Resynchronization of Circadian Oscillators and the East-West Asymmetry of Jet-Lag

PH 567: Nonlinear Dynamics Course Project

Roshni Singh, Drishti Baruah, Sachin Teli, Mahadevan Subramanian, Aneesh Bapat

Introduction

- The circadian rhythm regulates the sleep-wake cycle and repeats on each rotation of the Earth roughly every 24 hours.
- It is regulated by certain clock cells in the brain called the Suprachiasmatic nucleus (SCN cells).
- We model the cells regulating the circadian rhythm as forced coupled oscillators using the Kuramoto model.

Introduction

- Jet-lag - mismatch between the external stimuli (daylight) and internal circadian clock.
- Desynchronization of the clock cells within the brain, followed by resynchronization.
- Study the phase space behaviour in order to explain the asymmetry in the severity of jet-lag.

The Kuramoto Model

- SCN cells modelled as coupled phase oscillators - Kuramato model.
- Large number of phase oscillators (N=$10^4 >> 1$) with random intrinsic frequencies
- $\frac{d\theta_i}{dt} = \omega_i + \frac{K}{N}\sum_{j=1}^{N} \sin\left(\theta_j - \theta_i\right) + F \sin\left(\sigma t - \theta_i + p(t)\right)$

The Kuramoto Model

- ω_i distribution - Lorentzian $g(\omega) = \frac{\Delta}{\pi\left[(\omega-\omega_0)^2+\Delta^2\right]}$.
- $p(t)$ - current time-zone and $\in [-\pi, \pi]$.
- Fast travel

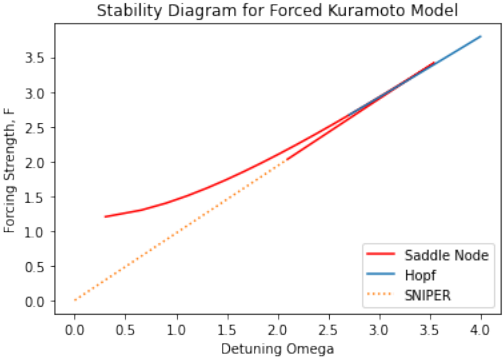
$$p(t) = \begin{cases} p_1, & \text{if } t \leq \tau \\ p_2, & \text{if } t > \tau \end{cases}$$

Ott-Antonsen Ansatz

- complex order parameter $z = \frac{1}{N}\sum_{j=1}^{N} e^{i[\theta_j(t)-\sigma t-p_2]}$
- $\dot{z} = \frac{1}{2}\left[(Kz+F) - z^2(Kz+F)^*\right] - (\Delta + i\Omega)z$ where $\Omega = \sigma - \omega_0$
- After travelling, $z = z_{\mathrm{st}} e^{i(p_1-p_2)}$
- $|z(t) - z_{\mathrm{st}}|$ as a measure of recovery in this model

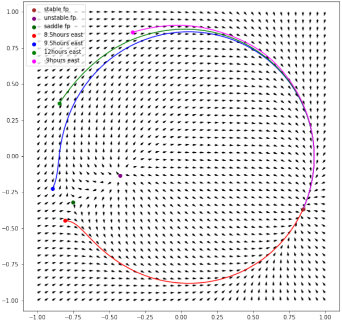
Plots

Figure 1: Bifurcation curves are shown with respect to the strength F and detuning of the external forcing, which have been nondimensionalized by the width of the distribution of the oscillators' natural frequencies.

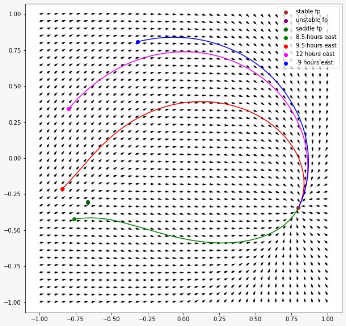


Plots

Figure 2: Trajectories of order parameters corresponding to different time-zone travels for types C & A dynamics.



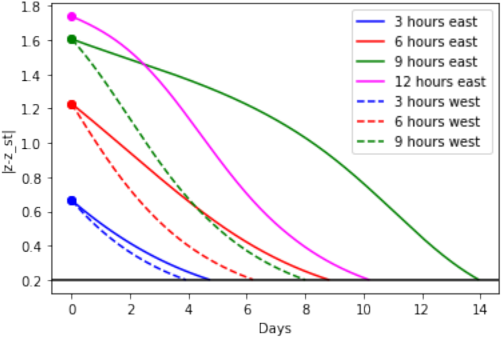
(a) $K = 10\Delta; F = 3.5\Delta; X = 1.4\Delta$



(b) $K = 4.5\Delta; F = 3.5\Delta; X = 1.4\Delta$

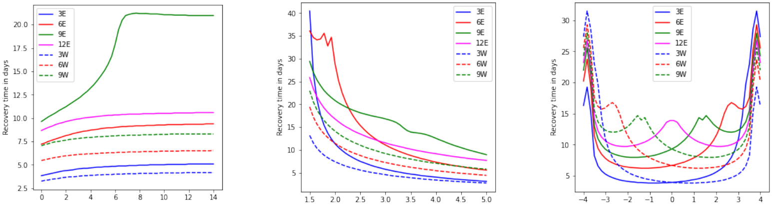
Plots

Figure 3: Recovery curve from several eastward & westward travels with different numbers of time-zones crossed



Plots

Figure 4: Recovery time dependence on K (a), F (b), and X (c) with the non-varying parameters kept at the reference values in equations. In each case, x-axis represents the corresponding parameter value.

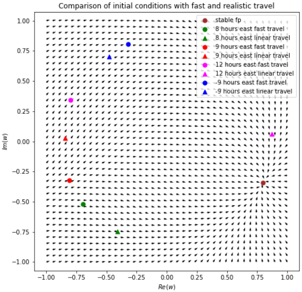


(a) K/Δ (b) F/Δ (c) Ω/Δ

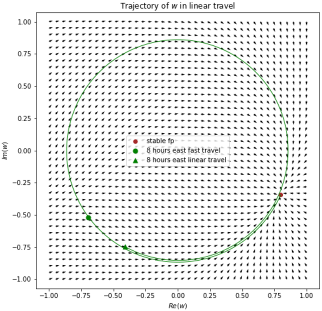
Open Questions - Relaxation of fast travel approximation

- $w = \frac{1}{N}\sum_{i=1}^{N} e^{(\theta_i - \sigma t - p(t))}$.
  $w(t) = z(t) \cdot e^{-i(p(t)-p_2)}$
  $\dot{w}(t) = \dot{z}(t) \cdot e^{-i(p(t)-p_2)} - i\frac{dp}{dt}w$
- we will always start our model with $w(0)$ being the stable node of the earlier model.
- $p(t) = \begin{cases} p_1 & t \leq 0 \\ p(t) & 0 \leq t \leq \tau \\ p_2 & t \geq \tau \end{cases}$

Open Questions - Relaxation of fast travel approximation



(a) Difference between initial conditions of linear vs fast travel



(b) Trajectory for linear travel

Open Questions - Changing the forcing function

- Consider the forcing function to be a periodic function clamped at zero. (intensity of light varies, peaking around noon, at night intensity is negligible)
- Cannot employ Ott-Antonsen ansatz for dimensionality reduction as the differential equation is nonlinear and has implicit functions of the phase.
- Would possibly have to numerically integrate for a very large number of oscillators to study the system.