Pabna Cadet College

First Fortnightly Examination - 2021 Solution

Subject: Statistics

Class: XI

Time: 40 minutes

Full Marks: 20

Answer all the question.

1. Information about Marks of students of two classes are

Class	No. of Students	$\bar{X}$	σ
XI	51	82	2
XII	52	86	5

    a. What is a measure of dispersion?   1

    b. When do we get the least possible value of variance? Explain in short.   2

    c. Estimate the combined standard deviation of the marks.   3

    d. Which class performed better? Justify mathematically.   4

2. Answer concisely

    a. If the unit of mean is cm, what is the unit of standard deviation?   1

    b. If $n = 10, \sum x_i = 110$, and $\sum x_i^2 = 1400$, find C.V.   3

    c. Find σ of -3, 0, 3.   1

    d. Find the C.V. of the series: $1, 3, 5, \cdots, (2n - 1)$ 5

Solution

    1.

    a. A measure of dispersion is a measures how values of a dataset vary within themselves.

    b. When all observations are same. See more

    c. Combined Standard Deviation

$n_1 = 51, n_2 = 52$

$\bar{X}1 = 82, \bar{X}_2 = 86$

$\sigma_1 = 2, \sigma_2 = 5$

Combined Mean, $\bar{X}_c = \frac{51 \times 82 + 52 \times 86}{51 + 51} = 84.02$

$d_1 = \bar{X}_1 - \bar{X}_c = -2.02 \quad d_2 = \bar{X}_2 - \bar{X}_c = 1.98$

Combined Variance,

$$
\begin{aligned}
\sigma_c^2 &= \frac{n_1(\sigma^2 + d_1^2) + n_2(\sigma^2 + d_2^2)}{n_1 + n_2} \\
&= \frac{51(4 + 4.08) + 52(25 + 3.92)}{51 + 52} \\
\therefore \sigma_c^2 &= 18.6 \\
\sigma_c &= 4.31
\end{aligned}
\tag{1}
$$

$\therefore$ Combined standard deviation, $\sigma_c = 4.31$

    d. Performance

$CV_{XI} = \frac{2}{82} \times 100 = 2.43$

$CV_{XII} = \frac{5}{86} \times 100 = 5.81$

$\therefore$ Class XI is better.

    2.

    a. cm (N:B: unit of σ^2 is cm^2)
    b. CV

$$
\begin{aligned}
\sigma^2 &= \frac{1400}{100} - (\frac{110}{10})^2 \\
&= 140 - 121 \\
&= 19 \\
\therefore \sigma &= 4.36
\end{aligned}
$$

(2)

$\bar{X} = 11$

$\therefore CV = \frac{4.36}{11} \times 100 = 39.64\%$

    c. Standard Deviation of -3, 0, 3

$$
\begin{aligned}
\sigma^2 &= \frac{\sum x_i^2}{n} - (\frac{\sum x_i}{n})^2 \\
&= \frac{9 + 9}{3} - 0 \\
&= 6 \\
\therefore \sigma &= 2.45
\end{aligned}
$$

(3)

    d. CV of the series $1, 3, 5, \cdots, (2n - 1)$

$$
\begin{aligned}
\sigma^2 &= \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2 \\
&= \frac{\frac{1}{3}n(4n^2 - 1)}{n} - \left(\frac{n^2}{n}\right)^2 \\
&= \frac{1}{3}(4n^2 - 1) - n^2 \\
&= \frac{4n^2 - 1 - 3n^2}{3} \\
&= \frac{n^2 - 1}{3} \\
\therefore \sigma &= \sqrt{\frac{n^2 - 1}{3}} \\
\bar{X} &= \frac{\sum x}{n} = \frac{n^2}{n} = n \\
\therefore CV &= \frac{\sigma}{\bar{X}} \times 100 \frac{\sqrt{\frac{n^2-1}{3}}}{n} \times 100 \\
&= \sqrt{\frac{n^2 - 1}{3n^2}} \times 100
\end{aligned}
$$

(4)

—Good Luck—