Efficient Quasi–Monte Carlo Integration by Adjusting the Derivation-sensitivity Parameter of Walsh Figure of Merit

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Quasi–Monte Carlo Integration over $[0..1)^s$

Approximate

$$I_f := \int_{[0..1)^s} f(x)\, dx$$

by

$$I_f^P := \frac{1}{|P|} \sum_{x \in P} f(x)$$

with some good finite subset $P \subset [0..1)^s$

$$\mathrm{err}_f^P := I_f^P - I_f$$

Walsh figure of merit

Let c be a parameter. If $P \subset [0..1)^s$ is a digital net, then

$$\left|\mathrm{err}_f^P\right| \leq \|f\|_c W_c(P)$$

Good digital net P has small $W_c(P)$

Walsh figure of merit

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Walsh figure of merit

If $P \subset [0..1)^s$ is a digital net, then

$$L_c W_c(P) \leq \mathrm{err}_g^P \leq \|g\|_c W_c(P)$$

where

$$g(x) = \exp\left(-2^c \sum_{0 \leq i < s} x_i\right)$$

Aim of this research

We provide digital nets P with small $W_c(P)$
for $4 \leq s \leq 16$ and $|P| = 2^m$ where $8 \leq m \leq 30$
How to choose c?

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Derivation-sensitivity Parameter c

Parameter c controls the space $\mathcal{F}_c \subset C^\infty[0..1]^s$ of the functions f where

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Choose c according to the dimensionality s and the size m

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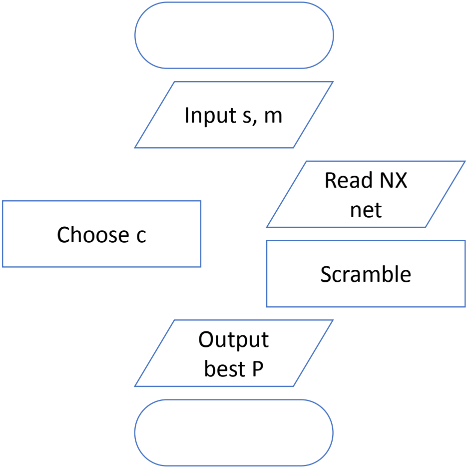
Algorithm



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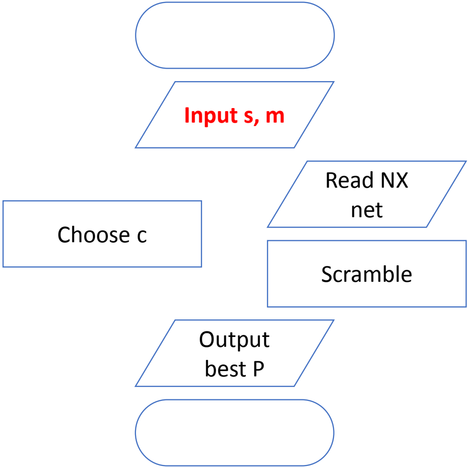

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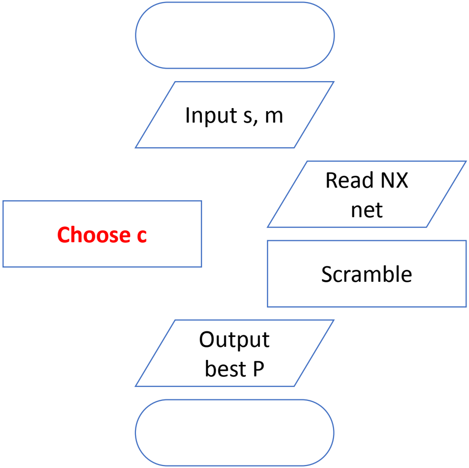

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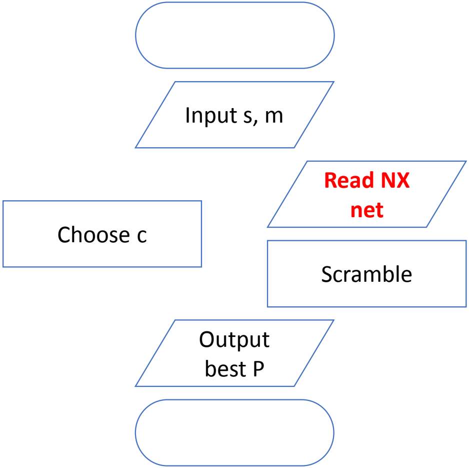

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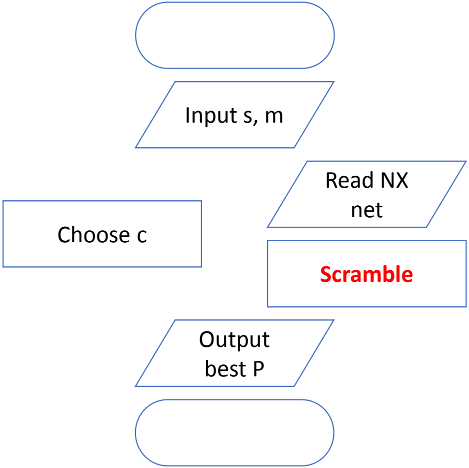

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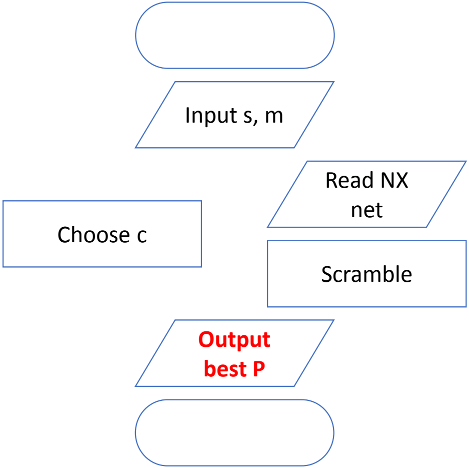

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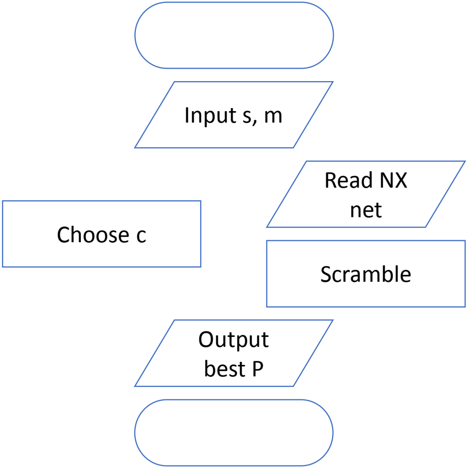

Result

- Digital nets obtained by this method is available on GitHub: http://github.com/majiang/adjust-dsp
- Example code is provided as a D library: http://code.dlang.org/packages/digitalnet

You can perform efficient QMC over tonight!

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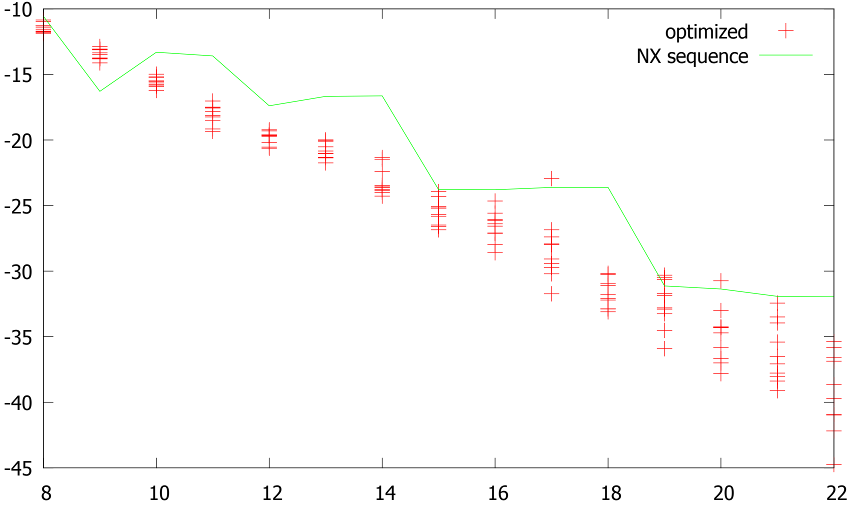
Result: slope of $(m, \lg W_{c(s,m_0)}(P_m))$

m_0	$s=4$	5	6	7	8	9	10	11	12	13	14	15	16
8	−2.30												
9	−1.97			−1.95	−1.97		−1.97						
10	−1.85	−1.81	−1.80	−1.74	−1.70	−1.72	−1.68	−1.68	−1.69	−1.70	−1.74	−1.72	−1.76
11	−1.78	−1.71	−1.70	−1.64	−1.59	−1.59	−1.55	−1.53	−1.53	−1.52	−1.54	−1.51	−1.52
12	−1.72	−1.65	−1.64	−1.57	−1.52	−1.52	−1.47	−1.45	−1.45	−1.43	−1.45	−1.42	−1.41
13	−1.68	−1.60	−1.59	−1.52	−1.47	−1.46	−1.42	−1.40	−1.39	−1.37	−1.38	−1.36	−1.35
14	−1.63	−1.55	−1.54	−1.48	−1.43	−1.42	−1.37	−1.36	−1.34	−1.33	−1.33	−1.31	−1.30
15	−1.59	−1.52	−1.51	−1.44	−1.40	−1.38	−1.34	−1.32	−1.31	−1.29	−1.29	−1.27	−1.26
16	−1.55	−1.48	−1.48	−1.41	−1.37	−1.35	−1.31	−1.29	−1.28	−1.26	−1.26	−1.24	−1.23
17	−1.50	−1.45	−1.45	−1.39	−1.34	−1.32	−1.28	−1.26	−1.25	−1.23	−1.23	−1.21	−1.20
18	−1.45	−1.42	−1.42	−1.36	−1.31	−1.29	−1.25	−1.24	−1.23	−1.21	−1.20	−1.19	−1.18
19	−1.39	−1.39	−1.39	−1.33	−1.29	−1.26	−1.23	−1.22	−1.20	−1.19	−1.18	−1.17	−1.16
20	−1.33	−1.36	−1.36	−1.31	−1.27	−1.24	−1.21	−1.20	−1.18	−1.17	−1.16	−1.15	−1.14
21	−1.25	−1.32	−1.34	−1.29	−1.24	−1.22	−1.19	−1.18	−1.16	−1.15	−1.14	−1.13	−1.12
22	−1.17	−1.29	−1.31	−1.26	−1.22	−1.20	−1.17	−1.16	−1.15	−1.13	−1.13	−1.12	−1.11
23	−1.08	−1.25	−1.28	−1.24	−1.20	−1.18	−1.16	−1.14	−1.13	−1.12	−1.11	−1.10	−1.10
24		−1.21	−1.25	−1.22	−1.19	−1.16	−1.14	−1.13	−1.12	−1.11	−1.10	−1.09	−1.08
25		−1.16	−1.22	−1.19	−1.17	−1.14	−1.13	−1.12	−1.11	−1.09	−1.09	−1.08	−1.07
26		−1.11	−1.19	−1.17	−1.15	−1.13	−1.11	−1.10	−1.09	−1.08	−1.08	−1.07	−1.06
27		−1.07	−1.16	−1.15	−1.13	−1.11	−1.10	−1.09	−1.08	−1.07	−1.07	−1.06	−1.06
28		−1.02	−1.13	−1.13	−1.11	−1.10	−1.09	−1.08	−1.07	−1.06	−1.06	−1.05	−1.05
29			−1.09	−1.11	−1.10	−1.09	−1.08	−1.07	−1.06	−1.06	−1.05	−1.05	−1.04
30			−1.06	−1.09	−1.08	−1.07	−1.07	−1.06	−1.05	−1.05	−1.04	−1.04	−1.04

Result: Error comparison



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- Adjusts the derivation-sensitivity parameter (D.S.P.) of Walsh figure of merit (WAFOM)
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Thank you for listening.