Problem 1

Use the Hebb rule to store the vectors $(1, 1, 1, 1)$ and $(1, 1, -1, -1)$ in an autoassociative neural net.

(a) Find the weight matrix with the diagonal non-zero terms, another matrix with the diagonal zero terms.

(b) Test two nets with the above matrices, using the following vectors:

$$v_1 = (1, 1, 1, 1), \tag{1}$$
$$v_2 = (1, 1, -1, -1), \tag{2}$$
$$v_3 = (1, 1, 1, 0). \tag{3}$$

(c) Again, find the weight matrix with the diagonal zero terms, and repeat the part (b).

(d) Discuss any difference you find in the response of the nets, with both weight matrices.

Problem 2

Consider the integers from 0 to 9, and define each integer as a vector with 64 elements, which contains the values -1 and $+1$; any integer is shown as a 8×8 matrix, where assigns $+1$ to each black point and -1 to each white point (Fig. 1). Therefore, ten patterns are generated.

(a) Propose a Hopfield net with an appropriate weight matrix to store all ten patterns.

(b) For each pattern, define a noisy pattern with 30% distortion. Test the ability of your proposed net with these noisy patterns.

(c) Discuss the obtained results from part (b), using the Hamming distance and the dependency of inputs.

Figure 1: The integers in matrix-like form, Problem 2.

Problem 3

Write a computer program to implement a bipolar BAM neural network. Allow for (at least) 15 units in the $\mathcal{X}$-layer and 3 units in $\mathcal{Y}$-layer.

(a) Use the program to store the following association:

$$A \to (-1, 1), \tag{4}$$
$$C \to (1, 1). \tag{5}$$

Now, test your program with some noisy version of the letter C.

(b) Use the program to store the patterns shown in Fig. 2, then answer the following questions:

- Is it possible to store all eight patterns at once?
- How many of the eight patterns can be stored at the same time?

- Test the net with the medium and low noisy versions of the eight patterns, (in low noisy version, 10 percents of the original pattern is distorted, and high noisy one is 30 percents the distortion amount.
- Determine the Hamming distances between the $\mathcal{Y}$-layer patterns associated with each of these letters. From the ratios of the Hamming distances, which pattern will be most likely to be stored successfully?
- How much are the results of the tests compatible with the expectation based on the Hamming distances?

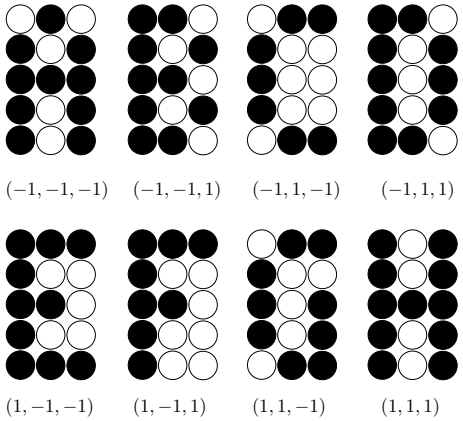

Figure 2: The eight patterns and their associations, Problem 3.