MA 399 Intro to Quantum Information Theory

Hayden Roszell

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Abstract

No idea what is happening in this class lol do this later

Contents

1 Intro to Linear Algebra

Let's jump right in. This section is an abbreviation of the introduction to linear algebra session.

A vector space is a group of objects (vectors) which may be added together and multiplied by compatible scalars from $\mathbb{R}$ or $\mathbb{C}$. In this class, we primarily care about vector spaces $\mathbb{C}^n$ from $\mathbb{C}$ and $\mathbb{R}^n$ from $\mathbb{R}$. Recall that $\mathbb{C}$ is the scalar field of complex numbers $a + bi$, where multiplication is defined by the rule $i^2 = -1$, and is equipped with:

(a) a complex conjugation operation – $\overline{a + bi} = (a + bi)^* = a - bi$ and

(b) a size function called the **modulus** – $|a + bi| = \sqrt{a^2 + b^2}$.

Note that the modulus is similar to magnitude.
To work with complex numbers, it's useful to have an understanding of the basic operations.

(a) To add/subtract complex numbers, add/subtract the corresponding real/imaginary parts. For example – $(a + bi) + (c + di) = (a + c) + (b + d)i$

(b) To multiply/divide complex numbers, multiply both parts of the complex number by the real number. For example – $(a + bi) * (c + di) = ac + adi + bcj - bd = (ac - bd) + (ad + bc)i$. This form will be useful for the duration of the class.

1.1 Representing Vectors in Complex Spaces

As mentioned, this class primarily works in complex number spaces. For this reason, having useful tools for representing vectors in these abstract spaces is useful. Vectors are represented in **bra-ket** notation, where *bra* represents a *row* vector, and *ket* represents a *column* vector.

Definition 1.1 If $|v\rangle \in \mathbb{C}^n$ is a *ket* vector which consists of n complex numbers,

$$|v\rangle = \begin{bmatrix} v_1 \\ v_2 \\ \dots \\ v_n \end{bmatrix} \text{ for } v_1, v_2, ..., v_n \in \mathbb{C} \tag{1.1}$$

♦

Definition 1.2 If $\langle v| \in \mathbb{C}^n$ is a *bra* vector which consists of n complex numbers,

$$\langle v| = \begin{bmatrix} v_1 & v_2 & \dots & v_n \end{bmatrix} \text{ for } v_1, v_2, ..., v_n \in \mathbb{C} \tag{1.2}$$

♦

Note that the the integer n in definitions 1.1 and 1.2 is called the **dimension** of the vector space $\mathbb{C}^n$.

Definition 1.3 A linear combination of $\{|v_1\rangle, ..., |v_n\rangle\} \subset \mathbb{C}^n$ is a single vector in the form $\lambda_1 |v_1\rangle + \lambda_2 |v_2\rangle + ... + \lambda_n |v_n\rangle$ for some $\lambda_1, \lambda_2, ..., lambda_k \in \mathbb{C}^n$. ♦