Homework 3

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Homework 3 webpage

1 Task 1

Source code

Simulations

1. I assume point M moves from A to B, so I actually have not OM, but rather AO in motion equation.

2. Find derivatives of motion and angle equations:
   Equation of motion:

$$s_r(t) = 6\pi t^2 \tag{1}$$
$$\dot{s_r}(t) = 12\pi t \tag{2}$$
$$\ddot{s_r}(t) = 12\pi \tag{3}$$

   Equation of rotation:

$$\phi(t) = \pi t^3/6 \tag{4}$$
$$\dot{\phi}(t) = \pi t^2/2 \tag{5}$$
$$\ddot{\phi}(t) = \pi t \tag{6}$$

3. Obtain positions of all points:

   (a) Position of point O_1:

$$\vec{r}_{O_1}(t) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{7}$$

   (b) Position of point O_2:

$$\vec{r}_{O_2}(t) = \begin{bmatrix} 2 \cdot R \\ 0 \end{bmatrix} \tag{8}$$

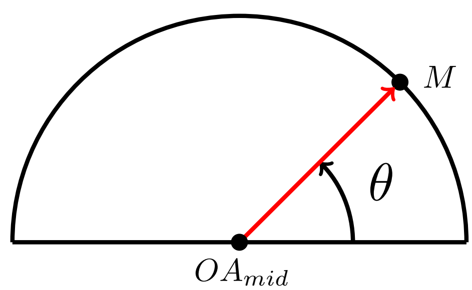
(c) Position of point O. It rotates around O_1:

$$\vec{r}_O(t) = \begin{bmatrix} \cos(\phi(t)) \\ \sin(\phi(t)) \end{bmatrix} \tag{9}$$

(d) Position of point A. It rotates around O_2:

$$\vec{r}_A(t) = \begin{bmatrix} 2 \cdot R + R \cdot \cos(\phi(t)) \\ R \cdot \sin(\phi(t)) \end{bmatrix} \tag{10}$$

(e) Position of point M. It follow semicircle from A to O. Using $s_r(t)$ we can find an angle θ for M as angle inside object D.



$$\theta(t) = s_r(t)/R = 6\pi t^2/R \tag{11}$$

$$\vec{r}_M(t) = \frac{\vec{r}_O(t) + \vec{r}_A(t)}{2} + R \cdot \begin{bmatrix} \cos(\theta(t)) \\ \sin(\theta(t)) \end{bmatrix} \tag{12}$$

4. Find velocities for M

(a) Relative velocity. We consider motion of M relative to OA_{mid}.

$$\vec{v}_M^{rel}(t) = \dot{\vec{\theta}}(t) \times (\vec{r}_M(t) - \vec{r}_{OA_{mid}}(t)) \tag{13}$$

(b) Transport velocity. Bar OA performs translational motion because it remains parallel all the time ($O_1A = O_2A$). As a result, it has the same speed at any point on OA and we can take any one. I took O.

$$\vec{v}_O(t) = \dot{\phi}(t) \times \begin{bmatrix} O_1O\cos\phi(t) \\ O_1O\sin\phi(t) \end{bmatrix} \tag{14}$$

(c) Absolute velocity.

$$\vec{v}_M(t) = \vec{v}_O(t) + \vec{v_M^{rel}}(t) \tag{15}$$

5. Find accelerations for M

   (a) Relative tangential acceleration. We consider motion of M relative to OA_{mid}.

$$\vec{a}^{rel}_{M\tau}(t) = \ddot{\vec{\theta}}(t) \times (\vec{r}_M(t) - \vec{r}_{OA_{mid}}(t)) \tag{16}$$

   (b) Relative normal acceleration. We consider motion of M relative to OA_{mid}

$$\vec{a}^{rel}_{Mn}(t) = \dot{\vec{\theta}}(t) \times (\dot{\vec{\theta}}(t) \times (\vec{r}_M(t) - \vec{r}_{OA_{mid}}(t))) \tag{17}$$

   (c) Transport acceleration.

$$\vec{a}_O(t) = \vec{a}_{On}(t) + \vec{a}_{O\tau}(t) = \dot{\phi}(t) \times \begin{bmatrix} O_1O\cos\phi(t) \\ O_1O\sin\phi(t) \end{bmatrix} + \ddot{\phi}(t) \times \begin{bmatrix} O_1O\cos\phi(t) \\ O_1O\sin\phi(t) \end{bmatrix} \tag{18}$$

   (d) Coriolis acceleration is zero because bar AB is doing translational motion and does not rotate.

   (e) Absolute acceleration.

$$\vec{a}_M(t) = \vec{a}_O(t) + \vec{a^{rel}_{M\tau}}(t) + \vec{a^{rel}_{Mn}}(t) \tag{19}$$

6. Find t, when M reaches O. It will happen when $\theta(t) = \pi$.

$$6\pi t^2/R = \pi \tag{20}$$

$$t = \sqrt{\frac{R}{6}} = \sqrt{3} \tag{21}$$

Answer

Answers were highlighted in the text with green background.

2 Task 2

Source code
Simulations

1. Find derivatives of motion and angle equations: Equation of motion

$$s_r(t) = 75\pi(0.1t + 0.3t^2) \tag{22}$$
$$\dot{s_r}(t) = 75\pi(0.1 + 0.6t) \tag{23}$$
$$\ddot{s_r}(t) = 75\pi(0.6) \tag{24}$$

Equation of angle

$$\phi(t) = 2t - 0.3t^2 \tag{25}$$
$$\dot{\phi}(t) = 2 - 0.6t \tag{26}$$
$$\ddot{\phi}(t) = -0.6 \tag{27}$$

2. Find position of points:
   (a) Point O_1:

$$\vec{r}_{O_1}(t) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{28}$$

   (b) Point O:

$$\vec{r}_{O_2}(t) = \sqrt{2}R \cdot \begin{bmatrix} \cos\left(\phi(t) + \frac{\pi}{4}\right) \\ \sin\left(\phi(t) + \frac{\pi}{4}\right) \end{bmatrix} \tag{29}$$

   (c) Point D (center of the disk):

$$\vec{r}_D(t) = R \cdot \begin{bmatrix} \cos\left(\phi(t) + \frac{\pi}{2}\right) \\ \sin\left(\phi(t) + \frac{\pi}{2}\right) \end{bmatrix} \tag{30}$$

   (d) Let us define angle θ as angle $\angle ODM$.

$$\theta(t) = \frac{s_r(t)}{R} = \frac{75\pi(0.1t + 0.3t^2)}{R} \tag{31}$$

   (e) Point M:

$$\vec{r}_M(t) = \vec{r}_D(t) + R \cdot \begin{bmatrix} \cos(\theta(t)) \\ \sin(\theta(t)) \end{bmatrix} \tag{32}$$

3. Velocities of M.

   (a) Relative velocity. (we fix the disk $\implies$ rotation around D)

$$\vec{v}_M^{rel}(t) = \dot{\vec{\theta}}(t) \times (\vec{r_M}(t) - \vec{r_D}(t)) \tag{33}$$

   (b) Transport velocity. (we fix the point M $\implies$ rotation around O_1)

$$\vec{v}_M^{tr}(t) = \dot{\vec{\phi}}(t) \times \vec{r_M}(t) \tag{34}$$

   (c) Absolute velocity of point M.

$$\vec{v}_M(t) = \vec{v}_M^{rel}(t) + \vec{v}_M^{tr}(t) \tag{35}$$

4. Accelerations of M.

   (a) Relative acceleration (we fix the disk).

$$\begin{aligned} \vec{a}_M^{rel}(t) = \vec{a}_{M\tau}^{rel}(t) + \vec{a}_{Mn}^{rel}(t) = \\ \ddot{\vec{\theta}}(t) \times (\vec{r}_M(t) - \vec{r}_D(t)) + \\ \dot{\vec{\theta}}(t) \times (\dot{\vec{\theta}}(t) \times (\vec{r}_M(t) - \vec{r}_D(t))) \end{aligned} \tag{36}$$

   (b) Transport acceleration (we fix the point).

$$\vec{a}_M^{tr}(t) = \ddot{\vec{\phi}}(t) \times \vec{r}_M(t) + \dot{\vec{\phi}}(t) \times (\dot{\vec{\phi}}(t) \times \vec{r}_M(t)) \tag{37}$$

   (c) Coriolis acceleration:

$$\vec{a}_M^{cor}(t) = 2\dot{\vec{\phi}}(t) \times \vec{v}_M^{rel}(t) \tag{38}$$

   (d) Absolute acceleration of point M.

$$\vec{a}_M(t) = \vec{a}_M^{rel}(t) + \vec{a}_M^{cor}(t) + \vec{a}_M^{tr}(t) \tag{39}$$

5. Find t, when M reaches O second time.

$$\begin{aligned} \theta(t) &= 2\pi \\ 0.3t^2 + 0.1t - \frac{2R}{75} &= 0 \\ t \approx 1.4781 \text{ (take positive root)} \end{aligned} \tag{40}$$

Answer

Answers were highlighted in the text with green background.