Justification Proof Search Implementation in Python

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Abstract

In short what's it all about.

Contents

Chapter 1

Introduction

1.1 Motivation

What's the motivation behind it? Not **MY** motivation, but the scientific motivation.

1.2 Goal

The initial goal was to extend an existing proof search engine Z3 [Microsoft Research] such that it could also handle Justification Logic. Deeper investigation into that project revealed that to make it handle also Justification Logic the given interface in Python would not work. Instead it would have to be look into the core of the programm which is written in C. The expenses it would require to get so much deeper into the material that the actual indented work would be only secondary. So instead of extending Microsoft Research project the actual goal changed to implementing a simplified proof search for Justification Logic. It meant that the implementation would be easier since it does not depend on anything else anymore. As a downside a lot of the functionallity that was hoped go get from Z3 would have to be implemented as well or left out.

The program should satisfy to following conditions:

Should this really be here in this chaper?

Input The formula to be proven as well as a list of formulas needed for the proof is given as string. It may be presumed that the string is exactly formatted in the way needed. It must not be checked for syntax error or general typing mistakes.

Output A simple *True* or *False* for the provability of the formula. [1]

1.3 Overview

The second chapter will present a short introduction to Justification Logic, but will go only as deep as needed to understand the problem as well as the develop algorithm.

[1] Optional the output could give information about how a proof was found if the formula is provable.

The heart of the third chapter will introduce the algorithm used in the program. Since this thesis concerns itself more with the practical side of implementation and not the theoretical side of mathematical logic theory there will be little proof here but instead many example to show how the algorithm works.

j-logic

Finally the last chapter will discuss the result of the work and give some ideas about how the work of a Justification Logic proof search implementation could be improved.

Chapter 2

Background

The theory of Justification Logic as it is used here requires little knowledge of the wide fields of Modal Logic. For the purpose of this proof search a few basic rules and definitions are sufficient to provide the reader with the needed knowledge.

From a more practical angle a basic data structure was also needed to represent formulas. Already widely known and used *Binary Syntax Trees* revealed to be just the perfect way to handle formulas.

2.1 Justification Logic

The theory presented here is oriented mainly on the work of Goetschi [2005] as well as the older reference Paper and also from the homepage Stanfort. This definitions and rules given here are not complete to the justification logic. Priority was given to those informations which are vital for the implementation. So however briefly and incomplete the theory is presented here full reference can be found in the named sources.

2.1.1 Origins

Justification Logic has its origins from the field of modal logic. In model logic $\Box A$ means that A is *know* or that we have *proof* of A. In justification logic the equivalent would be $t : A$ where t is a proof term of A. So we have the notion that *knowledge* or *proofs* may come from different sources. Justification logic lets us connect different *proofs* with a few simple operators and thus describe better the proof. It may be said that where in model logic the knowledge is implicit it is explicit in Justification Logic[1].

2.1.2 Rules and Definitions

Definition. [2]

[1] Goetschi [2005]
[2] Goetschi [2005] Page 17, incomplete

Is is ok to just leaf '?' and such out? How does it have to be referenced?

Justification terms *or just* ***terms*** *are syntactic objects given by the grammar*

$$t ::= c_i^j | x_i | (t \cdot t) | (t + t) | !t,$$

where i and j range over positive natural numbers, c_i^j denotes a (justification) constant of level j, and x_i denotes a (justification) variable.

2.2 Order of Operation Tree

Chapter 3

A Divide and Conquer Algorithm

3.1 Core Idea

To search a formula for is provability it had to be found a way which allows to do the same steps, now matter what form the formula actually has. A first attempt was to strictly use recursion. This method should have worked but it proved to be very difficult to implement, because there are so many different cases to consider in one recursion step. Also the stack created by this could become problematic for very large formulas.

Instead a *Divide and Conquer* approach is used. Diving will break even a large and complicated formula down to its most simple elements. Then these elements can be tested for their provability and in the conquer-step the results of the elements are put together giving the final result. Since the *Divide and Conquer* algorithm design pattern uses multi-branched recursion there still remains some recursion but as this takes place at a much deeper level the cases within a recursion are reduced as well as the size of the recursion stack.

3.2 Divide

The motivation behind the divide-step existed already long before the actual idea of the *Divide and Conquer* approach. Given a formula there would be no way to know what kind of formula it was or more precise: what operations were to be found within the *justification term*. The original goal was to find a way to restructure any given formula so that handling it would always need the same steps and not depend to much on what the formula looks exactly. I was looking for something like the CNF[1] and use it in a similar way as CNF is used in PSC[2]

[1] *conjunctive normal form*, a conjunction of clauses, where a clause is a disjunction of literals

[2] Proof Search Calculus

3.2.1 Atomize

Sumsplit

Simplify Bang

Remove Bad Bang

3.2.2 Get Must

3.3 Conquer

3.3.1 Configurations and Conditions

3.3.2 Merge

3.4 Implementation

Chapter 4

Results

4.1 Application

4.2 Enhancement

Bibliography

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Microsoft Research. Z3, high performance theorem prover. URL `http://z3.codeplex.com/`.

Artis Paper.

Plato Stanfort.

Todo list