Assignment 4

Computational Intelligence

Team Members		
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1 Maximum Likelihood Estimation – BACKGROUND

2 Maximum Likelihood Estimation of Model Parameters

2.1

Regarding scenario 2, we found out that anchor $i = 0$ is exponentially distributed (fig. 1), whereas the other anchors are gaussian distributed (f.e. anchor 1 in fig. 2).

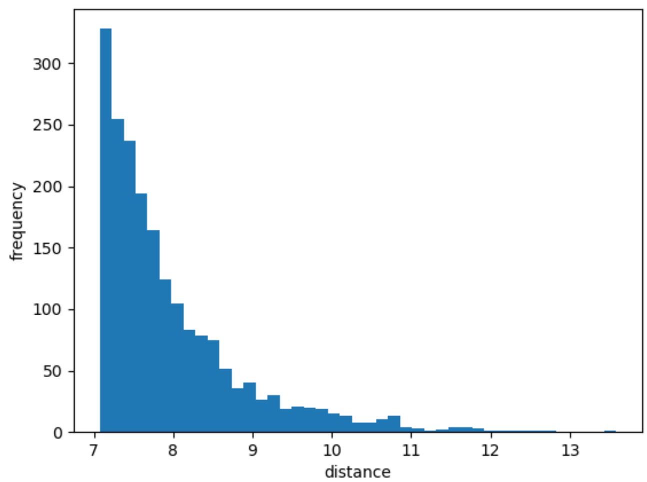

Figure 1: Histogram of exponential distributed measurements for anchor 0.

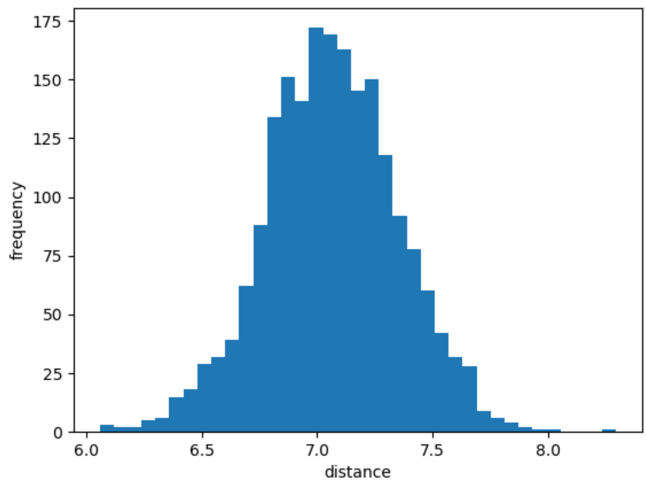

Figure 2: Histogram of gaussian distributed measurements for anchor 1 (similar distributed as 2 and 3).

2.2

$$p(r_i|p) = \begin{cases} \lambda_i e^{-\lambda_i (r_i - d_i(p))} & r_i \geq d_i(p) \\ 0 & \text{else} \end{cases}$$

$\vec{x} = \vec{r}_i - \vec{d}_i$

(2000x1) measurements

$n = 2000$

Likelihood function, since the meosurements are indipendently and identically-distributed random variables:

$$L(\lambda_i, x_1 .. x_n) = \lambda^n \exp\left(-\lambda_i \sum_{j=1}^{n} x_j\right)$$

The Log-Likelihood function is:

$$\ell(\lambda_i, x_1 .. x_n) = n \ln(\lambda) - \lambda \sum_{j=1}^{n} x_j$$

Maximum likelihood

$$\hat{\lambda}_i = \arg\max_{\lambda} (\lambda_i, x_1 \dots x_n)$$

$$\frac{d}{d\lambda_i} \ell(\lambda_i, x_1 .. x_n) \overset{!}{=} 0$$

$$\frac{n}{\lambda_i} - \sum_{j=1}^{n} x_j = 0$$

$$\boxed{\hat{\lambda}_i = \frac{n}{\sum_{j=1}^{n} x_j}}$$

Figure 3: Analytic derivation of the maximum likelihood solution for the exponential distribution.

2.3

Scenario 1: Estimation of variance parameters σ_i^2 for 4 anchors:
[0.09089433] [0.08836127] [0.08704619] [0.09239089]

Scenario 2: Estimation of λ_0 for the exponentially distributed anchor measurements and the variance parameters σ_i^2 (i = 1,2,3):
[0.12557937] [0.08369921] [0.0933427] [0.08984957]

Scenario 3: Estimation of λ_i (i = 0,1,2,3) for the exponentially distributed anchor measurements:
[0.12504257] [0.12516356] [0.12545039] [0.12577921]

3 Estimation of the Position

3.1 Least-Squares Estimation of the Position

3.1.1 Equivalence of least squares estimator to ML estimator

$$\hat{\underline{p}}_{ML} = \underset{\underline{p}}{\operatorname{argmax}}\ p(\underline{r}\,|\,\underline{p}) = \underset{\underline{p}}{\operatorname{argmax}} \prod_{i=1}^{N_a} p(r_i\,|\,\underline{p})$$

Logarithm

$$\overset{\ln}{=} \underset{\underline{p}}{\operatorname{argmax}} \sum_{i=1}^{N_a} -\frac{1}{\sqrt{2\pi}\,\sigma_i}\,\frac{\left(r_i - d_i(\underline{p})\right)^2}{2\,\sigma_i^2}$$

$$\simeq \underset{\underline{p}}{\operatorname{argmin}} \sum_{i=1}^{N_a} \left(r_i - d_i(\underline{p})\right)^2 = \underset{\underline{p}}{\operatorname{argmin}} \left\| \underline{r} - d(\underline{p}) \right\|^2$$

Figure 4: Analytic derivation of the equivalence of least squares estimator to ML estimator.

3.1.2 Gauss-Newton implementation

We implemented the Gauss-Newton algorithm and ran evaluates our estimation algorithm with tol = 10^{-11} and max-iter = 10^{11}.

3.1.3 Evaluation of the estimation algorithm

Scenario 1:
Error mean: -0.03333375255793272 Error var: 0.047513610107281445

Scenario 2:
Error mean: 0.13822625457856347 Error var: 0.2743078055741306

Scenario 3:
Error mean: -0.17933954759700965 Error var: 1.3370474393938168

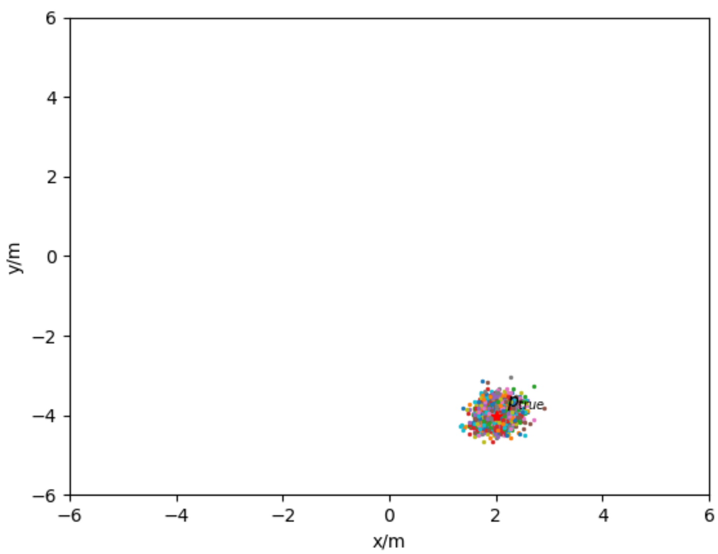

Figure 5: Scatter plot of the estimated positions for scenario 1.

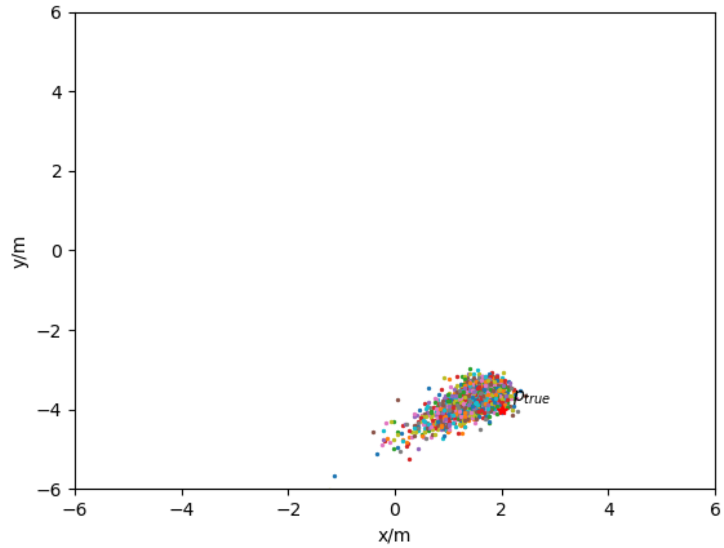

Figure 6: Scatter plot of the estimated positions for scenario 2.

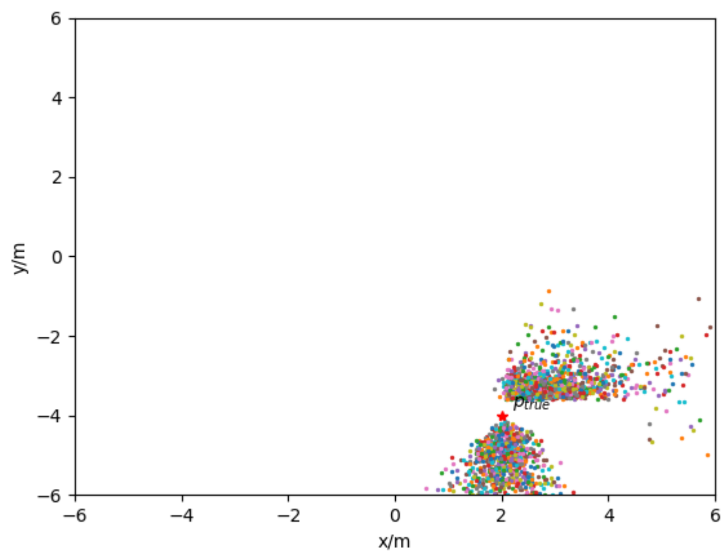

Figure 7: Scatter plot of the estimated positions for scenario 3.

In figure 5 we can see that the estimated positions are gaussian distributed around the true value for gaussian distributed measurements. However, in figures 6 to 7 we see non-gaussian distributions, as the measurements are partly exponentially distributed for scenario 2 and completely exponentially distributed in scenario 3. The least-squares approximation holds only for gaussian distributed data, which explains the increased variance of the estimation error for scenarios 2 and 3 (figures 9 and 10).

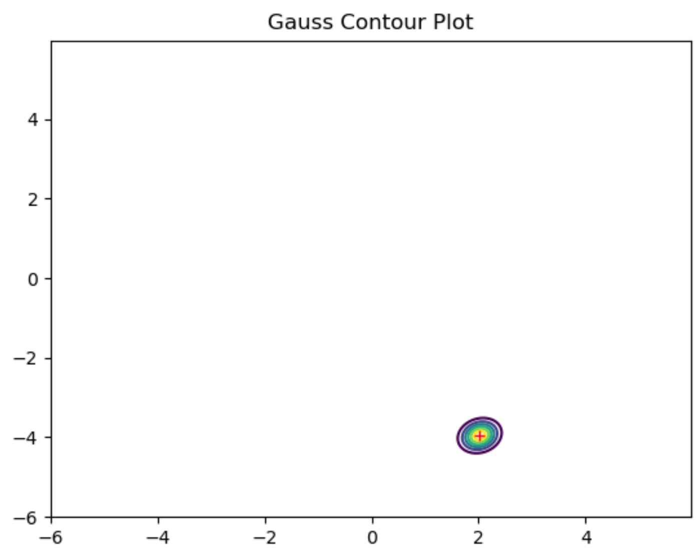

Figure 8: Gauss contour plot of the estimated positions for scenario 1.

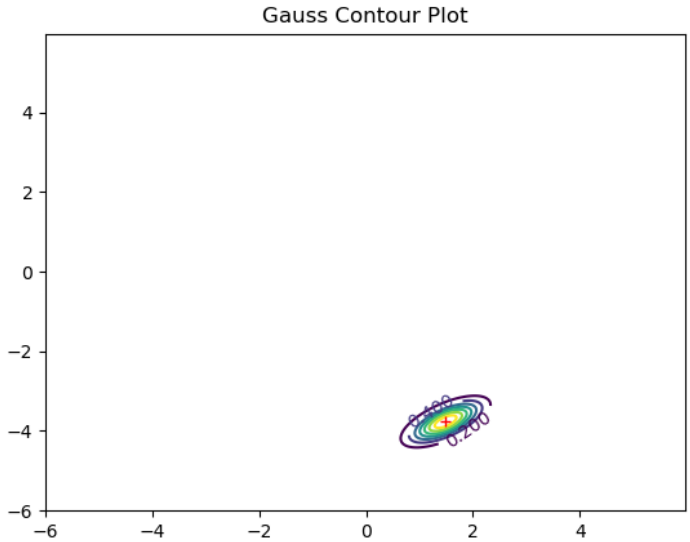

Figure 9: Gauss contour plot of the estimated positions for scenario 2.

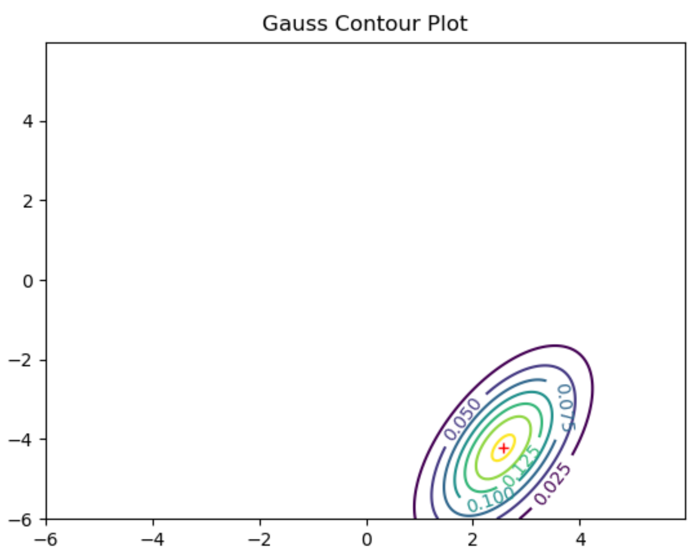

Figure 10: Gauss contour plot of the estimated positions for scenario 3.

In figure 11 we can see that the probability of large estimations errors is very small, as the probability converges to 1 at an estimation error of 0.1. For scenario 3 (figure 13)we see an increased range of estimation errors including values up to 4.

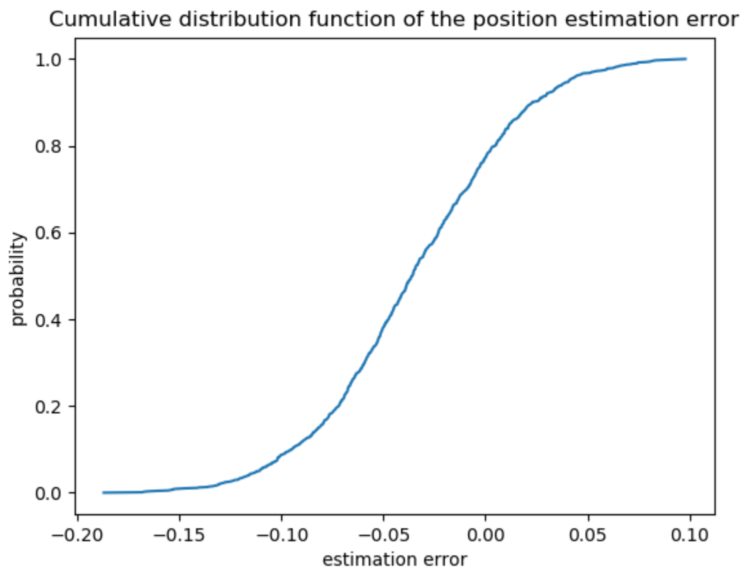

Figure 11: CDF plot of the estimation error for scenario 1.

3.1.4 Neglection of the exponentially distributed anchor

Neglecting the exponentially distributed anchor leaves us still with 3 equations and two unkwons (x and y position to be estimated). This means the estimation problem is still solvable and we expect better and gaussian distributed estimates of the position. However, simulations have not been carried out.

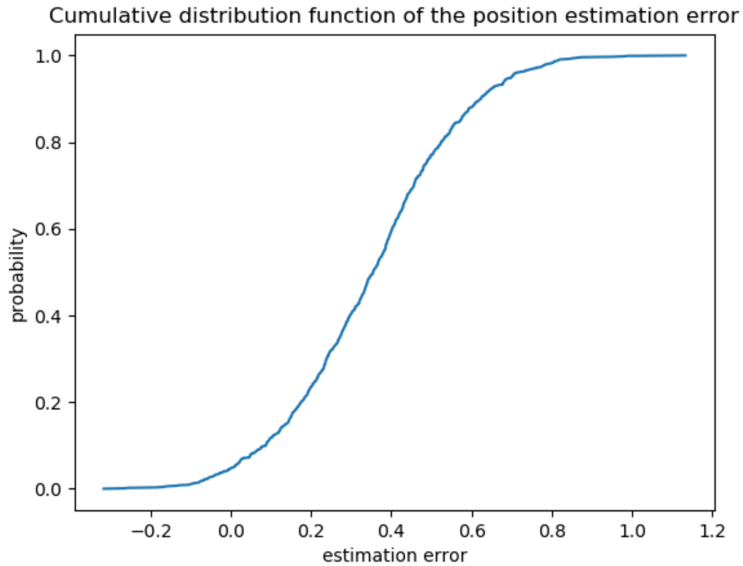

Figure 12: CDF plot of the estimation error for scenario 2.

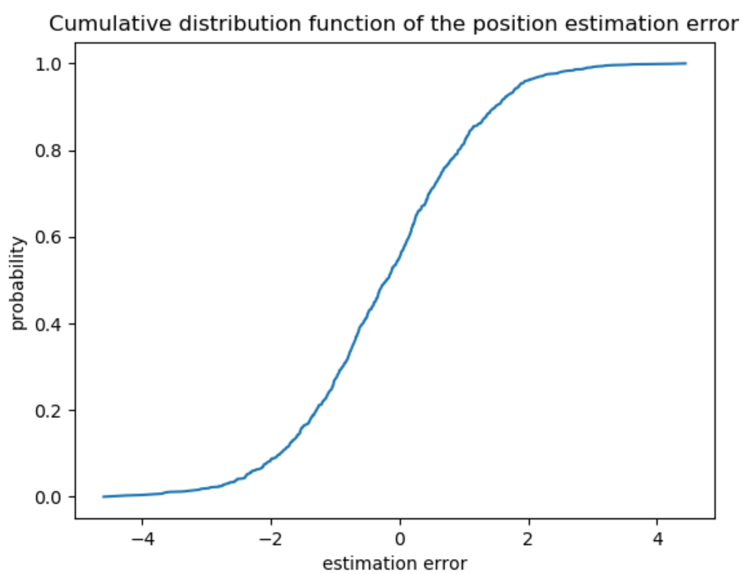

Figure 13: CDF plot of the estimation error for scenario 3.