)$ is done to consider only x part of position change change.

- v. Y coordinate can be just taken for particular x from trajectory.
- vi. Normal acceleration can be calculated through current velocity and x coordinate:

$$a_n(t) = v(t)^2 \kappa(x(t)) \quad (14)$$

- (c) Part 1: accelerate until velocity has reached maximum

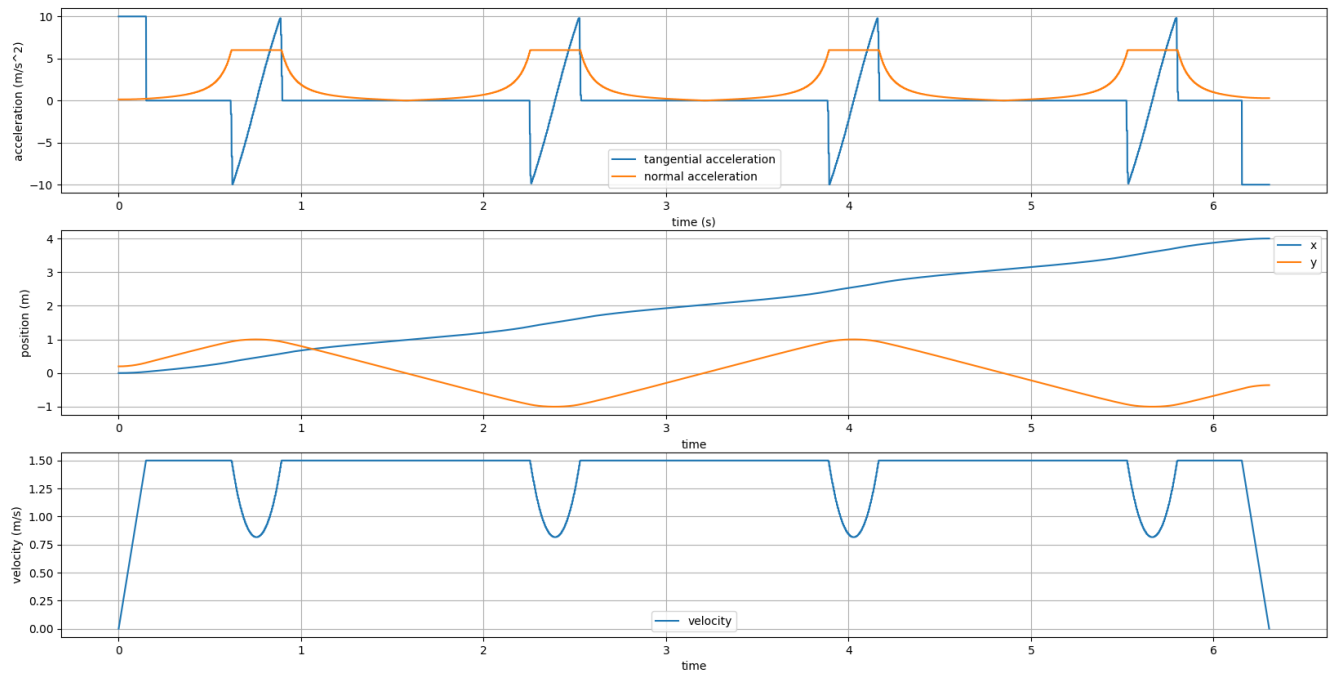
$$a_{\tau}(t) = a_{\tau max} \quad (15)$$

- (d) Part 2: Follow acceleration profile for x 's until we reach x_{decel}

$$a_{\tau}(t) = a_{\tau}(x_{cur}) \quad (16)$$

- (e) Part 3: decelerate until velocity has reached 0

$$a_{\tau}(t) = -a_{\tau max} \quad (17)$$



Answer:

For my simulation approximate time to follow this trajectory was: 6.3 seconds.