Maths for Physics 2

Assignment 3

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CHOSEN PARTIAL DIFFERENTIAL EQUATION: Diffusion Equation

1. **Explain the significance of the equation in terms of the Physics it represents.**

The diffusion equation is a parabolic partial differential equation. In the one-dimensional case it is represented by:

$$\frac{\partial u}{\partial x} = D\frac{\partial^2 u}{\partial x^2}$$

This is a special case of the Diffusion-conduction equation with the initial conditions being set. u is a function of x,t i.e.: $u(x, t)$.

The function u describes the macroscopic behavior of a system, such as the Temperature, diffusion rates, velocity and so on of particles in the system undergoing Brownian motion, resulting from the random movements and collisions of the particles. The behaviour of these particles follow the Fick's Laws of Diffusion and are strongly correlated via the Fokker-Planck equation.

The solution to the differential equation is mathematically a Gaussian distribution and the problem of solving the differential equation is a classical Initial Value Problem.

The 1-D form of the diffusion equation is also known as the heat equation. The heat equation is of high significance in both the field of mathematics and physics, as well as several other fields.. It plays a major role in the understanding of random walks and Brownian Motion as mentioned earlier. It is extensively used to study heat flow as well as particle diffusion(the specific application I have used this exercise to evaluate).It is also use in optics and image processing to resolve pixelation. An extension of it into imaginary time also has expanded its applications to the field of Quantum physics and computing.

Impedance spectroscopy in material science. State change problem at higher.

2. **Solve it analytically using the method of separation of variables. Choose appropriate boundary conditions**

Scanned and attached to the last pages of this pdf.

3. **Using the finite difference method, solve the above equation numerically (write a Python code)**

In code pasted in moodle

4. **Compute and plot the error (if any) between your numerical and analytical results.**

Computed and plotted as part of code from question 3(pasted in moodle). Graph as visible in figures given below.