Homework 7

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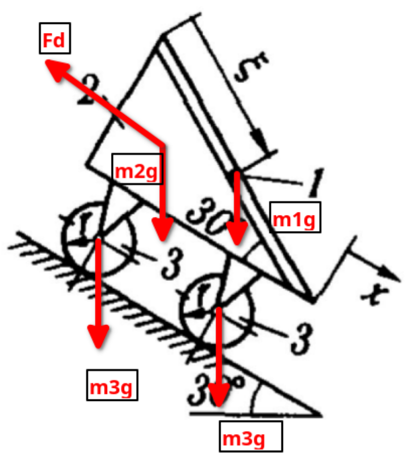
Source code

Task 1



1. RO: System of 4 bodies: particle 1 (translatory), body 2 (trnaslatory), wheel 3_1, wheel 3_2 (planar)
2. Method: analytic mechanics - Euler-Lagrange 2_{nd} order
3. Kinematics analysis: 2 dof - $q_1 = x$, $q_2 = \xi$
4. Force analysis: active forces are only: $\vec{P}_1$, $\vec{P}_2$, $\vec{P}_{3_1}$, $\vec{P}_{3_2}$, $\vec{F}_d = -b\vec{v}$
5. Solution:

   (a) Euler-Lagrange equations:

$$\begin{cases} \frac{d}{dt}\frac{\partial T}{\partial \dot{q}_1} - \frac{\partial T}{\partial q_1} = -\frac{\partial \Pi}{\partial q_1} + Q_{q_1} \\ \frac{d}{dt}\frac{\partial T}{\partial \dot{q}_2} - \frac{\partial T}{\partial q_2} = -\frac{\partial \Pi}{\partial q_2} + Q_{q_2} \end{cases} \tag{1}$$

(b) Kinetic energy energy:

$$J_3 = \frac{1}{2} m_3 r^2 \tag{2}$$

$$T = \frac{m_1}{2}(\dot{x}^2 + \dot{\xi}^2 + 2\dot{x}\dot{\xi}\cos(\alpha)) + \frac{m_2}{2}\dot{x}^2 + 2(\frac{m_3}{2}\dot{x}^2 + \frac{1}{2}J_3(\frac{\dot{x}}{r})^2) \tag{3}$$

$$T = \frac{m_1}{2}(\dot{x}^2 + \dot{\xi}^2 + 2\dot{x}\dot{\xi}\cos(\alpha)) + \frac{m_2}{2}\dot{x}^2 + \frac{3m_3}{2}\dot{x}^2 \tag{4}$$

(c) Potential energy:

$$\Pi = -P_1(x\sin(\alpha) + \xi\cos(\alpha)) - P_2 x \sin(\alpha) - 2P_3 x \sin(\alpha) \tag{5}$$

(d) Generalized forces:
I compute generalized forces through work:

$$\begin{cases} \delta A_{q_1} = Q_{q_1} \cdot \delta q_1 \\ \delta A_{q_2} = Q_{q_2} \cdot \delta q_2 \end{cases} \tag{6}$$

The only non potential forces on generalized coordinates would be:

$$\begin{cases} Q_{q_1} = -b\dot{x} \\ Q_{q_2} = 0 \end{cases} \tag{7}$$

(e) Compute everything:

$$\frac{\partial T}{\partial \dot{q_1}} = (m_1 + m_2 + 3m_3)\dot{x} + \dot{\xi} m_1 \cos(\alpha) \tag{8}$$

$$\frac{\partial T}{\partial \dot{q_2}} = m_1\dot{\xi} + m_1\dot{x}\cos(\alpha) \tag{9}$$

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q_1}} = (m_1 + m_2 + 3m_3)\ddot{x} + \ddot{\xi} m_1 \cos(\alpha) \tag{10}$$

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q_2}} = m_1\ddot{\xi} + m_1\ddot{x}\cos(\alpha) \tag{11}$$

$$\frac{\partial T}{\partial q_1} = 0 \tag{12}$$

$$\frac{\partial T}{\partial q_2} = 0 \tag{13}$$

$$\frac{\partial \Pi}{\partial q_1} = (P_1 + P_2 + 2P_3)\sin(\alpha) \tag{14}$$

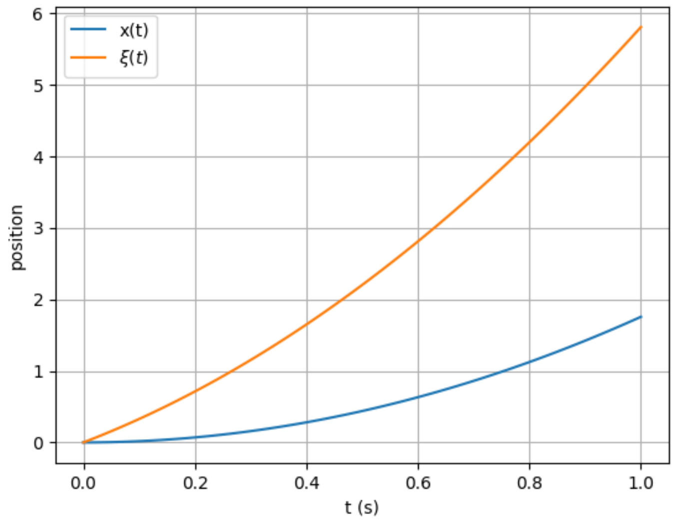
$$\frac{\partial \Pi}{\partial q_2} = P_1\cos(\alpha) \tag{15}$$

(f) Substitute into (1) and chill:

$$\begin{cases} (m_1 + m_2 + 3m_3)\ddot{x} + \ddot{\xi} m_1 \cos(\alpha) = (P_1 + P_2 + 2P_3)\sin(\alpha) - b\dot{x} \\ m_1\ddot{\xi} + m_1\ddot{x}\cos(\alpha) = P_1\cos(\alpha) \end{cases} \tag{16}$$

This is already enough to complete this task, only thing left is to solve this system of equations.

(g) Plot of $x(t)$ and $\xi(t)$:



Answer:

$$
\begin{aligned}
(m_1 + m_2 + 3m_3)\ddot{x} + \ddot{\xi} m_1 \cos(\alpha) &= (P_1 + P_2 + 2P_3)\sin(\alpha) - b\dot{x} \\
m_1 \ddot{\xi} + m_1 \ddot{x} \cos(\alpha) &= P_1 \cos(\alpha)
\end{aligned}
$$