Research 1

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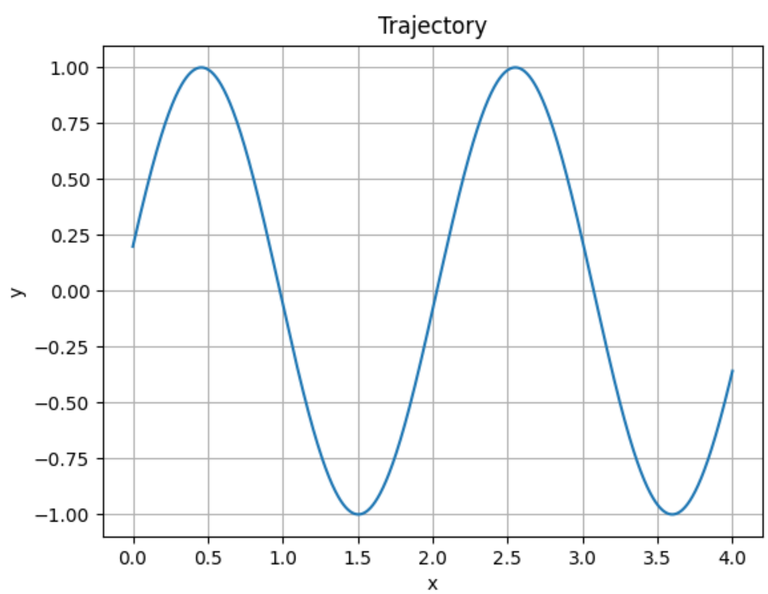
Solution

Disclaimer

1. Introduce basic equations:

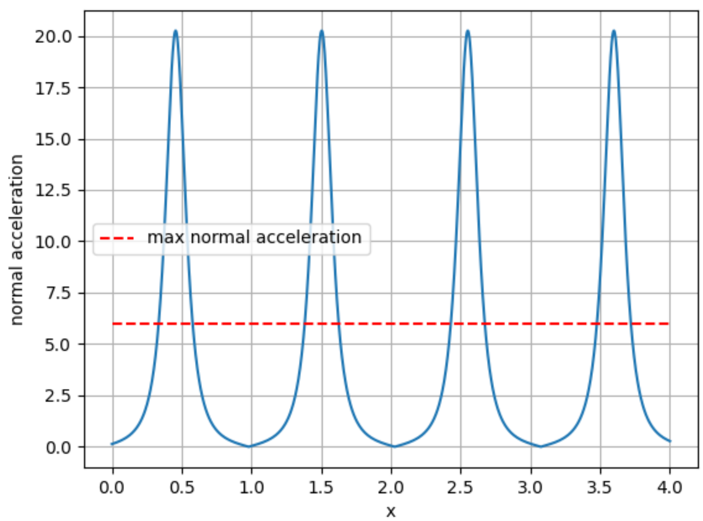
$$y = A\sin(O_m x + \theta_0) \tag{1}$$
$$y'_x = AO_m\cos(O_m x + \theta_0) \tag{2}$$



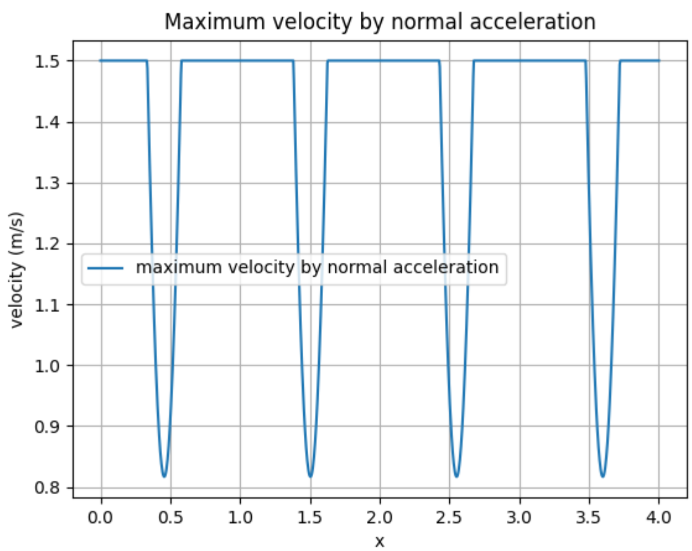
2. Find natural form of motion:

$$\sigma(x) = \int_{X_{min}}^{x} \sqrt{1 + y'_x(x)^2}dx \tag{3}$$
$$\dot{\sigma}(x) = \sqrt{1 + y'_x(x)^2} \tag{4}$$

3. As we prepared inital formulas, we can proceed with explanations.
4. Find how limitation on normal acceleration affects the motion.
   (a) Curvature formula:
   $$\kappa(x) = \frac{|y''_x(x)|}{\sqrt{1 + y'_x(x)^2}} \tag{5}$$
   (b) Let us imagine we are moving with v_{max}, what our normal acceleration would be throughout the x
   $$a_n(x) = v_{max}^2 \kappa(x) \tag{6}$$



   (c) As we see that normal acceleration exceeds the limitation, we could reverse calculations and obtain the maximum velocity we could take through the curve.
   $$v_{max}(x) = \sqrt{\frac{a_n(x)}{\kappa(x)}} \tag{7}$$



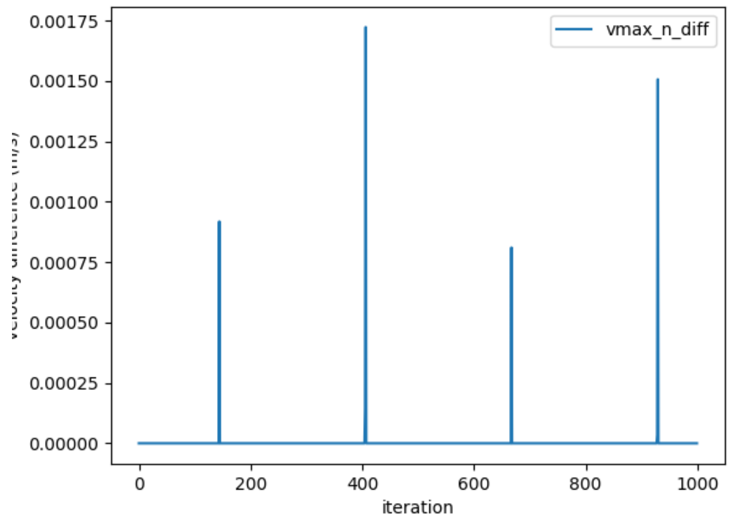
5. Find how limitation on tangential acceleration affects the motion.

   (a) We can observe that at some point we cannot accelerate and decelerate on the given dx interval. I would thank Ilya Miloshin for explanation of this case.

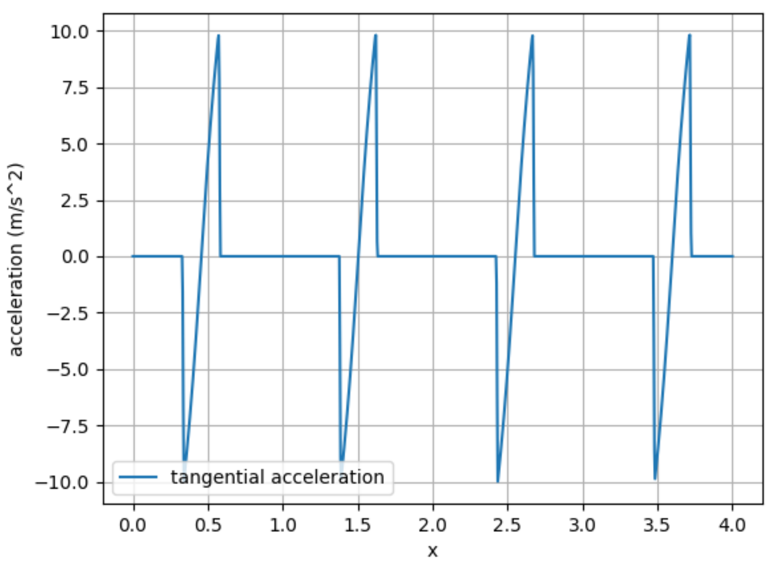
$$a_\tau(t) = \frac{v'_{max}(x(\sigma(t)))}{dt} = \frac{dv_{max}}{dx}\frac{dx}{d\sigma}\frac{d\sigma}{dt} \tag{8}$$

$$\frac{dx}{d\sigma} = \frac{1}{\sqrt{1+{y'_x}^2}} \tag{9}$$

   (b) Actually, the difference between velocity limited by normal acceleration and further by tangential component was not big.



6. Tangential acceleration
   By differentiating the velocity by x, we can find the tangential acceleration for each x.



7. Acceleration and deceleration
   So far there was no talk about start and finish of trajectory.

(a) We have to start from 0 velocity and end with 0 velocity.

(b) We will use trapezoidal profile during simulation to accelerate and decelerate.

(c) Position to start deceleration:

$$t_{decel} = \frac{v_{max}}{a_\tau} \approx 0.15 \tag{10}$$

$$s_{decel} = a_{\tau max} * t_{decel}^2/2 \tag{11}$$

(d) So, when we are left with s_{decel} distance (we will convert to x_{decel} for simplicity in code), we start deceleration.

8. Simulation

(a) We will use trapezoidal profile for acceleration and deceleration

(b) General approach:

i. Choose appropriate acceleration

ii. Store current parameters to use in next iteration

iii. Velocity simulation:

$$v(t) = v(t - dt) + a(t) \cdot dt \tag{12}$$

iv. Position simulation:

$$x(t) = x(t - dt) + v(t) \cdot dt \cdot \left(\sqrt{1 + {y_x'}^2}\right)^{-1} \tag{13}$$

Multiplication by $\dot{\sigma}(x)$ is done to consider only x part of position change change.

v. Y coordinate can be just taken for particular x from trajectory.

vi. Normal acceleration can be calculated through current velocity and x coordinate:

$$a_n(t) = v(t)^2 \kappa(x(t)) \tag{14}$$

(c) Part 1: accelerate until velocity has reached maximum

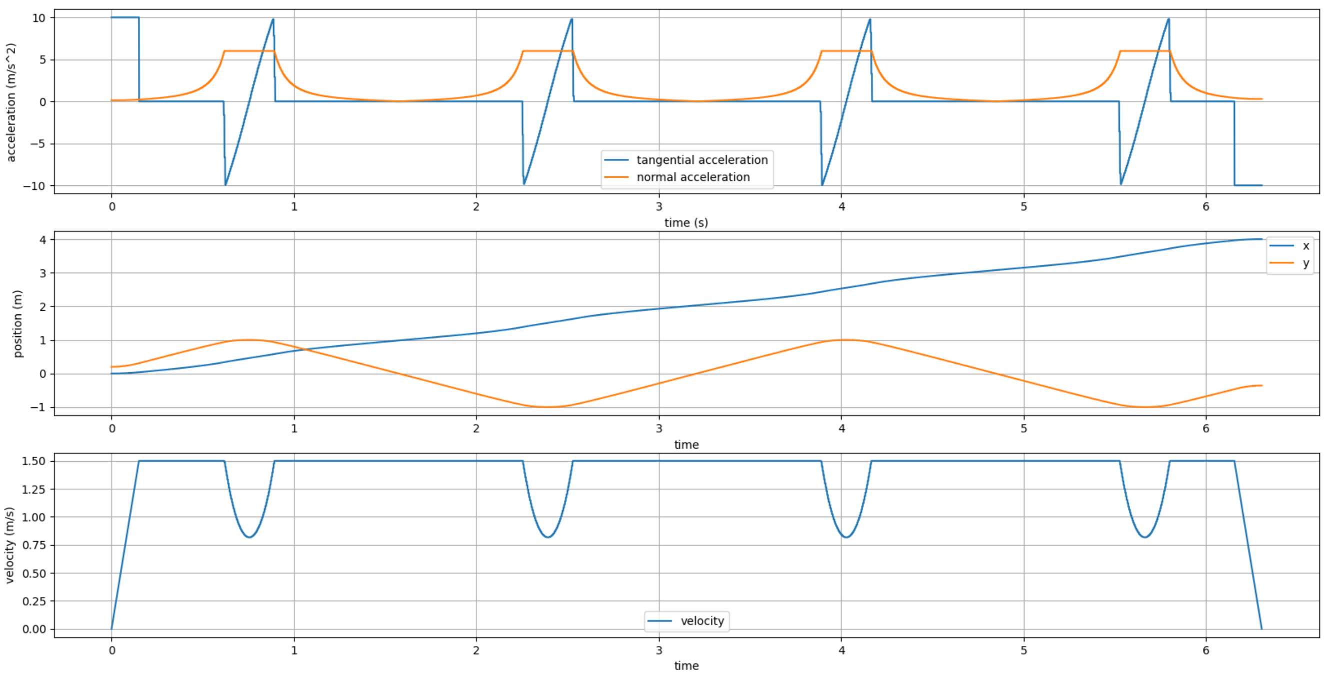
$$a_\tau(t) = a_{\tau max} \tag{15}$$

(d) Part 2: Follow acceleration profile for x's until we reach x_{decel}

$$a_\tau(t) = a_\tau(x_{cur}) \tag{16}$$

(e) Part 3: decelerate until velocity has reached 0

$$a_\tau(t) = -a_{\tau max} \tag{17}$$



Answer:

For my simulation approximate time to follow this trajectory was: 6.3 seconds.