

Efficient Quasi-Monte Carlo Integration by Adjusting the Derivation-sensitivity Parameter of Walsh Figure of Merit

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Quasi-Monte Carlo Integration over $[0..1)^s$

Approximate

$$I_f := \int_{[0..1)^s} f(x) dx$$

by

$$I_f^P := \frac{1}{|P|} \sum_{x \in P} f(x)$$

with some good finite subset $P \subset [0..1)^s$

$$\text{err}_f^P := I_f^P - I_f$$

Walsh figure of merit

Let c be a parameter. If $P \subset [0..1)^s$ is a digital net, then

$$|\text{err}_f^P| \leq \|f\|_c W_c(P)$$

Good digital net P has small $W_c(P)$

Walsh figure of merit

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Walsh figure of merit

If $P \subset [0..1)^s$ is a digital net, then

$$L_c W_c(P) \leq \text{err}_g^P \leq \|g\|_c W_c(P)$$

where

$$g(x) = \exp \left(-2^c \sum_{0 \leq i < s} x_i \right)$$

Aim of this research

We provide digital nets P with small $W_c(P)$
for $4 \leq s \leq 16$ and $|P| = 2^m$ where $8 \leq m \leq 30$

How to choose c ?

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Derivation-sensitivity Parameter c

Parameter c controls the space \mathcal{F}_c of the functions f where

$$|\text{err}_f^P| \leq \|f\|_c W_c(P)$$

holds

We choose c according to the dimensionality s and the size m

Derivation-sensitivity Parameter c

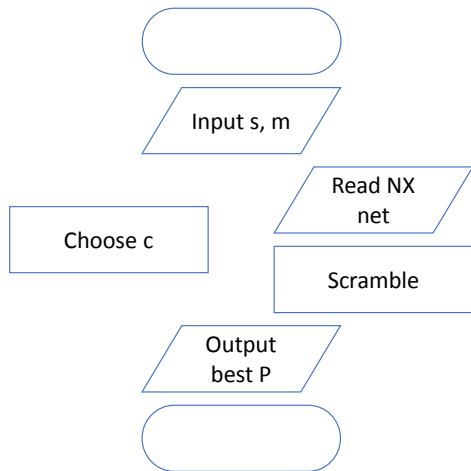
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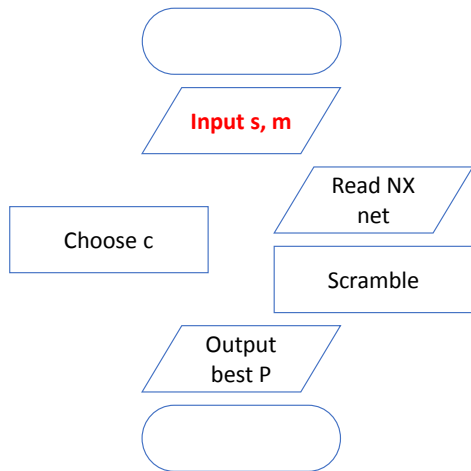
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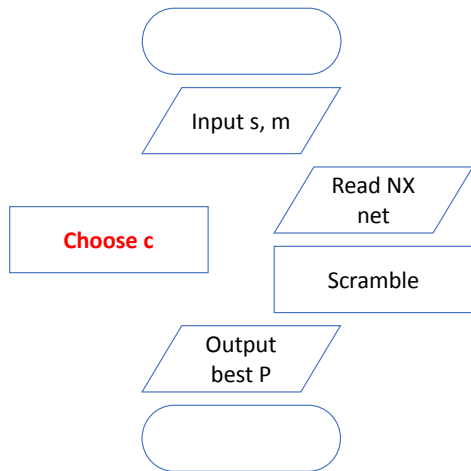
Algorithm



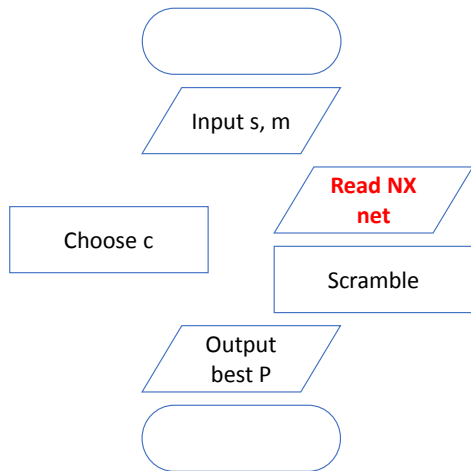
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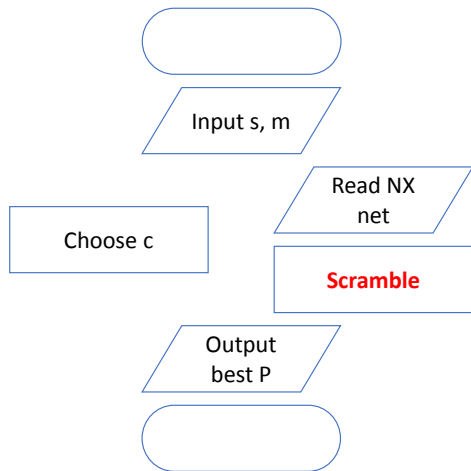
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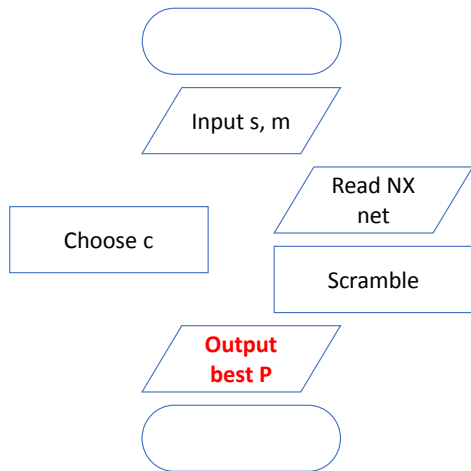
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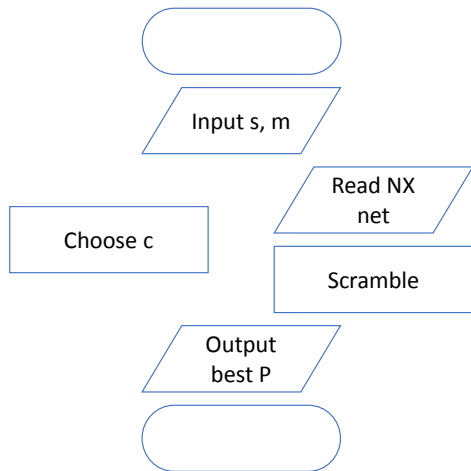
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Result

- Digital nets obtained by this method is available on GitHub: <http://github.com/majiang/adjust-dsp>
- Example code is provided as a D library: <http://code.dlang.org/packages/digitalnet>

You can perform efficient QMC over tonight!

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Result: slope of $(m, \lg W_{c(s, m_0)}(P_m))$

m_0	$s = 4$	5	6	7	8	9	10	11	12	13	14	15	16
8	-2.30												
9	-1.97			-1.95	-1.97		-1.97						
10	-1.85	-1.81	-1.80	-1.74	-1.70	-1.72	-1.68	-1.68	-1.69	-1.70	-1.74	-1.72	-1.76
11	-1.78	-1.71	-1.70	-1.64	-1.59	-1.59	-1.55	-1.53	-1.53	-1.52	-1.54	-1.51	-1.52
12	-1.72	-1.65	-1.64	-1.57	-1.52	-1.52	-1.47	-1.45	-1.45	-1.43	-1.45	-1.42	-1.41
13	-1.68	-1.60	-1.59	-1.52	-1.47	-1.46	-1.42	-1.40	-1.39	-1.37	-1.38	-1.36	-1.35
14	-1.63	-1.55	-1.54	-1.48	-1.43	-1.42	-1.37	-1.36	-1.34	-1.33	-1.33	-1.31	-1.30
15	-1.59	-1.52	-1.51	-1.44	-1.40	-1.38	-1.34	-1.32	-1.31	-1.29	-1.29	-1.27	-1.26
16	-1.55	-1.48	-1.48	-1.41	-1.37	-1.35	-1.31	-1.29	-1.28	-1.26	-1.26	-1.24	-1.23
17	-1.50	-1.45	-1.45	-1.39	-1.34	-1.32	-1.28	-1.26	-1.25	-1.23	-1.23	-1.21	-1.20
18	-1.45	-1.42	-1.42	-1.36	-1.31	-1.29	-1.25	-1.24	-1.23	-1.21	-1.20	-1.19	-1.18
19	-1.39	-1.39	-1.39	-1.33	-1.29	-1.26	-1.23	-1.22	-1.20	-1.19	-1.18	-1.17	-1.16
20	-1.33	-1.36	-1.36	-1.31	-1.27	-1.24	-1.21	-1.20	-1.18	-1.17	-1.16	-1.15	-1.14
21	-1.25	-1.32	-1.34	-1.29	-1.24	-1.22	-1.19	-1.18	-1.16	-1.15	-1.14	-1.13	-1.12
22	-1.17	-1.29	-1.31	-1.26	-1.22	-1.20	-1.17	-1.16	-1.15	-1.13	-1.13	-1.12	-1.11
23	-1.08	-1.25	-1.28	-1.24	-1.20	-1.18	-1.16	-1.14	-1.13	-1.12	-1.11	-1.10	-1.10
24		-1.21	-1.25	-1.22	-1.19	-1.16	-1.14	-1.13	-1.12	-1.11	-1.10	-1.09	-1.08
25		-1.16	-1.22	-1.19	-1.17	-1.14	-1.13	-1.12	-1.11	-1.09	-1.09	-1.08	-1.07
26		-1.11	-1.19	-1.17	-1.15	-1.13	-1.11	-1.10	-1.09	-1.08	-1.08	-1.07	-1.06
27		-1.07	-1.16	-1.15	-1.13	-1.11	-1.10	-1.09	-1.08	-1.07	-1.07	-1.06	-1.06
28		-1.02	-1.13	-1.13	-1.11	-1.10	-1.09	-1.08	-1.07	-1.06	-1.06	-1.05	-1.05
29			-1.09	-1.11	-1.10	-1.09	-1.08	-1.07	-1.06	-1.06	-1.05	-1.05	-1.04
30			-1.06	-1.09	-1.08	-1.07	-1.07	-1.06	-1.05	-1.05	-1.04	-1.04	-1.04

Summary

- Adjusts the derivation-sensitivity parameter (D.S.P.) of Walsh figure of merit (WAFOM)
- Provides digital nets $P \subset [0..1)^s$ with $|P| = 2^m$ for $4 \leq s \leq 16$ and $8 \leq m \leq 30$
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Thank you for listening.