

# Resynchronization of Circadian Oscillators and the East-West Asymmetry of Jet-Lag

PH 567: Nonlinear Dynamics Course Project

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# Introduction

- The circadian rhythm regulates the sleep-wake cycle and repeats on each rotation of the Earth roughly every 24 hours.
- It is regulated by certain clock cells in the brain called the Suprachiasmatic nucleus (SCN cells).
- We model the cells regulating the circadian rhythm as forced coupled oscillators using the Kuramoto model.

# Introduction

- Jet-lag - mismatch between the external stimuli (daylight) and internal circadian clock.
- Desynchronization of the clock cells within the brain, followed by resynchronization.
- Study the phase space behaviour in order to explain the asymmetry in the severity of jet-lag.

# The Kuramoto Model

- SCN cells modelled as coupled phase oscillators - Kuramoto model.
- Large number of phase oscillators ( $N=10^4 \gg 1$ ) with random intrinsic frequencies
- $$\frac{d\theta_i}{dt} = \omega_i + \frac{K}{N} \sum_{j=1}^N \sin(\theta_j - \theta_i) + F \sin(\sigma t - \theta_i + p(t))$$

# The Kuramoto Model

- $\omega_i$  distribution - Lorentzian  $g(\omega) = \frac{\Delta}{\pi[(\omega - \omega_0)^2 + \Delta^2]}$ .
- $p(t)$  - current time-zone and  $\in [-\pi, \pi]$ .
- Fast travel

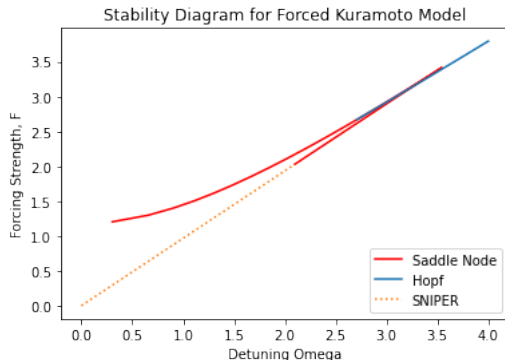
$$p(t) = \begin{cases} p_1, & \text{if } t \leq \tau \\ p_2, & \text{if } t > \tau \end{cases}$$

# Ott-Antonsen Ansatz

- complex order parameter  $z = \frac{1}{N} \sum_{j=1}^N e^{i[\theta_j(t) - \sigma t - p_2]}$
- $\dot{z} = \frac{1}{2} [(Kz + F) - z^2(Kz + F)^*] - (\Delta + i\Omega)z$  where  $\Omega = \sigma - \omega_0$
- After travelling,  $z = z_{\text{st}} e^{i(p_1 - p_2)}$
- $|z(t) - z_{\text{st}}|$  as a measure of recovery in this model

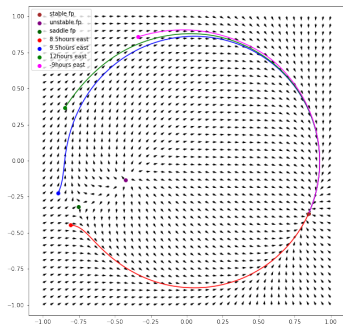
# Plots

Figure 1: Bifurcation curves are shown with respect to the strength  $F$  and detuning of the external forcing, which have been nondimensionalized by the width of the distribution of the oscillators' natural frequencies.

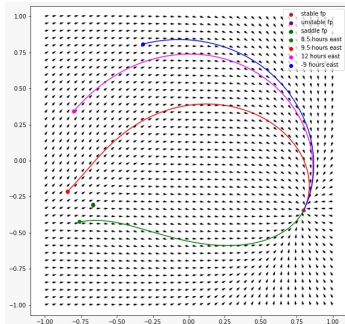


# Plots

Figure 2: Trajectories of order parameters corresponding to different time-zone travels for types C & A dynamics.



(a)  $K = 10\Delta$ ;  $F = 3.5\Delta$ ;  $X = 1.4\Delta$

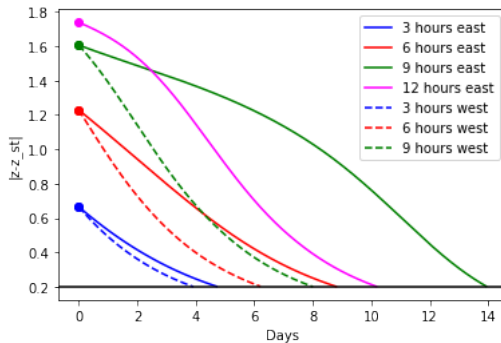


(b)  $K = 4.5\Delta$ ;  $F = 3.5\Delta$ ;  $X = 1.4\Delta$



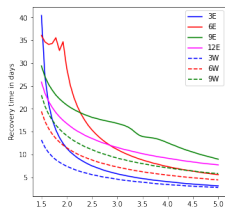
# Plots

Figure 3: Recovery curve from several eastward & westward travels with different numbers of time-zones crossed

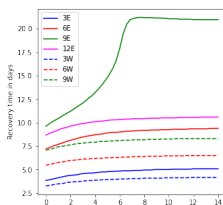


# Plots

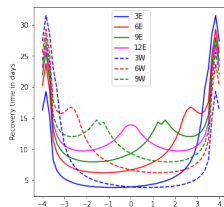
Figure 4: Recovery time dependence on  $K$  (a),  $F$  (b), and  $X$  (c) with the non-varying parameters kept at the reference values in equations. In each case, x-axis represents the corresponding parameter value.



(a)  $K/\Delta$



(b)  $F/\Delta$



(c)  $\Omega/\Delta$

# Open Questions - Relaxation of fast travel approximation

- $w = \frac{1}{N} \sum_{i=1}^N e^{(\theta_i - \sigma t - p(t))}.$

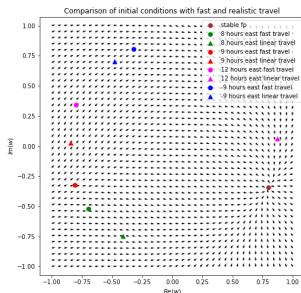
$$w(t) = z(t) \cdot e^{-i(p(t) - p_2)}$$

$$\dot{w}(t) = \dot{z}(t) \cdot e^{-i(p(t) - p_2)} - i \frac{dp}{dt} w$$

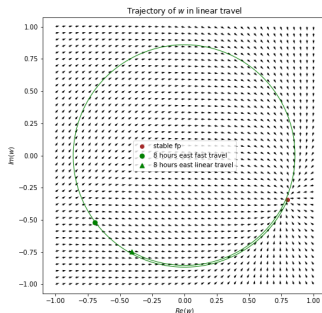
- we will always start our model with  $w(0)$  being the stable node of the earlier model.

- $$p(t) = \begin{cases} p_1 & t \leq 0 \\ p(t) & 0 \leq t \leq \tau \\ p_2 & t \geq \tau \end{cases}$$

# Open Questions - Relaxation of fast travel approximation



(a) Difference between initial conditions of linear vs fast travel



(b) Trajectory for linear travel

# Open Questions - Changing the forcing function

- Consider the forcing function to be a periodic function clamped at zero. (intensity of light varies, peaking around noon, at night intensity is negligible)
- Cannot employ Ott-Antonsen ansatz for dimensionality reduction as the differential equation
- Would possibly have to numerically integrate for a very large number of oscillators to study the system.