Homework 6

Lev Kozlov

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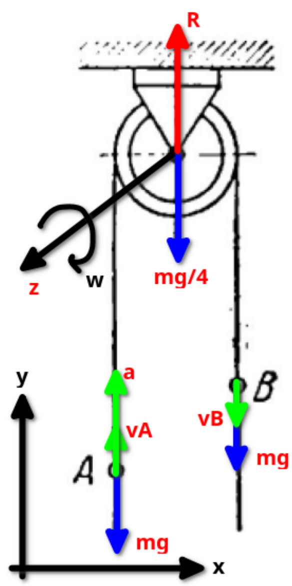
Task 1



1. RO: system of pulley, two masses
2. Motion: A, B - rectilinear
3. Conditions:

	$initial$	$final$
y_A	y_A^0	?
y_B	y_B^0	?
v_A	0	$v+a$
v_B	0	$-v$
a_A	$-g$	$-g$
a_B	$-g$	$-g$

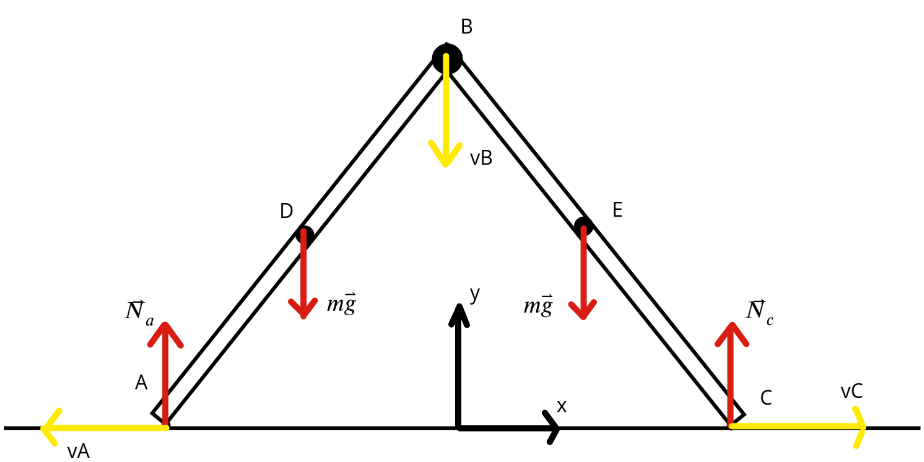
4. Force analysis:
$m_A\vec{g}$, $m_B\vec{g}$, $m_{pulley}\vec{g}$, $\vec{R}$

5. Method: Theorem of change of angular momentum of the system
6. Solution:
   (a) Find momentum around axis z:
   $$-m_a g \cdot r + m_b g \cdot r = 0 \tag{1}$$
   We know that $\sum M_z(\vec{F}_i) = 0$ which makes easier to use the theorem.
   (b) Theorem application:
   $$0 = L_{pulley} + L_A + L_B \tag{2}$$
   $$0 = I\omega + m \cdot v_a \cdot r + m \cdot v_b \cdot r \tag{3}$$
   $$0 = \frac{mr^2}{4}(\frac{v_b}{r}) + m \cdot (v_b + a) \cdot r + m \cdot v_b \cdot r \tag{4}$$
7. Answer:

$$v_b = -\frac{4 \cdot a}{9}$$

mass B will start to move upwards

Task 2



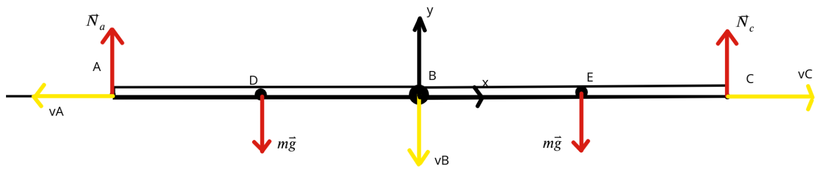
1. RO: system of 2 rods AB, BC
2. Method: Theorem of change of kinetic energy (correlation between displacement and velocity)

3. Force analysis:

$$\vec{N}_a, \vec{N}_c, m_{AB}\vec{g}, m_{BC}\vec{g}$$

There are no forces along x-axis $\implies$ B will hold its x position

4. Conditions:



	$initial$	$final$
x_b	0	0
y_b	h	0
x_c	$\sqrt{4l^2 - h^2}$	$\sqrt{4l^2 - h^2} + h$
y_c	0	0
x_a	$-\sqrt{4l^2 - h^2}$	$-\sqrt{4l^2 - h^2} - h$
y_a	0	0

5. Solution:

   (a) Kinetic energy:

$$T_{AB} = \frac{1}{2} I\omega_{AB}^2 \tag{5}$$
$$T_{BC} = \frac{1}{2} I\omega_{BC}^2 \tag{6}$$

   (b) Inertia of the rod:
   I will use Huygens–Steiner theorem to find moment of inertia.

$$I = m_{AB}l^2 + m_{AB}\rho^2 \tag{7}$$

   (c) Angular velocity of the rods:
   IC of velocity for rod AB at the final will be at A, BC at C.

$$v_B = \omega_{AB} \cdot 2l \tag{8}$$
$$v_B = \omega_{BC} \cdot 2l \tag{9}$$

   (d) Work done by external forces:

$$A_{if} = mg\frac{h}{2} + mg\frac{h}{2} \tag{10}$$

(e) Equation of change:

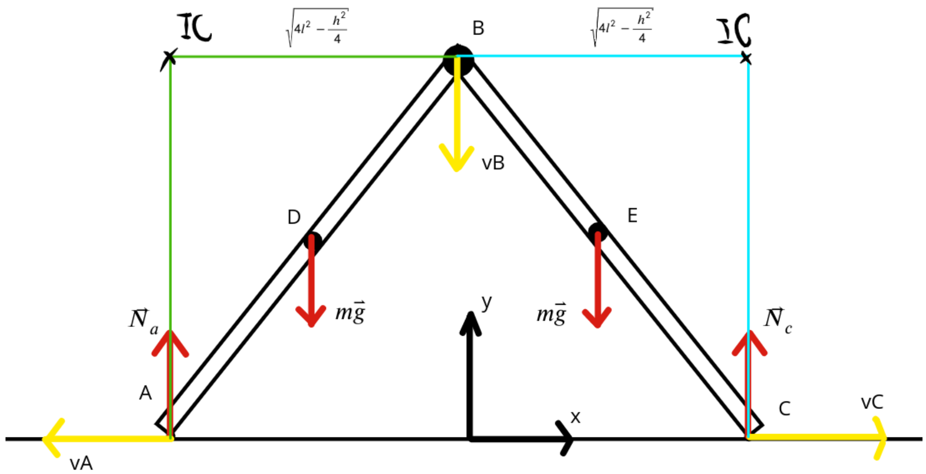
$$T_{AB} + T_{BC} = A_{if} \tag{11}$$

$$\frac{1}{2} I \cdot (\frac{v_B}{2l})^2 + \frac{1}{2} I \cdot (\frac{v_B}{2l})^2 = mg\frac{h}{2} + mg\frac{h}{2} \tag{12}$$

$$v_B = 2l\sqrt{\frac{gh}{l^2 + \rho^2}} \tag{13}$$

Task 2 (next part)

This part is pretty much the same as the previous one, but with a different final conditions.



1. Conditions:

	$initial$	$final$
x_b	0	0
y_b	h	$h/2$
x_c	$\sqrt{4l^2 - h^2}$	$\sqrt{4l^2 - h^2} + h/2$
y_c	0	0
x_a	$-\sqrt{4l^2 - h^2}$	$-\sqrt{4l^2 - h^2} - h/2$
y_a	0	0

2. Angular velocities of the rods:
   IC is shown on picture above:

$$v_B = \omega_{AB} \cdot \sqrt{4l^2 - \frac{h^2}{4}} \tag{14}$$

$$v_B = \omega_{BC} \cdot \sqrt{4l^2 - \frac{h^2}{4}} \tag{15}$$

3. Work done by external forces:

$$A_{if} = mg\frac{h}{4} + mg\frac{h}{4} \tag{16}$$

4. Equation of change:

$$T_{AB} + T_{BC} = A_{if} \tag{17}$$

$$\frac{1}{2}I \cdot (\frac{v_B}{\sqrt{4l^2 - \frac{h^2}{4}}})^2 + \frac{1}{2}I \cdot (\frac{v_B}{\sqrt{4l^2 - \frac{h^2}{4}}})^2 = mg\frac{h}{4} + mg\frac{h}{4} \tag{18}$$

$$v_B = \frac{1}{2}\sqrt{16l^2 - h^2}\sqrt{\frac{gh}{2(l^2+\rho^2)}} \tag{19}$$

Answer:

1. $$v_B = 2l\sqrt{\frac{gh}{l^2+\rho^2}}$$

2. $$v_B = \frac{1}{2}\sqrt{16l^2 - h^2}\sqrt{\frac{gh}{2(l^2+\rho^2)}}$$