

Delaunay Triangulation

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Abstract

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Contents

Contents	i
1 Introduction	1
2 Background	3
2.1 Graph Theory	3
2.2 Geometry	3
2.2.1 Circumcircles and Circumspheres	3
2.3 Delaunay Triangulation and Tessellation	3
3 Algorithms and Data Structures	5
3.1 Hash Map	5
3.2 Triangle Mesh	5
3.3 Quad-Edge Data Structure	5
3.4 Radix Sort	5
3.5 Linear Morton Sort	5
3.6 Linear Floating-Point Quad-Tree	5
3.7 Bowyer-Watson Algorithm	5
3.8 Guibas-Stolfi Incremental Algorithm	5
3.9 Guibas-Stolfi Divide-and-Conquer Algorithm	5
3.10 Dwyer Algorithm	5
4 Design and Implementation	7
4.1 API	7
4.2 Robustness	7
4.3 Incremental Algorithm	7
4.4 Divide-and-Conquer Algorithm	7
4.5 Multidimensional Tessellation	7
5 Tests and Testscenes	9
5.1 Uniform Rectangular Distribution	9
5.2 Gaussian Distribution	9
5.3 Robustness Tests	9
6 Benchmarks	11
7 Examples	13
7.1 Image Mosaic and Tessellation	13
7.2 Fluids for Ray Tracing	13
7.3 Finite Element Method	13
7.4 Pareto Frontiers	13
8 Results	15
9 Conclusions	17

References

19

1 Introduction

2 Background

2.1 Graph Theory

2.2 Geometry

2.2.1 Circumcircles and Circumspheres

2.3 Delaunay Triangulation and Tessellation

Triangulation, Tessellation, Simplicialization, Subdivision, Mesh Generation Two and Three Dimensions

3 Algorithms and Data Structures

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3.2 Triangle Mesh

3.3 Quad-Edge Data Structure

3.4 Radix Sort

3.5 Linear Morton Sort

3.6 Linear Floating-Point Quad-Tree

3.7 Bowyer-Watson Algorithm

3.8 Guibas-Stolfi Incremental Algorithm

3.9 Guibas-Stolfi Divide-and-Conquer Algorithm

3.10 Dwyer Algorithm

4 Design and Implementation

4.1 API

4.2 Robustness

4.3 Incremental Algorithm

4.4 Divide-and-Conquer Algorithm

4.5 Multidimensional Tessellation

5 Tests and Testscenes

5.1 Uniform Rectangular Distribution

5.2 Gaussian Distribution

5.3 Robustness Tests

6 Benchmarks

7 Examples

7.1 Image Mosaic and Tessellation

7.2 Fluids for Ray Tracing

7.3 Finite Element Method

7.4 Pareto Frontiers

8 Results

9 Conclusions

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