slfdde method

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1 System definition

This method computes numerical solution of system of linear fractional differential equations of matrix form:

$$\mathrm{D}_{t_0}^{\alpha}\mathbf{x}(t) = \mathbf{A}(t)\mathbf{x}(t) + \mathbf{B}(t)\mathbf{x}(t-\tau) + \mathbf{f}(t),$$

$\mathbf{A}(t), \mathbf{B}(t)$ are matrices $(n \times n)$ depending on time t,
$\mathbf{f}$(t) is column vector $(n \times 1)$ depending on time t,
$\mathbf{x}(t)$ is column vector $(n \times 1)$ of variables in time t,
$\mathbf{x}(t-\tau)$ is column vector of variables in time $t-\tau$,
α is order of the system and
t_0 is initial time.

2 Input values

To call this method, use command:

```
[t,x]=sfdde(met,limit,t0,T,tau,h,alfa,x0,A,B,f)
```

- *met* stands for method used for numerical integration, $met = 0$ to use explicit rectangular method and $met = 1$ to use implicit rectangular method.
- *limit* is upper bound for case of a function that is unbounded as $t \to 0$.
- $t0$ is initial time.
- T is end time.
- *tau* is time delay.
- h is step between any time t_j and t_{j+1}.
- *alpha* is order of system.
- $x0$ is initial function.
- A, B are matrices.
- f is vector of functions.

2.1 Example

Investigate behaviour of system described by following system of equations. Use initial time $t_0 = 0$, end time $T = 70$ with delay $\tau = \pi$. Step in the time series is $h = 0.01$, order of system is $\alpha = \frac{2}{5}$ and method used for integration is explicit rectangular.

$$\mathrm{D}^{\alpha}_{t_0}\begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -2 & 0 & 0 \\ -2 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -2 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \end{pmatrix}\begin{pmatrix} x_1(t-\tau) \\ x_2(t-\tau) \\ x_3(t-\tau) \\ x_4(t-\tau) \end{pmatrix}$$

with initial function

$$\mathbf{x}_0(t) = \begin{pmatrix} sin(t)cos(t) \\ sin(t)cos(t) \\ cos^2(t) - sin^2(t) \\ cos^2(t) - sin^2(t) \end{pmatrix}, \quad t_0 - \tau \leq t \leq t_0. \tag{1}$$

Input for such a system is:

```
met=0;t0=0;T=70;tau=pi;h=0.01;alfa=2/5;
x0=@(t)[sin(t)*cos(t);sin(t)*cos(t);
   (cos(t))^2-(sin(t))^2;(cos(t))^2-(sin(t))^2];
A=@(t)[0 0 1 0;0 0 0 1;0 -2 0 0;-2 0 0 0];
B=@(t)[0 0 0 0;0 0 0 0;-2 0 0 0;0 -2 0 0];
f=@(t)[0;0;0;0];
```

3 Output and solution

After computation, method returns:

- time series t, row vector $(1 \times N)$,
- vector of solution $\mathbf{x}(t)$, matrix $(n \times N)$.

To plot $i-$th component of variable $\mathbf{x}$ depending on time, use command:

```
plot(t,x(i,:))
```

To plot phase portrait of variables depending on each other, use command:

```
plot(x(i,:),x(j,:))
```

Variables x_1 and x_3 of system [1] depending on time are shown in figures [1] and [2]. Figure [3] shows phase portrait of variables x_1 and x_3.

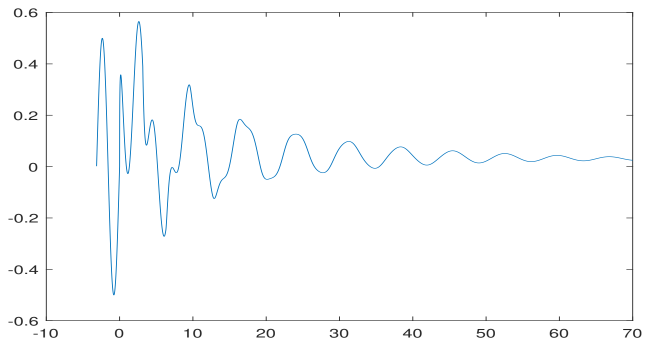

Figure 1: x_1 depending on time

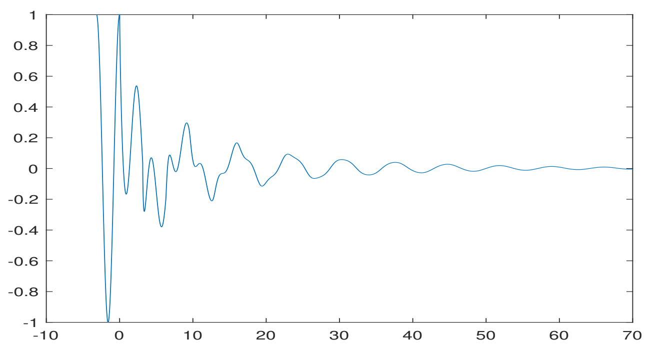

Figure 2: x_3 depending on time

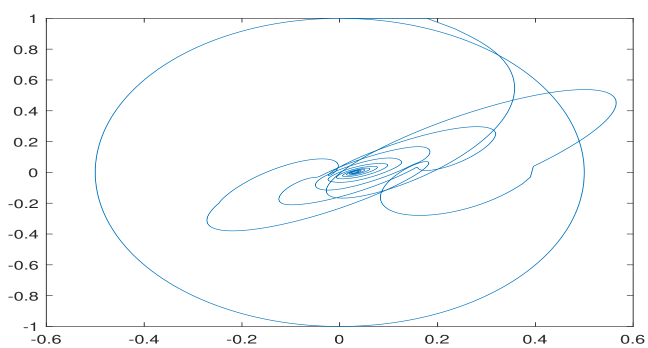

Figure 3: Phase portrait of x_1 and x_3