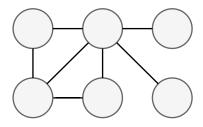
On the Prediction Instability of Graph Neural Networks

Max Klabunde and Florian Lemmerich

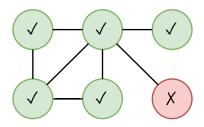


Faculty of Computer Science and Mathematics

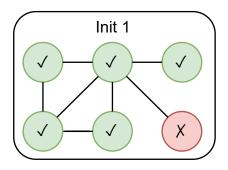




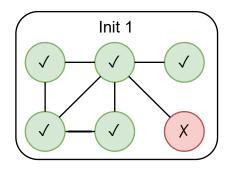


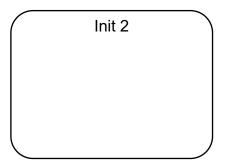




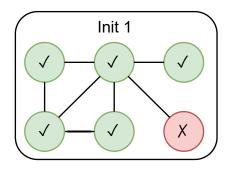


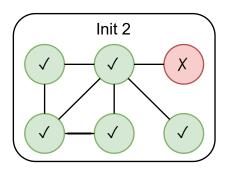




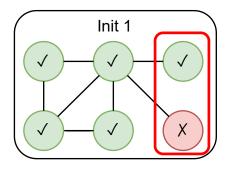


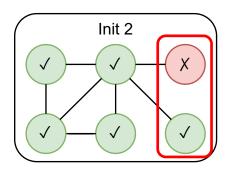




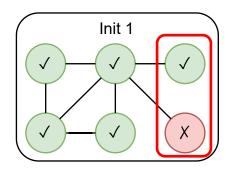


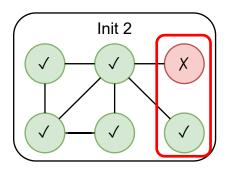








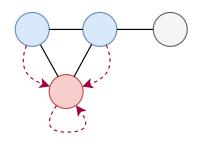




Individual predictions rely on random factors

Graph Neural Networks (GNNs)





► Focus: Graph Convolutional Networks (GCN)¹ and Graph Attention Networks (GAT)²

¹Kipf and Welling 2017.

²Veličković et al. 2018.

Context



- ► Fully reproducible training may be infeasible³
- Previous works mainly focus on large non-graph models⁴
- Work on graph models focused on older unsupervised methods with geometrical perspective⁵

³Zhuang et al. 2022.

⁴Summers and Dinneen 2021.

⁵Wang et al. 2022; Schumacher et al. 2021.

Outline

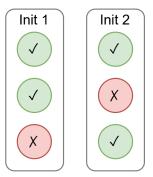


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How to measure prediction instability



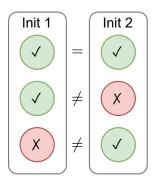
Measure with disagreement: share of predictions that are different between two models



How to measure prediction instability



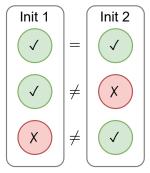
Measure with disagreement: share of predictions that are different between two models



How to measure prediction instability



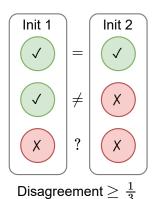
Measure with disagreement: share of predictions that are different between two models



Disagreement $=\frac{2}{3}$

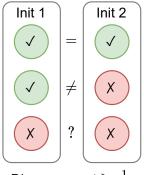
Error rate affects possible disagreement



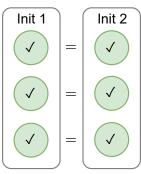


Error rate affects possible disagreement





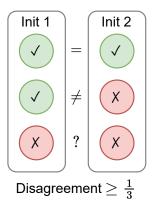
Disagreement $\geq \frac{1}{3}$

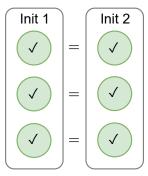


Disagreement $= \frac{0}{3}$

Error rate affects possible disagreement







Disagreement $= \frac{0}{3}$

► Normalization possible but similar results



Dataset	Model	Accuracy	d	d_{norm}	d_{False}
CiteSeer	GAT GCN				
Pubmed	GAT GCN				
CS	GAT GCN				
Physics	GAT GCN				
Computers	GAT GCN				
Photo	GAT GCN				
WikiCS	GAT GCN				



Dataset	Model	Accuracy	d	d_{norm}	d_{False}
CiteSeer	GAT GCN	69.0 ± 1.0 69.2 ± 0.7			
Pubmed	GAT GCN	$\begin{array}{c} 75.7 \pm 0.6 \\ 76.8 \pm 0.5 \end{array}$			
CS	GAT GCN	$\begin{array}{c} 90.7 \pm 0.5 \\ 90.7 \pm 0.5 \end{array}$			
Physics	GAT GCN	92.0 ± 0.7 92.7 ± 0.3			
Computers	GAT GCN	81.0 ± 1.5 81.2 ± 0.9			
Photo	GAT GCN	90.3 ± 0.8 90.8 ± 0.5			
WikiCS	GAT GCN	$\begin{array}{c} 79.6 \pm 0.3 \\ 79.4 \pm 0.2 \end{array}$			



Dataset	Model	Accuracy	d	d_{norm}	d_{False}
CiteSeer	GAT GCN	$\begin{array}{c} 69.0 \pm 1.0 \\ 69.2 \pm 0.7 \end{array}$			
Pubmed	GAT GCN	$\begin{array}{c} 75.7 \pm 0.6 \\ 76.8 \pm 0.5 \end{array}$			
CS	GAT GCN	90.7 ± 0.5 90.7 ± 0.5	J., = J.		
Physics	GAT GCN	92.0 ± 0.7 92.7 ± 0.3	$3.8 \pm 0.8 \\ 1.6 \pm 0.4$		
Computers	GAT GCN	81.0 ± 1.5 81.2 ± 0.9	$9.5 \pm 2.2 \\ 9.9 \pm 1.9$		
Photo	GAT GCN	$\begin{array}{c} 90.3 \pm 0.8 \\ 90.8 \pm 0.5 \end{array}$	$4.4 \pm 1.1 \\ 3.7 \pm 0.8$		
WikiCS	GAT GCN	79.6 ± 0.3 79.4 ± 0.2	0.0 = 0.0		



Dataset	Model	Accuracy	d	d_{norm}	d_{False}
CiteSeer	GAT GCN	69.0 ± 1.0 69.2 ± 0.7	$\begin{array}{c} 10.5\pm1.7 \\ 7.1\pm1.0 \end{array}$		
Pubmed	GAT GCN	$\begin{array}{c} 75.7 \pm 0.6 \\ 76.8 \pm 0.5 \end{array}$			
CS	GAT GCN	90.7 ± 0.5 90.7 ± 0.5	3.7 ± 0.5 3.3 ± 0.6		
Physics	GAT GCN	$\begin{array}{c} 92.0 \pm 0.7 \\ 92.7 \pm 0.3 \end{array}$	3.8 ± 0.8 1.6 ± 0.4		
Computers	GAT GCN	81.0 ± 1.5 81.2 ± 0.9	9.5 ± 2.2 9.9 ± 1.9		
Photo	GAT GCN	$\begin{array}{c} 90.3 \pm 0.8 \\ 90.8 \pm 0.5 \end{array}$	4.4 ± 1.1 3.7 ± 0.8		
WikiCS	GAT GCN	79.6 ± 0.3 79.4 ± 0.2	3.8 ± 0.5 3.3 ± 0.4		



Dataset	Model	Accuracy	d	d_{norm}	d _{False}
CiteSeer	GAT GCN	69.0 ± 1.0 69.2 ± 0.7		$\begin{array}{c} 15.4 \pm 2.5 \\ 10.3 \pm 1.6 \end{array}$	
Pubmed	GAT GCN	$\begin{array}{c} 75.7 \pm 0.6 \\ 76.8 \pm 0.5 \end{array}$	3.7 ± 1.4 2.4 ± 0.7	$6.4 \pm 2.7 \\ 4.1 \pm 1.4$	
CS	GAT GCN	$\begin{array}{c} 90.7 \pm 0.5 \\ 90.7 \pm 0.5 \end{array}$	3.7 ± 0.5 3.3 ± 0.6	17.3 ± 2.0 15.4 ± 2.7	
Physics	GAT GCN	92.0 ± 0.7 92.7 ± 0.3		19.7 ± 4.2 8.6 ± 2.7	
Computers	GAT GCN	81.0 ± 1.5 81.2 ± 0.9	9.5 ± 2.2 9.9 ± 1.9	21.6 ± 5.6 24.2 ± 4.9	
Photo	GAT GCN	$\begin{array}{c} 90.3 \pm 0.8 \\ 90.8 \pm 0.5 \end{array}$	4.4 ± 1.1 3.7 ± 0.8	18.9 ± 4.9 17.5 ± 3.7	
WikiCS	GAT GCN	79.6 ± 0.3 79.4 ± 0.2	3.8 ± 0.5 3.3 ± 0.4	$\begin{array}{c} 8.6 \pm 1.3 \\ 7.6 \pm 1.0 \end{array}$	



Dataset	Model	Accuracy	d	d_{norm}	d_{False}
CiteSeer	GAT GCN	69.0 ± 1.0 69.2 ± 0.7	10.5 ± 1.7 7.1 ± 1.0	15.4 ± 2.5 10.3 ± 1.6	22.3 ± 3.8 15.1 ± 2.4
Pubmed	GAT GCN	$\begin{array}{c} 75.7 \pm 0.6 \\ 76.8 \pm 0.5 \end{array}$	3.7 ± 1.4 2.4 ± 0.7	6.4 ± 2.7 4.1 ± 1.4	8.0 ± 3.3 5.6 ± 2.2
CS	GAT GCN	$\begin{array}{c} 90.7 \pm 0.5 \\ 90.7 \pm 0.5 \end{array}$	3.7 ± 0.5 3.3 ± 0.6	17.3 ± 2.0 15.4 ± 2.7	22.0 ± 3.6 19.9 ± 4.1
Physics	GAT GCN	92.0 ± 0.7 92.7 ± 0.3	3.8 ± 0.8 1.6 ± 0.4	19.7 ± 4.2 8.6 ± 2.7	$\begin{array}{c} 25.7 \pm 6.4 \\ 12.2 \pm 4.3 \end{array}$
Computers	GAT GCN	$\begin{array}{c} 81.0 \pm 1.5 \\ 81.2 \pm 0.9 \end{array}$	9.5 ± 2.2 9.9 ± 1.9		29.6 ± 7.3 31.9 ± 6.0
Photo	GAT GCN	$\begin{array}{c} 90.3 \pm 0.8 \\ 90.8 \pm 0.5 \end{array}$	4.4 ± 1.1 3.7 ± 0.8		26.0 ± 6.9 24.1 ± 5.5
WikiCS	GAT GCN	79.6 ± 0.3 79.4 ± 0.2	3.8 ± 0.5 3.3 ± 0.4	8.6 ± 1.3 7.6 ± 1.0	$\begin{array}{c} 11.7 \pm 1.8 \\ 10.1 \pm 1.4 \end{array}$



Dataset	Model	Accuracy	d	d_{norm}	d_{False}
CiteSeer	GAT GCN	$\begin{array}{c} 69.0\pm1.0 \\ 69.2\pm0.7 \end{array}$	10.5 ± 1.7 7.1 ± 1.0	15.4 ± 2.5 10.3 ± 1.6	22.3 ± 3.8 15.1 ± 2.4
Pubmed	GAT GCN	$\begin{array}{c} 75.7 \pm 0.6 \\ 76.8 \pm 0.5 \end{array}$	3.7 ± 1.4 2.4 ± 0.7	6.4 ± 2.7 4.1 ± 1.4	8.0 ± 3.3 5.6 ± 2.2
CS	GAT GCN	$\begin{array}{c} 90.7 \pm 0.5 \\ 90.7 \pm 0.5 \end{array}$	3.7 ± 0.5 3.3 ± 0.6	17.3 ± 2.0 15.4 ± 2.7	22.0 ± 3.6 19.9 ± 4.1
Physics	GAT GCN	$\begin{array}{c} 92.0 \pm 0.7 \\ 92.7 \pm 0.3 \end{array}$	3.8 ± 0.8 1.6 ± 0.4	$\begin{array}{c} 19.7 \pm 4.2 \\ 8.6 \pm 2.7 \end{array}$	25.7 ± 6.4 12.2 ± 4.3
Computers	GAT GCN	81.0 ± 1.5 81.2 ± 0.9	9.5 ± 2.2 9.9 ± 1.9		29.6 ± 7.3 31.9 ± 6.0
Photo	GAT GCN	$\begin{array}{c} 90.3 \pm 0.8 \\ 90.8 \pm 0.5 \end{array}$	4.4 ± 1.1 3.7 ± 0.8	$18.9 \pm 4.9 \\ 17.5 \pm 3.7$	26.0 ± 6.9 24.1 ± 5.5
WikiCS	GAT GCN	79.6 ± 0.3 79.4 ± 0.2	3.8 ± 0.5 3.3 ± 0.4	8.6 ± 1.3 7.6 ± 1.0	11.7 ± 1.8 10.1 ± 1.4

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Setup

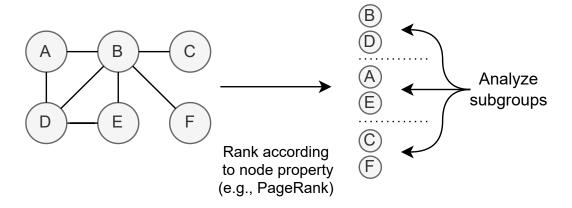


- Centrality, clustering etc. could influence stability
- ► PageRank, clustering coefficient, k-core, class label

Setup

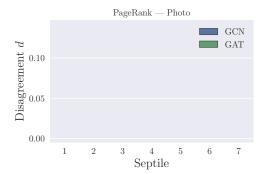


- Centrality, clustering etc. could influence stability
- PageRank, clustering coefficient, k-core, class label



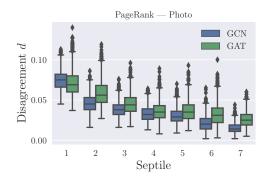
Results: PageRank





Results: PageRank

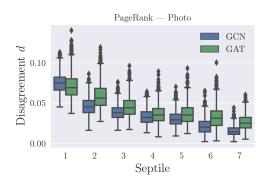


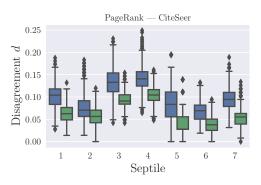


► Central nodes are more stable

Results: PageRank

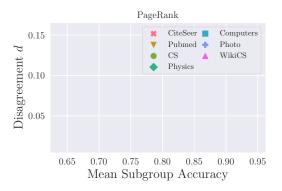




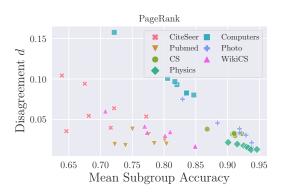


- ► Central nodes are more stable
- ► Less clear effects for other properties

Error Rate and Disagreement are correlated UNIVERSITÄT PASSAU



Error Rate and Disagreement are correlated UNIVERSITÄT



- Central nodes are more stably predicted
- ► Normalized disagreement explains this effect

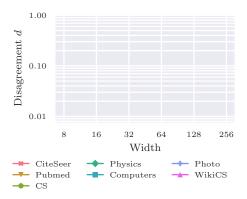
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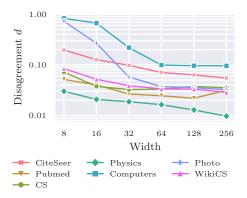


► Change configuration of model along one axis, e.g., width



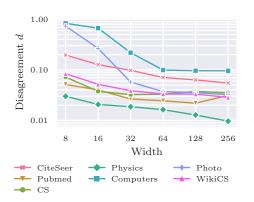


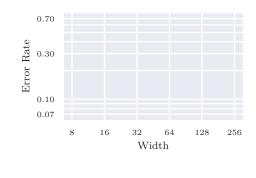
Change configuration of model along one axis, e.g., width





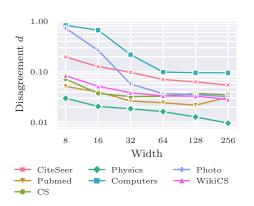
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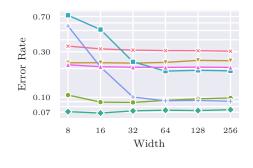






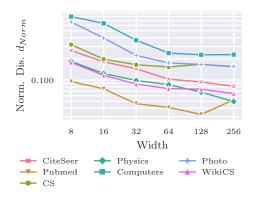
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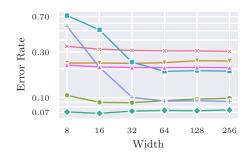






► Change configuration of model along one axis, e.g., width





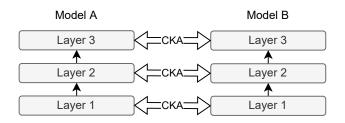
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Where do instabilities arise?



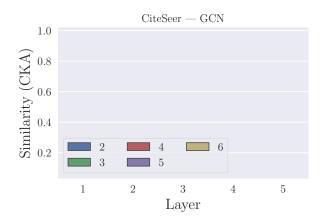


 Compare representations of layers over varying depth with Centered Kernel Alignment (CKA)⁶

⁶Kornblith et al. 2019.

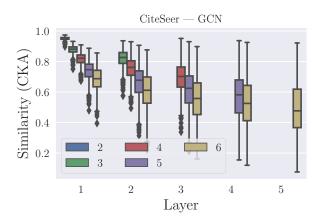
Where do instabilities arise?





Where do instabilities arise?





Deep layers and deep models are less stable

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Summary



- Deep neural networks are not stable: GCN and GAT are no exception
- ► Data properties influence stability
- Selecting good hyperparameters can positively influence stability
- ▶ Internal representations mirror instability of predictions



Max Klabunde



Florian Lemmerich



Github: mklabunde/ gnn-prediction-instability

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