Homework 5

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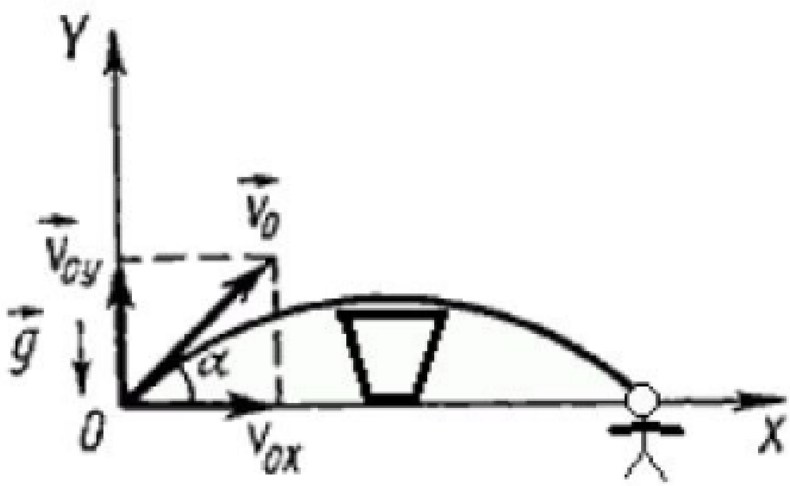
Source code

Task 1

Too lazy to draw in tikz, so here you are.



Part 1: find the angle to shoot the officer

1. RO: particle - planar motion
2. Condition:

	$initial$	$final$
t	0	?
x	0	L
x'	$v_0 \cdot \cos(\alpha)$	$v_0 \cdot \cos(\alpha)$
x''	0	0
y	0	0
y'	$v_0 \cdot \sin(\alpha)$	?
y''	$-g$	$-g$

3. Force analysis: $\vec{G}$

4. Solution:

   (a) Equations by axis:

   $$\begin{cases} mx'' = 0 \\ my'' = -mg \end{cases} \tag{1}$$

   Integration yields:

   $$\begin{cases} x' = c_1 \\ y' = -gt + c_3 \end{cases} \tag{2}$$

   Another integration:

   $$\begin{cases} x = c_1 t + c_2 \\ y = -\frac{1}{2}gt^2 + c_3 t + c_4 \end{cases} \tag{3}$$

   (b) Substitution of initial values:

   $$\begin{cases} c_1 = v_0 \cdot \cos(\alpha) \\ c_2 = 0 \\ c_3 = v_0 \cdot \sin(\alpha) \\ c_4 = 0 \end{cases} \tag{4}$$

   (c) Combining:

   $$\begin{cases} L = v_0 \cdot \cos(\alpha) t \\ 0 = -\frac{1}{2}gt^2 + v_0 \cdot \sin(\alpha) t \end{cases} \tag{5}$$

   (d) Result: Python says there are two solutions: $\alpha = 0.0097$ and $\alpha = 1.561$. And I have no doubts to not trust Python.

Part 2: find the max height of the cargo ship can be to make this shot

1. As there are two angles that satisfy the first part, we need to find the max height for each of them.

2. Analysis of equation for y-axis:

$$y = -\frac{1}{2}gt^2 + v_0 \cdot \sin(\alpha)t \tag{6}$$

3. As y is parabola, we can simply find its maximum height by finding extrema:

$$t_{max} = \frac{v_0 \cdot \sin(\alpha)}{g} \tag{7}$$

$$y_{max} = y(t_{max}) \tag{8}$$

4. Result:
For the first case: $y_{max} = 3.64555853045729$
For the second case: $y_{max} = 38574.3360928457$

Part 3: find an angle α, if you take into consideration the air resistance

1. RO: particle - planar motion

2. Condition:

	$initial$	$final$
t	0	?
x	0	L
x'	$v_0 \cdot \cos(\alpha)$	?
x''	0	0
y	0	0
y'	$v_0 \cdot \sin(\alpha)$	?
y''	$-g$	$-g$

3. Force analysis: $\vec{G}$, $\vec{F_c}$

4. Solution:

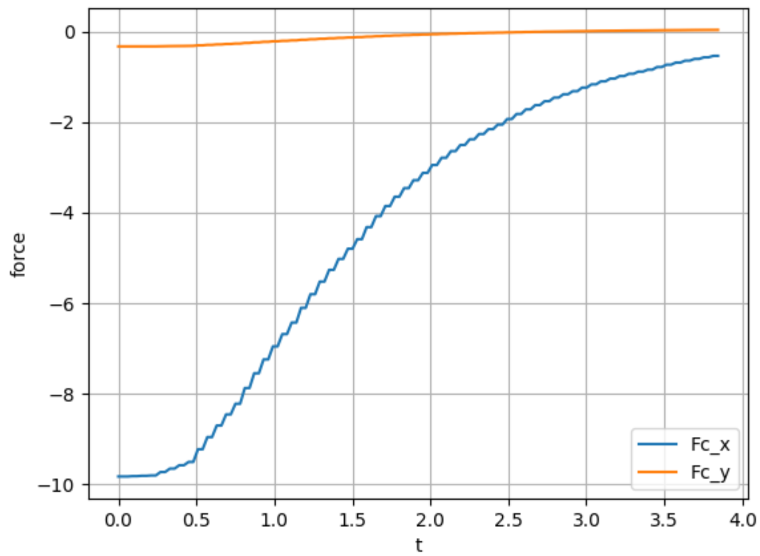
   (a) Equations by axis:

$$\begin{cases} mx'' = -k\sqrt{x'^2 + y'^2}x' \\ my'' = -mg - k\sqrt{x'^2 + y'^2}y' \end{cases} \tag{9}$$

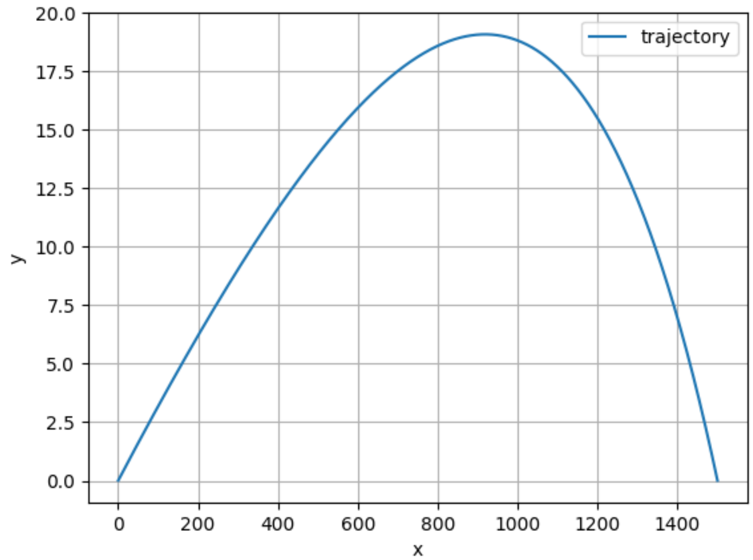
   (b) Too hard to integrate by hands, so I'll use Python. Everthing is the same as in part 1, but with different result.

(c) Result:
An angle to shoot officer is ≈ 0.0324

5. Air resistance force:



6. Trajectory with resistance:



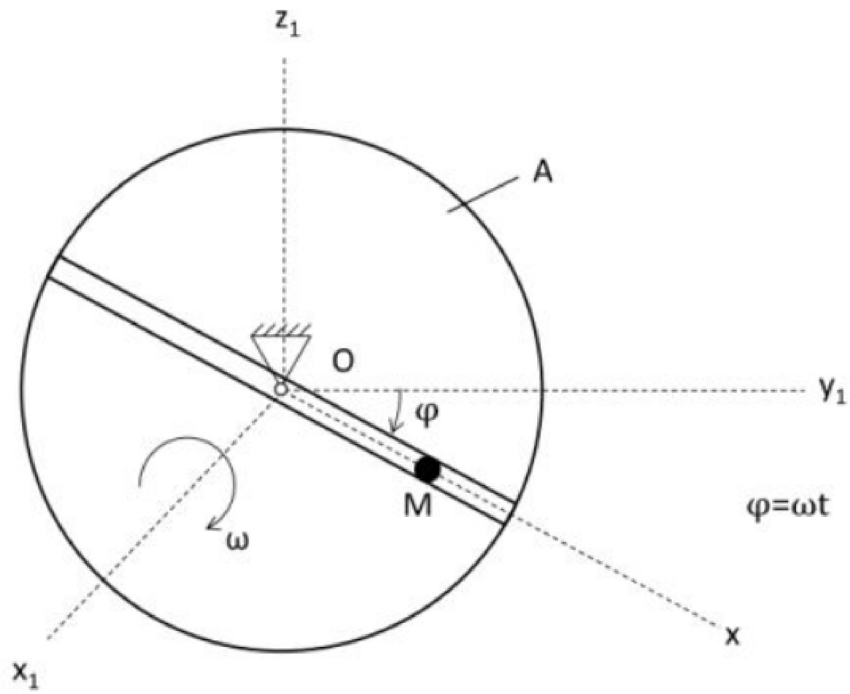
1. $\alpha = 0.0097$,
   $\alpha = 1.561$

2. $y_{max} = 38574.336$

3. $\alpha = 0.0324$

Task 2

simulation



1. RO: particle M - translatory motion, disk A - rotation
2. Condition:

	initial	*final*
t	0	?
x	0	r
x'	0.4	?
x''	0	0

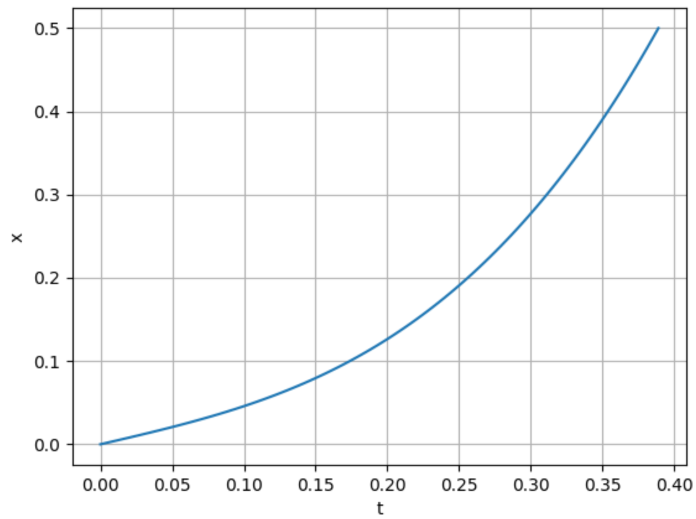
3. Force analysis: $\vec{G}$, $\vec{N}$
4. Solution:
   (a) Equation on x(not static) axis:

$$mx'' = \sum F_x + \Phi_{trx} + \Phi_{corx} \tag{10}$$

$$mx'' = mg\sin(\omega t) + m\omega^2 x \tag{11}$$

   As coriolis acceleration is $\vec{a}_{cor} = 2(\vec{w}_{tr} \times \vec{v}_{rel})$, we see that its projection on x axis is 0.

(b) I will not directly solve this equation here, because it just a matter of math (wolfphram) solution



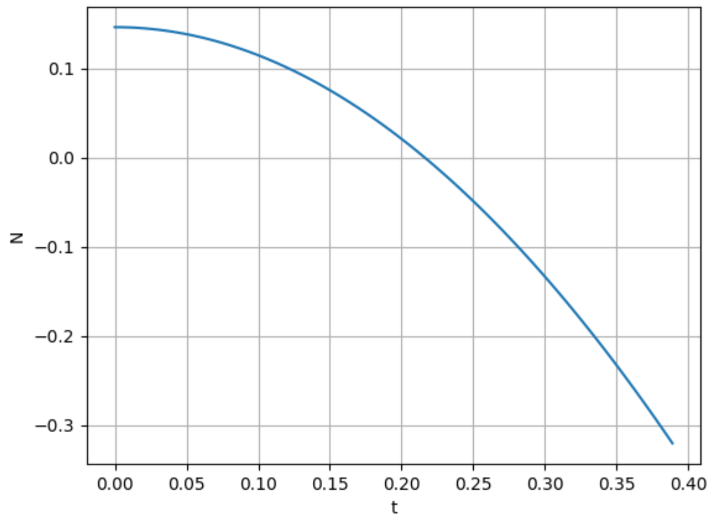
(c) At next point we will find t until which we have to simulate everything:

$$x(t) = r \tag{12}$$

(d) Equation on y(not static) axis:

$$ma_{cor} + N - mg\sin(\omega t) = 0 \tag{13}$$
$$\tag{14}$$

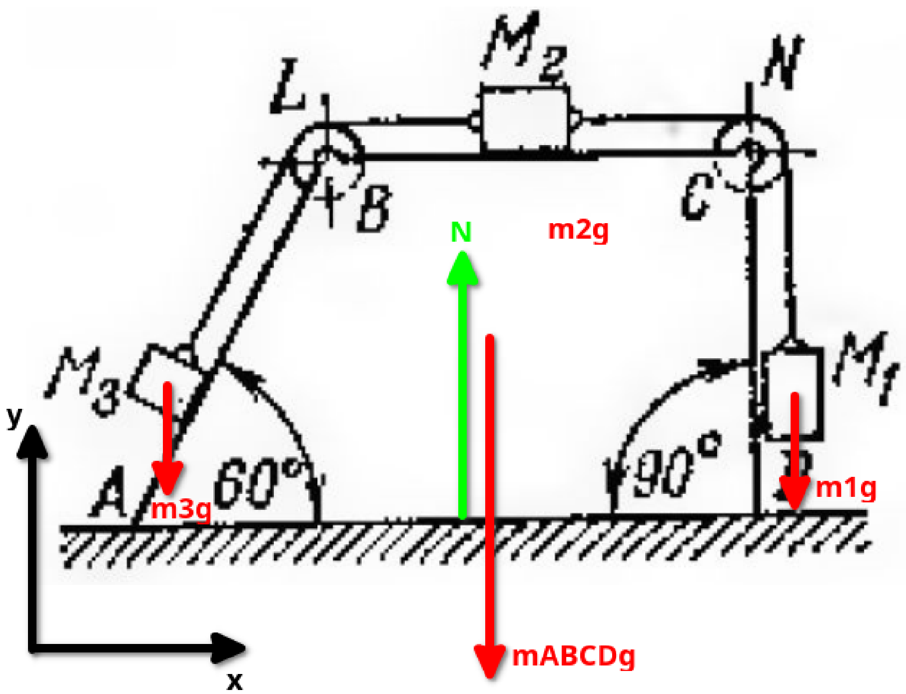
This equation is enough to find N for each moment of time.



Answer:

$t_{final} \approx 0.38s$

Task 3



1. I will solve the task as it was supposed by "Meshcherskiy", so mass of $ABCD$ will be 100.
2. RO - system of 4 bodies, $ABCD$, body 1, body 2, body 3 - translatory motion.
3. Method: CoM
4. Conditions:
   I will write only for x axis as it is enough to complete this task

	initial	final
x_{ABCD}	x_0	$x_0 + s$
x_{body1}	x_1	$x_1 + s$
x_{body2}	x_2	$x_2 + s + d$
x_{body3}	x_3	$x_3 + s + d\cos(\pi/3)$
x'	0	0
x''	0	0

5. Forces analysis: $\vec{G}_{ABCD}$, $\vec{G}_{body1}$, $\vec{G}_{body2}$, $\vec{G}_{body3}$, $\vec{N}$ (of the whole system)
6. Solution:

(a) Writing this equation for x axis:

$$Mx_c'' = 0 \tag{15}$$
$$Mx_c' = 0 \tag{16}$$
$$Mx_c = 0 \tag{17}$$
$$\tag{18}$$

Which means that between initial and final position center of mass did not move.

(b) Equation connecting initial and final position:

$$Mx_c^{init} = Mx_c^{final} \tag{19}$$

(c) Using CoM:

$$\sum m_i x_i^{init} = \sum m_i x_i^{final} \tag{20}$$
$$m_1 s + m_2 d + m_2 s + m_3 d \cos(\pi/3) + m_3 s + m_{ABCD} s = 0 \tag{21}$$

Answer:

$$s = -\frac{4}{29}$$