Introduction to Scientific Computing

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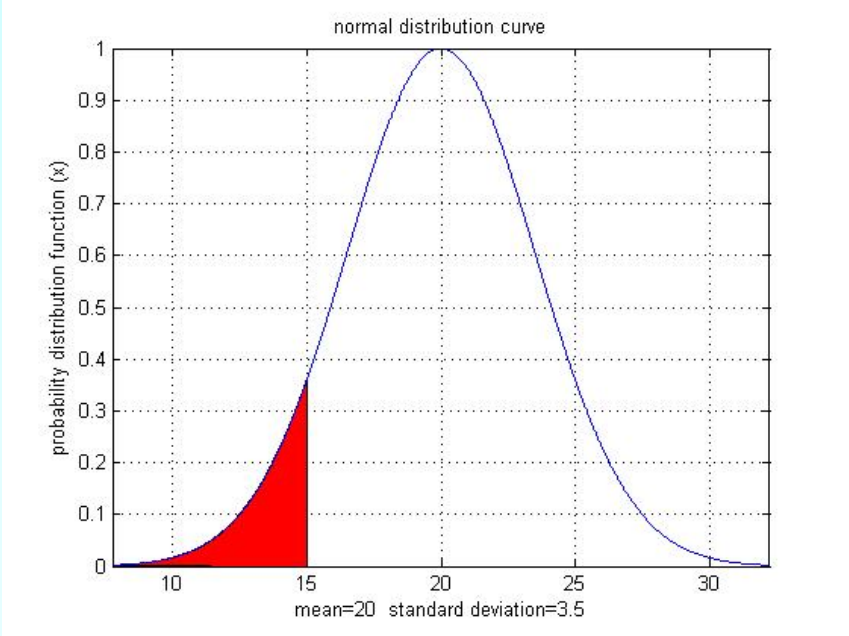
Textbook: TEXTBOOK: NUMERICAL METHODS WITH APPLICATIONS

Why use Numerical Methods?

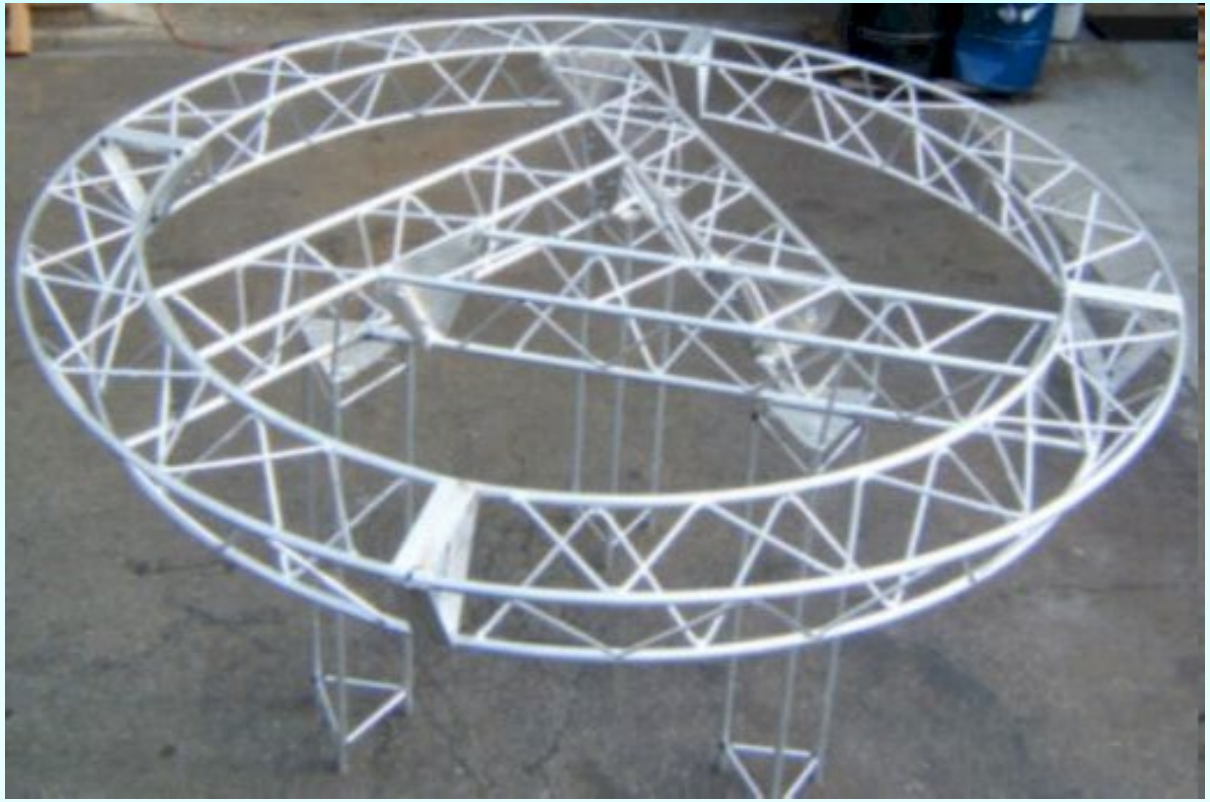
- To solve problems that cannot be solved exactly

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{u^2}{2}} du$$



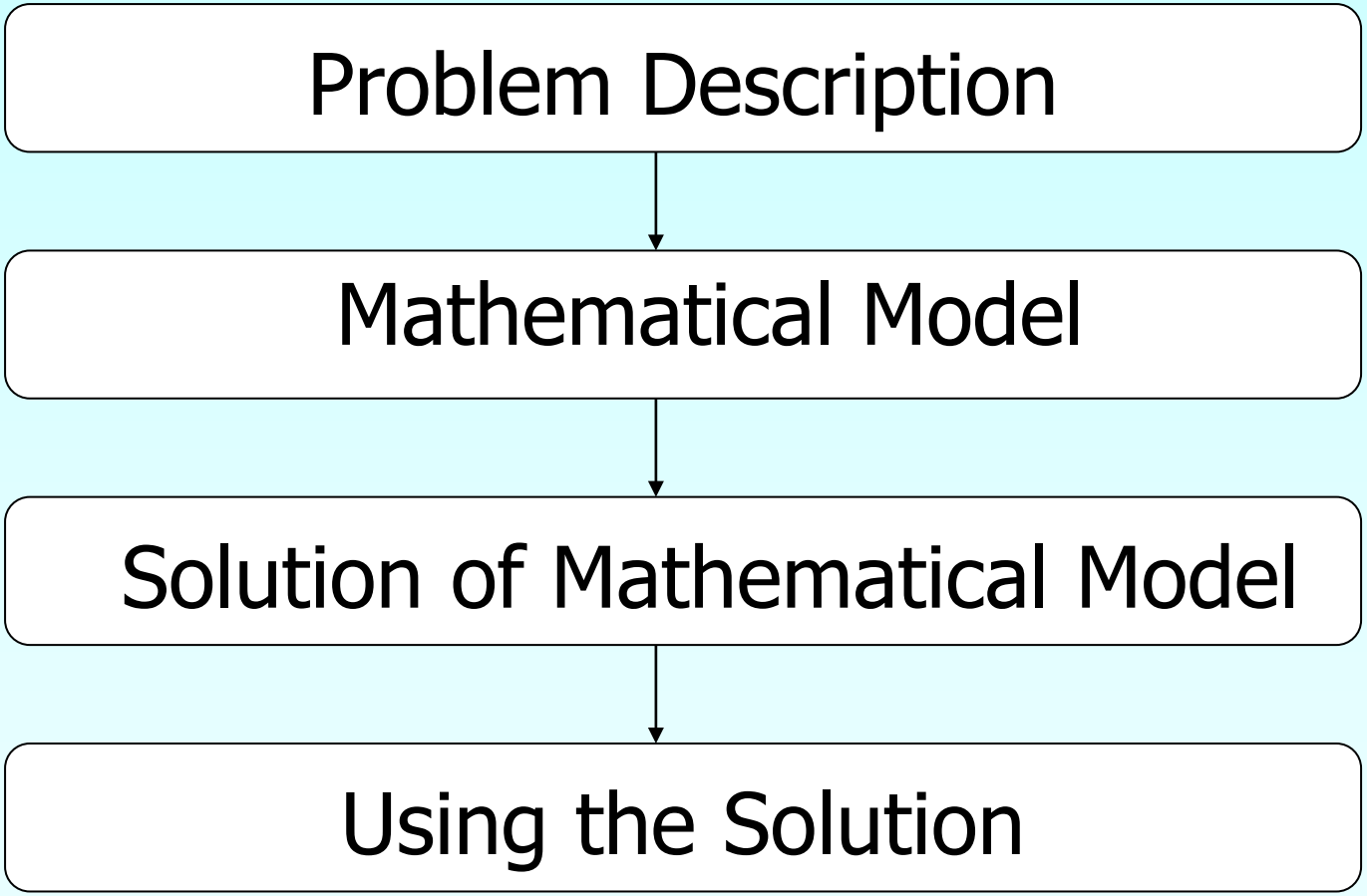
Why use Numerical Methods?

- To solve problems that are intractable!
  - No efficient algorithm is available



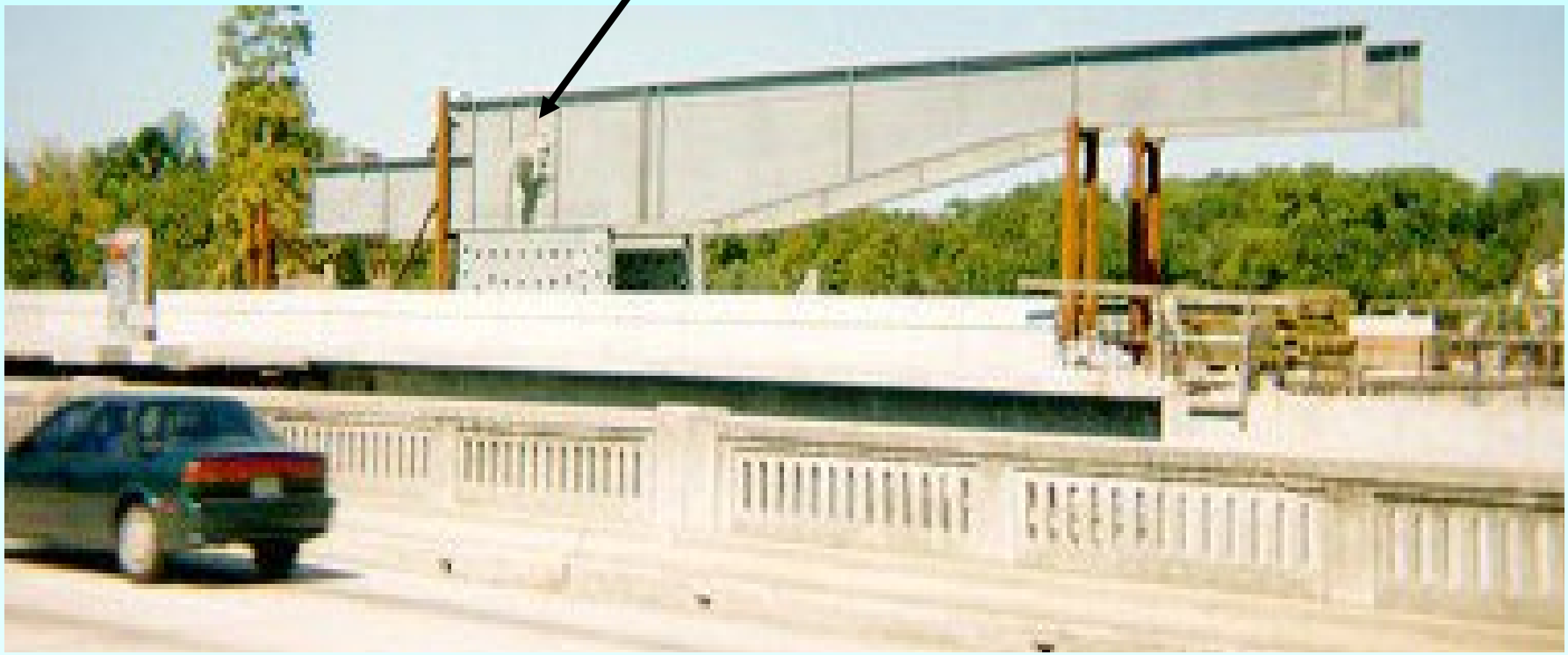
Steps in Solving an Engineering Problem

How do we solve an engineering problem?

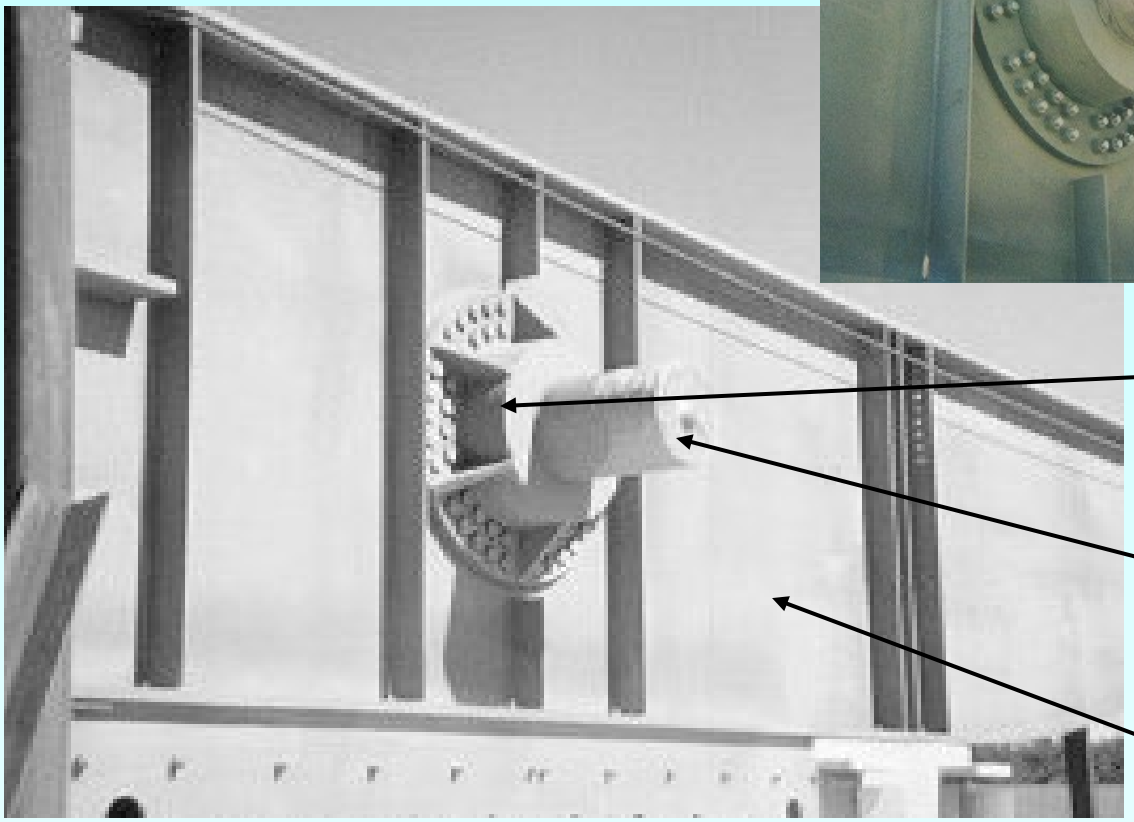


Example of Solving an Engineering Problem

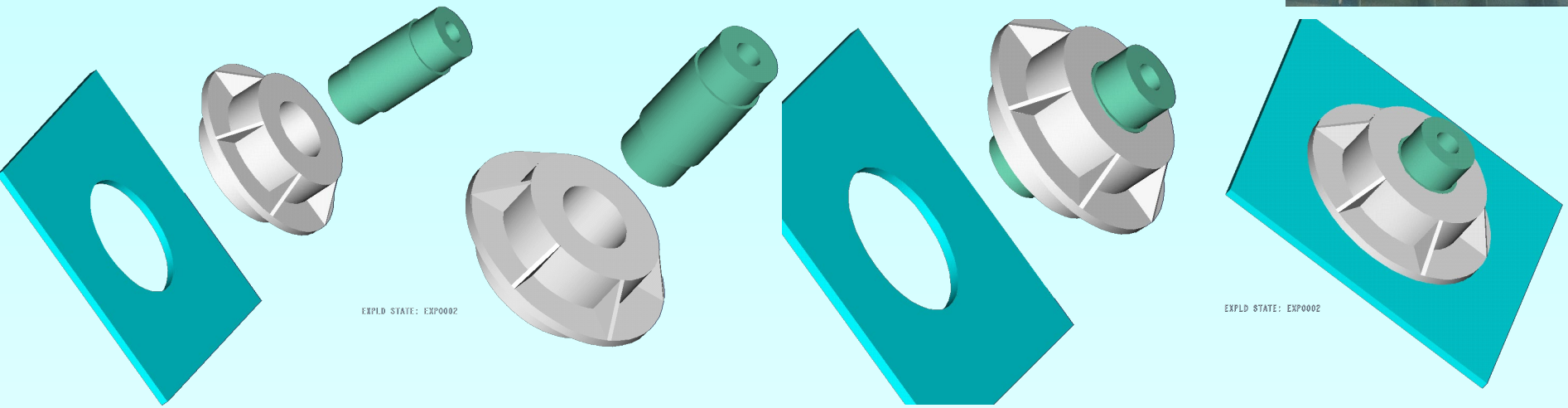
Bascule Bridge THG



Bascule Bridge THG



Trunnion-Hub-Girder Assembly Procedure



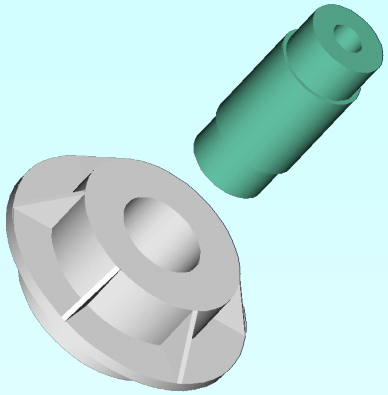
Step1. Trunnion immersed in dry-ice/alcohol

Step2. Trunnion warm-up in hub

Step3. Trunnion-Hub immersed in dry-ice/alcohol

Step4. Trunnion-Hub warm-up into girder

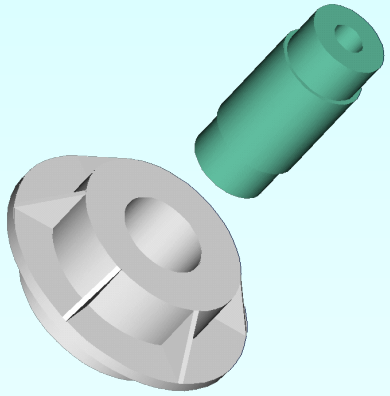
Problem



After Cooling, the Trunnion Got Stuck in Hub

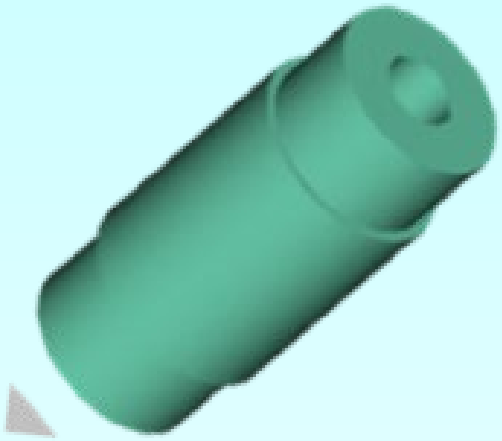
Why did it get stuck?

Magnitude of contraction needed in the trunnion was 0.015” or more. Did it contract enough?



Consultant calculations

$$\Delta D = D \times \alpha \times \Delta T$$



$$D = 12.363"$$

$$\alpha = 6.47 \times 10^{-6} in/in/^{o} F$$

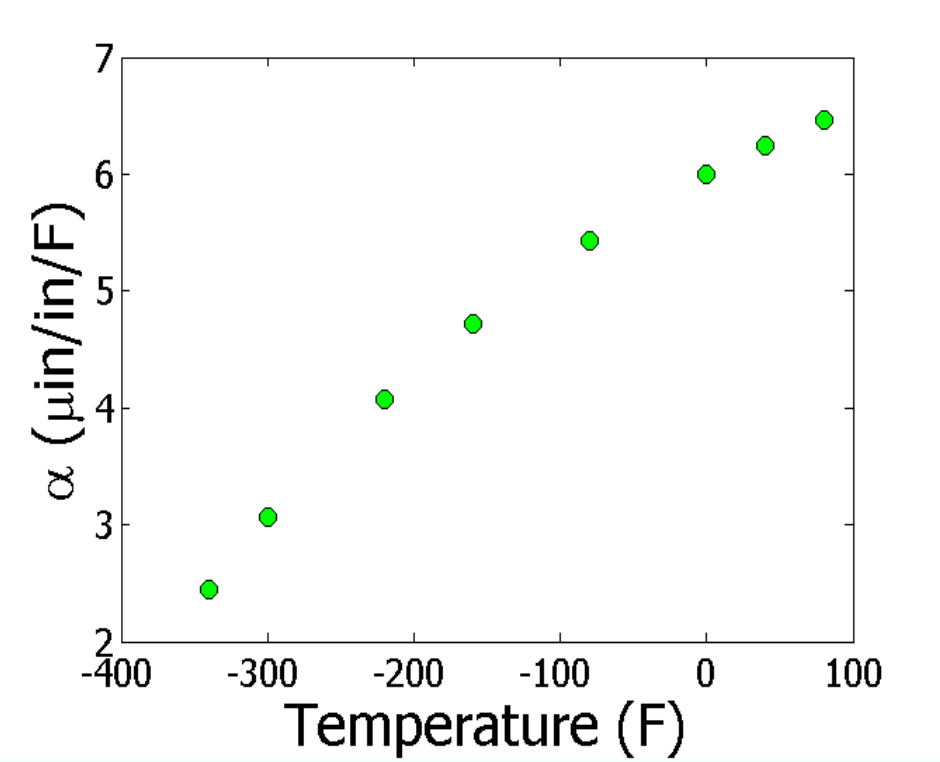
$$\Delta T = -108 - 80 = -188^{o} F$$

$$\Delta D = (12.363)(6.47 \times 10^{-6})(-188)$$

$$= -0.01504"$$

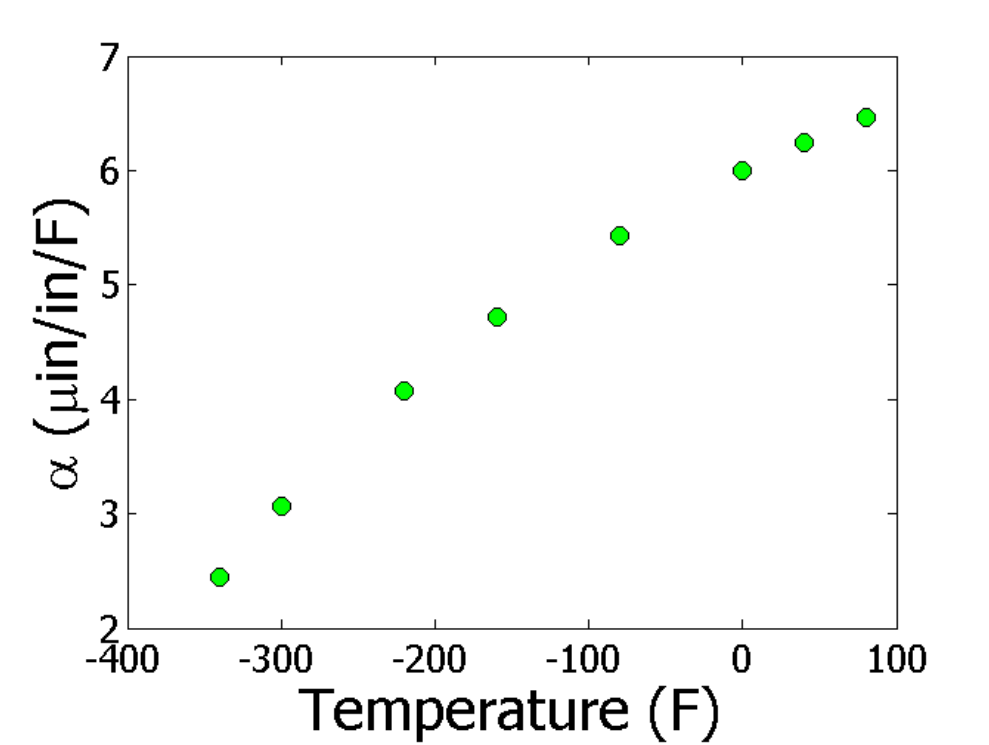
Is the formula used correct?

$$\Delta D = D \times \alpha \times \Delta T$$



T(°F)	α (μin/in/°F)
-340	2.45
-300	3.07
-220	4.08
-160	4.72
-80	5.43
0	6.00
40	6.24
80	6.47

The Correct Model Would Account for Varying Thermal Expansion Coefficient

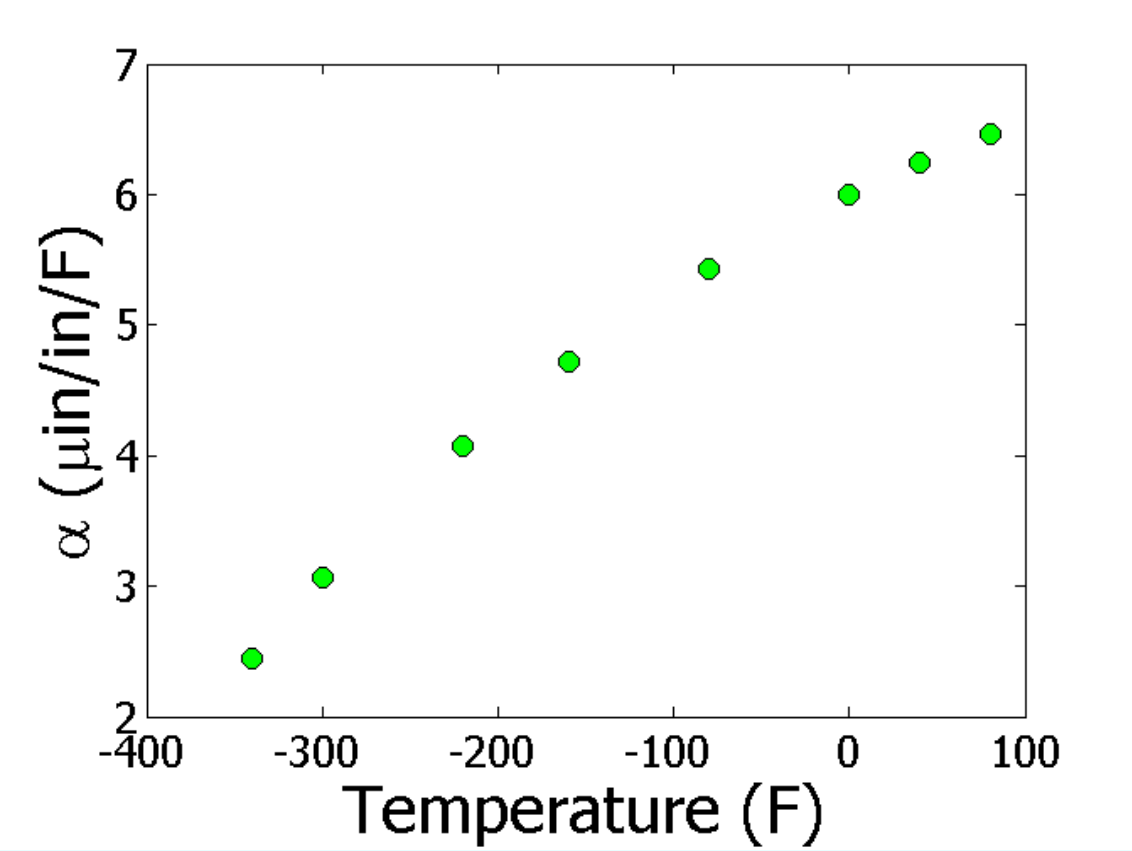


$$\Delta D = D\int_{T_a}^{T_c} \alpha(T)dT$$

Can You Roughly Estimate the Contraction?

$$\Delta D = D\int_{T_a}^{T_c} \alpha(T)dT$$

T_a=80°F; T_c=-108°F; D=12.363”



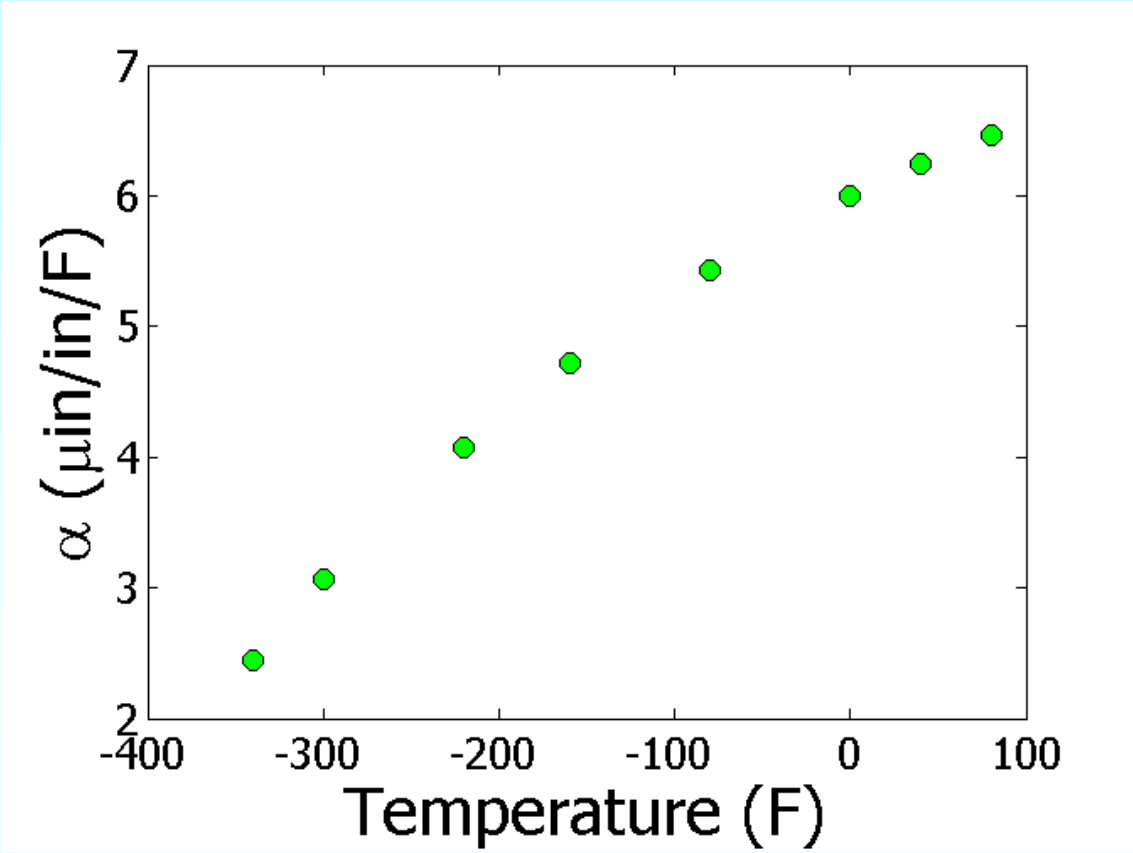
Can You Find a Better Estimate for the Contraction?

$$\Delta D = D\int_{T_a}^{T_c} \alpha(T)dT$$

$T_a = 80^oF$

$T_c = -108^oF$

$D = 12.363"$

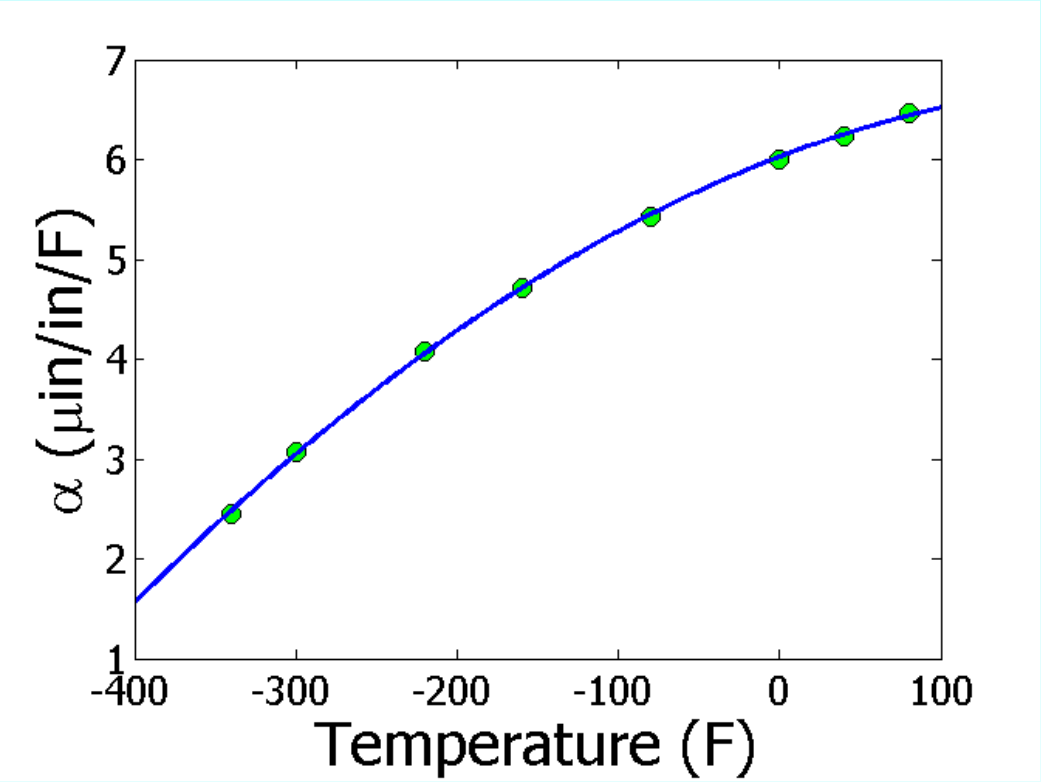


Estimating Contraction Accurately

Change in diameter (ΔD) by cooling it in dry ice/alcohol is given by

$$\Delta D = D\int_{T_a}^{T_c} \alpha(T)\,dT$$

$$T_a = 80^{o}\text{F}$$
$$T_c = -108^{o}\text{F}$$
$$D = 12.363"$$



$$\alpha = -1.2278\times10^{-5}T^{2} + 6.1946\times10^{-3}T + 6.0150$$

$$\Delta D = -0.0137"$$

So what is the solution to the problem?

One solution is to immerse the trunnion in liquid nitrogen which has a boiling point of -321°F as opposed to the dry-ice/alcohol temperature of -108°F.

$$\Delta D = -0.0244"$$

Revisiting steps to solve a problem

1) Problem Statement: Trunnion got stuck in the hub.

2) Modeling: Developed a new model

$$\Delta D = D\int_{T_a}^{T_c} \alpha(T)dT$$

3) Solution: 1) Used trapezoidal rule OR b) Used regression and integration.

4) Implementation: Cool the trunnion in liquid nitrogen.

THE END

Introduction to Numerical Methods

Mathematical Procedures

Mathematical Procedures

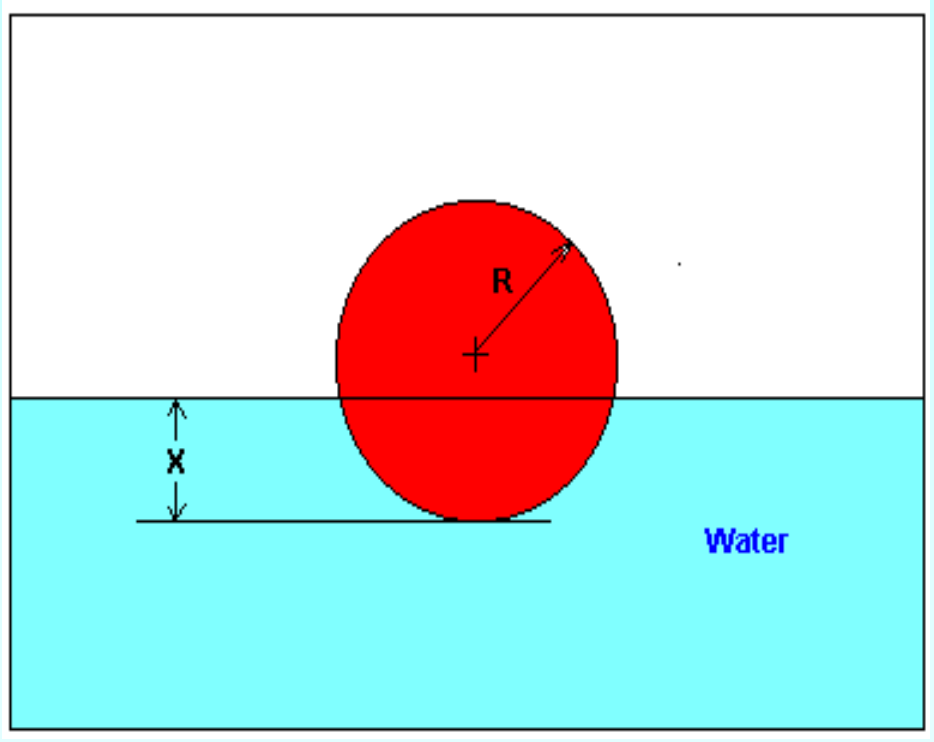
- Nonlinear Equations
- Simultaneous Linear Equations
- Differentiation
- Curve Fitting
  - Interpolation
  - Regression
- Integration
- Ordinary Differential Equations
- Other Advanced Mathematical Procedures:
  - Partial Differential Equations
  - Optimization
  - Fast Fourier Transforms

Nonlinear Equations

How much of the floating ball is under water?

Diameter=0.11m

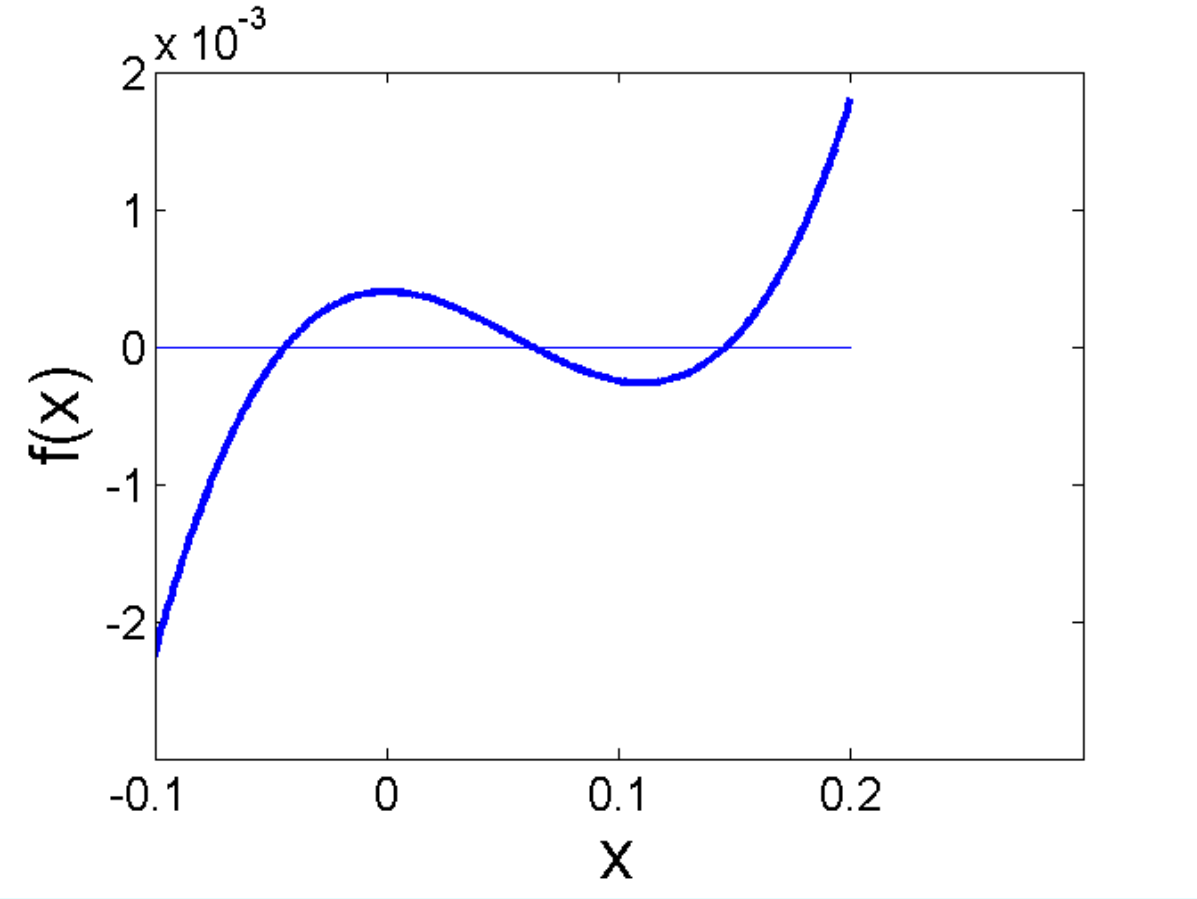
Specific Gravity=0.6



$$x^3 - 0.165x^2 + 3.993 \times 10^{-4} = 0$$

Nonlinear Equations

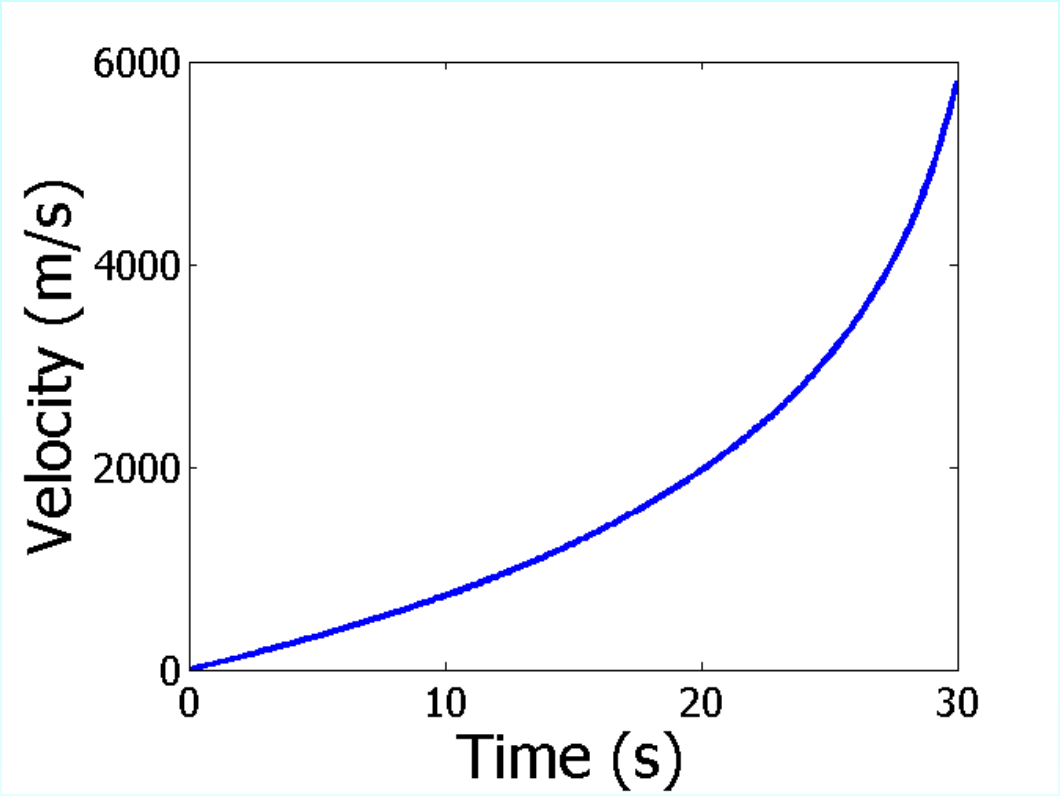
How much of the floating ball is under the water?



$$f(x) = x^3 - 0.165x^2 + 3.993 \times 10^{-4} = 0$$

Differentiation

What is the acceleration at t=7 seconds?



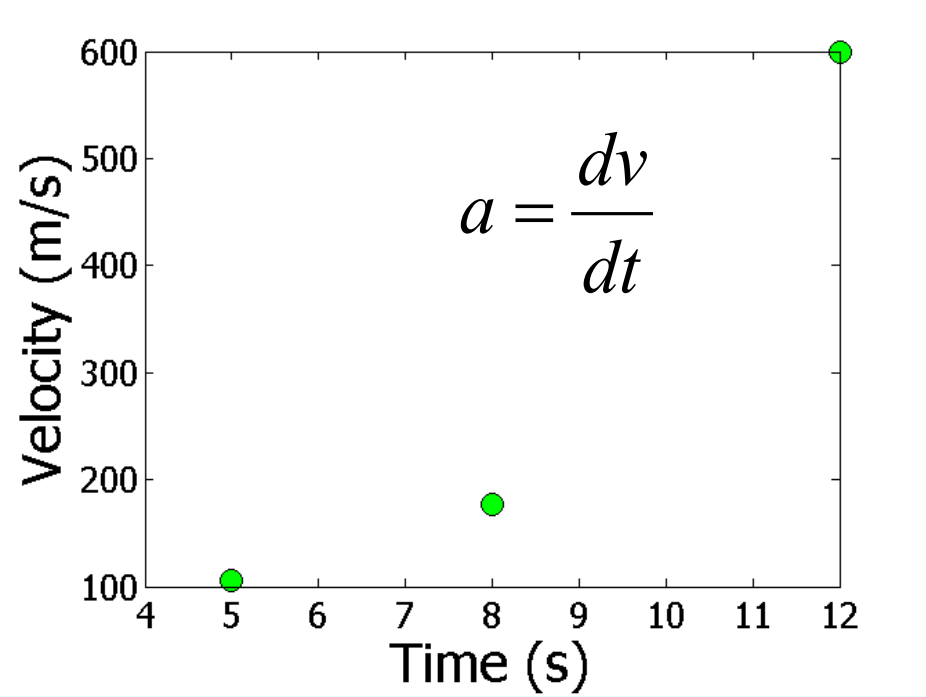
$$v(t) = 2200 \ln\left(\frac{16\times10^4}{16\times10^4 - 5000t}\right) - 9.8t$$

$$a = \frac{dv}{dt}$$

Differentiation

What is the acceleration at t=7 seconds?

Time (s)	5	8	12
Vel (m/s)	106	177	600



Simultaneous Linear Equations

Find the velocity profile, given

Time (s)	5	8	12
Vel (m/s)	106	177	600

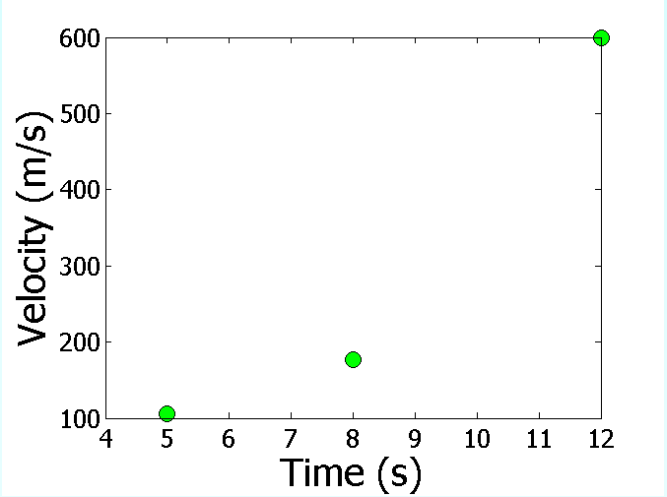
$$v(t) = at^2 + bt + c,\ 5 \le t \le 12$$

Three simultaneous linear equations

$$25a + 5b + c = 106$$

$$64a + 8b + c = 177$$

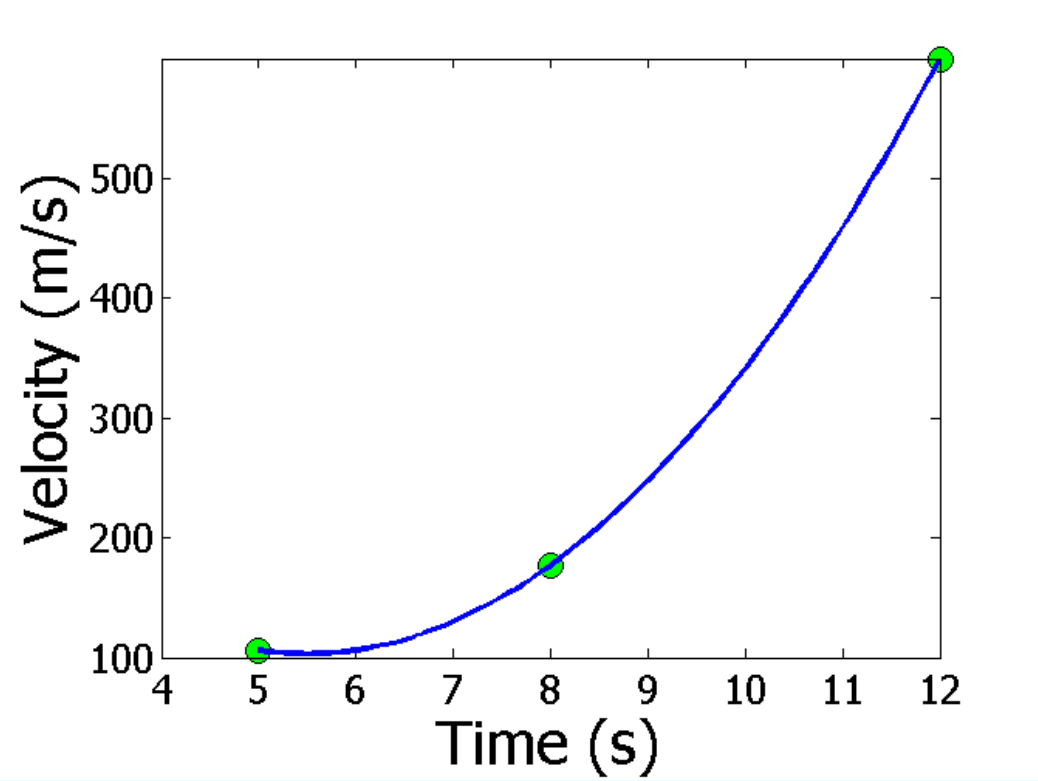
$$144a + 12b + c = 600$$



Interpolation

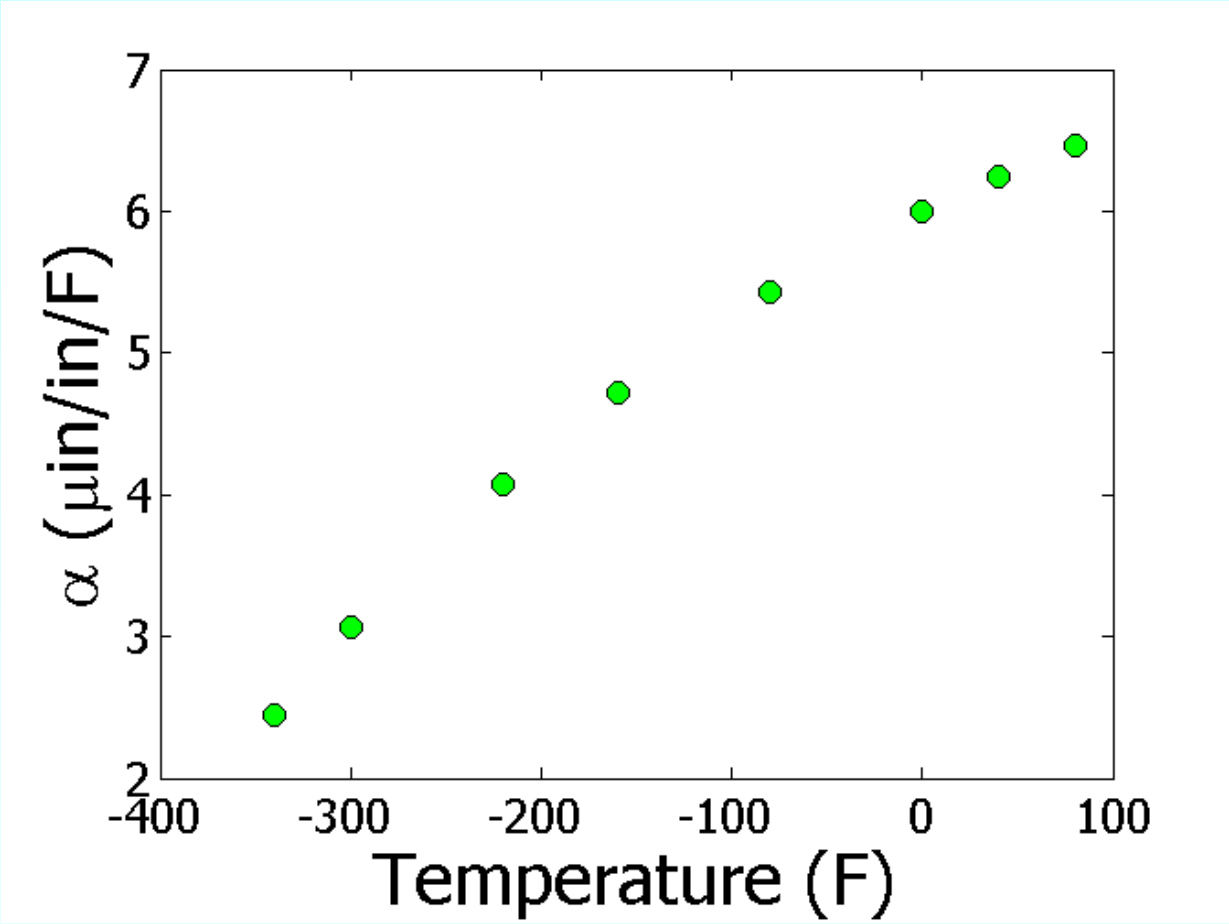
What is the velocity of the rocket at t=7 seconds?

Time (s)	5	8	12
Vel (m/s)	106	177	600

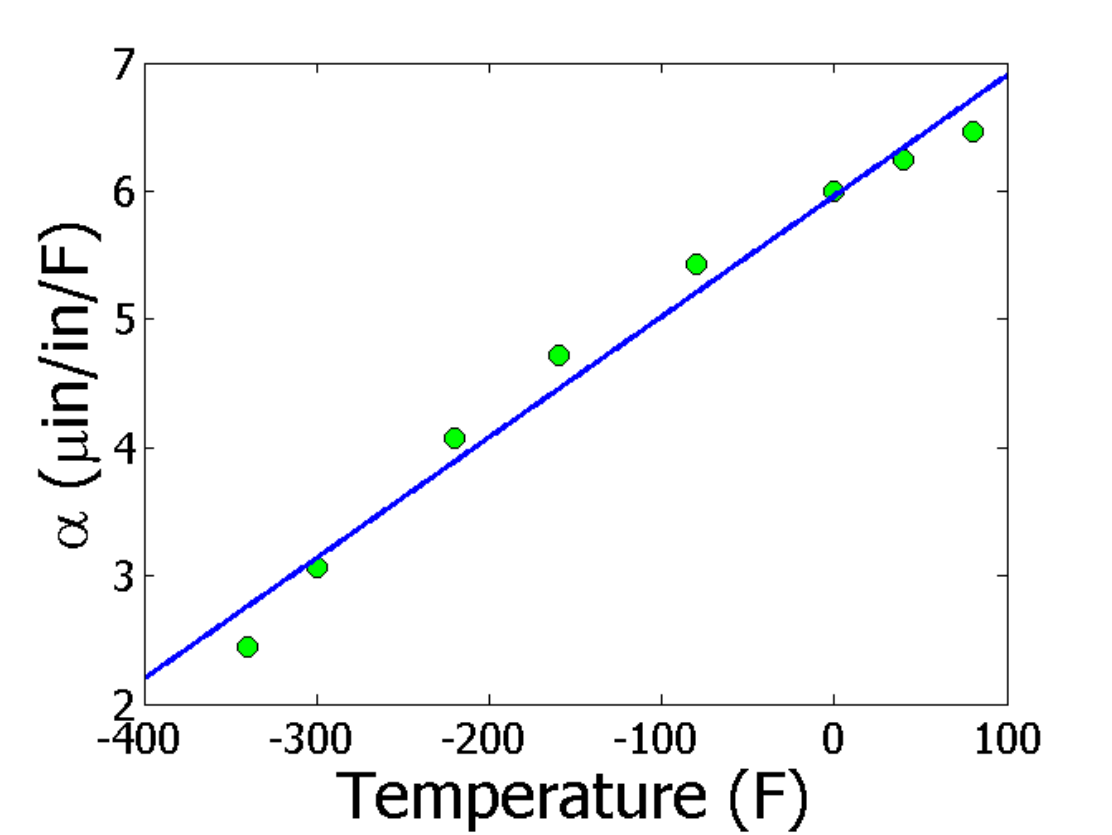


Regression

Thermal expansion coefficient data for cast steel



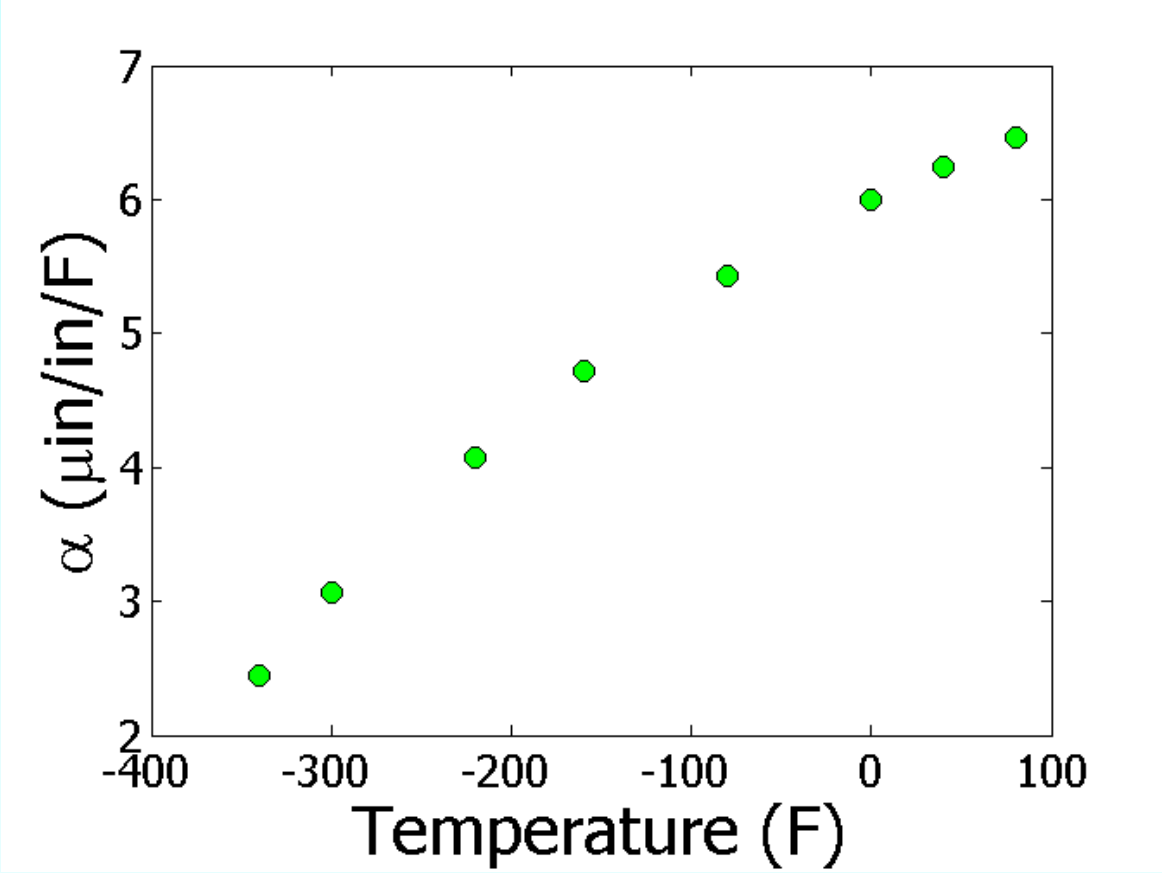
Regression (cont)



Integration

Finding the diametric contraction in a steel shaft when dipped in liquid nitrogen.

$$\Delta D = D \int_{T_{room}}^{T_{fluid}} \alpha \, dT$$



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