

11/08/20

classmate

Date _____

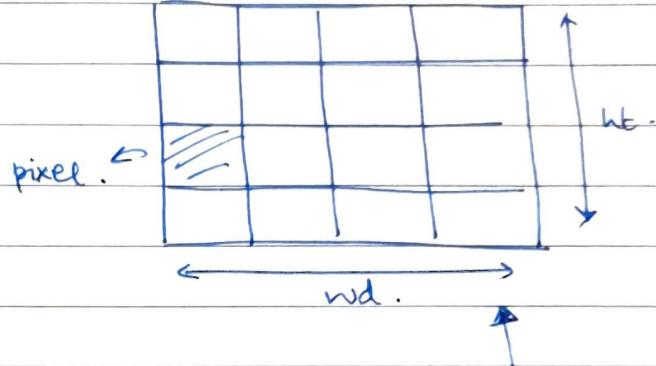
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Digital Image Processing.

Digital Images are an array of numbers representing colour intensities.

Uses 8 bits to rep. a pixel's intensity.

black \leftrightarrow 0
white \leftrightarrow 255.



Grey scale \rightarrow 8 bits / pixel \times 1 2D channel.

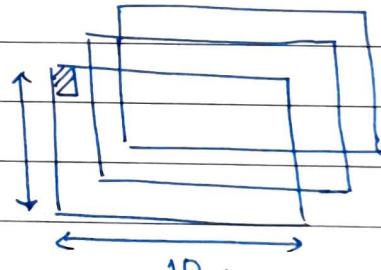
→ Images are rendered from the video card reader.

→ More common: RGB notation / representation.

3 - 2D channels.

24 bits / pixel

$\therefore 8 + 8 + 8$ [for 3 channel]



In fMRI :-

56 2D channels used.

- Classifying Images:

→ Via source:-

a) EM spectrum :-

- {
 - ~ γ rays : PET scans.
 - ~ x-rays : greyscale \Rightarrow surveillance use colour changing techniques.
 - ~ ultraviolet : currency forging dipped in material.

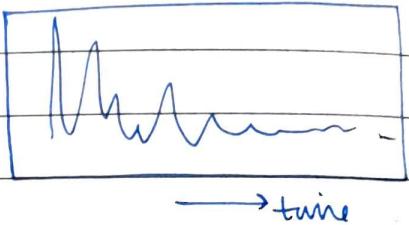
most dominant way of generating images

- ~ visible spectrum
- ~ Microwaves :- can help detect images by avoiding obstructions i.e. clouds.
- ~ Radio waves : fMRI.

b) Ultrasound :-

- Sonography : bouncing back of waves based on distance.
- Spectrogram : based on ~~as~~ audio freq/ intensities generate graphs.

For different words being said, different plots can be obtained.



c) Electronics :-

- Electron beams shot on surfaces (focused) to get a microscopic view.
- The changes in the properties of the beam is what turns into an image.

d) Computer generated :-

- Reflection images : commonly formed & perceived by human eyes.
- Absorption images :- primarily about internal structure (just structures) bone damages etc.
- Emission images : primarily about internal properties tissues cartilage etc.

Ex: Barium milkshake : emits radiations & using this diff kinds of analysis info is deduced.

X-rays are opposite. In this sense :: here the source lies outside the body.

- Enhancement : highlighting certain properties of image
- 3D images : Stereo images
 - ↳ use specialized cameras to mimic the human eyes
 - ↳ Given a L & R pair we can figure out the depth of the image.
- Multiview images : capturing a 3D image in its entirety.
 - Diff b/w multiview & stereo → can view with complete details in multi .



Processing :

- creating newer images .] algos .
- retrieve its attributes
- consumer based views .
 - ↳ making analysis .
 - ↳ raw human eyes .
 - ↳ consumption by machine based processes .

I First half of course :-

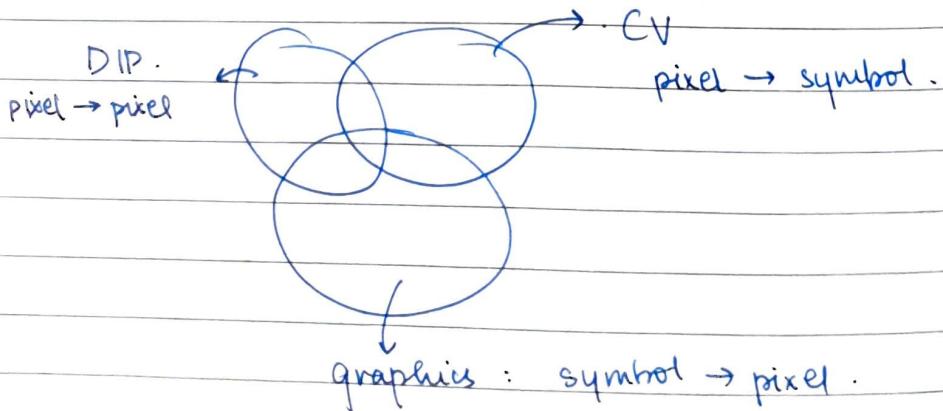
- work on enhancement.
- restoration.
- enhance colour images.
- Morphological proc.

II : Second half :

- Segmentation : define groups
 - Desc: roundedness, elongation
 - Object recog.
-] work on image attributes

Misc:

- contrast detection.
- feature detce (panorama).
- B/W to colour algorithms.
- Image restoration



- Libraries :- scikit-image
openCV-python
- pandas
 scikit-learn.

12/08/20 : Tutorial :

→ select my roll number 2018101041

→ proceed like a normal repo after that

→ keep committing & pushing code : remote repos !

Configure : `git config --global user.name <username>`
`" " -- global user.email <email ID>`

Colab :-

- run bash commands using !
- `pip install opencv-contrib-python`.

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TA hours:- Saturday 4:00 - 4:45.

→ Consumers of an image processing algo :- Humans.

• Human eye:

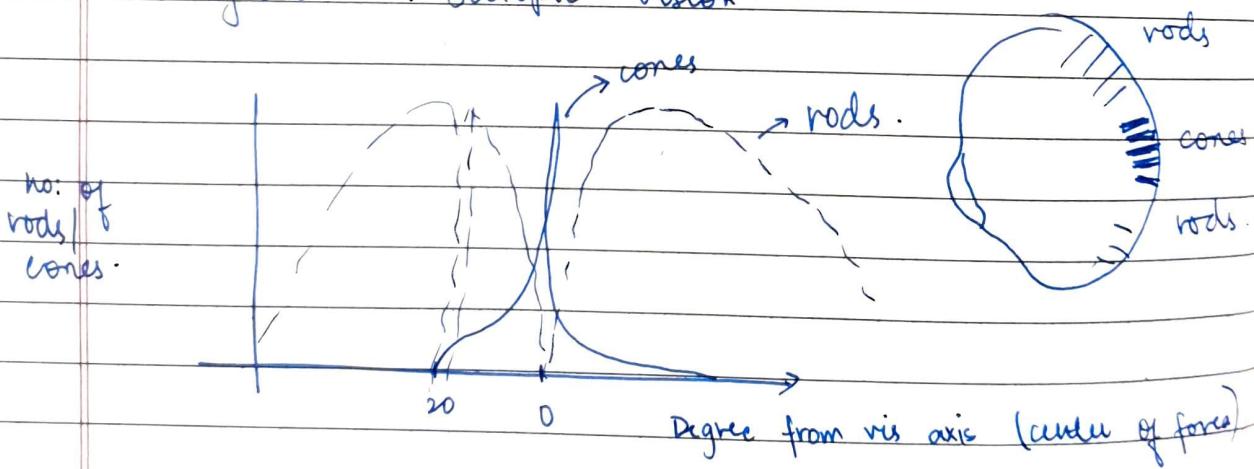
- Ball of diameter 20 mm.
- Light enters the eye, passes through the lens.
- Chamber filled with liquid vitreous humor.
- Cornea shrinks & expands based on amount of light.
- Lens absorbs IR, UV rays (capability dec. with age)

Image formation occurs on the retina.

Light receptors : cones & rods.

- Located in fovea region.
- respond to bright light.

→ Rods - generally become active at low-light illumination.
Cones : scotopic vision.



→ Fovea considered to be a square sensor array (1.5 mm x 1.5 mm)

Eye lens is flexible (variable focal length). with the help of ciliary muscles. (14-17 mm).

→ The unit Lambert is used to measure intensity $(10^{-6} - 10^4)$ range. But this range cannot be perceived simultaneously.

→ brightness perceived

Extent to which subjective brightness can be perceived
→ adaptation range.

Brightness disc:

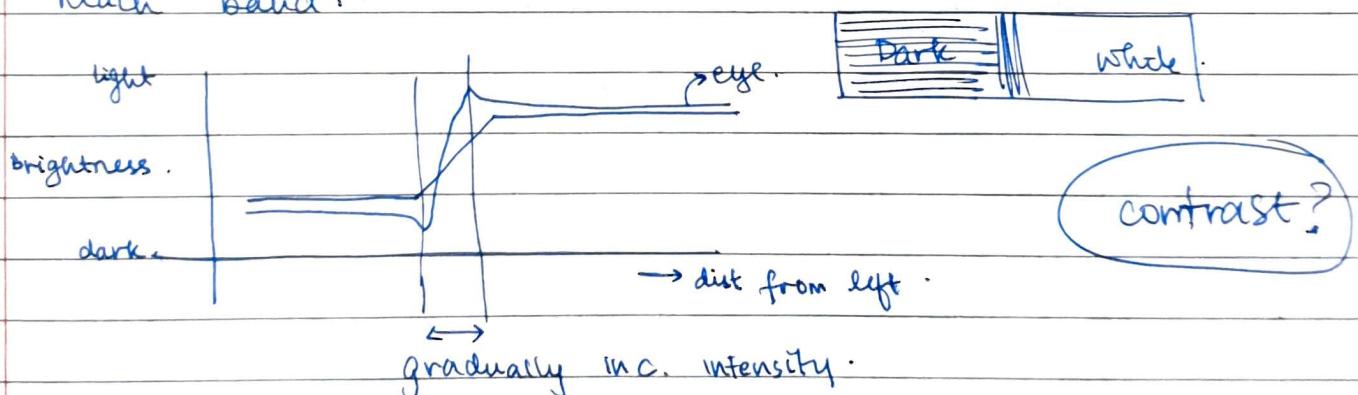
For lower intensities

human eye is very sensitive

i.e.: $\frac{\Delta I_c}{I}$ → small weber ration \Rightarrow good disc.
Large weber ration \Rightarrow poor disc.

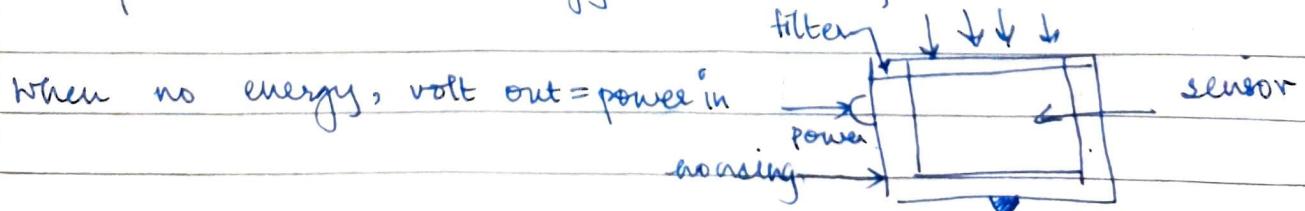
→ Psychovisual Effects:

- Change in 20% intensity may not be perceived as 50%.
- Mach band:



• Digital Image Acquisition :

Principle:- convert energy to intensity (numeric).



→ Scanners :-

(DOC)
Analog to
digital
converted

Light is shined on the doc → reflected to a series of mirrors → passes through after a lens → converted to voltage → acquire image.

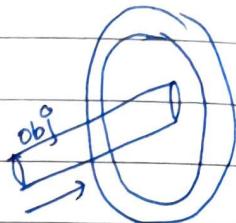
→ linear array acquisition :-



→ single single rows.

→ 3D image

Each portion acquires a cross section.



Integrate all to obtain image.

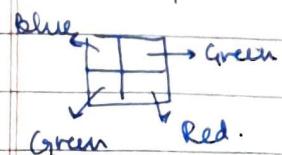
- Light can also be particle (photon) nature.



It is the motion of e^- that produces the electricity & change in voltage. [Used to exploit & convert light energy to voltage].

Light sensing diodes are sensitive to 'light' but not to 'colour'.

- To capture colour, use the Bayer Filters



using same sensor array should work to reduce area & cost.

Relatively more set green sensors \rightarrow human eye sensitive to shades of green.

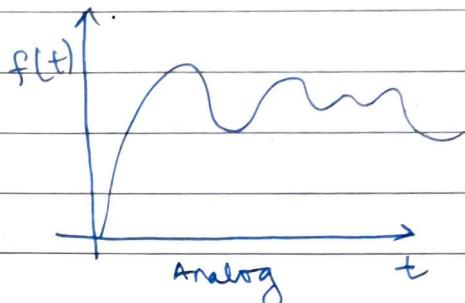
\rightarrow Demosaicing: process of getting colours from ind. grey scale & ind. colours.
 \rightarrow borrow colour from neighbours -

Image Sampling & Quantization:

Images are essentially signals.

Notion:

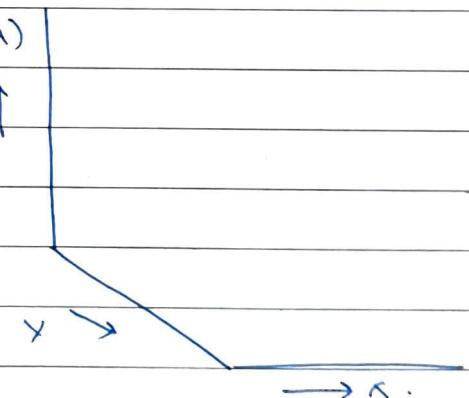
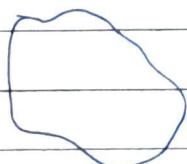
Images are a function of discretized space.



Intensity at a particular point is a $f(x, y)$.

Forms a neat 3-D space. $I = f(x, y)$

The signal is captured at certain quantized timestamps
 ie: sampling.



\rightarrow What is a resolution of a sensor?

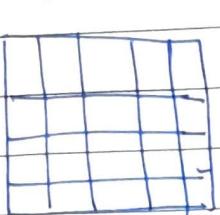
A: The total no: of photosites (however some photosites)
 are dead or useless

Size of the sensor plays a huge role: Graph paper.

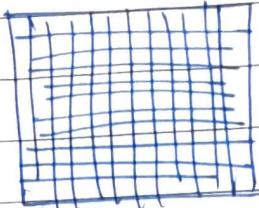
Sampling :- measuring the frequencies at regular intervals of time.

Quantization : Expressing these freq. as Quanta.

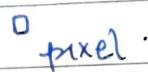
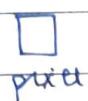
- Resolution isn't the only deciding factor
: A pixel's size is also important.



coarser look



better



→ Aspect Ratios:

$$\text{Ratio of width} \quad \text{height} = \text{AR}.$$

→ Spatial quantization :-

Variation in no. of sensors ($x \times x$)



sampling

Intensity quantization :- intensities merge.
(color distort)

8 bits/pixel, 4 bits/pixel etc

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Image : a function defined over a discrete domain

$$\text{i.e.: } f(x, y) = z.$$

Domain: (x, y)

Range: (Intensity)

→ Sampling :- quantization in spatial domain

values only in certain spatial quant.

→ Range Sampling : 8 bits/pixel 2 etc.

→ Other factors: Temporal Sampling
aperture shutter speed ISO

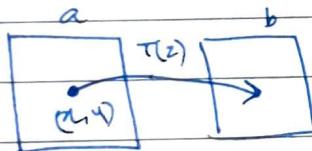
Smoothed version of ocean \Rightarrow got by shutter speed



• 2 Paradigms in image processing:

- Spatial domain: manipulate pixels in SP.
- Transform domain.

→ Intensity Transform : Point to Point :-



$$a(x,y) = z$$

$$b(x,y) = z' = T(z) = T(a(x,y))$$

Attacks the intensity.

Eg: converting to negative of the image to clearly observe / focus some areas.

Img 1: r

$$T(r) = s = (L-1) - r \quad \rightarrow \text{in neg image}$$

→ Brightening / Darkening image

$$\text{Dark} : - T(z) = z - k \quad [\text{red. intensity}]$$

$$\text{Bright} : - T(z) = z + k \quad [\text{inc. intensity}]$$

} ensure
 $T(z)$ remains
within domain

If at all it exceeds: do clamping: set equal to end values



→ Scaling to colour map range. (linear mapping).

For eg: 4 bit/pixel $0 \rightarrow 0 \quad 15 \rightarrow 255$

Doing so \Rightarrow we're also increasing no. of bits.

So far all transforms are linear intensity transforms

$$T(z) = k_1 z + k_2$$

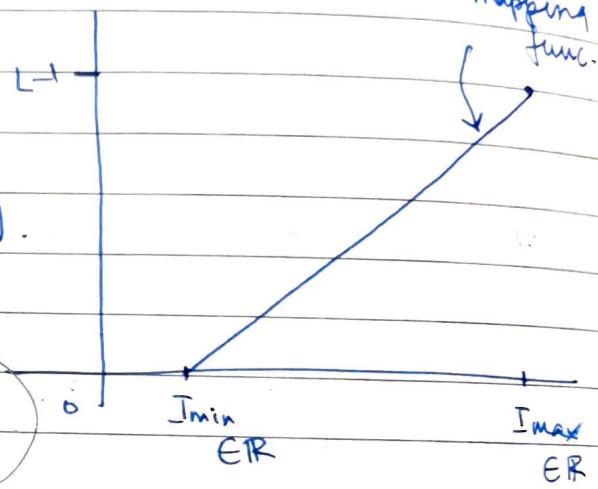
∴ Mathematically:-

Normalize

$$J = \text{round} \left(255 \times \left(\frac{I - \min(I)}{\max(I) - \min(I)} \right) \right)$$

$$J = \frac{255}{\max - \min} I + \min$$

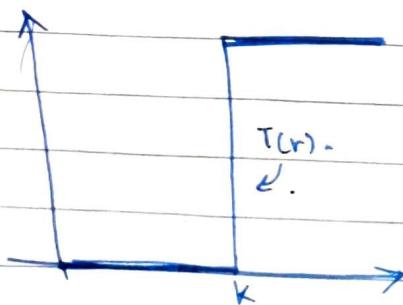
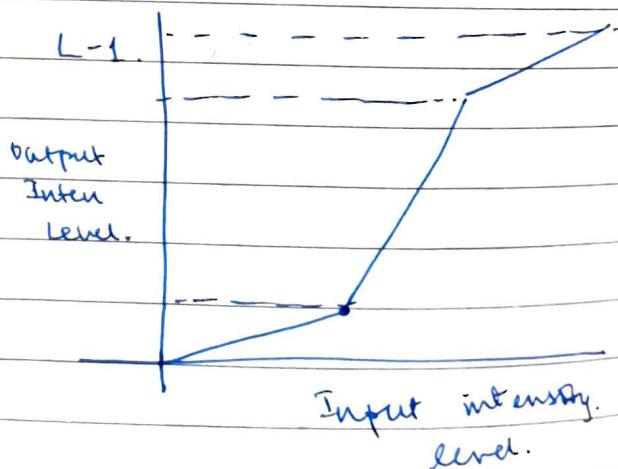
$$J = \frac{255}{\max - \min} I - \frac{255 \min}{\max - \min}$$



→ Piecewise Linear Transform:

Extreme form of this kind of piece wise function is called THRESHOLDING.

if $(\text{Inp } I) < \kappa \Rightarrow \text{set } 0$
else $\Rightarrow \text{set } 255$



$$T(r) =$$

$$(r_1, s_1) \quad (k, 0) \\ (r_2, s_2) \quad (k, 255)$$

explosive part that can appropriate write di

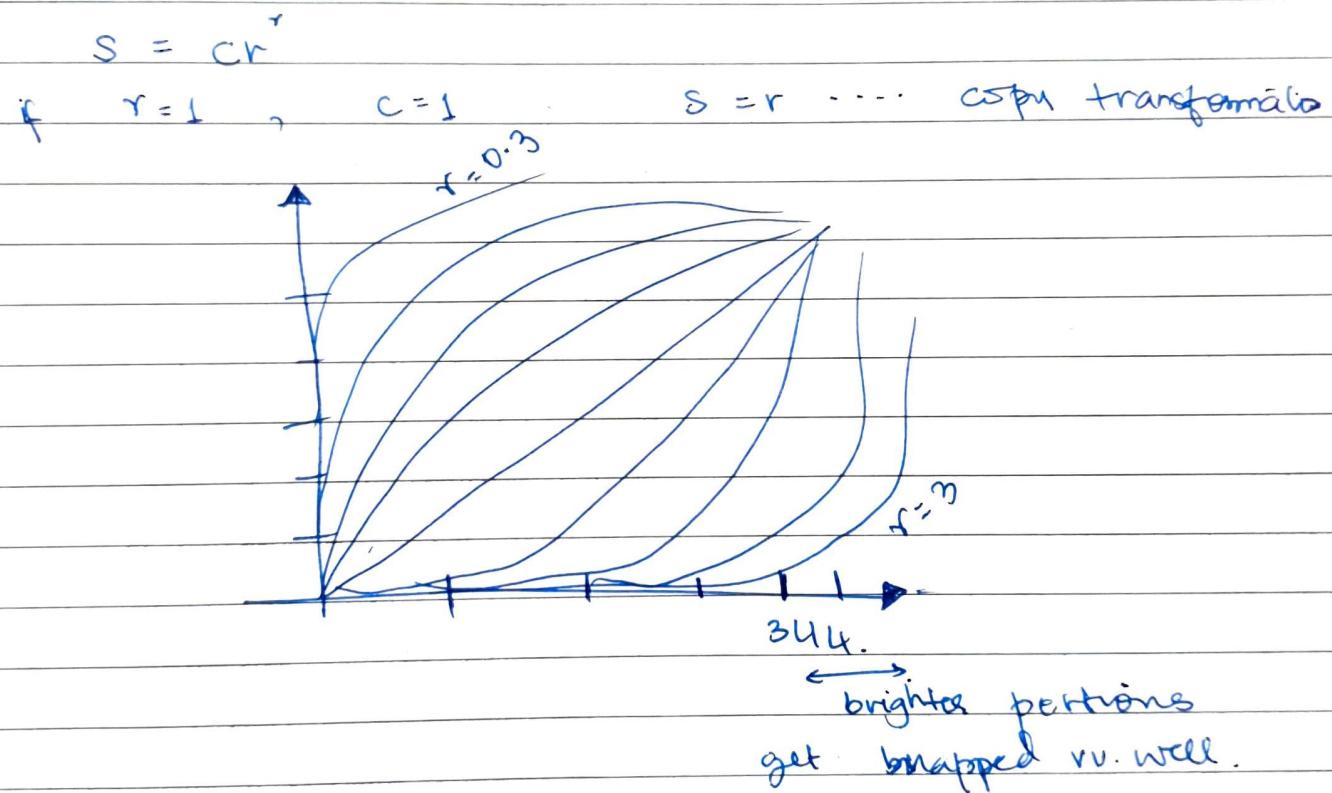
Non-linear Intensity Transformation :

- Too many values.
- Range is very large eg: 0 - 255.

Trick → Use log transformations :-

$$T(r) = C \log(1+r). \quad [\text{Non Linear}]$$

→ Power Law Transformations :-



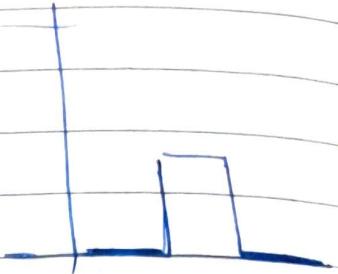
→ Larger no. of white intensities get compressed.

→ One kind of haze removal can be done in this way ↑

The value of c is fractional

If say $T(r) = cr^3 \rightarrow c = \frac{1}{r^3}$
for $r=255^3 = 16M$.

Intensity slicing :-



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LECTURE 4:

3 approaches to spatial domain processing

→ pDint to point:-

- linear intensity
- piece wise : thresholding
- power law transform.

Bit Plane Slicing

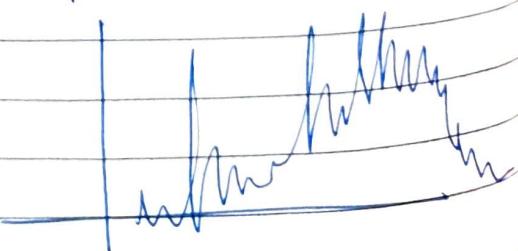
Each pixel represented by 8 bits.

- collect least significant bits \Rightarrow you'll get bitplane-1.
- We observe that 8th bit plane is a close approximation to the original image.
[Revisit topic \Rightarrow image compression]
- uint8 for numpy will be stored.
- Histogram: visualization + representation.

$$hr(i) = ni$$

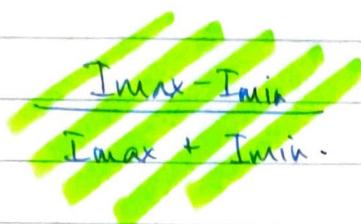
i = intensity values

ni = no: of pixels with intensities i



We can view histograms & gauge the intensities or contrast in the img.

contrast =

$$\frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$


\approx

$$\in [0, 1]$$

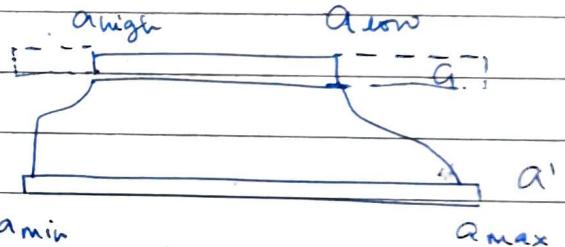
- After image processing we can look at the histograms & then predict whether or not the proc was good.
- Higher contrast \Rightarrow spread across intensities

* reconcile image & intensity distribution.

- contrast stretching:

$$y = () (r) + c$$

$$f_{oc}(a) = a_{\min} + \frac{(a - a_{\min})}{(a_{\max} - a_{\min})} \cdot (a_{\max} - a_{\min})$$



histogram thus gets stretched or spread apart.

i.e. spread of line stretching :

- If single pixel with 255: stretching happens only at left
- If single pixel with 0: stretching happens only at right
- Similarly: if 1 pixel 0 & 1 pixel 255
X no contrast stretching

- Another method:-

$$A_{\text{low}} = \{ i \mid H(i) \geq M.N \} \quad \text{using this set}$$

$$A_{\text{high}} = \{ i \mid H(i) \leq M.N \} \quad \text{some } \%$$

31, 51.

- Intensity aware operation:

→ histogram equalization.

Intention Transform an image such that the histogram of the transformed image is equalized

→ normalized histogram → probability dist.

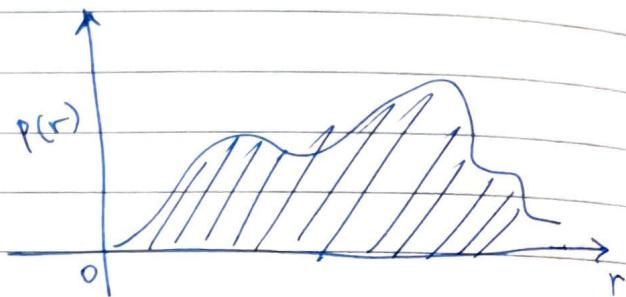
→ For the purpose of analysis, we can consider continuous distribution.

Consider R be the R.V for the intensities

$$\int_{-\infty}^{\infty} p_R(r) = 1.$$

Practically :-

$$\int p_R(r) = 1.$$



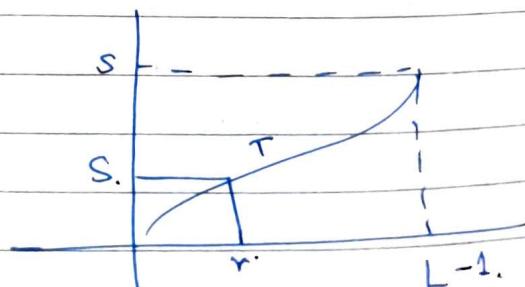
→ Let R be the R.V for the image. Consider another R.V S .

We want a uniform dist. in S .

Need a transformation T st: $s = T(r)$.

For S to have uniform dist, we should have the

$$P_S(s) = \frac{1}{L-1}.$$

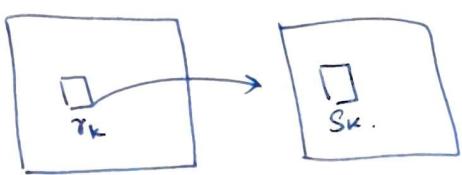


- T is usually preferred to be monotonic.

2 R.V related by a monotonic transformation then

$$P_S(s) = P_R(r) \frac{dr}{ds}$$

rate at which it changes wrt target



B
3 = 253

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$$\text{ie: } P_s(s) ds = P_r(r) dr.$$

Sub for $P_s(s)$:-

$$\frac{1}{L-1} ds = P_r(r) dr.$$

$$ds = (L-1) P_r(r) dr.$$

Int both sides:-

$$\int ds = \int (L-1) P_r(r) dr.$$

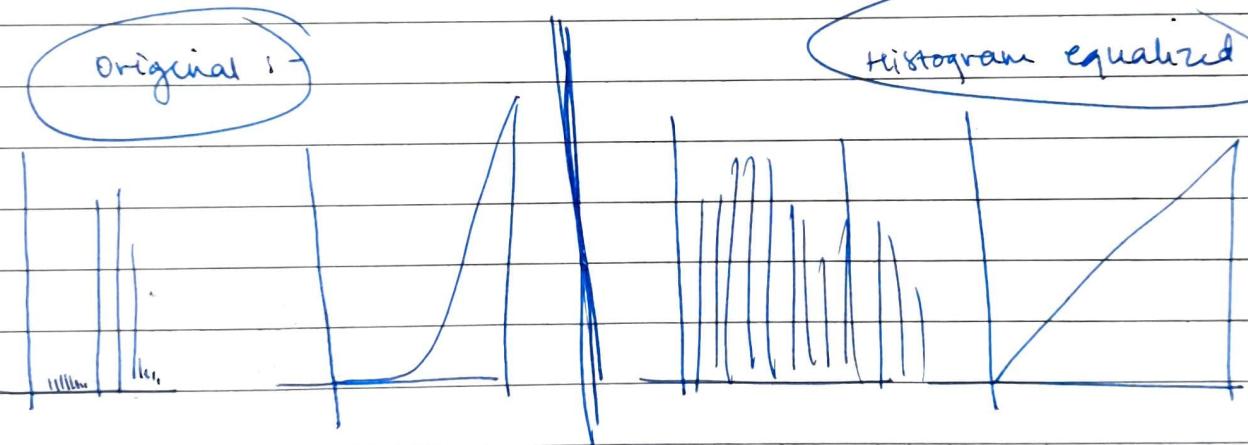
$$s = (L-1) \int_0^r P_r(w) dw.$$

$$s = T(r) = (L-1) \int_0^r P_r(w) dw$$

use the CDF to subst. & compute.

That is:

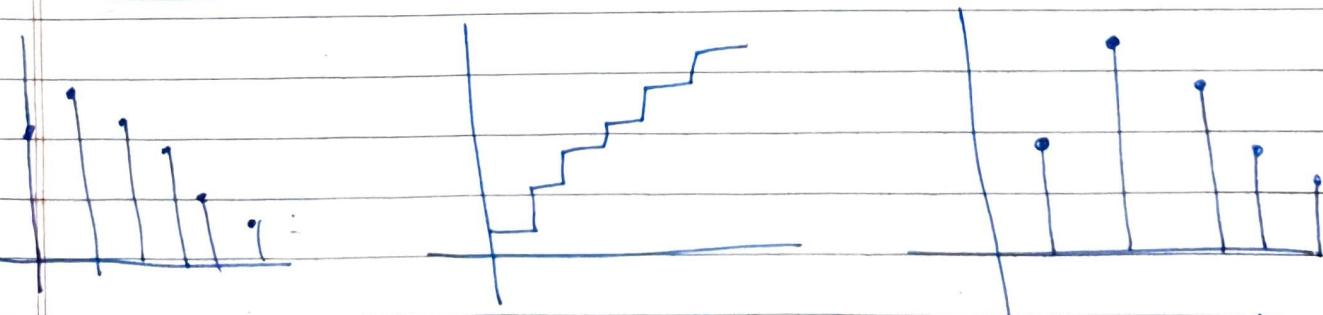
Original :-



histogram equalized

Histogram equalization

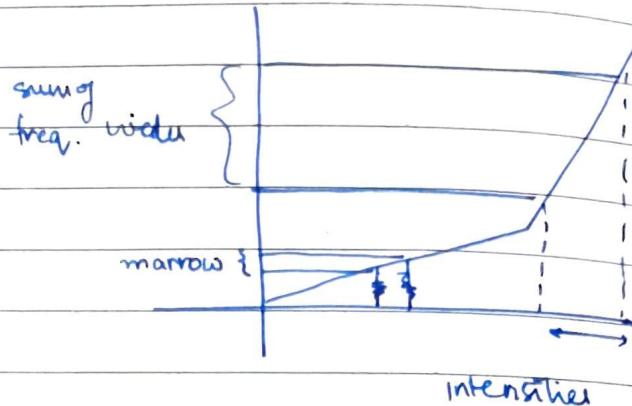
→ enhancement procedure



$$s_k = \text{round} \left[(L-1) \sum_{j=0}^k P_r(w_j) \right]$$

Equalized.

when a set of intensities maps to a wider band \Rightarrow low contrast.



→ Contrast enhancement & histogram equalization may differ from situation to situation.

range of contrast & histogram are roughly same

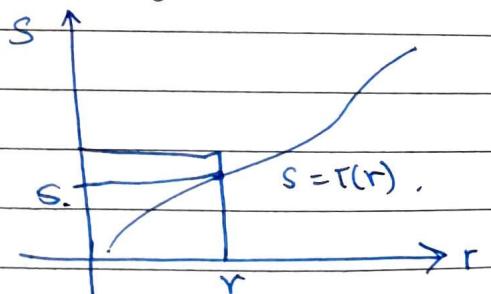
- Histogram Specification:

Prob. dist over s

$P_s(s) \rightarrow$ uniform.

& T is the transfer func.

originally:

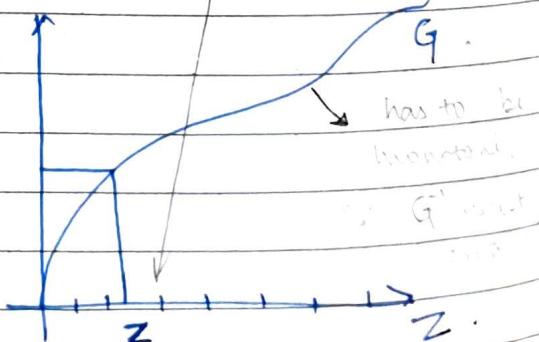


Let us have a new transfer function that we define.

- Specify a custom curve G .

\exists some value of z that maps to s

i.e.: $s = G(z)$ CDF of refiner image



Consider strictly monotonic increasing.

$$\Rightarrow z = G^{-1}(s) = G^{-1}(T(r))$$

GIMP tool for image processing.

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Now, we have z as a function of r .

$$\text{ie: } z = G^{-1}(T(r))$$

$$\text{ie: } z = U(r)$$

Histogram specification can have a curve from a reference image.

→ The output image would then have the same intensity style & intensity profile as the ref image.

Histogram Equalization is an example of global to point processing.

→ Histograms:

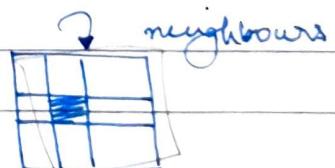
- More of a visualizations.
- useful statistic of image intensities.
- Not dependent on image size.

Dis:-

- No spatial info.
- Intensity Centric

• Neighbourhood To Point:

Neighbourhood based transition of this process.



Map a group of say x by x pixels to the same n by n pixel group.

↳ manipulate this value & proceed.

Conditional Image Enhancement:

$$P(r_i) = n_i/n$$

$$m(r) = \sum_{i=0}^{L-1} p(r_i) r_i$$

$$\sigma^2(r) = \sum_{i=0}^{L-1} (r_i - m)^2 p(r_i)$$

}

Stat params.

Objective : Enhance dark image areas leaving light areas unchanged.

- Step 1: Identify 'dark pixels' & 'light pixels',
 some special proc to check.
- Step 2: Enhance dark pixels.

① Using some kind of an if condition check all pixels.
 → use some $| m_{say} \leq k_1 m(r) |$ helps ~~for~~ find
 considered pixels

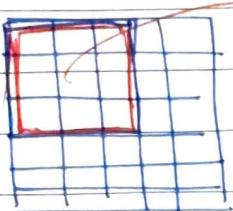
Here 'enhance' means contrast enhance.

$$\left[\sigma_{say} \leq k_2 \sigma(r) \right] \rightarrow k_3 \sigma(r) \leq \sigma_{say} \leq k_2 \sigma(r)$$

$\hookrightarrow c_1$

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- Spatial Domain Filtering
(Intensity transform eg.)



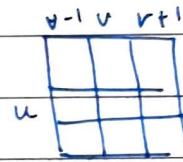
Take mean of all 9 values
or replace with avg of all.
round (avg all neighbours)

High difference of intensities will be replaced by closer intensities.

→ Weight mask, kernel, filter.

Mask Co-efficient sums to = 1.

i.e. $H = \begin{bmatrix} 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \end{bmatrix}$



Transform :-

$$I' = \sum_{i=-1}^1 \sum_{j=-1}^1 I(u+i, v+j) \cdot H(i, j)$$

$i+1 \rightarrow j+1$

→ With a larger averaging filter size :-

$3 \times 3 \rightarrow 5 \times 5 \rightarrow 7 \times 7$

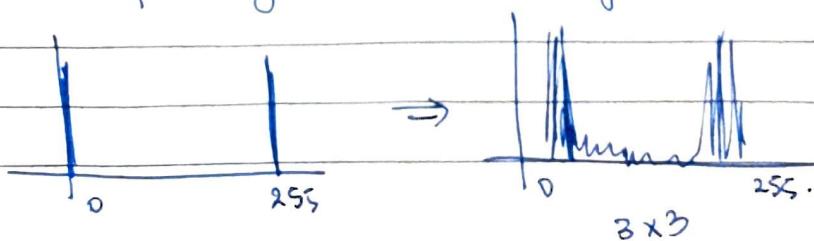
blurred & blurred

Image enhancement?

For most natural photographs \Rightarrow blurring \because more smaller values.

Smoothening : Forms first step in many image proc pipelines

→ Binary image \Rightarrow blurring \Rightarrow intensities spread apart.



- 1) Visually you can get the effect of larger filters by smoothening ^{repeatedly} with smaller filters.
 - 2) Op. with smaller filters would be faster as well.
- Weighted averaging:

e.g.: req. to remove undesirable probabilities.

Giving central pixel more value \rightarrow gaussian filters.

1	2	1
2	4	2
1	2	1

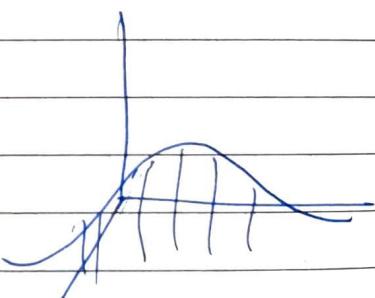
higher weighted

Gaussian function :- param μ, σ

$$F(\mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Sample a gaussian.

Specify the $(s, \text{neighbourhood})$



Pick a value for filter size such that it covers a good range of the image.

As $\sigma \uparrow$, blurring \uparrow .

\therefore if σ is small, the center pixel given max wt.

• Derivatives -

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \quad f'(x)|_{x=a}$$

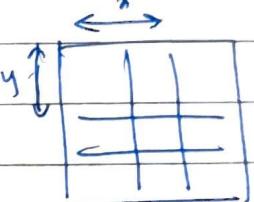
2D :-

$$\frac{\partial f(x,y)}{\partial x} = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x, y) - f(x, y)}{\Delta x}$$

$$\frac{\partial f(x,y)}{\partial y} = \lim_{\Delta y \rightarrow 0} \frac{f(x, y + \Delta y) - f(x, y)}{\Delta y}$$

Digital first approximation -

first $\frac{\partial f(x,y)}{\partial x} \approx f[x+1,y] - f[x,y]$



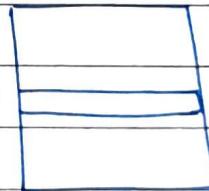
Second $\frac{\partial^2 f(x,y)}{\partial x^2} \approx (f[x+1,y] - f[x,y]) - (f[x,y] - f[x-1,y])$

This is essentially ~~the~~ a 2×1 rectangular filter.

$$\begin{bmatrix} -1 & 1 \end{bmatrix}$$

center

Run the filter across the rows here
The resultant image will then
contain the first derivatives.



Rapid change in intensity : corresponds to an edge.

1st derivative : tells us where there are edges.

2nd derivative : tells us the location of edges.

Symmetric Diff. : $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x-h)}{2h}$

$x : \begin{bmatrix} -1 & 0 & 1 \end{bmatrix}$

$y : \begin{bmatrix} -1 & 0 & 1 \end{bmatrix}$

- Gradient in 'x' tells where all intensity is changing as you go from L - R.
- Likewise same thing for top - bottom also.

Can ∴ record the presence of images

- Prewitt edge filter.

$$G_x = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

$$G_y = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}$$

edge ⊥ to gradient .

1/09/20

• Recap:

- Mean/Avg filter.

- Operation: specify a mask: neighbourhood & their entries

- Filter entries $\sum = 1$.

Mask size: perform image smoothing with larger filter size - blurring in creases.

Repeated averaging: increased blurring.

↳ used to approx gaussian filter by using smaller set of cheaper filters.

→ Gaussian filter:

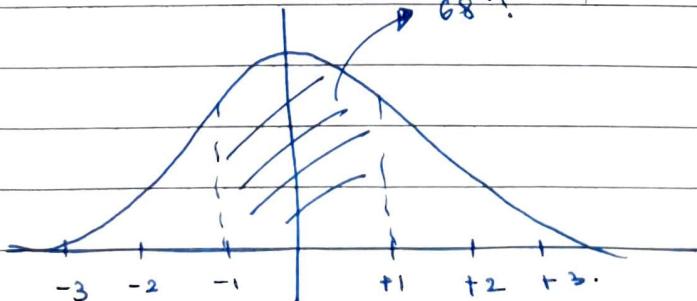
- arriving at the Gaussian ... how?

$$\text{PDF: } g_\sigma(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}}.$$

$$\int g_\sigma(x) dx = 1.$$

→ Taking 1 unit σ : 68%.

3σ: 99.7%.



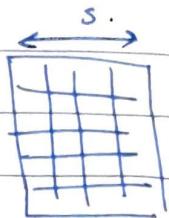
If you take upto 3σ: you'll get the filter more or less.

→ How to decide the size of the filter.

To select $\neq s$

$$S = 3\sigma + 3\sigma + 1\sigma$$

in case you want $\leftarrow S = 7\sigma$



This would represent the filter more or less faithfully.

To avoid strange boundary effects - take odd sized filters.

→ Gaussian's co-efficient can be obtained via the Pingala / Pascal's triangle.

$$\sum \text{coeff for } N = k : 2^k$$

For 1D For eg $S = 7 \times 7 \rightarrow$ select row 7.
 $\frac{1}{64} [1 \ 6 \ 15 \ 20 \ 15 \ 6 \ 1] = V^T$.

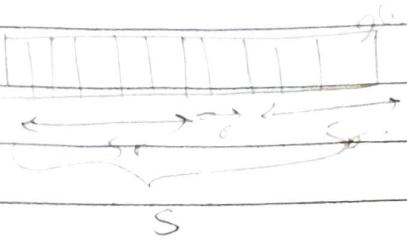
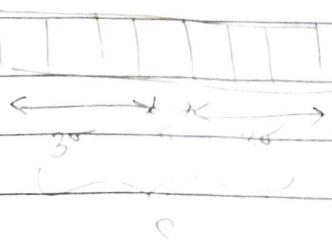
For 2D filter, take outer product of $V \times V^T$.
 i.e. $\frac{1}{64} [V] \times \frac{1}{64} [V^T]$.

hence we can obtain the digital approx to gaussian filter given a value of σ .

Peak / central val of gaussian is got by setting $x=0$

$$\frac{1}{\sqrt{2\pi}\sigma} = \frac{C_{N/2}}{2^N}$$

$$N = S - 1$$



• Edge detection:

- Area in our visual cortex dedicated to see intensity variation.
- Edges are a key IP op.
- Sudden changes/ discontinuities to intensities.

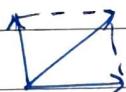
Obtain greater support for edge we use prewitt edge filters.

Edge is \perp to gradient / intensity change.

→ Given an image

Obtain the $\frac{\partial f}{\partial x}$ $\frac{\partial f}{\partial y}$ image.

At any pt.



magnitude
loc. [edges are strongest]

& orientation info.
[captures direction... gen. manner]
in which grad changing

$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

• 2 gradients:

<

For a function: sum of its derivatives = Laplacian.

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

Laplacian
op.

$$\nabla^2 f = f(x+1, y) +$$

0	1	0
1	-4	1
0	1	0

$\Sigma = 0$

Sobel



Sobel X : filter :-

Sweep from L-R. finds intensity changes vertical edge.

~ Sobel Y : filter :-

Intensities in vertical direction.

Unlike edge filter \neq sum to 1. sum to 0 ✓

These filters are symmetric: to avoid giving preference to any 1 particular direction.

Sobel is not sensitive, & so is less noisy.

• Gradient Filters \rightarrow Image Enhancements . Image Sharpening

$I(u, v)$ get $\nabla^2 I(u, v)$

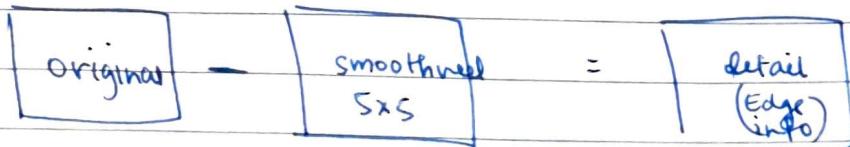
Add to old image $\nabla^2 I(u, v)$ ~~$I(u, v)$~~ + $I(u, v)$.

Result \rightarrow Sharper.

Only vis $\nabla^2 I(u, v) + 128$.

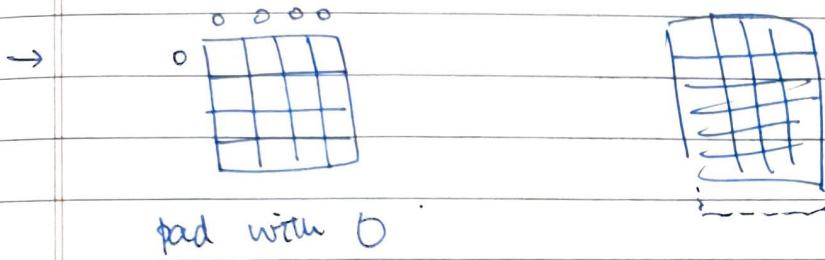
• High Boost Filtering:-

what does
blurring ie:
take away



Can scale the laplacian also & attain the sharpening
 ∇^2 : unsharp masking
 blur method : highboost filtering.

- Corner cases & Padding:



replicate border entries.

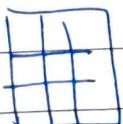
Do not cause much issue unless object of interest is on the boundary.

Advantage: 5×5 image $\xrightarrow{\text{filter}}$ 5×5 image.
 valid filter $5 \times 5 \rightarrow 3 \times 3$.

SO FAR ALL LINEAR FILTERS.

- Non-linear Spatial filters:

\rightarrow Max Filter :



max of $\in \boxed{\oplus}$

can be used to get rid of pepper noise.

\rightarrow min filter

can be used to get rid of salt noise.

Tend to become little dull.

Median filter: 50% filter. can get rid of salt & pepper noise. adaptive.

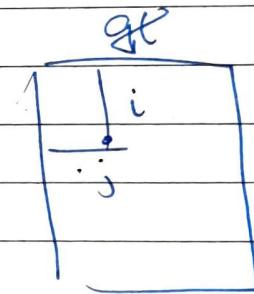
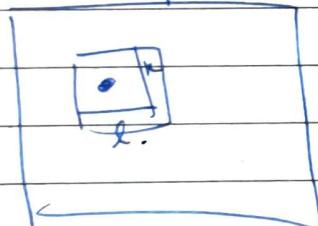
! Mean extremely prone to outliers.

All rank order filters.

- Edge preserving smoothing: BILATERAL FILTER.
Record pos. where edges present
↳ don't do smoothing in those areas.

Add edges to smoothed image.

Non spatial filter



$$g(i,j) = \frac{\sum_{k,l} f(k,l) d(i,j,k,l)}{\sum_{k,l} d(i,j,k,l)}$$

4/09/20

Bilateral Filtering

Recap:

→ faster

Mean: blurs, removes simple noise, × details

Median: preserves some details, removing strong noise

Gaussian: blurs, preserves details only for σ .

→ better than mean

→ Edge Preserving Filtering

Smooth isotropically: only at edges:-

Ex: Consider 1 row:-

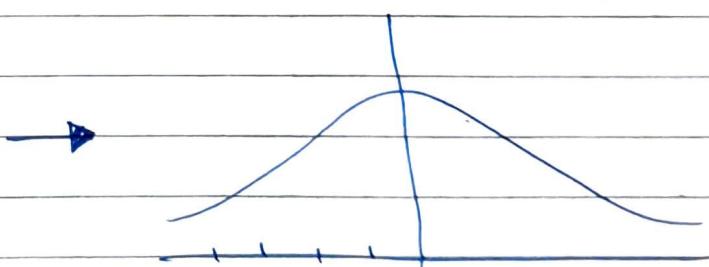
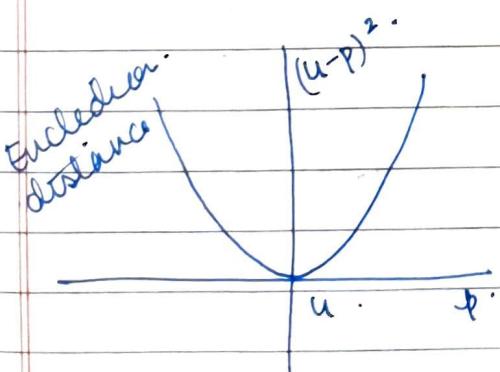
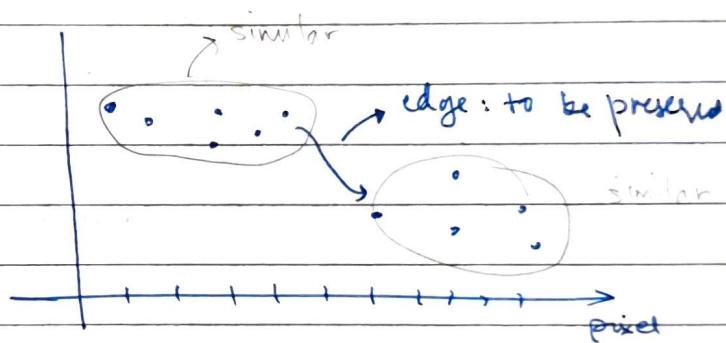
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Assignment

Now consider the

gaussian

$$w_c(p) = \exp\left(-\frac{(u-p)^2}{2\sigma^2}\right)$$

↑
rate at
which
it falls



The neighbourhood values pull down the image.

∴ the two edge diff. values come closer & closer.

The contribution to sum by wts is diff. as
distance increases from $\neq u$.

Since with intensities : called photometric weights.

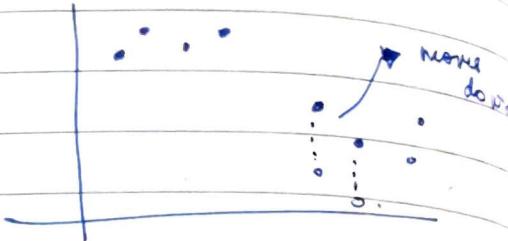
As a result, introduce another weight :-

$$W_s(p) = \exp \left(- \frac{(I(u) - I(p))^2}{2\sigma_s^2} \right).$$

Look at intensities also now:

for all intensities, they're pulled down.

For similar intensities, wts ≈ 1 .



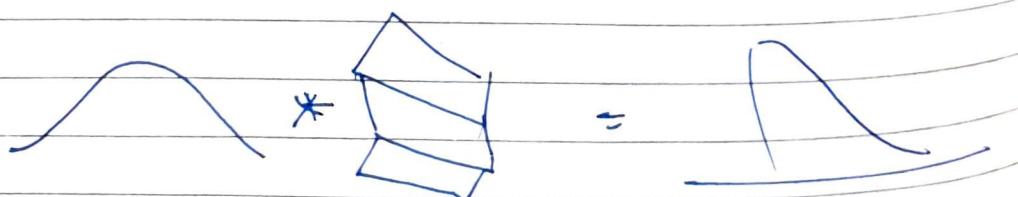
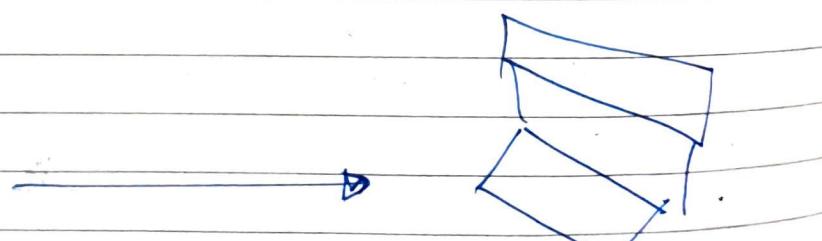
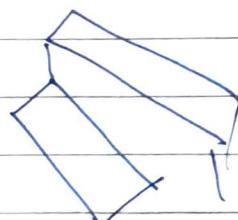
This helps us preserve the edges.

$$W_{bi}(p) = W_s(p) \times W_c(p).$$

I.e.: $I(u) = \sum_{p \in N(u)} \frac{-\|u-p\|^2}{2\sigma_c^2} e^{-\frac{|I(u)-I(p)|^2}{2\sigma_s^2}} (I(p))$

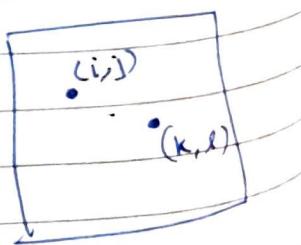
Denoise \leftarrow - $\frac{\|u-p\|^2}{2\sigma_c^2}$ - $\frac{|I(u)-I(p)|^2}{2\sigma_s^2}$ (I(p)) feature preservation

→ Filtering Process.



→ Same extension to 2D also: [domain kernel]

$$d(i,j,k,l) = \exp \left(- \frac{(i-k)^2 + (j-l)^2}{2\sigma_a^2} \right)$$

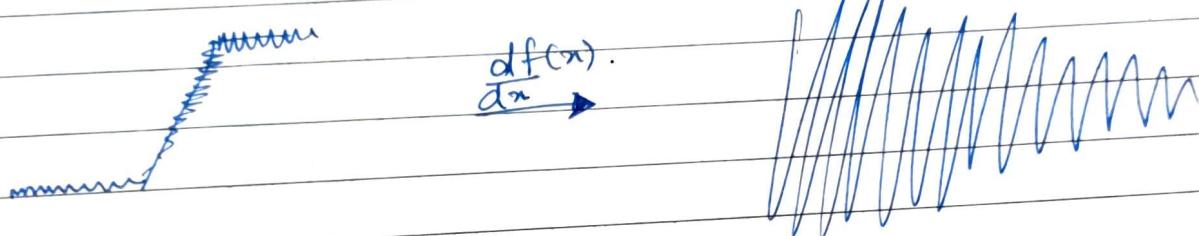


$$g(i,j,k,l) = \exp\left(-\frac{\|f(i,j) - f(k,l)\|^2}{2\sigma^2}\right).$$

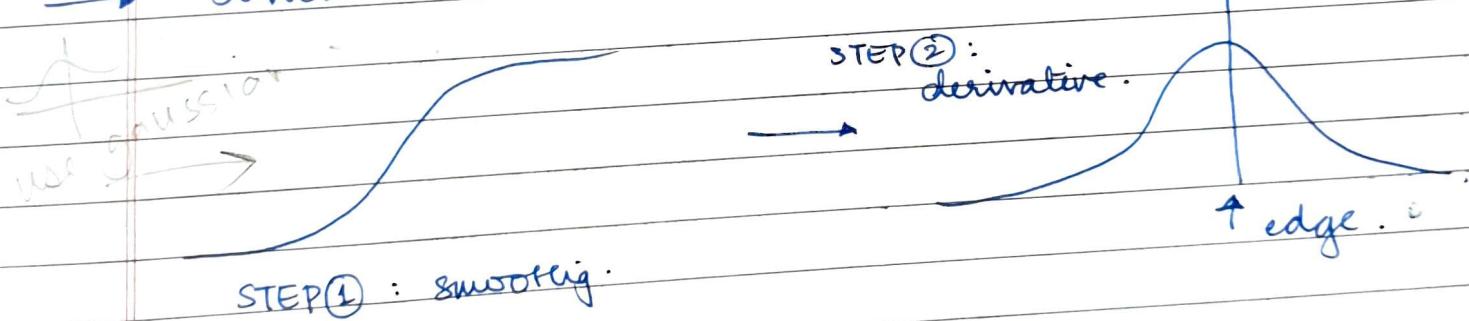
- data dependent,
- range filter
- kernel depends on image content

Progressively decreases the difference b/w regions with similar intensities.

→ Effect of noise :



→ Solution : smooth first :

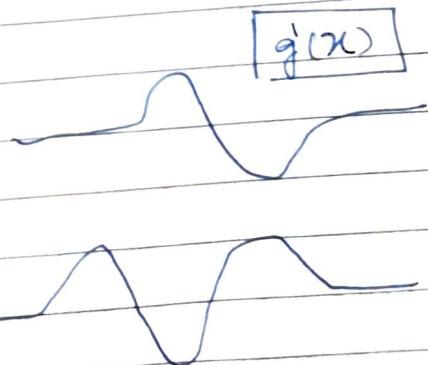


To save one step:- since we're working with linear filters:-

$$\frac{d}{dx}(f * g) = f * \frac{d}{dx}(g).$$

Use the kernel as $\frac{d}{dx}(g)$

Further for $g''(x)$: Laplacian of gaussian.
↳ Noise suppression.



$\nabla^2 G$ is small \Rightarrow Linear

Approx Log with DoG \rightarrow difference of two gaussians
 choose σ_1, σ_2 $\nabla^2 G \approx G_1 - G_2$. appropriately
 Band Pass Filter.

- All the previous operations were called as linear filters since :-

Scaling :- $T(x) = y$; $T(ax) = aT(x) = ay$.

Additivity :- $T(x_1 + x_2) = T(x_1) + T(x_2) = y_1 + y_2$.

Operation of shift & slide :-

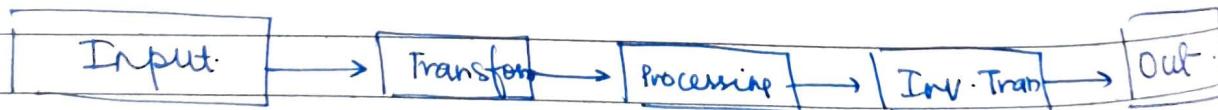


: correlation

If we horizontally & vertically shift the filter value \Rightarrow convolution

- whatever we have seen until now are all directly manipulative pixels in spatial domain.

TRANSFORM :



→ Periodic signals :-

$$n(t) = A \cos(\omega t)$$

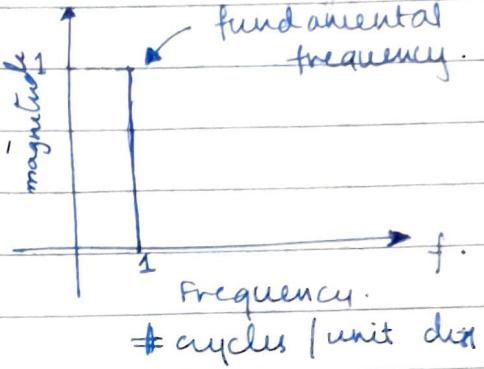
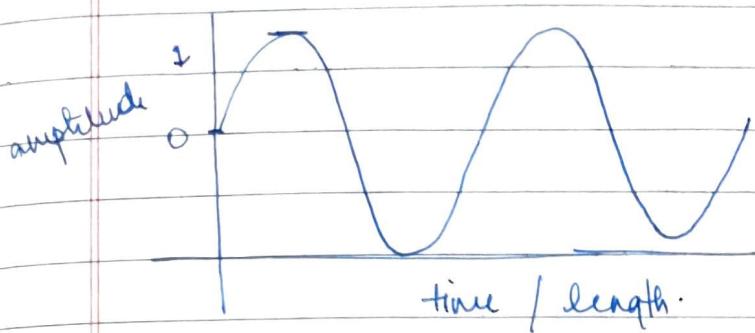
Angular freq :- $\omega = 2\pi f$ radians/s.

$$= \frac{2\pi}{T} \quad T: \text{Time period}$$

A cost

ACOS ωt --- signalperiod = $\frac{2\pi}{\omega}$

Image can be considered as signals.

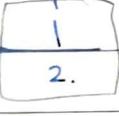


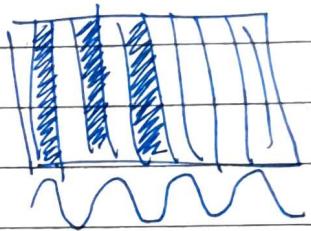
spatial domain : How much diff to travel before signal repeats.

Eg:- $I(x, y) = 128 \sin(2\pi x/16)$.

spatial
128x128 signal

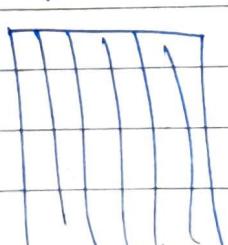
Smallest period we can have

is  2 pixels spatial period.

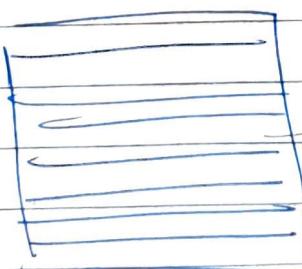


If $f = \frac{1}{16} \Rightarrow 16$ pixels \rightarrow it repeats.

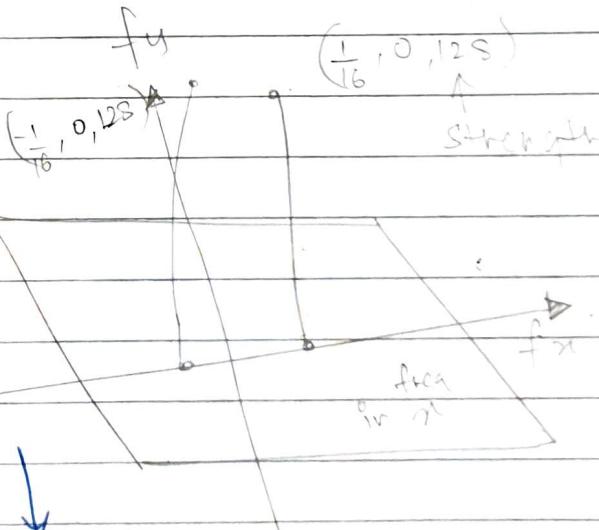
The direction of the wave indicates direction of frequency.



f_x .



f_y .



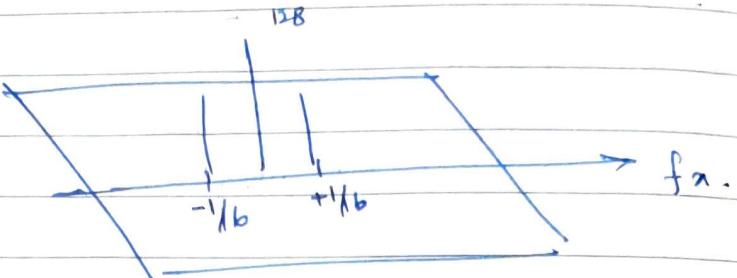
08/09/20

Note: as $f \downarrow$ pts. on f_m & f_N come closer
ie: if $f_m = \frac{1}{16}$ $\text{----} \uparrow \uparrow \uparrow \uparrow$ if $f_N = \frac{1}{4}$ $\text{---} \uparrow \uparrow \uparrow \uparrow$

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Combination of wave patterns:

$$128 + 64 \sin\left(\frac{2\pi x}{16}\right)$$

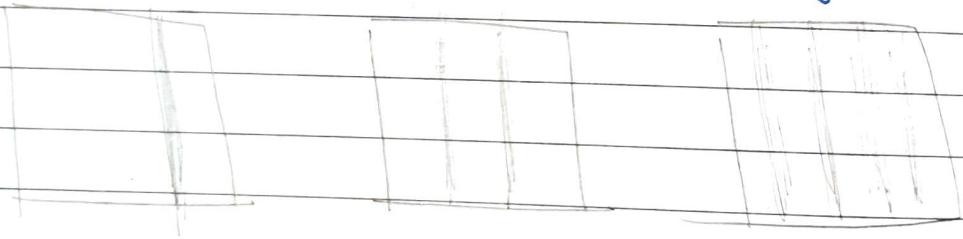


Consider :- intensity images for :

$$s(x, y) = \sin [2\pi (u_0 x + v_0 y)]$$

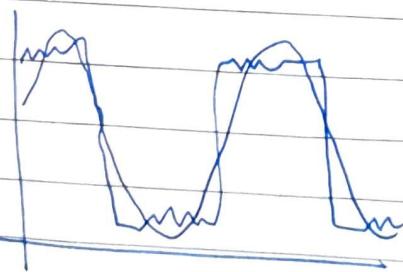
then AS v_0 increases
freq. decreases

u_0 : freq in x v_0 : freq in y.



• Fourier series : approximate periodic signals with sines & cosines.

e.g.:



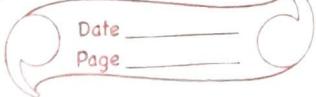
$$y(t) = \sum_n a_n \sin(nf \cdot 2\pi t) +$$

$$\sum_n b_n \cos(nf \cdot 2\pi t)$$

$$\omega = \frac{2\pi}{T}$$

$$a \sin(n \omega t)$$

Exp. of periodic wave \rightarrow Fourier series CLASSMATE



Eg

Converting square wave to sum of sines & cosines.

In ppl, take any wave which has a certain period & represent it as a weighted sum of its harmonics.

Euler's identity

$$e^{it} = \cos t + i \sin t. \quad i = \sqrt{-1}$$

Plotting a complex sinusoid, cosines are reflection on real axis & sines are reflection along imaginary axis.

anticlockwise

$$\cos t = \frac{e^{it} + e^{-it}}{2}$$

clockwise

$$\sin t = \frac{e^{it} - e^{-it}}{2i}$$

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{\frac{i 2 \pi n t}{T}}$$

$$c_n = \frac{1}{T} \int_0^T f(t) e^{-\frac{i 2 \pi n t}{T}} dt$$

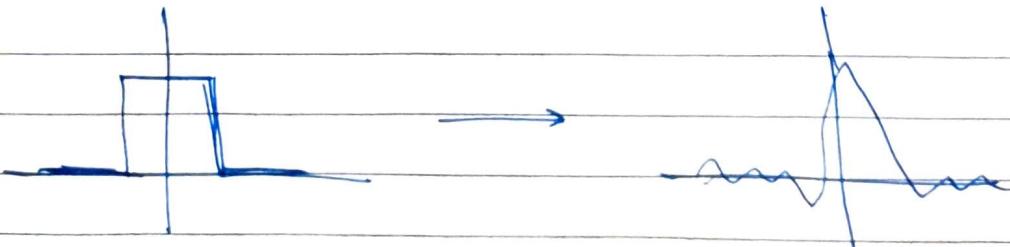
amplitude / extent to which it's present in signal.

- Fourier Transform : (If $f(t)$ is non-periodic)

Approx non-periodic signals & approx them with complex sinusoids.

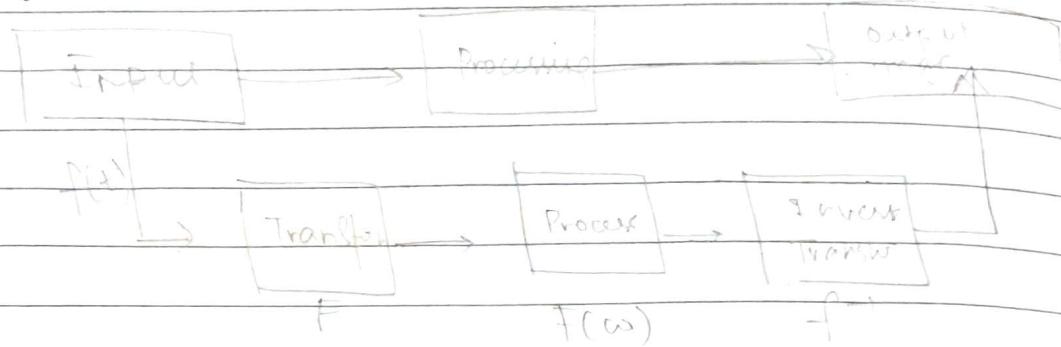
Transform :-

$$F(w) = \int_{-\infty}^{\infty} f(t) e^{-iwt} dt \quad \text{For signal } f(t)$$



Magnitude Spectrum

$$\sqrt{R^2 + I^2}$$

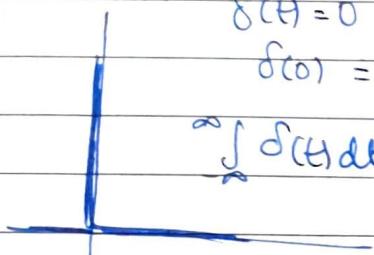


• Impulse function :-

$$\delta(t) = 0 \text{ for } t \neq 0$$

$$\delta(0) = \infty$$

$$\int \delta(t) dt = 1$$

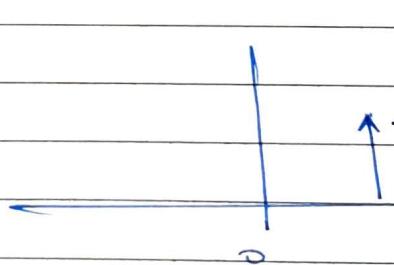


discrete impulse function



Continuous:

(1) can shift by T



→ Properties :-

$$(1) \int_a^b \delta(t) dt = \begin{cases} 1 & a < 0 < b \\ 0 & \text{otherwise} \end{cases}$$

$$(2) \int_a^b \delta(t) f(t) dt = \begin{cases} f(0) & a < 0 < b \\ 0 & \text{otherwise} \end{cases}$$

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt$$

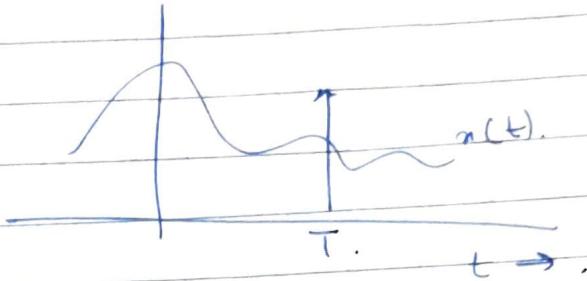
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$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{i\omega t} d\omega$$

sifting property :-

$$\int_a^b \delta(t-t') x(t) dt = x(t) \quad a < t < b.$$

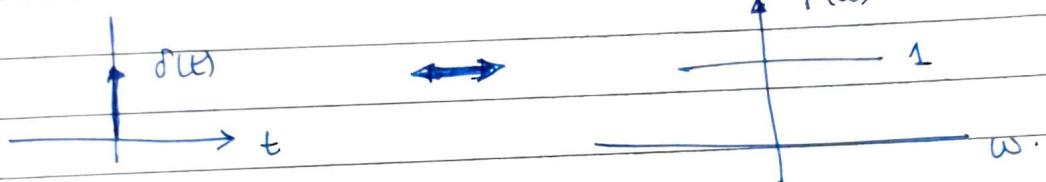
roundabout way.



→ FT. of impulse function :-

$$F(\omega) = \int f(t) e^{-i\omega t} dt$$

$\delta(t)$ ↓
 $e^{-i\omega(0)} = 1$



→ Symmetry property :-

$$F(f(t)) = F(\omega)$$

$$F[F(t)] = 2\pi f(-\omega)$$

$$\int e^{i\omega_0 t} \times e^{i\omega' t} dt$$

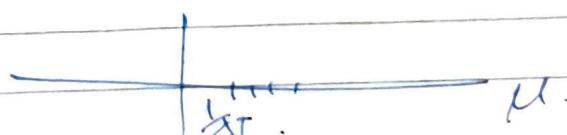
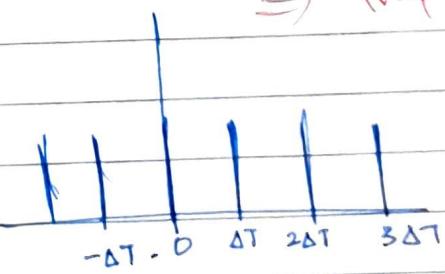
$$\int e^{-(\omega - n\omega_0)t} dt$$

→ FT of impulse train :-

Fourier of impulse train
⇒ impulse

$$S_{\Delta T}(t) = \sum_{n=-\infty}^{\infty} \delta(t - n\Delta T)$$

$$f(t) = \frac{1}{\Delta T} \sum_{n=-\infty}^{\infty} \delta(t - \frac{n}{\Delta T})$$

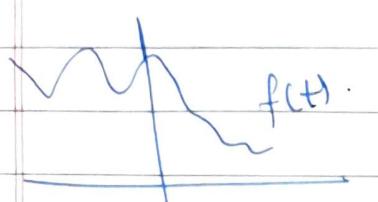


$$\mu = 2\pi w_+$$

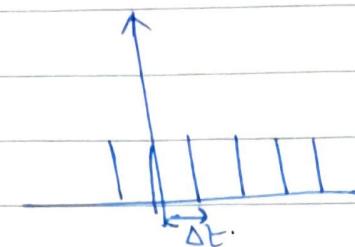
- Sampling = $f(t) \times$ Impulse train

while sampling; the continuous analog signal is multiplied with impulse train.

i.e.:



signal.

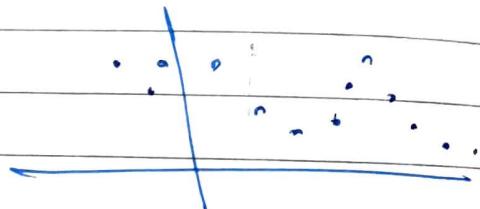


$$s_{\text{sr}}(t) = \sum \delta(t - n\Delta t)$$

Impulse Train

convert a set of discrete samples into cont. version

$$\tilde{f}(t) = \sum_{n=-\infty}^{n=\infty} f_n \delta(t - n\Delta t)$$



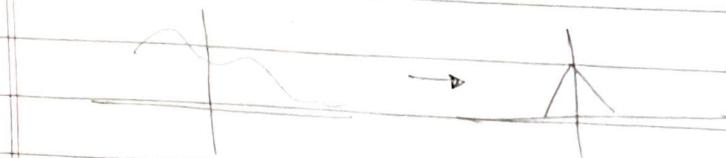
- FT of a sampled function:

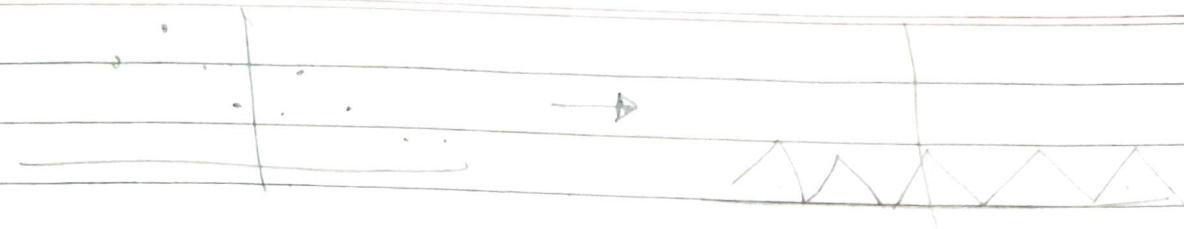
$$\tilde{f}(t) = \sum_{n=0}^{n=(m-1)\Delta t} f_n \delta(t - n\Delta t)$$

Sum of shifted copies of the f

$$\tilde{F}(\mu) = \frac{1}{\Delta t} \sum_{n=-\infty}^{\infty} F\left(\mu - \frac{n}{\Delta t}\right).$$

$$\tilde{F}(\mu) = \frac{1}{\Delta t} \sum_{n=-\infty}^{n=\infty} F\left(\mu - \frac{n}{\Delta t}\right) : \text{periodic continuous}$$





$$\tilde{F}(M) = \sum_{n=0}^{N-1} f_n e^{-j2\pi M n \Delta T}$$

$$x \longrightarrow x$$

22/09/20

Converting non-periodic to complex sinusoids to model real life situations.

$$f(w) = \int_{-\infty}^{\infty} f(t) e^{-jwt} dt. \quad f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(w) e^{jwt} dw.$$

→ Sampling : $s_{\Delta t}(t) = \sum_{n=-\infty}^{\infty} \delta(t - n\Delta t)$.

$$f(t) = (f_n \times \delta(t - n\Delta t)) \text{ sum.}$$

→ Discrete FT:-

$$F[m] = \sum_{m=0}^{M-1} f_m e^{-j2\pi m \frac{w}{M}}$$

M: length of signal

$$\frac{m}{M} = \frac{1}{\Delta t}.$$

↳ w direct dep on Δt

$$F[w] \rightarrow \operatorname{Re}\{F[m]\}, \operatorname{Im}\{F[m]\}.$$

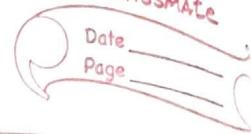
Phase = $\tan^{-1}\left(\frac{Im}{Re}\right) \dots$ strength of the particular angular freq.

$$\text{Amplitude} = \sqrt{Re^2 + Im^2}.$$

$$\text{Power } F = \text{Power } f$$

records of
energy
power

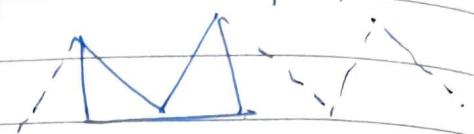
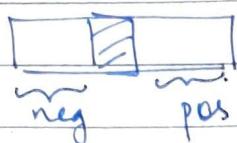
Multiplying $f[n] \times e^{\frac{j2\pi k n}{N}}$ complex sum classmate
 & obtain shifted DFT result.



Squaring & adding intensities :- power / intensity.

→ Center shifting : move neg & positive freq.

freq. shifting purpose



Can also perform x by comp sinusoid before DFT.

Sampling & Freq. intervals :

compression in time domain

⇒ Expansion in freq. domain

$$f[n] = \underbrace{\Delta t}_{0} \quad \underbrace{\Delta t}_{1} \quad \underbrace{\Delta t}_{2} \quad \dots \quad \underbrace{\Delta t}_{M-1}$$

∴ total length = $T = (M-1) \Delta t$ total time period.

$$F[m] = \underbrace{\Delta u}_{0} \quad \underbrace{\Delta u}_{1} \quad \underbrace{\Delta u}_{2} \quad \dots \quad \underbrace{\Delta u}_{(M-1)}$$

$$\rightarrow \Delta u = (M-1) \Delta u = \frac{1}{\Delta t} \dots \text{Range of freq.}$$

$$\rightarrow \Delta u = \frac{1}{(M-1) \Delta t} = \frac{1}{T} \dots \text{freq. resolution of DFT}$$

Eg:- Discrete signals :

$$\begin{array}{ccccccc} 1 & 2 & 4 & 4 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 0 & 1 & 2 & 3 & 4 \end{array}$$

$$F[m] = \sum_{n=0}^{M-1} f(n) \cdot e^{-j \frac{2\pi m n}{M}}$$

$$F[0] = 1 + 2 + 4 + 4 = 11$$

$$F[1] = \sum_{n=0}^{M-1} f(n) \cdot e^{-j \frac{2\pi n}{4}}$$

$$\begin{aligned} 0 &\rightarrow 1 \cdot 1 \\ 1 &\rightarrow -j \cdot 2 \\ 2 &\rightarrow +j \cdot 4 \\ 3 &\rightarrow +j \cdot 4 \end{aligned} = -3 + 2j$$

At $m=0, n=0$, Max value

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$$y = (N-1) \quad n = M-1$$
$$F[m, n] = \sum_{y=0}^N \sum_{n=0}^{M-1} f[n, y] e^{-j 2\pi \left(\frac{m n}{M} + \frac{n y}{N} \right)}$$

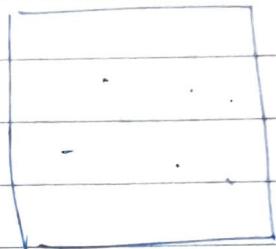
High peak at $(0,0)$ & at wave vector

As freq increases, spatial plots decrease in size.

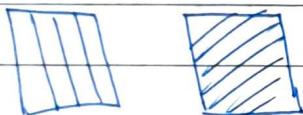


\Rightarrow

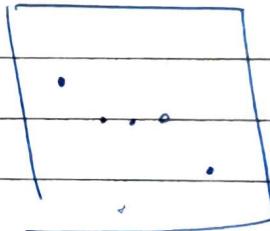
...



$|F[m, n]| \rightarrow$ DFT magnitude



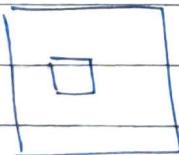
\rightarrow



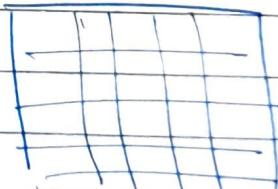
$$F(f_1 + f_2) = F(f_1) + F(f_2)$$



e.g:-

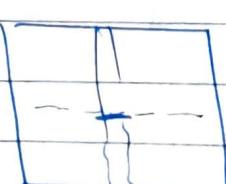
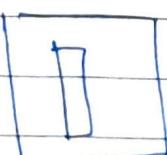


$|F|$.



Brighter

log
conversion

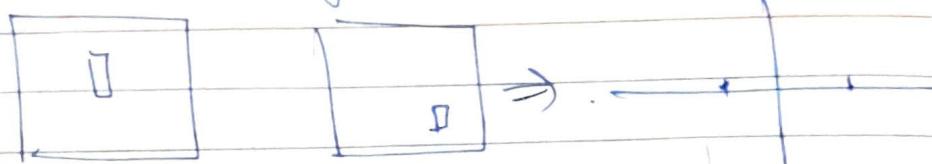


log

M: wt

N: wd

→ Translation & mag remain same.

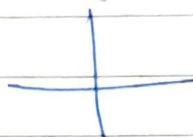


→ Alphabets Design patterns

B Q



H T



→ Phase: offset relative to the original
 $A \sin(\omega t + \phi)$... shift left

→ Phase & spectra both matter for reconstruction.

on performing

Setting	phase ie: $ A(\Omega, \Psi) $	spot magnitude	to constant ... reconstruction
inv. F	$\Omega(\Omega, \Psi) = 0$	phase = 0	reconstruction not possible

b: positional info
N: strength

Given that we can reconstruct the image with a smaller set of frequencies → compression

$$\begin{array}{lll} \text{Complexity of DFT} & = O(n^2) \\ \text{This red} = \text{FFT} & = O(N \log N). \end{array}$$

Total loss
is

Retaining lower change in frequencies ⇒ Smoothing
Apply a mask ST. only lower freq. retained

Ideal low pass :- $H(u,v) = \begin{cases} 1 & D(u,v) \leq D_0 \\ 0 & D(u,v) > D_0 \end{cases}$

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As radius inc, smoothing decreases.

I-O transition causes the ringing effect.

Instead : use gaussian. \rightarrow Gaussian low Pass Filters.

$H_{hp}(u,v)$ = $1 - H_{lp}(u,v)$. \rightarrow Edge detection

low pass :- $D(u,v) = [(u - M/2)^2 + (v - N/2)^2]^{1/2}$

Gaussian : $H(u,v) = e^{-D^2(u,v)/2\sigma^2}$

QUIZ:

128 x 128 image

Range

0 - 64

64

= 256

6 bits/pixel \rightarrow 1.

6 x

128 x 128

128 x 128 x 6

1	5	9	6	3	5	6
---	---	---	---	---	---	---

1	6	6	5	5	5	6
---	---	---	---	---	---	---



→ Adding 50% of the lowest 3 4 5 freq. is not bad & gives us most of the detail.
& high detail \Rightarrow high freq.

Edges cause high freq.

4	1	3	2	2	2	2	3	(L-1) $\sum_{j=0}^L p_r(x_j)$
3	1	1	1	2	2	2	2	
0	1	5	2	0	2	4	3	$(L-1) = 5-1$
1	1	2	2	2	2	3	3	= 4

0 1 2 3 4 5

$$\text{PDF} \quad \frac{1}{16} \quad \frac{2}{16} \quad \frac{6}{16} \quad \frac{2}{16} \quad \frac{1}{16} \quad \frac{1}{16} \quad 4 \ 2 \ 4 \ 3$$

$$\frac{1}{16} \quad \frac{2}{16} \quad \frac{6}{16} \quad \frac{2}{16} \quad \frac{1}{16} \quad \frac{1}{16} \quad 4 \ 2 \ 2 \ 2$$

$$\text{CDF} \quad \frac{1}{16} \quad \frac{3}{16} \quad \frac{12}{16} \quad \frac{16}{16} \quad \frac{18}{16} \quad \frac{15}{16} \quad 0 \ 2 \ 5 \ 3$$

$$\frac{1}{16} \quad \frac{3}{16} \quad \frac{12}{16} \quad \frac{16}{16} \quad \frac{18}{16} \quad \frac{15}{16} \quad 2 \ 2 \ 3 \ 2$$

~~$0 = \mathcal{S}\left(\frac{1}{16}\right) = \frac{1}{4} = 0 \quad \frac{1}{16}$~~

~~$2 = \mathcal{S}\left(\frac{2}{16}\right) = 2 = 2 \cdot \frac{0}{16} \quad \frac{1}{16}$~~

~~$2 = \mathcal{S}\left(\frac{2}{16}\right) = 2 = 2 \cdot \frac{0}{16} \quad \frac{1}{16}$~~

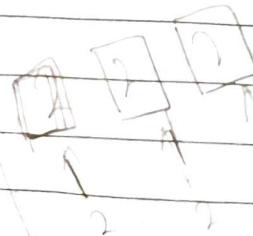
~~$3 = \mathcal{S}\left(\frac{3}{16}\right) = 2 + \frac{1}{16} = 2 \frac{1}{16} = \frac{3}{16}$~~

~~$4 = \mathcal{S}\left(\frac{4}{16}\right) = 2 + \frac{1}{16} = 2 \frac{1}{16} = \frac{5}{16}$~~

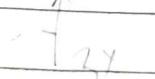
~~$0 \ 5 = \mathcal{S}\left(\frac{5}{16}\right) = 6 \ 5$~~

0 2
0 1
1 2
2 3
3 4
4 5
5 6
6 7
7 8
8 9
9 10
10 11
11 12
12 13
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92 93
93 94
94 95
95 96
96 97
97 98
98 99
99 100

2 pixels



3 3



3 3

$$f(x,y) = \frac{1}{MN} \sum_{n=1}^N \sum_{m=1}^M F[m,n] e^{j2\pi \left[\frac{nx}{M} + \frac{ny}{N} \right]}$$

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IDFT

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15/09/20 : Image Enhancement & Filtering in Freq domain

$$\rightarrow I \xrightarrow{\text{DFT}} F(u,v) H(u,v) \xrightarrow{\text{IDFT}} I_{\text{DFT}}$$

x in the transform domain =

ie: one can convert $H(x,y) \xrightarrow{\text{IDFT}} h(x,y)$
to convolution on the $I * h(x,y)$.

\rightarrow The radius is the dial that is used to increase power
ie: amount of total freq. content present in the image.

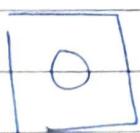
Jump in freq. is not that much.

10, 30, 60, 160, 360

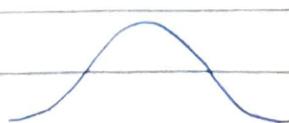
87, 93.1, 95.2 ... 99.2

\rightarrow Ringing effect is due to the higher level harmonics

For eg:



Mark difference.



Gaussian low pass filter instead!

Gaussian smoother has much
→ smoother effect occurring without spreading

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Additional high freq. components
do not exist. Ringing effect dec.

Application of low pass filtering

- Text breaks can be fixed with the help of LPF.

Low pass: define a mask such that only those frequencies that you want are retained.

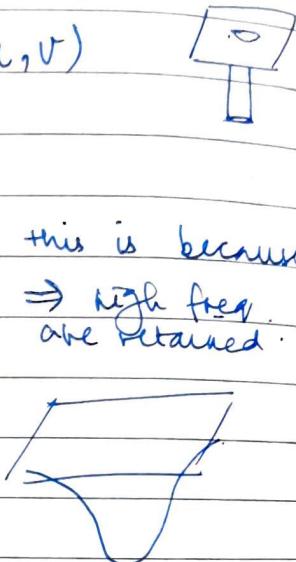
→ High pass :- $H_{HP}(u,v) = 1 - H_{LP}(u,v)$

image
sharpening

Edge info inc and is more clearly visible.

As $\Delta \uparrow$, edge info increases, this is because only higher frequencies are retained \Rightarrow high freq. are retained.

Again, we have gaussian HP:-



- Laplacian in Freq. Dom:

$$\mathcal{F} \left(\frac{d^n f(x)}{dx^n} \right) = (ju)^n F(u).$$

$$\begin{aligned} \mathcal{F} \left(\frac{\partial^2 (f(x,u))}{\partial x^2} + \frac{\partial^2 (f(x,u))}{\partial y^2} \right) &= (ju)^2 F(u,v) + (ju)^2 F(u,v) \\ &= -(u^2 + v^2) F(u,v) \end{aligned}$$

- Noise rejection filter is a bandpass filter
- removes all the frequencies except CFS

→ Notch Reject Filter (N Pass Filter)

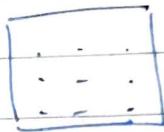
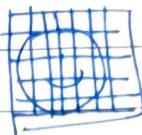
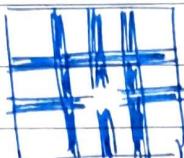
characteristic of retaining central freq.

Specify a custom mask.

IDFT or DPP of filter \Rightarrow noise
+ filter = original.

→ Artefact removal.

Create a mask:



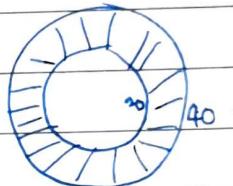
grid pattern
freq. domain

$$\text{Unsharp} : I - I_{\text{LOF}}$$

$$\text{High boost} : I + C \sqrt{I}$$

→ Band Reject Filters

After sufficient observation we see that we do not want the freq. values $20 < n < 40$



→ ~~Fresnel~~ Homomorphic filtering:

depends on intrinsic

$$I(x,y) = \underbrace{M(x,y)}_{\text{material}} \cdot \underbrace{L(x,y)}_{\text{light}} : \text{prep. of material} \times \text{light}.$$

$$\log I(x,y) = \log(M(x,y)) + \log(L(x,y)).$$

Perform freq. domain processing individually

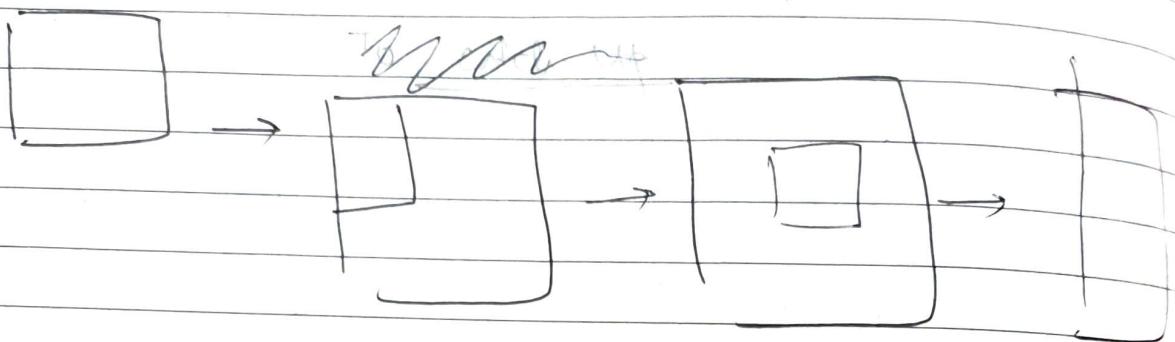
→ Performing circular conv. \rightarrow wrap around error.
Simple recipe:- zero padding.

$$I \xrightarrow{\text{DFT}} F$$

$$FH.$$

steps: Given $N \times N$ image f .

- 1) Pad f_p to size P, Q $P = 2M, Q = 2N$.
- 2) Mult f_p by $(-1)^{n+q}$... centering
3. $F_p = \text{DFT}(\tilde{f}_p)$



Perform filtering, inv, reshif^{free}:

Algo:-

read . im

Sobel = 3×3

imf = fft (im, 402, 402)

sf = fft (sobel, 402, 402)

F-fH = ifft(imf) * ifftshift(sobel)

ifft (FH)



Only linear spatial filters can be transformed to frequency domain for processing.



Not non-linear (min, max, median)

Guide process of spatial filter design.

Gabor filters, wavelets, Shape descriptors.

- Morphological Processing: (study of shape)

- Binary images : thresholding : one kind of contrast stretching.

Used in plant phenotyping.

- shape of leaves
- count number/kinds of leaves.

- Background subtraction

- Morph. Operators:

while performing : area: \exists a region of interest.

0: background

1: foreground.

set of pixels

interest : fg. everything else : bg.

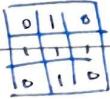
- Structuring El:- → Binary numbers

Box.



shape.

disc



Specify

$se = \text{strel}(3, 3, \text{disc})$

size

GE
size
shape
origin

Pencil & eraser tips are examples of this

Origin : default = centre. (can be top left).

1: active el

0: don't care conditions

First Type: Erosion

Thinning kind of op for binary structure

22/09/20

Bin Images :- Objects / Regions
 → 0 bg 1 fg

diff b/w regions
 filters - always
 binary

Thickening or erasing

Shape of the tip : determined by structuring el.

Effect of struct el. in erosion is ll^t to filtering.

Algo: If 3 location ST: all the locations completely get covered by SE then retain, else erode.
 → Reduces no. of foreground pixels

→ Output of erosion = $f(s \cdot El \text{ used})$
 ↗ origin of the s.el.

Erosion shrinks actual obj but tends to enlarge
 ↗ holes within the object

Counting coins eg: Binary (threshold) → Erode the edges

As disk size increases, erosion increases

→ Erosion can be considered similar to (Min filter)

→ $I_3 = \text{imeroode}(I_2, SE)$ → code.

- Simple application of pattern matching:
Find all locations where \exists a collection of pixels that matches SE.

- Eg: Erosion warp:-

- Shrinks foreground
- Foreground holes enlarged
- Small obj / noise increase

useful to remove noise

- Dilation :

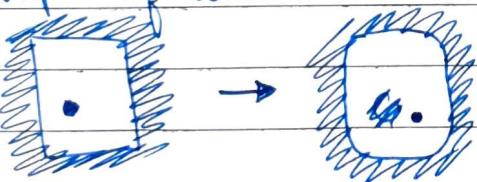
Opposite of ~~st.~~ erosion

Algo: If \exists at least 1 pixel that touches the filter/SE, set all to FG.

Thickening operation

Holes shrink, Foreground thickens.

SE shape of disk is used \Rightarrow rounding of corners occurs.



a) Disk SE

Diameter of circle is smaller for the black dot

b) Multiple dilations applied.

Dilation can be considered as a (Max Filter)

- ~~Text~~ Text breaks can also be fixed via the

- Expands fg.

fg holes shrink.

Text breaks fix -

LPF, Dilation

→ Applications of erosion & dilation :- boundary detection



D = Input

Dilation

Steps :-

1. Dilate Input
2. Dilate - Input

↳ Result is only boundary

Can use erosion also to do so.

1	1	1
1	1	1
1	1	1

→ 8-connectivity

All 8 pixels have some kind of connectivity

8: thicker

4: thinner, diagonal
connect not considered
here.

0	1	0
1	1	1
0	1	0

All operations now are just combinations of the 2 fundamental operations

• Opening

Same structure

Slide the SE inside the foreground, remove pixels that don't fall

Can be considered as erosion followed by dilation

Helps preserve the shape of the object except for the narrow parts

preserves width aspect ratio better

remove isolated islands
smooth portions.

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App: used in filtering out certain shapes.

! Opening is ~~idempotent~~: repeated application has no effect.

Opening = Erosion \rightarrow Dilation with same structural element.

• Closing:

→ This removes gaps closed

Do dilation followed by erosion

Applicta Skeletonization looks better for closed Images.
backbone is much better for closed images.

Here, slide the structural element outside this line

Opening \Rightarrow dual \Rightarrow closing. (I)

narrow
in (etc) get
filled

Opening foreground \Leftrightarrow closing bg ($1-I$)

eg \rightarrow fingerprint scanner
Opening + closing] remove all texture patterns

Dilation & closing : adding fg pixels
Erosion & opening : removing

\ominus : dilation

\oplus : erosion

\ominus : opening

\bullet : closing

\oplus : dilation

$$(f \ominus s)(x) \leq f(x)$$

counts $(f \ominus s)(x) \leq (fos)(x) \leq f(x) \leq (f \bullet s)(x) \leq (f \oplus s)(x)$

erosion opening closing dilation

sets $F(f \ominus s) \subseteq F(fos) \subseteq F(f) \subseteq F(f \bullet s) \subseteq F(f \oplus s)$

- Erosion on gray value images

- min filter → eroding a gray image
- images tend to get darker. → pulled to darker

- Dilation on gray

- max filter → pulled to lighter
- More uniform intensity
 - small isolated intensities getting magnified

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HAM

MISS.

- HIT & MISS TRANSFORM:

→ Eg: finding right angle convex corners.

→ St. els:  → don't cares

Rotate to get all corners & convert to AllB1c1l1D.

Slide st. el across the entire binary image.

Eg:-

	1	

0	1	0
0	0	0

a) Isolated points

b) end points on thin lines

	1	
1		
	1	

1		
	1	
1		

*	0	1
1	1	0
*	1	*

Locate triple junctions

- Distance transform.

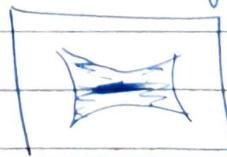
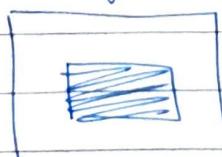
for bin images :- compute the dist to bg.

Chessboard dist :- $p_1: (x_1, y_1)$ $p_2: (x_2, y_2)$.

$$D(p_1, p_2) = \min \{ |x_1 - x_2|, |y_1 - y_2| \}.$$

This dist transform is made use of in skeletonization.

Can be defined as points of local maxima



Dist transform



Skeleton

Skeleton :- points of local maxima within the image

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Types of skeletonization

- by thinning : not reversible.
- chamfer (3,4) no pruning : fully reversible
- Chamfer (3,4) with pruning (not full rev).

- Much more compact representation. (signature)
- properties of the shape
- Can be used in pattern matching.

Finding Components

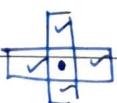
Quick motivation { Consider the different touch screen devices, want to detect no: of fingers touched
Or gesture recognition

Plant phenotyping \rightarrow shape & no: of leaves

(granulometry \rightarrow pack ~~with~~ grains of same size

Connected ness :-

4-connected :-



only top bottom RL.

8-connected ,



All 8 squares

\rightarrow

2 Pass algo for conn. comp. labelling .

white as
the object

for #

If left pixel is entered already, up label
to next pixel

If left pixel is bg, give new label #

complexity :- $O(MN)$ + disjoint

DTS: $O(V+E)$

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For next row, cp top labels.

If left pixel & top pixel are same, choose smaller ID & make the larger, the child of smaller.

- 1st pass generates img with multiple IDs.
- 2nd pass : replace child label with root label.

Flood fill:

Bucket fill tool in MS paint. Fill pixels based on the given pixels & boundaries

Specify a seed component :- recursive algorithms.

Based on connectedness the flood fill occurs

GEOMETRIC OPS:

→ Manipulates the ht & wd of the image / shape.
change in geometry.

Eg: translation, rotation, scaling etc.

Procedure for translation of images now :-

$$x \rightarrow f_x(x, y) = x'$$

$$y \rightarrow f_y(x, y) = y'$$

$$\text{And } I'(x, y) = I(f_x(x, y), f_y(x, y))$$

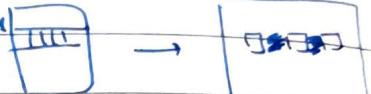
Ex:- Scale, Rotate, Flip, Translate, Affine

- Translate :-

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} dx \\ dy \end{bmatrix}$$

- Scaling : $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

Stretching creates holes in the image
 contraction : collision amongst intensities



- Sol:-
- subsampling : single pixel selection
 - interpolation : (avg. of neighbouring image)
 - replication - copy
 - interpolation -

- Shearing :-

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

add small value of y to x .

- Rotation :- $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

- Image warping :-

$$x' = x + r * \sin(\pi * n)$$

Affine geo. maps
2D transformation
 $\begin{matrix} x \\ y \end{matrix} \rightarrow \begin{matrix} x' \\ y' \end{matrix}$
 $\begin{matrix} x \\ y \\ 1 \end{matrix} \rightarrow \begin{matrix} x' \\ y' \\ 1 \end{matrix}$

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• Homogeneous coords.

converting all ops. to point-matrix mult.

Translation problem with mat mult.

∴ Add a dummy coord.

Combinations
can be done with
single matrix

Add r
dummy
axis
here

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \rightarrow \text{gets desired result.}$$

Homograph

$$\begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & \dots & h_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

→ Affine combination of scaling, rotate, transform.

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Affine \Rightarrow includes all

Inverse of
transformation
is inverse
mapping.

→ Filters, Point Op:

- modify colour vals.
- Domain (mostly fixed).

→ Geo op:

- modify position of pixels.
- colours kept (mostly) same

Defining functions modifying w-ordinates.

i.e.: $\begin{cases} x \rightarrow f_x(x, y) = x' \\ y \rightarrow f_y(x, y) = y' \end{cases} \quad I$

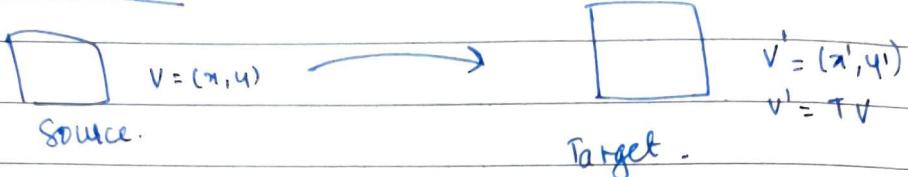
• Affine combination: $\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$

easier to perform operations.

Inverse of transform matrix is :- inverse mapping
And hence \Rightarrow affine mapping is invertible

- Interpolation Methods:

\rightarrow forward mapping :- Forward mapping



Issue :- Non integer v' values cause issues.
Co-ordinates may go beyond image bound.

Dimensionality & - Discretization issues.

If we want to retain the entire image content, then we have to determine the size beforehand.

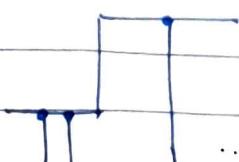
\rightarrow To overcome this we use backward mapping:

Go from the target image to source.

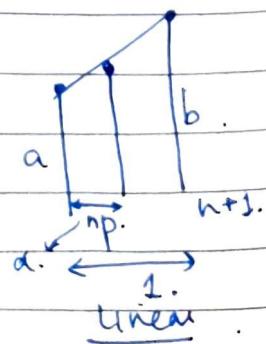
Sample value obtained via interpolation [\therefore original float interpolate]

\rightarrow Interpolation functions :-

$$I_D = (1-\alpha) a + \alpha b$$

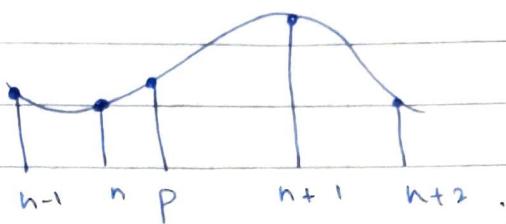


... 1D nearest neighbours.



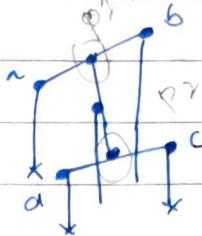
Cubic Interpolation

Intensities vary as a cubic function of the neighbours.



Bilinear :-

$2\alpha, 2\beta$ values



: new colours
'can' be introduced.

NN is quite step-y :- hence \exists some artifacts.

e.g.: imrotate (Im, 30°, loose/crop) → retain shape
anti clockwise

G.Transform

• Mapping Type of transform.

change shape

→ rotate

• Interpolation

→ Cross dissolve :

Morphing one persons image onto another

$$\frac{dt}{dt}$$

$$I(t) = (1-t)*S + t*T$$

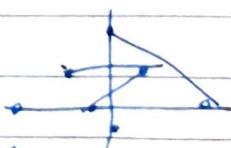
source $t=0$ } param.
target $t=1$ }

1. t: no. of steps. For each step we get an intermediate image.

Ternie : ghosting effect.

Nash based :- at every stage : valid image based on anchor points

$$x'_s y'_s \rightarrow x'_t y'_t : \text{use cross dissolve.}$$



Anchor points to be selected for both source & target.

Image Reg, Morphing - Given OP I, find T. classmate
begin with anchor point

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Geo Op Uses

- correct distortions introduced during morphing
- Transformations: special effects
- Registration: register images taken of the same scene at diff. times

$$\text{Intensity} = \frac{(R+G+B)}{3}$$

1

COLOR IMAGE PROCESSING:

VIBGYOR band
ultraviolet → infrared] white light on a prism.
400nm - 700nm. visible.

→ Phy. Stys:- colour depends on: source, surface, reflectance

Radiance (W), Luminance candela/m², Brightness: intensity

- Cones responsible for colour vision. ^{→ photopic.} LMS RGB
- Perception of a colour is a combination of illumination & reflectance. $T(r, \lambda) = I_l(n, \lambda) * R_f(r, \lambda)$.
- Intensity is a weighted function of RGB. $0.299R + 0.587G + 0.144B$.
- Perception of colour is nervous system consequences,
~~not inherent in wavelengths~~

Diff people diff. levels of colour receptors. Tetrachromat
Mantis Shrimp is one species.

$$\text{Perception} = f(\text{light, object})$$



Perceived colour

$$= f(\text{Light}, \text{Obj})$$

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Eg: Red objects in blue light appears black.

Brighter colours in colour channels.

Eg: Red strawberries brightest.

Paints & printing use: subtractive colour scheme.

$$x - x$$

→ Radiance: total energy flow from light source (W)

Luminance: amt of energy an obs. perceives $\frac{\text{W}}{\text{m}^2}$.

Brightness: intensity: sub. measure.

- Colour as a physiopsychological phenomenon.

- What we perceive as colour depends on illumination, reflecting wavelengths. ie: $I(x,y) = I_l(x,y) * \text{Reft}(x,y)$.

- Cone sensitivity: diff & diff extens.

ie: Stimulus $\xrightarrow[\text{cones.}]{\text{processed by}}$ Product / Result

Perception: $0.299 R + 0.587 G + 0.114 B$ wts.

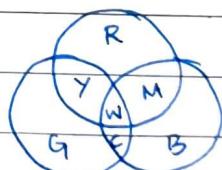
- Primary colours:

- Additive (CRT displays).

- offer good contrast to dark screens.

- combines on a surface that

- reflects all light falling on it.



- Subtractive filtering: CMY \rightarrow primary cols.

$$\boxed{\square \square} - \boxed{\square \square} = \boxed{\square \square} \downarrow \text{Green}$$

RG BG

$$\begin{aligned} M &= R+B \\ C &= B+G \\ Y &= G+R \end{aligned}$$

Van Krevel White

1.0 0.0 0.0

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→ CMYK in printers cause inks don't emit, they only absorb. cannot get true colours easily.

$$\text{cyan} = W-R \quad \text{Magenta} = W-Y \quad \text{Yellow} = W-B$$

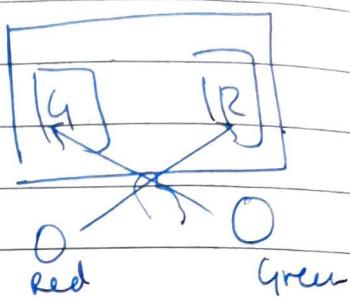
$$\begin{bmatrix} C \\ M \\ Y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

\downarrow : Neg. of intensity image

→ Projector systems

LMS response for eyes

Color blindness \rightarrow chromatic adaptation



Colour Models

① RGB :- $f(n,y) = \alpha_1 R + \alpha_2 G + \alpha_3 B$. $\alpha_i \in [0,1]$ $\sum \alpha_i = 1$.
Perceptually non uniform. Cannot differentiate b/w green.

② Chromaticity diagram CIE :-
separate the luminance + chrominance.
y chosen so matches lum eff funcn of
human eye. \downarrow chrom. code.

- Mc Adams ellipses :- cannot diff b/w colours in ellipse
& relative to centre.

Define L*a*b* colour space from chromaticity diag &
Mc Adams ellipses (L^*, a^*, b^*)

$$\text{Color distance } (C_1, C_2) = \|C_1 - C_2\|$$

$$= \sqrt{(L_1^* - L_2^*)^2 + (a_1^* - a_2^*)^2 + (b_1^* - b_2^*)^2}$$

Poor choice of colour map down what you want to convey !

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$$O \rightarrow O \text{ whitening } Y = AX.$$

$$\text{Chose } A^T A = \Sigma^{-1} \dots \text{ cov-matrix.}$$

HSI : hue, sat, intensity

H: dominant wt. perceived by person \rightarrow angle ①

S: white light mixed with hue. pure: saturated

I: Number of intensity. $[0,1] \rightarrow 255$

Saturation : Relative purity.

HSI : color description

RGB : generation

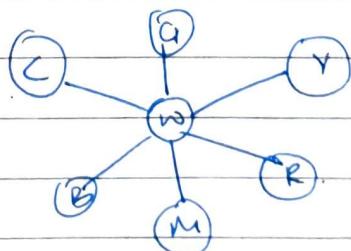
09/10/20.

\rightarrow CRT displays : additive
 ↳ contrast on dark images.

\rightarrow CIE lab space changes in CIE space lin. wrt human perception.

\rightarrow Color Gamuts : - colour printers, colour monitors.
 - choose colour gamut correctly for faithful rep
 - area: range of colours

\rightarrow Prim + Sec :-



: Pure colours
 True circles.

- At the lim of the circle are pure hues ($\text{sat} = 1$).
- Hue dominates less as move to center.
- Center no saturation

RGB to HSV :-

$$R' = R / 255$$

$$G' = G / 255$$

$$B' = B / 255$$

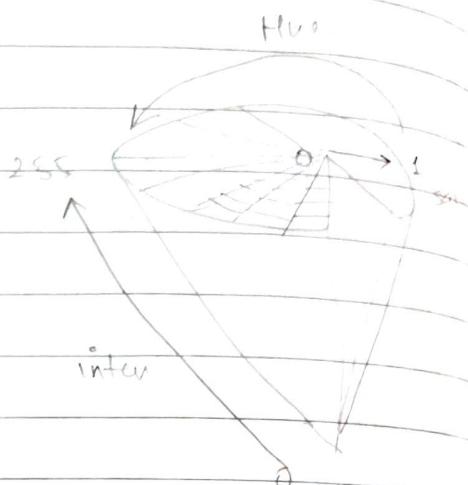
$$C_{\max} = \max(R', G', B')$$

$$C_{\min} = \min(R', G', B')$$

$$\Delta = C_{\max} - C_{\min}$$

$$H = \begin{cases} 0 & \Delta = 0 \\ \end{cases}$$

$$S = \begin{cases} 0 & C_{\max} = 0 \\ \frac{\Delta}{C_{\max}} & C_{\max} \neq 0 \end{cases}$$

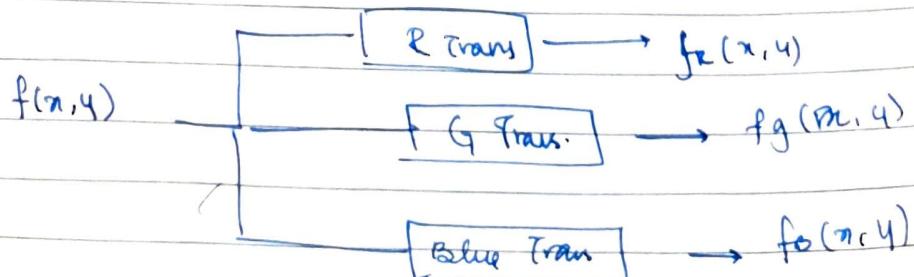


~~V~~ = C_{\max} . ie: HSV & HSI diff is :-

$$\text{for } I = \frac{R' + G' + B'}{3}$$

Extrapolate the triangular cross-section in the GUI.

- White balancing: ambient lighting.
- Pseudo colour :- use floating point values thereby increasing the space significantly.
- Items close to camera :- white, far : darker
- Depth image : rounding off uniform bg.
- Colour transforms used in security check. Diff objs. show up differently.

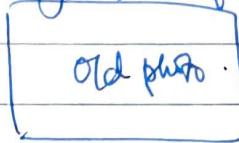


Sinusoidal transformations. Look at diff combinations of R, G, B to detect explosives, map to diff things.

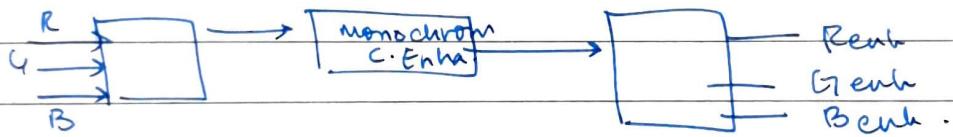
→ Also used for satellite imagery. Multispectral im proc.

Other imp aspect while dealing with RGB is
A: transparency. $\alpha \downarrow$ trans. \uparrow . Alpha channel.

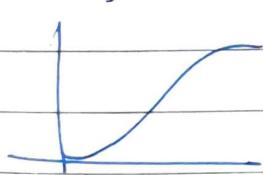
→ Applications: Filters, Vintaging effect, Vignetting effect.



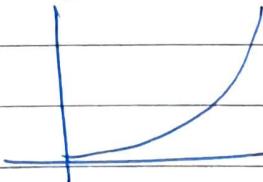
→ Contrast enhancement.



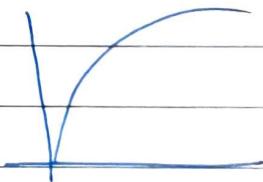
→ blurry \rightarrow normal.



bright \rightarrow normal



dark \rightarrow normal



• Hist Eq. Colours:

Opt1: Equalize each channel independently. And then interpolate.

Opt2: Take Avg. hist of R, G, B - Equalize wrt. Avg hist.

Opt3: Ref Image = I of HST. (may not correspond to perception of interest)

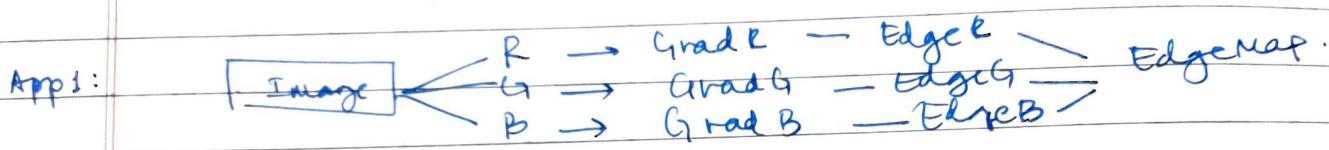
My opinion: Avg Opt2 best.

- Grayscale quantization :
Fewer colours - quantization.

Color quantization on RGB :- K-means ~~on~~ RGB.
Center of this vol. is the reference colour.

- Median filtering . now its not purely pepper noise.
- Iscan adaptive filtering
- RGB or HSV ... what to choose

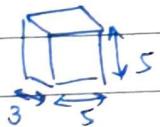
- Colour Edge Detection :



Locations in image where there is abrupt change in Intensity.

CNN : first set of filters learnt: low level features.

Filter



13/10/20

IMAGE SEG:

Partitioning an image into a collection of connected set of pixels.

- Into regions cover the image
- Structures (line) \rightarrow line curve.
- 2D shapes - circles, ellipses, squares.

18/10/20

Sobel : smooths / avg. the image followed
by edge filters (..., avoids noise)

classmate



Edge based

3 basic grey level discontinuities: detection of discont.

- Points / corners
 - edges
 - lines
- } abrupt change in intensity

→ result has edges as well

→ To detect a single point whose int. differs from neighbours. Fires maximally at point based regions.

→ LED blinks. Blinking, LED detectors

→ Image gradients & edges: in X direc & Y direc

1st derivative: edge detection 2nd deriv: edge detection

Edge detection: prewitt, Sobel, laplacian etc.

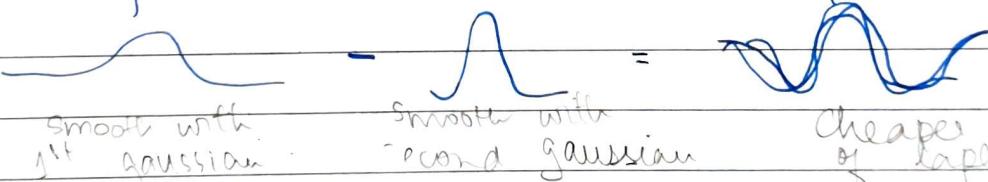
Combined edge image

$$\text{or } \rightarrow I_{\text{grad}} = I_{\text{gx}} + I_{\text{gy}}$$

$$\rightarrow \sqrt{g_x^2 + g_y^2}$$

$$\theta: \text{orientation} \quad \tan^{-1} \left(\frac{\partial f}{\partial y} \middle/ \frac{\partial f}{\partial x} \right)$$

→ Laplacian of Gaussian :-



- Oriented line detection: can use sobel ~~&~~ X, Y to obtain horizontal & vertical edges.

$$\rightarrow \text{For } +45^\circ \quad \begin{matrix} -1 & -1 & 2 \\ -1 & 2 & -1 \\ 2 & -1 & -1 \end{matrix} \quad \begin{matrix} 2 & 1 & 1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{matrix}$$

H.T To detect more detailed dimensions - lines, circles, arcs, curves etc. :- Hough Transform.

Can use Gr_x

$$\begin{bmatrix} -1 & -1 & 1 \\ 2 & 2 & 2 \\ -1 & -1 & 1 \end{bmatrix}$$

horizontal

Gr_y

$$\begin{bmatrix} -1 & 2 & 1 \\ -1 & 2 & -1 \\ 1 & -2 & 1 \end{bmatrix}$$

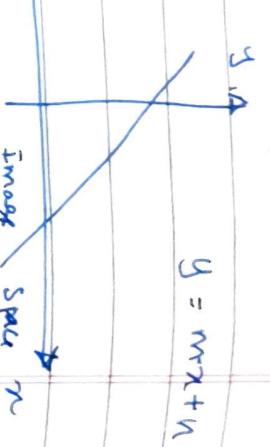
directly

LECTURE 17:

- Hough Transform:
Use for lines, curves, algebraic curves.
arbitrary but specific shapes.

Relies on notion of parameter spaces.

Consider we have an image
performed edge detection &
thresholding



\therefore pixels corresponding to edges exist in this image.

Line in image can be parameterized by slope.

$$y = mx + n$$

At least 2 points or slope & intercept characterise the line.

Line \leftrightarrow Point.
Points \leftrightarrow Lines (many).
Co-linear pts \leftrightarrow Intersecting lines.

(point of intersection in the line).

→ HT: Given an edge point P do no. of lines passing through it (say m, n)

Each of these lines can be represented as point in param space

If P 2 points (m_0, n_0) & (m_1, n_1) .



The lines intersect at the points given by lines joining $\Lambda \times \beta$ in the Hough Parameter Space.

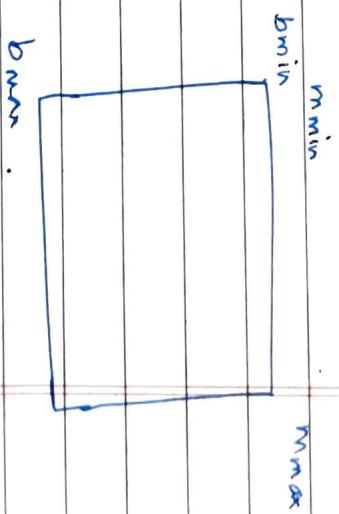
$\boxed{\ast \ast \ast}$ to find all possible lines : v. high complexity .

Image space

→ Line algo alternative (HT).

Initialize an accumulator array $A(m, b)$ to 0.

For each el. increment $A(m, b)$ to 0.
that loc. that satisfy the
 $b = -m * n + y$
in this mapping :



∴ All pts for an edge will inc. a particular m, b location.

After doing this the local maxima correspond to lines in edge image

Once we know these vals, we can draw our lines by setting top left as origin

\nearrow single line of peak
corresponds to the line detected

Issues :-

→ Slope of line is $-\infty < m < \infty$.

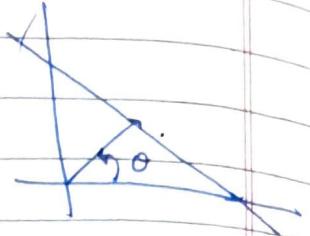
Param space is infinite.

→ Rep of $n = k$ doesn't exist.

Sol:- use normal eqn. of line

$$s = x \cos \theta + y \sin \theta$$

Now $s \& \theta$ are our params equivalent to $m \& n$.



\Rightarrow Point in img space becomes [sin cos]

$$0 < s < D$$

$$-\pi < \theta < \pi$$

FINITE SPACE

\therefore new space can represent all lines

Algo: Edge image $E(i,j) = 1$ for edges).

1. discretize θ & s in increments of $d\theta$ & ds
to set the accumulator array. $s \in \dots D$

2. for each pixel $E(i,j) = 1$ $h = 1, 2, \dots T$

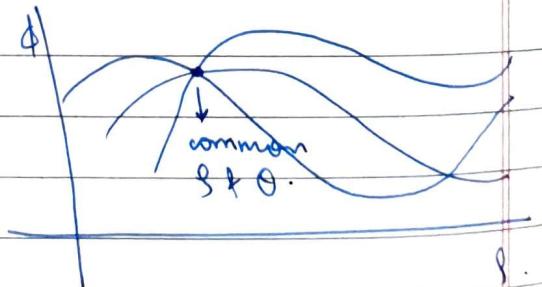
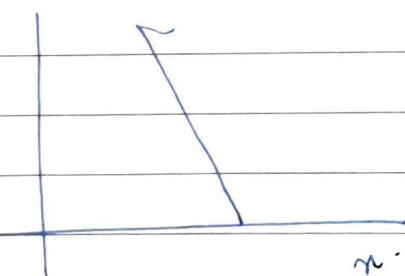
$$1. s = j \cos(h * d\theta) + i \sin(h * d\theta)$$

2. Find closest int k closest to this ~~s~~.

Increment $A(h, k)$.

3. Find local maxima now!

e.g.: y



Dealing with problems

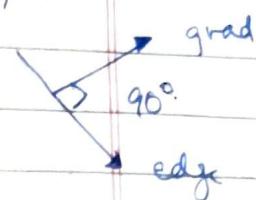
HT Speed Up:-

→ Edge image $\xleftarrow[\text{mag}]{\text{loc}}$ orientation

∴ Orientation of grad $\tan^{-1}\frac{(Ay)}{Ax}$

∴ uncertainty about θ is gone.

∴ fix θ in param space



Generalizations:

→ locate circles etc.

Circle: $(x_1-a)^2 + (x_2-b)^2 = r^2$ 3 params a, b, r .

3D accumulator array.

& for each point on circle edge (x, y)

increment $A[a, b, r]$:- point on circle vote

for that particular combination.

param $a = x + r \cos \theta$ $b = y + r \sin \theta$

Optimization: Only $1/8$ of circle needs to participate.

Conc: As long as \exists a parametric form for a shape, we can use Hough Transform [As dim increases accumulator complexity increases].

Problems: - It's blind

- susceptible to noise

→ * Map a difficult pattern to a simple peak detection problem.

⇒ Segmenting out lines :- corresponding to lines & not lines

- Works through occlusion

- Any part of st. line can be mapped.

Harris Corner Detection

Ham prone to noise

Visual system uses discontinuities as proof

Idea: Easily recognize point by looking through small window

- move in constant intensity: no change (anywhere)
- Edge: along edge no change in intensity.
- Corner: significant change in movement.
(Intersection of edges)

z must exist long enough to change

$$\text{Math: } E(u,v) = \sum_{x,y} w(x,y) [I(x+u, y+v) - I(x,y)]^2$$

For the shift $[u, v]$

windows func. shifted intensity

squared change in intensity

→ will apply to above 3 cases & see results

Now, for small shifts, $[u, v]$ we have bilinear exp.

$$I(x+u, y+v) \approx I(x,y) + (u I_x + v I_y) + \frac{1}{2} (I_x^2 u^2 + I_y^2 v^2 + 2 I_x I_y u v)$$

$f(x+u)$

$$\therefore E(u,v) = [u \ v] M [u \ v]^T$$

$= f(x) + u f'(x) + \frac{1}{2} u^2 f''(x)$

where M is a matrix computed from image derivatives

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

Take M & use to classify image points using eigen values of M . λ_1, λ_2 .

★ λ_1, λ_2 are small in flat region: \rightarrow since I_x^2, I_y^2 grad in x, y direction won't change in x, y direction. E almost constant in all directions.

* Edge: Either $\lambda_2 \gg \lambda_1$ or $\lambda_1 \gg \lambda_2$.
Either x & y grad.

* Corner: $\lambda_2 \approx \lambda_1$ large $\lambda_1 \approx \lambda_2$.
E inc in all directions.

Now, to come up with a cornerness response:-

$$R = \det M - k(\text{trace } M)^2$$

λ_1, λ_2 $\lambda_1 = \lambda_2$

- If R is very large \rightarrow corner.
- If |R| very small \rightarrow flat.
- If R large negative \rightarrow corner

→ Find pts with large corner response & take pts that are local maxima.

Use being able to warp - my images is useful.

- Compute cornerness response
- Perform thresholding
- Take pts of local maxima

LECTURE 18

Image segmentation: partitioning image into collection of connected set of pixels
 → Regions, linear structures etc.

- When to say an image is partitioned
- Partitioning image I into set of regions S satisfying
- 1. $\bigcup S_i = S$ partitions cover whole image
 - 2. $S_i \cap S_j = \emptyset$... No regions intersect
 - 3. $\forall S_i P(S_i) = \text{True}$ Homogeneity predicate (criteria \exists)
 - 4. $P(S_i \cup S_j) = \text{False}$ Union of adjacent regions

Used to define a valid segmentation.

→ 2-class segmentation:

Convert a grayscale to binary image

Easy way: thresholding: $g(x,y) = \begin{cases} 1 & f(x,y) > T \\ 0 & f(x,y) \leq T. \end{cases}$

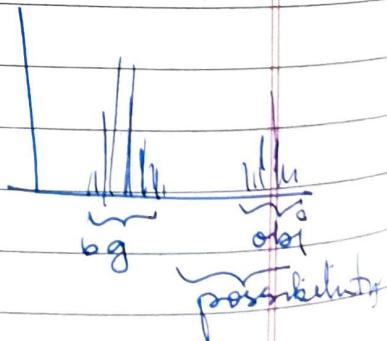
T const over entire image: global

T changes variable thresh. / adaptive thresh.

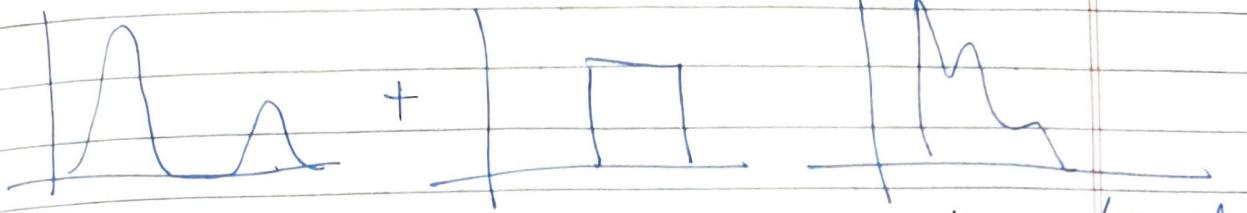
How to find T : explore the intensity histograms
 Good initial first step.

Objective is to find T .

~ With increase in a lot of noise, we cannot distinguish b/w the foreground & background.



same is the issue with illumination & reflectance.



Uneven/skewed.

Can't identify f_g & b_g from this as we go from L to R.

→ Thresholding forms a pipeline in many processes.

• Global thresh Iterative:

1. Select an initial estimate as T .
2. Segment image using T .
 - pixels G_1 & G_2 : 2 groups.
3. Compute avg int for these 2 m_1, m_2 .
4. Set new thresh $T_{\text{new}} = (m_1 + m_2)/2$.
5. If $|T_{\text{new}} - T| < \epsilon$ Stop.
6. else set $T = T_{\text{new}}$.

≈ This procedure is similar to the k-means algo
finding the mid pt. of the line joining the 2.

If intensity peaks/clusters are well separated then threshold can be found.

T is usually located at one of the valleys.
Same issues of clustering come up here too.

- Run with multiple random initializations to see what happens.
- If B_g similar to ROI → problematic.

Otsu's Method

Based on histograms.

Automatically finds optimal thresh maximizing b/w class variance.

→ Prelims..

- Mean & var of intensities in an image
 - Get probabilities / normalized histogram perspective
- $$M = \sum_{j=1}^n j P_j$$
- Var measures : spread around the mean:-
characteristic : better contrast compared to low var
Diff of int wrt mean \Rightarrow ... high cont



high interclass variance
low intraclass variance

→ Variance : measure of region homogeneity useful predicate. High homogeneity will be low var

Otsu : Find the threshold that minimizes intra class variance.

Algo: 1. compute normalized hist -

2. components as $P_i : i=0 \dots L-1$

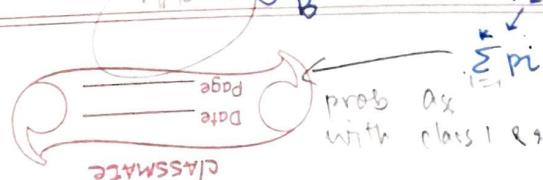
$\sum_{i=0}^k (m_i - \bar{m})^2 P(i)$ 3. $k, 0 < k < L-1$ select thresh.

4. C_1 : set of pixel levels $[0, 1, 2 \dots k]$

C_2 : set of pixels $[k+1 \dots L-1]$

5. Obtain thresh that max between class var

$$\text{max } \sigma_B^2(k) = P_1(k) (m_1(k) - \bar{m})^2 + P_2(k) (m_2(k) - \bar{m})^2$$



$$\begin{aligned} \sigma_B^2(k) &= \sum_{i=1}^k P_i P(C_i | r_i) \cdot P(r_i) \\ &= P_1(k) P_2(k) [m_1(k) - \bar{m}]^2 \end{aligned}$$

Picking a value of k
such that $m_1 \times m_2$
are equidistant from M_G .



m_G : fixed \rightarrow avg. of entire image.

If so $\Rightarrow P_1(k) = P_2(k)$.

Compute for each value of k & for whatever k the σ^2 is minimized.

Otsu respects the distribution of both fg & bg

\because it considers the var b/w the classes unlike the iterative where avg separation

\rightarrow In the presence of noise: first you should smooth the image & then to Otsus or any method.

Dis: Otsu not good if bg is non uniform illumination bg. \therefore No single thresh may work.

Reason: Global thresholding [won't work above]

Sol: Subdivide image into multiple parts & use adaptive thresh / var thresh.

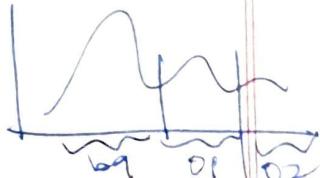
assm: int dist within each div $\stackrel{is}{\approx}$ good

Can be extended to computing T_{xy} , may for neighbourhood

$$g(x,y) = \begin{cases} 1 & f(x,y) > T_{xy} \\ 0 & f(x,y) \leq T_{xy}. \end{cases}$$

$T_{xy} = a \tau_{xy} + b m_x \text{ or } + b m_y$
func. of neigh stats

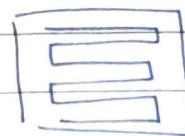
Dual thresh \rightarrow



→ can also compute image of local std. deviations

• Moving Average:

avoid
any sys
bias



→ processing
in zigzag
manner

Streaming data $n_1, n_2, \dots, n_N, n_{N+1}$

$$\mu_{N+1} = \frac{n_1 + n_2 + \dots + n_{N+1}}{N+1} \leftarrow \text{window size}$$

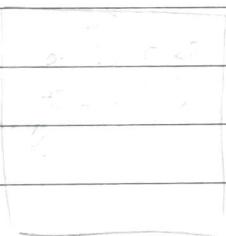
$$\mu_{N+1} = \frac{\mu_N N + n_{N+1}}{N+1}$$

Replace pixel value with moving avg.
Works v. well with doc. images.

→ lighting source causes spot shading

To apply local thresholding choose a window size
for the same.

Window << text. Fairly common in scanner type
applications



interpolation

sineoidal shading

→ moving average works well here

Tutorial

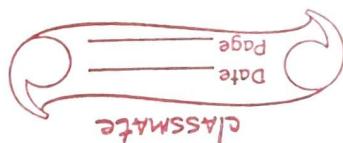
size of ROI / objects

small : adaptive

large : global

• Region Based Segmentation

→ Start with seed of points (like flood fill)
Region growing : find the common predicate
for the seed point. Some func. of RGB
vals.



lSe-Cel (sq-G) etc.

~~Region~~ → Region splitting & Merging

Define splitting & merging functions

Predicate (O_1, O_2) decides whether 2 segments to be given same label or not.

→ Clustering :-

We clustered based on colour earlier

Seg related to neighbourhood. (agglomerative, spectral etc.)

∴ include spatial coords.

∴ We cluster based on colours + spat.

feature vect : $[r, g, b, \alpha, u]^T$

• Humans Methods :- Gestalt principle

1. Similarity
2. Proximity
3. Closure
4. Continuation
5. Common fate
6. Symmetry
7. Parallelism
8. Familiarity



• LECTURE 19: RESTORATION:-

Some kind of degradation of image :- shook camera, inherent bkg, quality of sensors or gravitational waves.

→ Light field cameras :- capturing volume of light can then be sliced acc to different focus

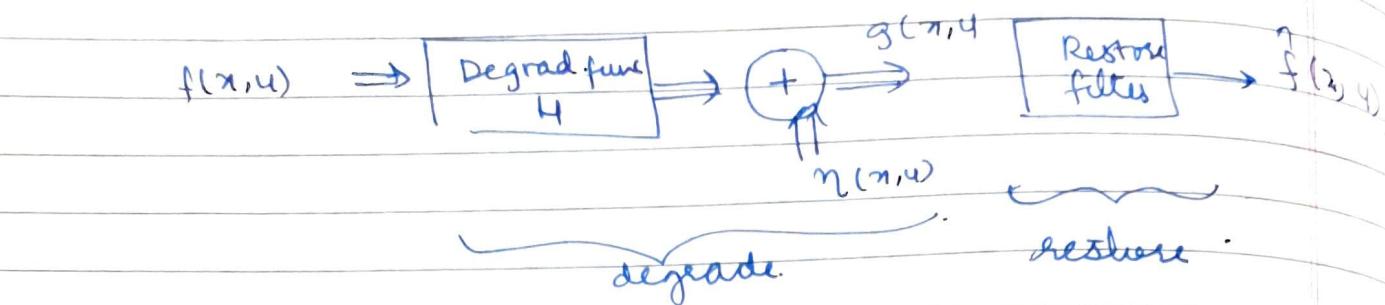
→ Camera shake, Haze removal, blur focus etc.

Enhancement
restoration

: subjective
: objective → original

→ Image restoration $\xrightarrow{\text{photometric (intensity)}}$ $\xrightarrow{\text{geometric}}$

- Model :-



$$\rightarrow g(x,y) = H[f(x,y)] + n(x,y).$$

$f(x,y)$ should be close to $f(x,y)$. Design some kind of a restored dist metric.

- Use a priori knowledge of degradation (known)
- Model degradation & apply H^{-1} reverse
- Formulate & evaluate

2 parts :-

→ Degrad fun H : Linear & Pos invariant
 → Noise : Indep of spatial loc & uncorrelated with image

Noise is not a func. of intensity of image.

→ If H is linear & pos invariant can be modelled

$$g(x,y) = h(x,y) \star f(x,y) + n(x,y)$$

action of $\star h$ can be modelled as a conv of $h \times f$.

∴ From fourier transform :-

$$G(u,v) = H(u,v) F(u,v) + N(u,v)$$

Can be obtained in the fourier domain

Step 1: Assume image degraded only by noise
 H identity $\therefore g(x,y) = f(x,y) + n(x,y)$

Now 3 diff types of noise models that can be applied.

Eg : $f(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z-\bar{z})^2}{2\sigma^2}}$... very convenient.

Rayleigh Noise : [Radar velocity image]

Corrupted by this. (useful for skewed histograms)

$$f(z) = \begin{cases} \frac{2}{b} (z-a) e^{-\frac{(z-a)^2}{b}} & z \geq a \\ 0 & z < a \end{cases}$$
$$\bar{z} = a + \sqrt{\pi b}/4 \quad \sigma^2 = \frac{b(4-\pi)}{6}$$

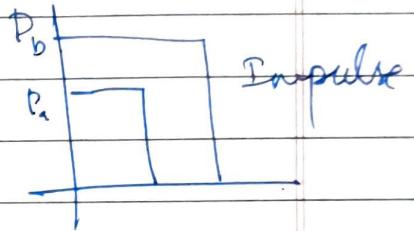
Earlang (Gamma Noise) : Images from laser.

Exponential noise

Uniform noise. (quantization, most unbiased)

defined over our dist. of intensities

$$P(z) = \begin{cases} P_a & z=a \\ P_b & z=b \\ 0 & \text{else} \end{cases}$$



Salt & pepper noise

$P_a = P_b \Rightarrow$ unipolar noise

Impulse \rightarrow sync in digitization

sensor malfunction

→ Not v. easy to identify noise model purely from appearance.

How to study system noise

a) Noise cancellation : capture 'flat environments' \rightarrow MLE.

b) Select model with better stat test scores AIC, LET etc.

Without access to camera & all,

- estimate from patches of const. intensity

- for impulse noise :- use mid-gray patch / area.

• Restoration by:

→ Arithmetic filter $f(m,n) = \frac{1}{mn} \sum g(s,t)$.

Trade off: small noise blurring

→ Geo mean = $\left[\prod_{(s,t) \in S_{xy}} g(s,t) \right]^{\frac{1}{mn}}$.

GM: preserves the detailed

Structure better than AM. but expensive.

→ Harmonic: $\hat{f}(x,y) = \frac{mn}{\sum_{(s,t) \in S_{xy}} \frac{1}{g(s,t)}}$

good for salt but
fails for pepper

→ Contra harmonic: = $\frac{\sum_{s,t} S_{xy} g(s,t)^{\alpha+1}}{\sum_{s,t} S_{xy} g(s,t)^{\alpha}}$

$\alpha > 0$ pepper $\alpha < 0$ salt
AM $\alpha = 0$ ~~HM~~ $\alpha = -1$

→ Spatially varying noise

To remove sinusoidally varying noise & all,
use band pass/reject filter.



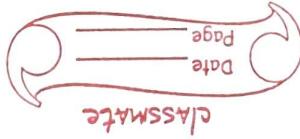
• Estimating Degradation Func.:

$$g = h * f + n$$

☰ three main ways to do this:-

- Observation: look, find & iterate [match the 2]
- Experimentation: important idea for calibration
- Math modelling.

If noise is absent degradation func \Rightarrow scaled impulse func.



→ Motion Blur :-

- Exposure
- If amt. of light hitting the sensor changes significantly over exposure period \rightarrow Motion Blur
- Person, camera moves, change in lighting

Filter $[I]$ on image get a motion blurred image.

Can assume linearity of motion

$$\text{coords} \rightarrow g(x,y) = \int_0^T f[x - x_0(t), y - y_0(t)] dt. \quad T: \text{period of motion}$$

some from
one other loc. also.

$$G(u,v) = F(u,v) \underbrace{\int_0^T e^{-j2\pi [u x_0(t) + v y_0(t)]} dt}_{H(u,v)}$$

uniform
motion
blur if:

$$\text{if } x_0 = \frac{at}{T} \text{ & } y_0 = \frac{bt}{T} : \text{ uniform velocity}$$

the blurring can be modelled,

→ Atmospheric Turbulence

$$H(u,v) = e^{-k(u^2+v^2)^{5/6}}$$

Recovering: even if we degradation func, we cannot get the undegraded image

~~if no noise~~

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)} \cdot F$$

$$\text{Sub from main: } \hat{F} = F + \frac{N}{H}$$

- ① - Problem $N(u,v)$ is random func $\not\rightarrow$ cannot compute FT.
- ② - If degradation has 0 or small val. $\frac{N(u,v)}{H(u,v)}$ dominates
ie: \hat{F} will be all zeros.

problem $g(x,y) \xrightarrow{FT} \frac{G(u,v)}{H(u,v)} = \hat{F}(u,v).$

INVERSE
FILTERING

$\hat{f}(x,y) \xleftarrow{IFT}$

∴, given original image. other option is to use Weiner filter

Assn: noise & image uncorrelated

Consider image & noise as RVs.

1 min

$$\therefore C^2 = E \{ (f - \hat{f})^2 \} \quad \text{estimate avg. error.}$$

At value
of min
of error
func.

$$\therefore F(u,v) =$$

stable
version of IF.

$$\left[\frac{|H(u,v)|^2}{H(u,v) |H(u,v)|^2 + S_n(u,v) / S_f(u,v)} \right] \xrightarrow{\text{power spec of degradation func}} G_u$$

Explicitly accounting for power spectrum by 2 things

$$S_n(u,v) = |N(u,v)|^2 = \text{noise} \quad (\text{auto corr of noise})$$

$$S_f(u,v) = |F(u,v)|^2 = \text{undegraded image}$$

chicken & Egg

∴ we approx * with K

Power spectrum: - Taking Fourier \otimes its magnitudes

LECTURE 2D.

- **Math model.**

- H : linear & pos invariant
- Noise is decoupled.

$$g(n, u) = H[f(n, u)] + \eta(n, u).$$

- when H is identity only exclude noise
- diff time of noise.

- To study noise model : do noise calibration.
- select model with better statistical score.

- if only img available.

- use estimate from patches of unknown

→ Restore in presence of only noise.

$$\hat{A}M : \hat{f}(n, u) = \frac{1}{mn} \sum g(i, u).$$

GM

$$H_M : \hat{f}(n, u) = \frac{mn}{\sum (n_i t_i) \sum \frac{1}{t_i} \sum g_i(u)}.$$

⇒ **Contraharmonic mean filter** (General purpose)

- Ω : order of filter.
- good for salt & pepper
- Elim . peper & $\Omega > 0$ salt $\Omega < 0$.
- $\Omega = 0$ AM ; $\Omega = -1$: HM

Eg: non-linear filters : max - min etc.

→ Periodic noise can be eliminated using band pass

How to model $H(u, v)$.

$$\text{Motion blur: } H(u, v) = \frac{T}{\pi} \sin [\pi (ua + vb)] e^{-j\pi (ua + vb)}$$

- Recovering the image in more complicated settings then:- estimate

$$\hat{f}(u, v) = \frac{G(u, v)}{H(u, v)}$$
- Recovering image in presence of noise + deg is not possible.

This where Wiener filter can be used to perform the inverse.

$$\hat{f}(u, v) = \left[\frac{1}{|H(u, v)|^2 + K} \right] G(u, v).$$

* param accounts for noise

→ what exactly is unknown in the math problem

$$g^{(n, 4)} = H [f^{(n, 4)}] + v^{(n, 4)}.$$

known | unknown

Problem

H, g

Recovery.

Blind recovery.

• particular family of H (partially) model H : f, g

Semi blind rec

Aystern identification.

All the above are called inverse problems.

All this is distortion due to interactivities

Geometric Distortion :

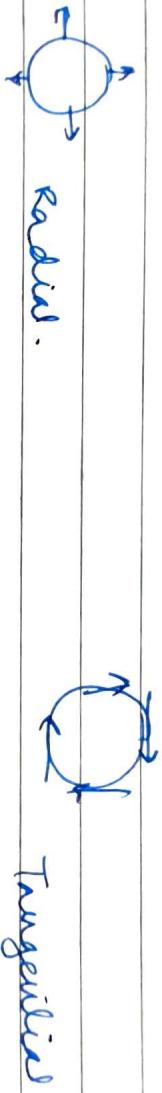
Pinhole camera :- Image a real world object.

→ Chromatic aberration :- Light of diff wavelengths don't converge at same point.

→ Radial & Tangential distortion.

→ Vignetting : don't uniformly receive light from all sources.

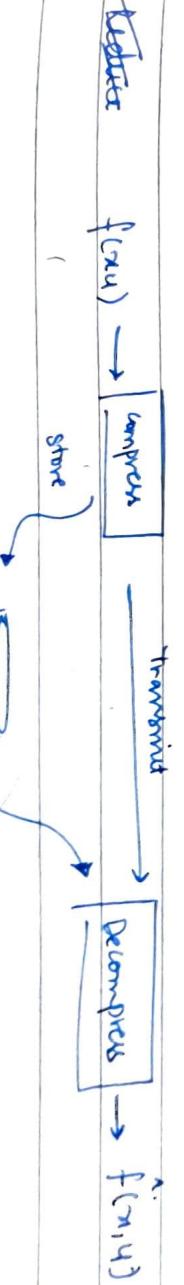
Lens flare : Starry interreflections of light within optical lens system.



Rectified using geometric camera calibration.

IMAGE COMPRESSION :

Aims to reduce the amount of data while preserving as much info as possible.



Transmit : Stream across some network or something

Obtain more or less same image

can be done by exploiting redundancies in image data.

- compression ratio:
extent to which we are able to reduce amt of data wr.



image carrying units

$$\text{Comp Ratio} : C_R = \frac{n_1}{n_2}$$

7 bits

10 bits

- 2 forms: uncomp : comp. :: $n_1 : n_2$
- the n_2 better is the compression achieved.

- Data redundancy.

$$R_D = 1 - \frac{1}{C_R}$$

$$C_R = \frac{40}{1} \quad \text{then} \quad R_D = 1 - \frac{1}{40} = 0.9.$$

⇒ 90% of the data in dataset 1 is redundant.

To put it on a 25GB blue ray disc, required compression factor = 53.6.

→ Types of Redundancy:

- ① Coding redundancy :-

Code is a list of symbols
Code word :- bunch of symbols used to rep some info
length : no. of symbols used

terminology :-

$N \times N$ image

r_k : k^{th} gray level.

$\lambda(r_k)$: # of bits of r_k .

$P(r_k)$: prob. :- histogram

$$\begin{aligned} \text{Lang} &= E(\lambda(r_k)) = \sum_{k=0}^{L-1} \lambda(r_k) P(r_k) \\ \text{Total} &= NM \cdot \text{Lang} \end{aligned}$$

Ques 1:

constant length code .
if constant length \Rightarrow 3 bits required .

Ques 2:

variable length encoding

long word $\in \{0, 1\}^n$ i.e. 2^n bits !

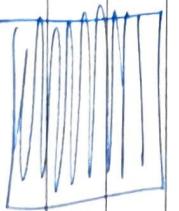
Now ~~#~~ bits :- $2.4 NM$ bits

$$C_e = \frac{3}{2.7} = 1.11. \quad R_D = 0.099.$$

② spatial / temporal Redundancy .

exists relationship with prob & code word .

↑ prob :- smaller length code word .



here everything is occurring frequently \Rightarrow cannot reduce code length .

Do a run-length encoding of the image .
represent each row by: (i, n) .

int \rightarrow no. of pixels .

eg : $(20, 256) \quad (170, 256) \quad \dots$ etc .

∴ 16 bits / row .
 $\Rightarrow 16 \times 265$ bits .

i.e.: $\frac{256 \times 256 \times 8}{16 \times 256} = 128 : 1$. compression

i.e.: (105, 20), (60, 4) . . .

e.g.: documents / uniform patches.

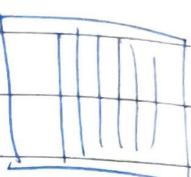
whenever \exists interpixel redundancy: pixel values correlated
i.e.: pixel can be reasonably predicted by
neighbours.

$$f(x) \circ g(n) = \int_{-\infty}^{\infty} f(x) g(x+a) da$$

auto corr $\Rightarrow f(x) = g(n)$.



no



yes

- with repetitive structure, auto correlate values ↑
- How self similar is the image vertically.
- can apply run-length encoding horizontally ↓
- autocorrelation: pre test for whether RLE can be applied

At times we may have to transform the image to obtain the interpixel value relation.

→ threshold & perform RLE e.g.: (1+10). bits/pair

for binary 0 or 1 for count

Temporal Redundancy

frame t, frame t+1 . . .

∴ store frame(t+1) - frame(t) and store .

RLE ($\delta +$) .

↓ residual

$$(n_w + n_u + 8)$$

or

$$H \times W \times 8$$

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③ Psychovisual Redundancy:

exploit the limitations of HVS. (peculiarities). throw info considered same by eye.

COMPRESSION CONTD:

Goal: reduce amt. of data req. to represent digital image.

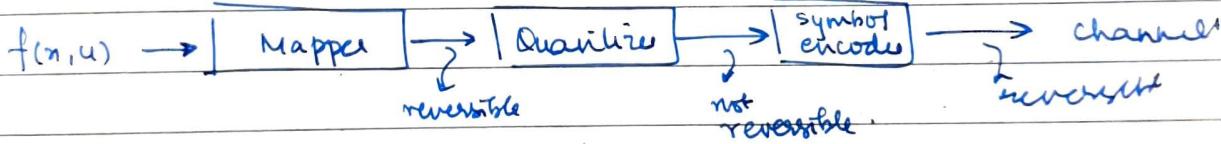
$$CR = \frac{n_1}{n_2} : n_1 : \text{before} \quad n_2 : \text{after}$$

cannot do any better than entropy

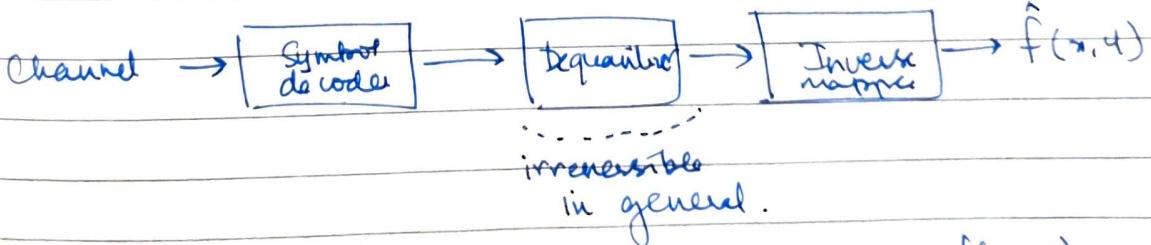
$$\text{Info} = -\log_2 p_i \text{ bits} \quad E = -\sum_i p_i \log p_i$$

Psychovisual: After quantizing with fewer bits, there may exist some discontinuities. \therefore can add small noise: feels random no:s to image

I.I Model:-



- Mapper: transform data to account for interpixel redundancies
- Quantizer: quantize to account for psychovis red.
- Encoder: acc for coding redundancies



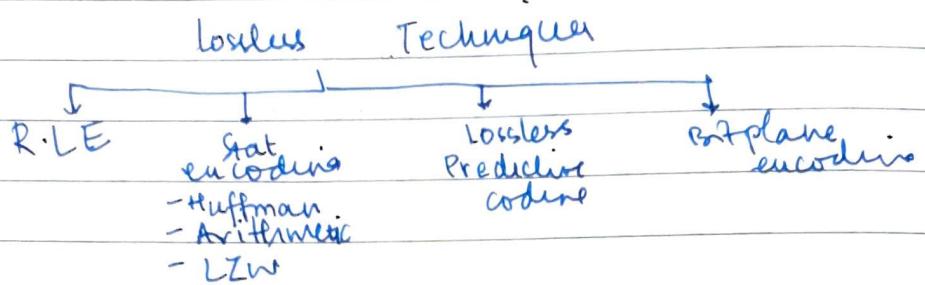
- As long as $f(x,u)$ looks close to $f(x,u)$ or it is good enough for image processing tasks we are good.
- If $f(x,u)$ is + in dim or width depth: more efficient representation for $f(x,u)$ \rightarrow processing saving

- Lossless Compression

$f(n,4) \rightarrow \text{Compress} \rightarrow \text{Decompress} \rightarrow f(x,4)$

$$e(n,4) = \hat{f}(n,4) - f(n,4) = 0$$

Achieves perfect compression



- Huffman Encoding: var length coding tech.
 - Source Symbol encoded one at a time 1-1 map
 - Optimal code

3 2 passes

Forward pass :-

Desc order [compute normalized histogram : probabilities
 combine last 2 prob to make a single
 repeat until only 2 prob remain]

Step 1: Is to form this sort of a prob tree

Eg. 01010, 011, 111, 100
 a_3 a_1 $a_2 a_2$ a_6 .

To encode, assign code symbols backward

Last 2 probs $\rightarrow 0, 1$.

then append 0, 1 to remaining, in the same manner (keep track of intensity value for corresponding vals)

weakness of huffman: encodes one at a time

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- Decoding can be done unambiguously using a lookup table.

• Arithmetical encoding: What does bunching or (range enc) mean?

Addresses bunching * encoding sequences of source symbols together.

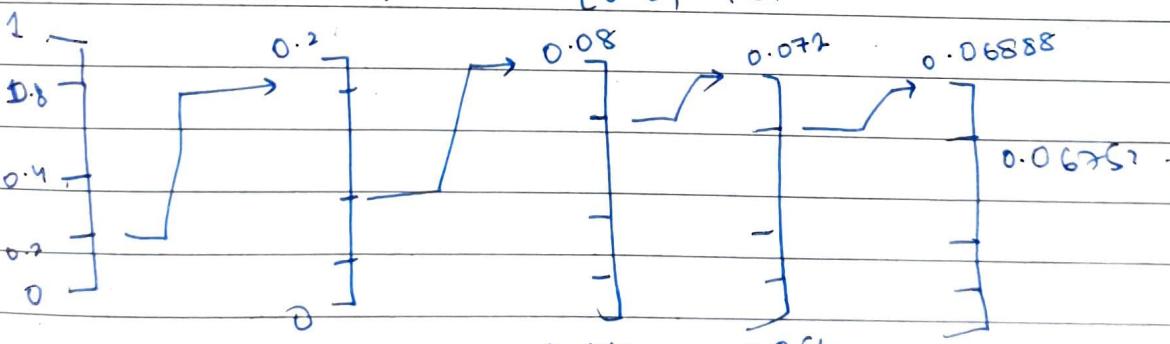
- No 1-1 mapping
- Slower than huffman but can achieve better compression

eg:	a_1	0.2
	a_2	0.2
	a_3	0.4
	a_4	0.2

Encode $a_1 a_2 a_3 a_3 a_4$

map to regions of prob

a_1	0.2	[0.0, 0.2)
a_2	0.2	[0.2, 0.4)
a_3	0.4	[0.4, 0.8)
a_4	0.2	[0.8, 1.0)



divide in same values

consider respective subintervals

∴ Resultant interval $[0.0675_2, 0.0688]$

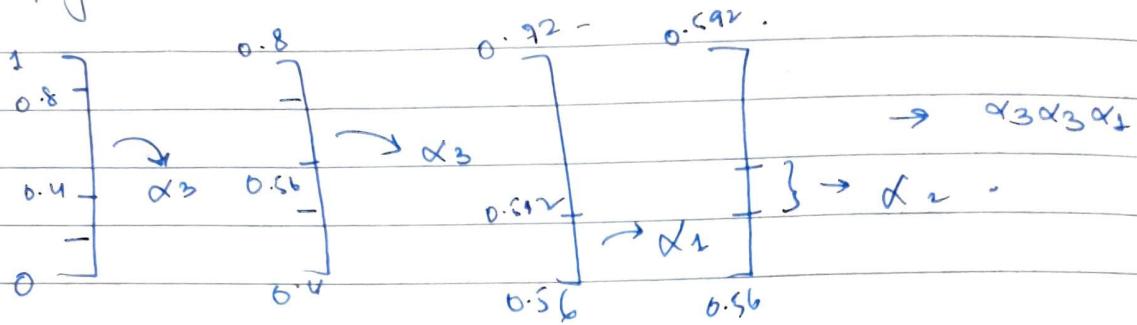
∴ Arithmetic encoding of symbol $\hat{=} 0.0688$

→ The sub interval required to represent the msg. ~~short~~ becomes smaller & smaller as the no: of symbols in the msg increases.

Decoding :-

Given $\alpha = 0.572$ & code length = 4

Again have access to probability table.



LZW coding :-

- Requires no prior know of symbol probabilities
- Assigns fixed length code words to var length symbol seq. No 1-1 corresponds.
Used in GIF, TIFF PDF.

- codebook / dict must be constructed
(data dependent manner. auxiliary table)
scanning image : row by row

0 0
()
;
255 255
266 ?

whatever gray level
seq. not in dict, assigned
to new entry.

39	.	126
39	.	126
.	.	.

39 39
yes
39-39 no ... add 39-39 → 286

39-126 ... add 39-126

- dict need not be sent with encoded input
- can be built on the fly.

CR = empty

repeat

P = next pixel

CS = CR + P

if CS is found

- no out

- CR = CS

else

out D(CR)

Add CS to D

CR = P

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→ RLE is good but cannot apply to not nice images. Sometimes very get bigger
can contains any type of data x higher ratios.

Can now miss & match these images.

→ Bit Plane Coding :-

process each bit plane individually.

- a) decompose to series of bin
- b) compress (using RLE.)

• Lossy Methods:

Block Truncation
Coding

lossy
Predictive
coding

Transform
Coding

Subband
coding

Fractal
Coding

Vect
Quant

$$e(x, u) = |f(x, u) - f(r, u)| > 0$$

→ Transform image to some other domain to red.
inter pixel redundancy.

e.g. throw away the higher freq

$$\frac{1}{N} \sum_{u=1}^{k-1} \sum_{v=1}^{L-1} T(u, v) h(x, u, v)$$

$T(u, v)$ can be computed using DFT, DCT, KLT...PCA

JPEG uses DCT

$$C(u, v) = \alpha(u) \alpha(v) \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} f(x, y) \cos\left(\frac{(2x+1)u\pi}{2N}\right) \cos\left(\frac{(2y+1)v\pi}{2N}\right)$$

One diff: no complex parts.

Entirely defined in terms of cosines.

WHT: Walsh Hadamard function:

uses
0-1.
basis

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function

- Take image & divide into sub patches.
- May be better to represent patches in compressed format better

Procedure :

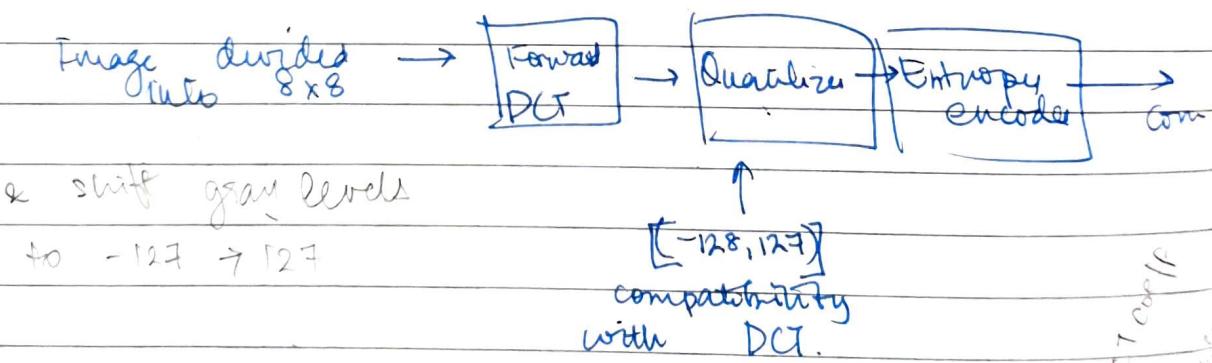
- Divide image into 8×8 patches
- For each patch encode using DCT
- Remove $\geq 70\%$ of co-efficients in transform domain

Taking RMSE, DCT is better: $DCT < WHT < DFT$

↳ encodes items at patch level better. Causes fewer artifacts at a patch level

Finding : 8×8 images are better

JPEG Compression:



→ Quantization table $C_q(u, v)$ round = $\frac{C_q(u, v)}{Q(u, v)}$

Given quality factor. Higher freq. entries made 0. bottom right matrix

→ Order co-efficients in zig-zag manner
∴ We can use RLE

↑ Quality factor, compression is very high but little noise may come in

JPEG insufficient for things like fingerprints. FBI uses WSQ. No "blocky" artifacts therefore.

→ Final step in JPEG:-

- Instead of using RGB it uses Y Co Cr
- It performs chroma subsampling: quantizes Y Co Cr aggressively. diff quant table.

JPEG supports diff modes:

- sequential : LR TB
- For web based we use progressive (multi resolution image). lower freq. sent first

Eg: GIF : lossy PNG : lossless

MPEG : exploits temporal redundancy.

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Comparing Images:

- Match patterns across image
- Similar version in HAM transforms S.T. structural maps to image & captured.

Q: How to tell if 2 images are same.

Obv: pixel-by-pixel comparison.

Noise, quantization etc. introduce differences.

- Depends on who is answering the question.
- To define what numerical diff is acceptable

$$\text{i.e. } V = \frac{\sum (a_i - b_i)^2}{MN} \quad V < \theta \Rightarrow \text{same.}$$

All factors need to be considered.

- pixel level it's much more noisy
- block matching are faster

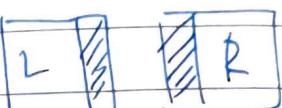
→ Better approach: template matching:

Identify similar sub images within a scene.

Applications:

- match R & L pic of stereo image
- find particular pattern in scene
- Track moving patterns (suspect's car)

Procedure forms basis of DIP & CV.



predictive matching /

block matching as done

in video frames → similar here.

• Basic idea

- None given pattern over search image
- Measure diff b/w template & sub images
- Record pos where diff \downarrow & size \uparrow
- Output is a set of locations

• Problems:

- what is distance / diff measure.
- what level of diff considered match.
- How results change when brightness / contrast change
(diff lighting conditions to be accounted for).

Score for each (r, s) plotted

e.g. measures :-

- sum of absolute diff :- $\sum_{i,j} |I(r+i, s+j) - f(i,j)|$
- Max of all such diff.
- Squared differences (N-dim Euclidean dist)

Output from template matching can either
be top left corner or origin off the patch.

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Squared distance :-

$$d_E^2(r,s) = \sum_{i,j \in R} (I(r+i, s+j) - R(i,j))^2$$

$$= \underbrace{\sum_{A(r,s)} I^2(r+i, s+j)}_{A(r,s)} + \underbrace{\sum_{B} R^2(i,j)}_{B} - 2 \sum_{C(r,s)} I(r+i, s+j) \cdot R(i,j)$$

const term
..., can be ignored

~~like a similarity~~ $C(r,s) \rightarrow$ linear cross correlation b/w $I \neq R$

~~measures sliding template across image for diff shifts~~

$$(I * R)(r,s) = \sum \sum I(r+i, s+j) R(i,j)$$

considered 0 outside bounds $\therefore \sum_{i,j \in R}$ only.

Now if A is constant, no matter where you consider a patch of template size :-

~~* Min value of $d_E^2(r,s)$ corresponds to max value of $(I * R)(r,s)$.~~

\rightarrow Range of $C(r,s)$ not bounded, ~~depends on template size~~ NOT invariant to changes in intensity amplitude.

• Norm cross correlation

$$C_N(r,s) = \frac{\sum I(r+i, s+j) \cdot R(i,j)}{[\sum I^2(r+i, s+j)]^{1/2} [\sum R^2(i,j)]^{1/2}}$$

$C_N(r,s)$ is a local distance measure $[0,1]$ range

$C_N(r,s) = 1$ indicates max match.

$C_N(r,s) = 0$ indicates images very dissimilar

\rightarrow Score indicates level of matching

\therefore Can set a threshold b/w $[0,1]$.

- Sum of abs diff : okay but affected by ~~global int changes~~
- Max diff : responds more to lighting than pattern
- Corr coeff

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→ Variant : zero normalized cross correlation

• Range : $[-1, 1]$

Subtract \bar{I} \bar{R} from the num & deno
atys. \rightarrow 0 mean.

$$\bar{I} = \frac{1}{K} \sum I(r+i, j+s) \quad \bar{R} = \frac{1}{K} \sum R(i, j)$$

$S = ZNCC(I, R)$... matrix

→ ~~Sum of absolute differences~~ ^{sq euclidean dist} \rightarrow
~~affected by global intensity changes~~

→ Ideal image :- only maxima are bigggggg.

Correlation coefficient produces the best results
ideally. robust measure in realistic lighting
condition

To obtain diff configurations :- try variants
of the template

If we get $[-1, 1]$...

^{opposite} \rightarrow exact match
exactly negative

- Template need not be rectangular.
- Can use circular, elliptical, or custom shaped
- Weight matrix stored in a ~~as~~ matrix while pattern matching. can weigh in diff values.

→ Matching & Rotation : computationally prohibitive

[~~for~~] Logarithmic polar space matching

Affine matching : do not want to match
median template. can transform & use

Binary Images:

- ~~Some~~ some missing pixels are okay. Output nice
- ~~problem~~ Very small rotations / distortions \Rightarrow large distances
 \therefore need more tolerant measure.

To account for things :-

Use distance transform: - distance to closest fg pixel

- Chamfer matching: template matching for binary images. \rightarrow Chamfer score: avg nearest dist from template points to image points.

i.e. wherever it matches best \Rightarrow dist least [from dist image]

→ computationally 'inexpensive'

- Dist image provides a smooth cost function. And as we move away from matching loc, score increases in smooth manner.

Score :- ① for each loc of template took at the

② look at dist transform values

$$Q = \frac{1}{k} \sum_{(i,j) \in R} D(r+i, s+j)$$

only for the i, j that
 ϵ to foreground in R
add it to the ~~sum~~ D .

Zero Q score \rightarrow max match

Large Q score \rightarrow large deviation

Insensitive to scale & rot

sensitive to small shape changes

large no. of template

robust to clutter

comp cheap

→ Paulid sort of classifier : cascade based obj detection
shape context

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Reducing Cost of Correlation:

- large costs of correlation matching.
- face recognition char Lucy] every single thing may be too good much.
- matching at the right scale may also be a cause of concern.
- Result for single scale search :- can sometimes result in false positives.
- cascade based object detection algorithm.

6/11/20

• REP & DESC:

(end product of many img proc pipelines)

- low-level img analysis - pre processing, extracting primitives
 - Mid level :- img rep + description stat. task spec. feature design . + ML
 - Deep learning : Obj understanding to image und. with data ↗
 - Segmentation, morph operations to define as - lines - circles - points etc.
↗ all in pixel level
- understanding
- + pixels
- Regions points lines
- ↗ extracting primitives
- eng / lang describe

- Representation : Image with one or more objects
- Encoding of object : can be a 1D vector (feature vec)
- vector to function as proxy : truthful but approx.

Descriptor :

- Only aspect of the object (Eg: its length, no: of wheels)
- suitable for classification
- Have desc. invariant to noise/translation

Image analysis $\frac{\text{rep Internal}}{\text{rep External}}$ \rightarrow shape or boundary of regions.

- \rightarrow Ext Rep :- Rep in terms of end words/polygons.
txt desc - perimeter, area anything
- \rightarrow Internal rep : Region :
Rep : pixels inside the region.
Desc : colors, Texture etc.

Eg: Understanding form filling :-

- Identifying regions & all external rep
- To understand who has filled what, diff b/w text & handwriting \rightarrow internal rep

Refining an object : - All 4 words or one word & ~~#~~ ^{ht 2nd} external descriptions.

- \rightarrow Semantic segmentation : to identify classes of objs.
- Instance seg : to also count them & IDs.
- Boundary rep set of pixels

- Boundary Following Algo.

Given a binary region R or its boundary

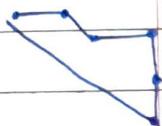
1. let the top left be starting point b_0 . label 1
2. Denote c_0 the west neighbor of b_0 . $c_0 : b_0$
3. Examine 8 neighbor of b_0 (starting at c_0) but
 b_1 be the first 1 pixel

Repeat . . .

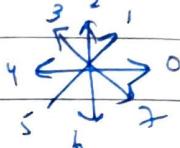
→ Boundary description :-

Chain Code (Freeman)

Boundary : A connected set of straight line segments of specified length & direction

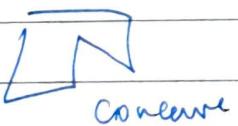


Encode as: 0, 7, 0, 6, 6, 3



Initialization depends.

→ Polygonal approx.



convex

Minimum perimeter polygon : stretch rubber band

10/11/20

Digital Video Processing:

- First moving pic were on film.
- because of bet
- does horse have all 4 hooves off ground.
- Mybridge experiment :- animals fire for each shot.
Standford won the bet & all 4 hooves were off.
- viewing these photos really fast.
- Film is an analog medium but it discards in time.
[]
- TV is a technology for transmission & reprod of moving picture.
- Rasterisation : allows images to be converted into ID signals for transmission. [signals continuous \rightarrow disc \uparrow]

Signal represented in Y U V format instead of RGB.
NTSC composite vid :- $x = Y + U \sin(\omega t + 33^\circ) + V \cos(\omega t + 33^\circ)$

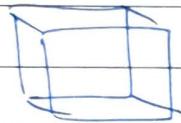
sin
cosine
change
tint
in colour

→ Storing TV signals on magnetic tape
Latest era : HD DVD, Blu-ray, HDV.

- Progressive Scanning
- Interlaced :- First draw odd lines then go back & draw even lines.
↓ diff to capture still frames from TV.

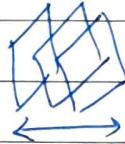
Digital video capture :- CCD array capture
Variant of JPEG used for video compression.

- CCD vs CMOS ... cheaper.
CMOS use 'rolling shutter'.
 - Sense exposed line by line (may be prob)
 - wavy looking image
- HVS extremely sensitive to motion. Capturing motion in fMRI ... interested in studying brain function.
 - Stack of frames in a video.
 - Take video & think as certain groups of frames.



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channel.



apply on
these
individual
frame.

- Gait based biometrics.
- Diversity of Motion :-
- True scale
- Localized? specific locations?

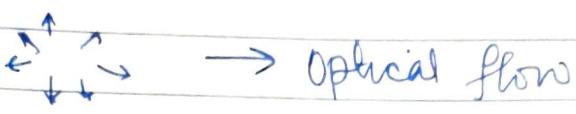
Predict how camera moved in 3D space.

- Time varying image formation model.
- 2 side ! $\underbrace{\text{geometric}}_{\text{movement \& strut}}$ & $\underbrace{\text{photometric}}_{\text{appearance}}$

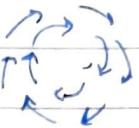
- Movie editing :-
camera panning, zoom in & out, combination.

→ Camera Motion:

a) Camera zoom

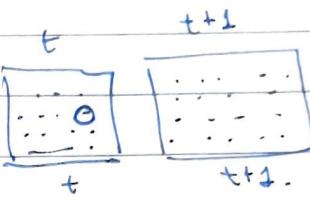


b) Camera rot around z-axis (roll.)



c) Flow view: if \exists some intensity at some loc (x,y) at time t , "we typically have coherent motion"

$I_t(x,y)$ goes where $I_{t+1}(x,y)$.



- Use flow vector to characterize this. Bunch of arrows.

- looking at flow field we can say about the motion.

$$\text{ie: } I_t + \Omega \rightarrow I_{t+1}.$$

• Real World Motion:

→ Rigid motion vs deformable motion.

→ Overlapping vs. non overlapping

Optical flow can't be used as a feature representation or descriptions (raw info).

Form histogram over Ω .

→ Along with RGB we want motion vectors as well.

Video acquisition \rightarrow camera / computer gen videos.

→ Removing unwanted tracking marks / watermarks by video inpainting. Exploit temporal nature of video.

Use values from surrounding pixels to change.

- 1) Use Isolate the text.
- 2) Then remove.

- Blocky artifacts Error concealments
removing info
- Superresolution :- LR seq \rightarrow HR seq.
intelligent upsampling

- Background subtraction :- identify bg objects in that image.

- sudden / gradual illumination changes.
- High freq rep. motion in bg.
- long term scene changes.

pixels \rightarrow sequences

Thresholding frame I_t , I_{t-1} , pixelwise thresh.
 $|I(x,y,t) - I(x,y,t-1)| > \text{Th.}$

Somewhat better way is to use mean filter.
 Estimate bg as mean of prev. frames.
 i.e. Obj not present all the time

Obj captured & stretched across.

Median fairly robust & gives better results.

→ If frames changing faster & will be higher