ICCV 2011 Tutorial: Non-Rigid Structure from Motion

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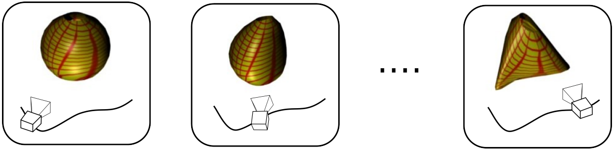

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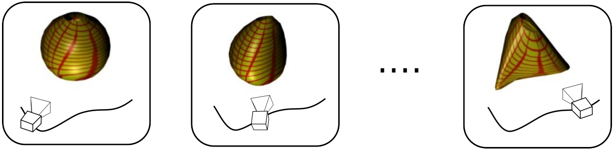

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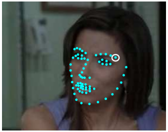

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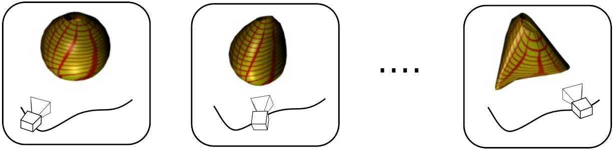

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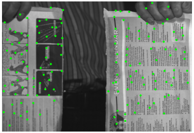
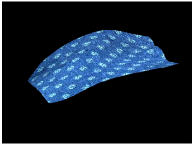
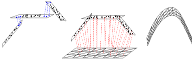
Physical priors

Single object (C_0 continuity).
Smooth surface (C_1 continuity).
Inextensibility, isometry.
Elasticity.
Temporal and spatial smoothness.
Piecewise planar/quadratic/rigid.







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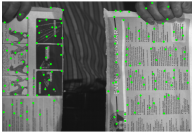
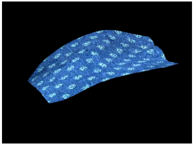
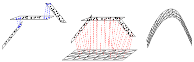
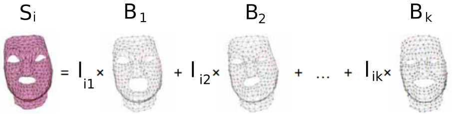






Statistical priors

Low rank shape basis.
Gaussian priors.
Dynamics.



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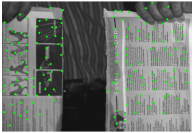

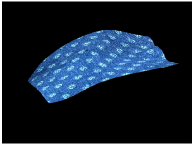

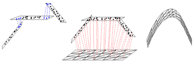

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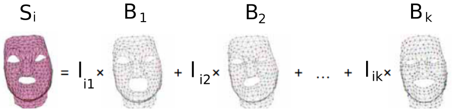
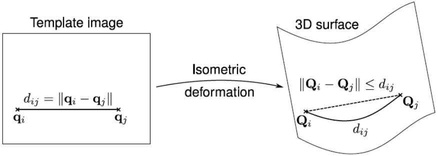
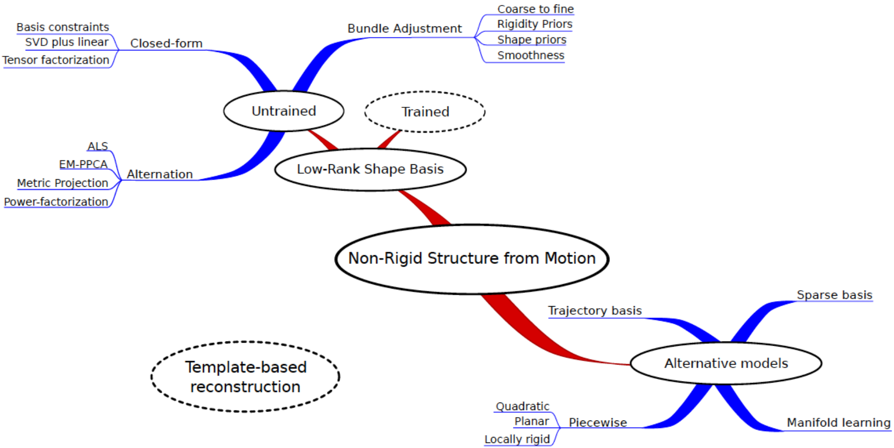


Known 3D template



A Taxonomy of Approaches to Non-Rigid 3D Reconstruction



Current Challenges I

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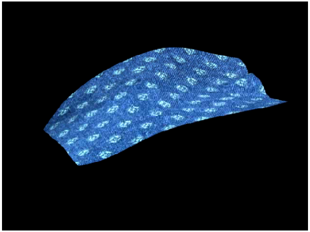

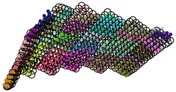

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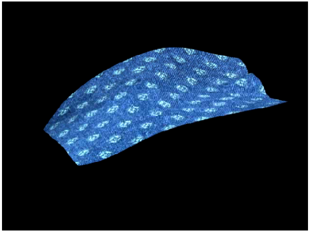
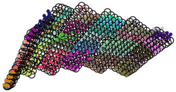
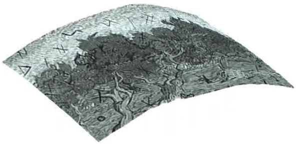


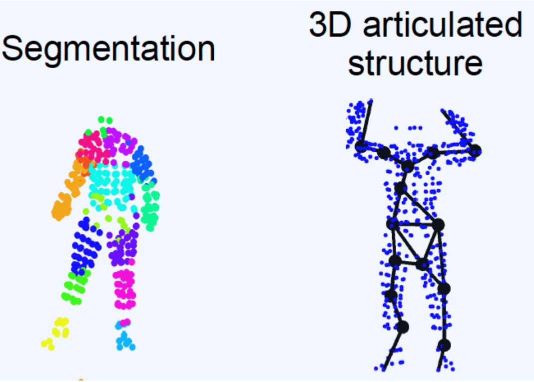


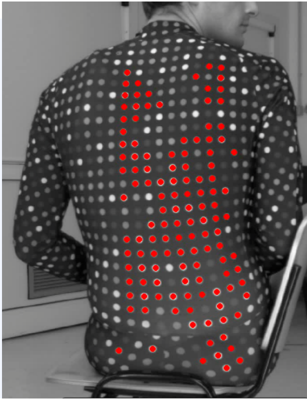
- **Dense: Optical Flow/ 3D Modelling**



Current Challenges II

- **Human Motion Modelling**





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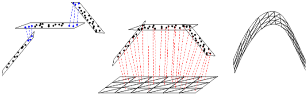

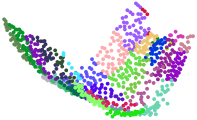

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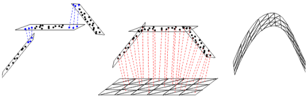
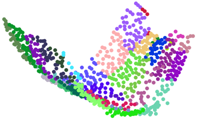
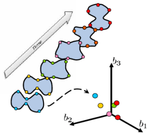
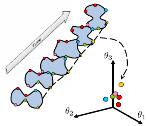




- **Trajectory Space** (Akhter et al. NIPS'08 PAMI'11), (Park et al. ECCV'10)





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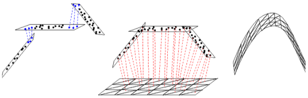

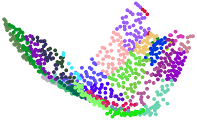

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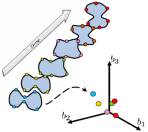
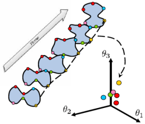
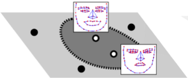
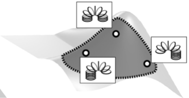




- **Manifold Learning** (Rabaud and Belongie CVPR'08)





Roadmap

- Piecewise Approaches to NRSfM.
- Trajectory Space.
- Smooth Time Shape Trajectory.

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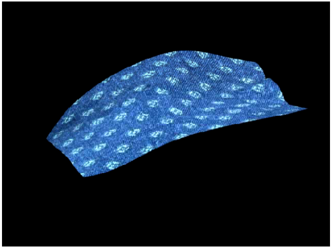
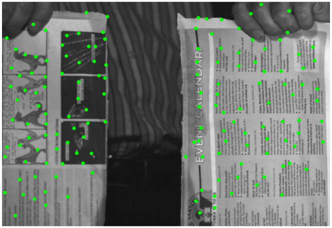
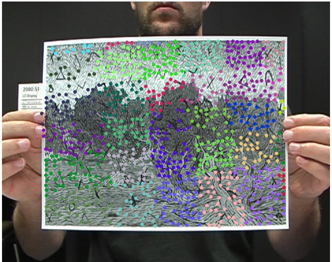
Piecewise Reconstruction of Deformable Surfaces

MOTIVATION: 3D Reconstruction of highly deformable surfaces.







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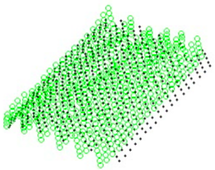
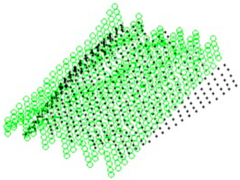


EM-PPCA 17.9% 3D error
(Torresani et al. PAMI 2008)



Metric Projections 18.6% 3D error
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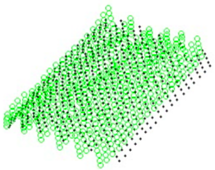

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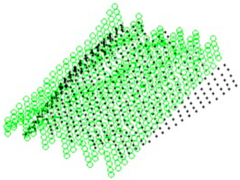

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- New family of solutions: **Piecewise modelling**.

Piecewise Reconstruction of Deformable Surfaces

- **Intuition:** Local models have fewer parameters therefore they are easier to optimise and less prone to overfitting.
- Spatial consistency is enforced between overlapping patches to create a continuous global surface.

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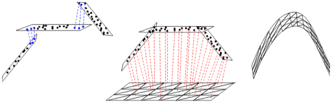
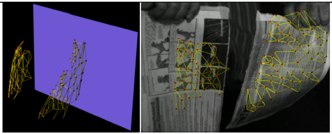
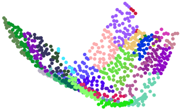
Piecewise Planar	"Triangle Soup"
 (Varol et al. ICCV'09)	 (Taylor et al. CVPR'10)

Locally Planar	Piecewise Quadratic
 (Collins and Bartoli VMV'10)	 (Fayad et al. ECCV'10)

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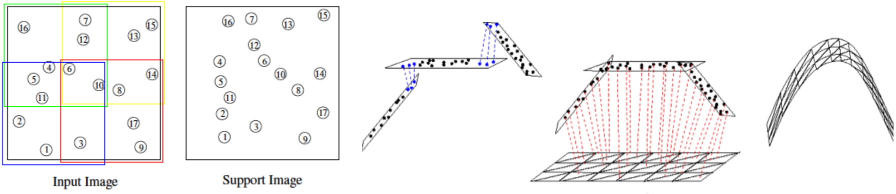

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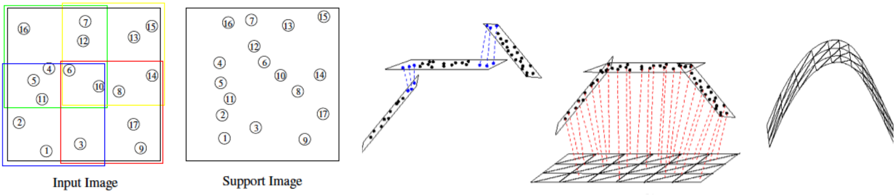

1. The first image is divided into overlapping patches.
2. Pair-wise correspondences are used to fit a homography to each plane.
3. The homographies are decomposed to reconstruct each plane in 3D.
4. Overlapping points are used to solve for the scale ambiguity and stitch patches.
5. A mesh is fit to the point cloud.

Piecewise Planar

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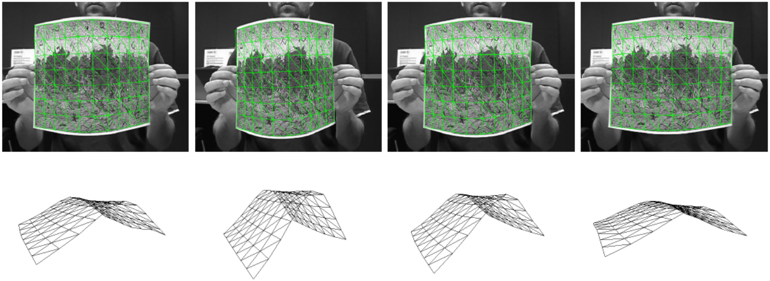
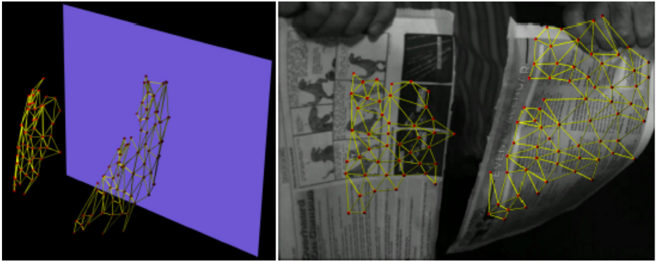


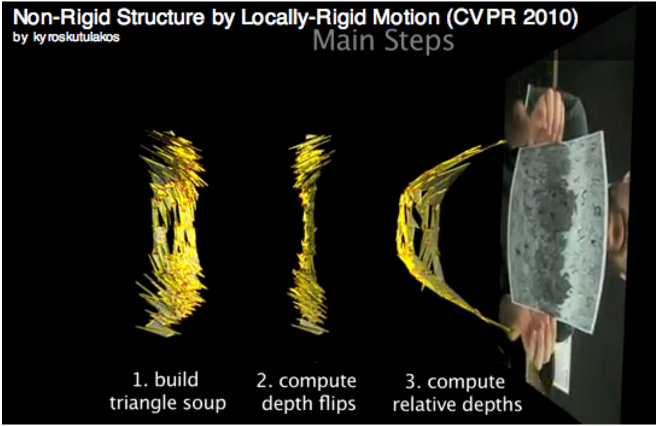
Non-Rigid Structure from Locally Rigid Motion

(Taylor et al. CVPR'10)



- Local rigidity is assumed for every triple of neighbouring points.

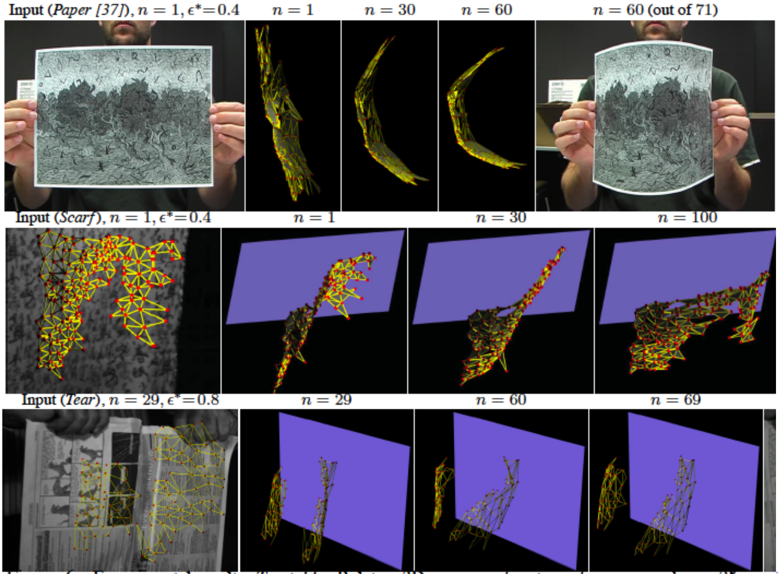
Non-Rigid Structure from Locally Rigid Motion



- Each triangle is reconstructed individually (linear)="triangle soup".
- Disambiguation step: reflection states (NP-hard) and relative depths.
- Implicitly assumes dense tracking data (local rigidity only valid when points are close).

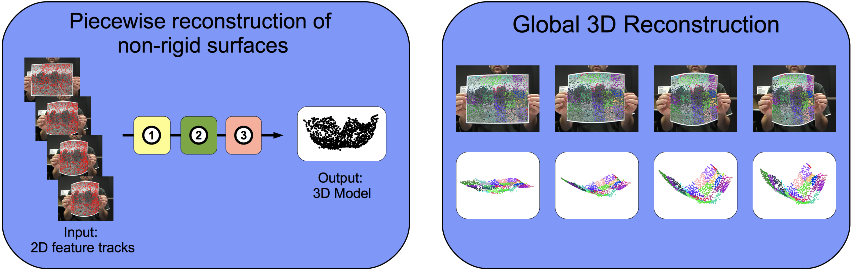
Non-Rigid Structure from Locally Rigid Motion

(Taylor et al. CVPR'10)



Piecewise Reconstruction of Deformable Surfaces

(Fayad-Agapito-Del Bue ECCV'10)



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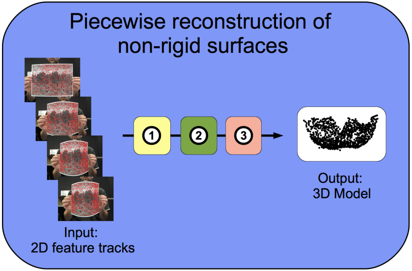

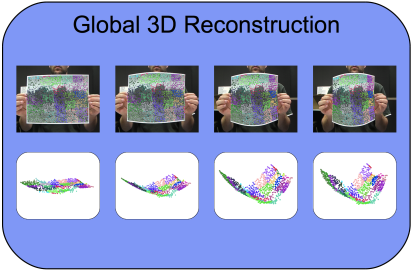

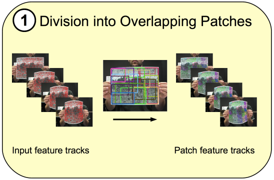

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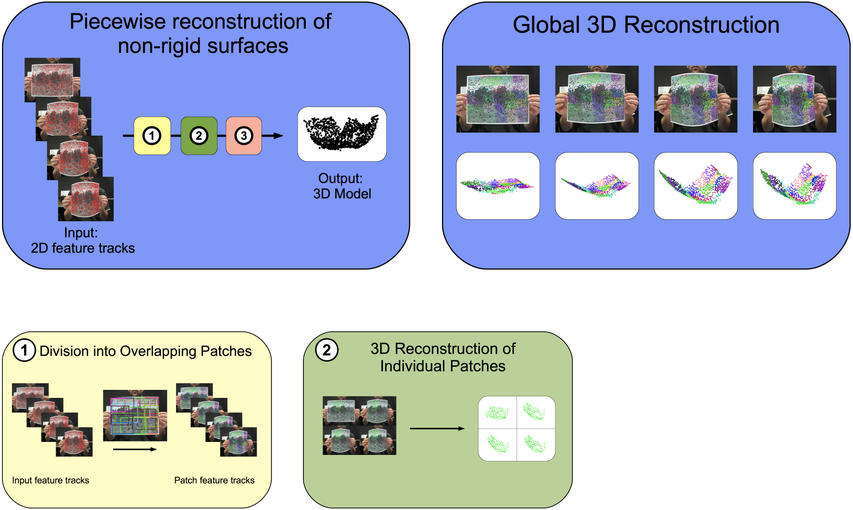

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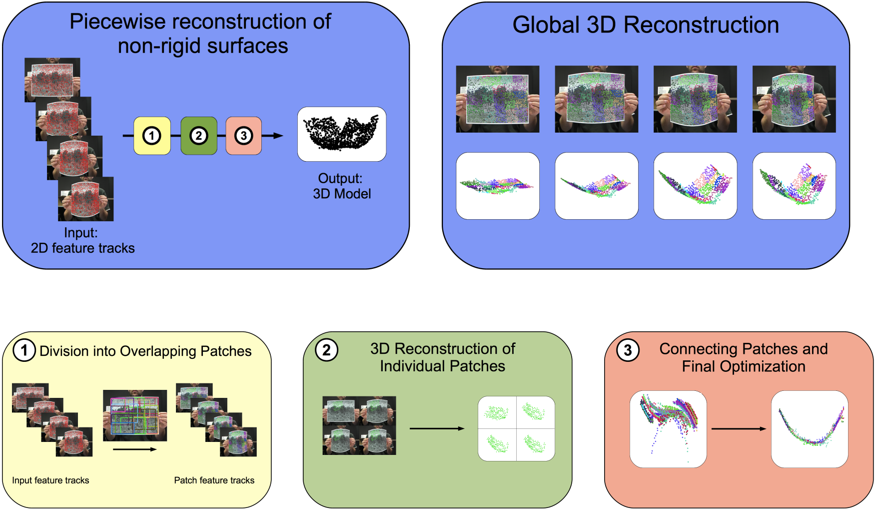

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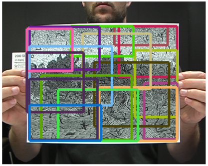
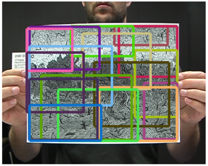


We contemplate three different scenarios:

1. Division into overlapping patches



We contemplate three different scenarios:

1. Known 3D reference template.

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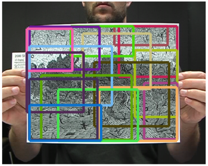
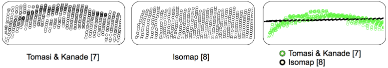


We contemplate three different scenarios:

1. Known 3D reference template.
2. Surface is planar.

"Flatten" surface using ISOMAP.



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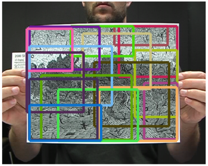

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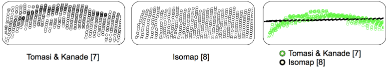
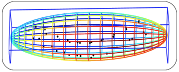
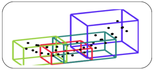


3. No a priori knowledge.

Fit an ellipsoid to data.



Ellipse and corresponding bounding box



Division into 4 overlapping patches

2. Reconstructing individual patches

To account for complex deformations we allow patches to deform according to the **quadratic deformation model** (Fayad et al. BMVC2009).

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$$\begin{array}{ccc} p_{x1} & & p_{xP} \\ p_{y1} & \cdots & p_{yP} \\ p_{z1} & & p_{zP} \\ \hline \end{array}$$

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$$
\begin{array}{ccc}
p_{x1} & & p_{xP} \\
p_{y1} & \cdots & p_{yP} \\
p_{z1} & & p_{zP} \\
\hline
p_{x1}^2 & & p_{xP}^2 \\
p_{y1}^2 & \cdots & p_{yP}^2 \\
p_{z1}^2 & & p_{zP}^2 \\
\hline
\end{array}
$$

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$$
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p_{z1} & & p_{zP} \\
\hline
p_{x1}^2 & & p_{xP}^2 \\
p_{y1}^2 & \cdots & p_{yP}^2 \\
p_{z1}^2 & & p_{zP}^2 \\
\hline
p_{x1}p_{y1} & & p_{xP}p_{yP} \\
p_{y1}p_{z1} & \cdots & p_{yP}p_{zP} \\
p_{z1}p_{x1} & & p_{zP}p_{xP}
\end{array}
$$

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$$\mathtt{S}_i = [\mathtt{L}_\mathtt{i}\mathtt{Q}_i\mathtt{M}_i]_{3\times 9} \left[\begin{array}{ccc} p_{x1} & & p_{xP} \\ p_{y1} & \cdots & p_{yP} \\ p_{z1} & & p_{zP} \\ \hline p_{x1}^2 & & p_{xP}^2 \\ p_{y1}^2 & \cdots & p_{yP}^2 \\ p_{z1}^2 & & p_{zP}^2 \\ \hline p_{x1}p_{y1} & & p_{xP}p_{yP} \\ p_{y1}p_{z1} & \cdots & p_{yP}p_{zP} \\ p_{z1}p_{x1} & & p_{zP}p_{xP} \end{array}\right]_{9\times P}$$

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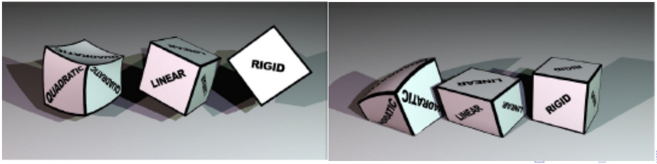
Deformation Modes



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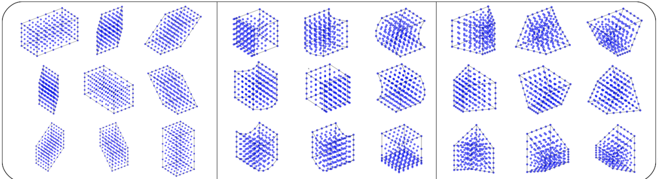
Linear ($\mathtt{L}_{3\times 3}$) Quadratic($\mathtt{Q}_{3\times 3}$) Mixed ($\mathtt{M}_{3\times 3}$)



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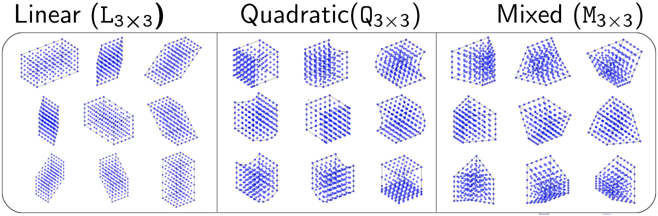

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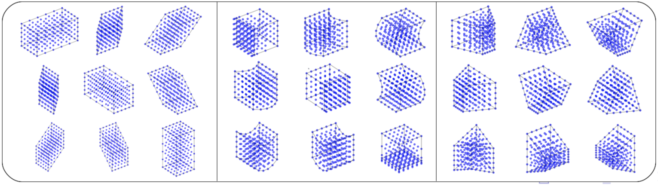
$$\mathtt{S}_i = [\mathtt{L}_\mathtt{i}\,\mathtt{Q}_i\mathtt{M}_i]\,\mathtt{S}$$

The 2D image coordinates of the points observed by an orthographic camera:

$$\mathtt{W}_i = \mathtt{R}_i\,[\mathtt{L}_\mathtt{i}\,\mathtt{Q}_i\mathtt{M}_i]\,\mathtt{S} + \mathbf{t}_i$$

Linear ($\mathtt{L}_{3\times3}$) Quadratic($\mathtt{Q}_{3\times3}$) Mixed ($\mathtt{M}_{3\times3}$)



2. Reconstructing individual patches: Parameter Estimation

The model parameters are estimated by minimizing the image reprojection error of all the points in all the views:

$$\min_{\mathbf{R}_i,\mathbf{t}_i,\mathtt{L}_i,\mathtt{Q}_i,\mathtt{M}_i} \sum_{i,j}^{F,P} \left|\left|\mathbf{w}_{ij} - \left(\mathtt{R}_i \left[\mathtt{L}_i \; \mathtt{Q}_i \; \mathtt{M}_i\right] \mathbf{S}_j + \mathbf{t}_i\right)\right|\right|^2$$

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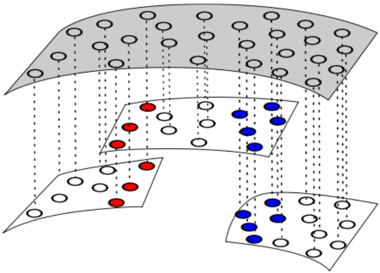
- Temporal smoothness priors are added for all the parameters.
- Minimized using sparse Levenberg-Marquardt.
- Initialized as a rigid object.

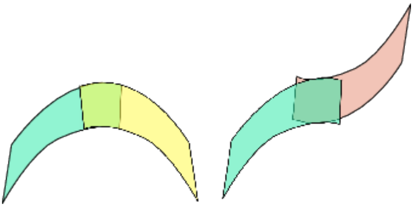
3. “Patch Stitching”

Overlap between neighbouring regions is used to enforce spatial consistency and create a continuous global surface.

Translation Ambiguity



Reflection Ambiguity



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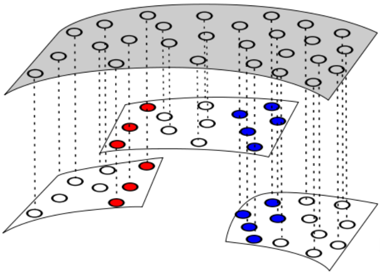
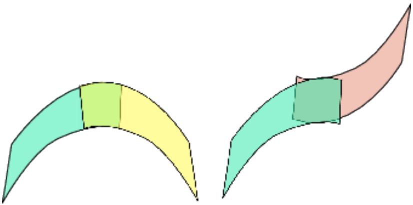


Align centroid of overlapping points along Z-axis

Reflection Ambiguity



Select reconstruction that minimizes distance between shared points

4. Final Optimization

$$\sum_{i,j}^{f,p} \sum_{n\in\Theta_j} \left|\left|\mathbf{w}_{ij}^{(n)} - \hat{\mathbf{w}}_{ij}^{(n)}\right|\right|^2 + \eta \sum_{k\in\Theta_j/\{n\}} \left|\left|\hat{\mathbf{X}}_{ij}^{(n)} - \hat{\mathbf{X}}_{ij}^{(k)}\right|\right|^2,$$

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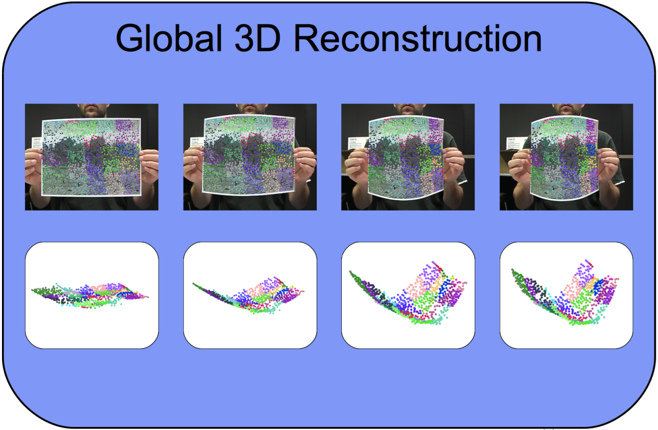
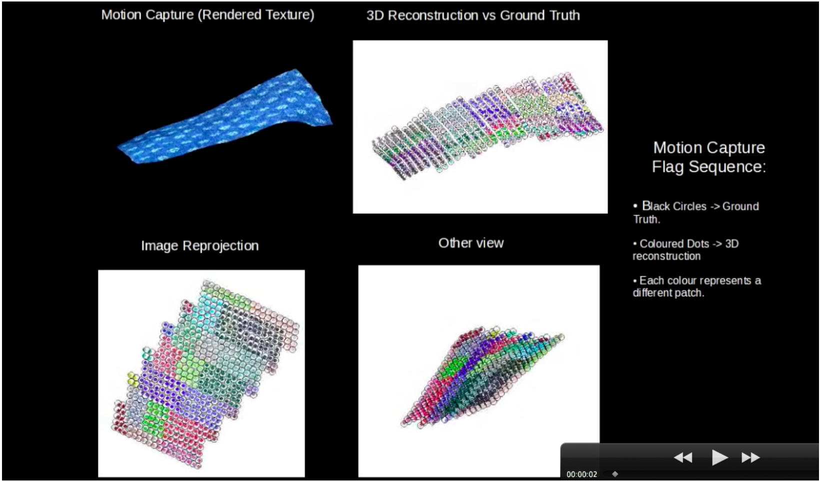


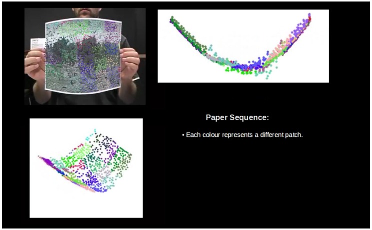
Experimental Results

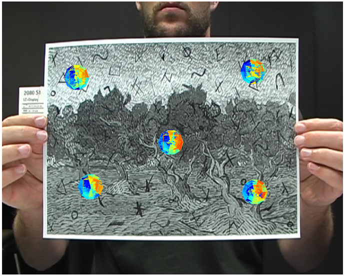
Flag Sequence



Experimental Results

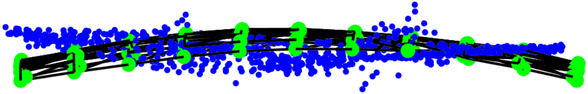
Paper Sequence





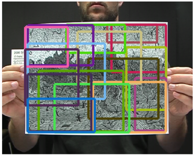
Experimental Results

Comparison with piecewise planar method



Network of Overlapping Models

- So far the division of the surface into patches is defined manually.



- Goal: to provide a **principled formulation** for patch division.
- What is the best division of points into patches?

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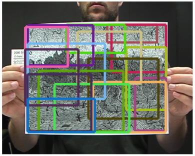


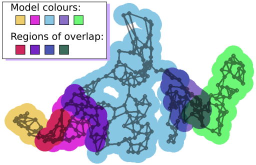
- Goal: to provide a **principled formulation** for patch division.

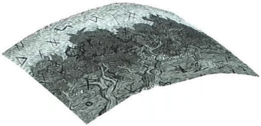
- What is the best division of points into patches?

- What is the **best assignment of points to models?**

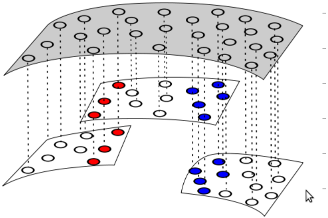
A new approach to 3d reconstruction

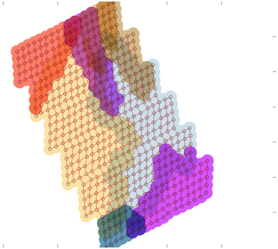
(Russell-Fayad-Agapito CVPR'11), (Fayad-Russell-Agapito ICCV'11)
Using overlapping local models to explain tracks and construct 3d models





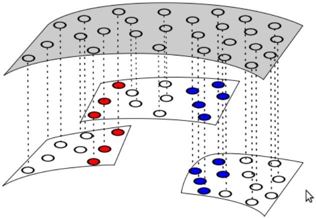
Overlapping Models

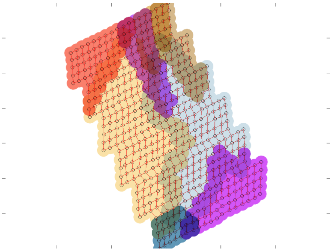




We segment *overlapping* models, and force the reconstructions to agree about the regions of overlap.

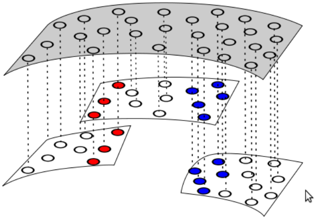
Overlapping Models

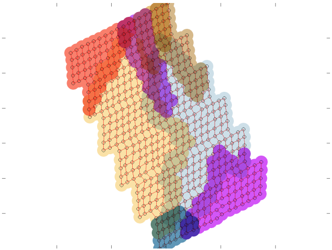




We segment *overlapping* models, and force the reconstructions to agree about the regions of overlap.
Bonus: Implicit higher-order smoothing can come from forcing models to explain the same data.

Overlapping Models





We segment *overlapping* models, and force the reconstructions to agree about the regions of overlap.
Bonus: Implicit higher-order smoothing can come from forcing models to explain the same data.
First proposed Fayad et al. ECCV'10 - they couldn't optimise this

Core algorithm

- A set of initial models is proposed by fitting the quadratic model to K-nearest neighbours of each point.
- Iterative algorithm.

```
Δ = −1;
while (Δ < 0) do
    CurrentError = GetError();
    Points = BestAssignment(CurrentModels);
    CurrentModels = BestFit(Points);
    NewError = GetError();
    Δ = NewError−CurrentError;
end
```

BestAssignment(CurrentModels) has an MDL term to encourage sparsity.

Energy Based Multiple Model Fitting for NRSFM

Russell-Fayad-Agapito CVPR 2011

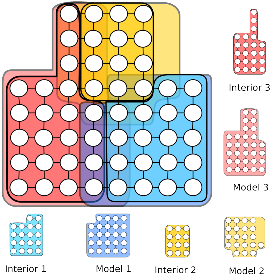
- The problem of 3D reconstruction of deformable surfaces is re-formulated as a labelling problem.
- The labels represent the parameters of a model that must be fitted to the data.
- The model parameters and the assignment of points to an instance are chosen to minimize some fitting error.
- Minimize **geometric model fitting cost**, subject to a spatial constraint: neighbouring points should belong to the same model.

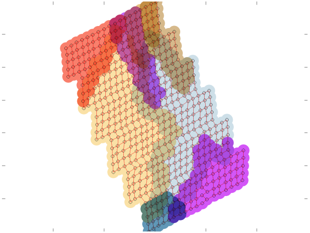
$$C(\mathbf{m}) = \sum_{p \in \mathcal{P}} \left(\sum_{\bigcup_{q \in \mathcal{N}_p} \{\alpha : q \in I_\alpha\}} U_p(\alpha) \right).$$

- We provide a joint optimization of the assignment of points to models and the fitting of models to points.

Energy Based Multiple Model Fitting for NRSFM

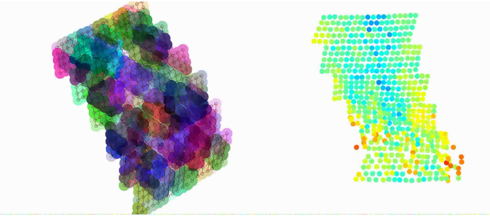
- Piecewise methods require overlapping points between patches.





- The classic labelling problem has to be modified: more than one label can be assigned to each point.
- We have modified α-expansion to allow points to belong to more than one model.
- Topological constraint: A point is an *interior point* of a model, if all of its neighbours belong to the same model
  Key idea: Every point must be an interior point of at least 1 model

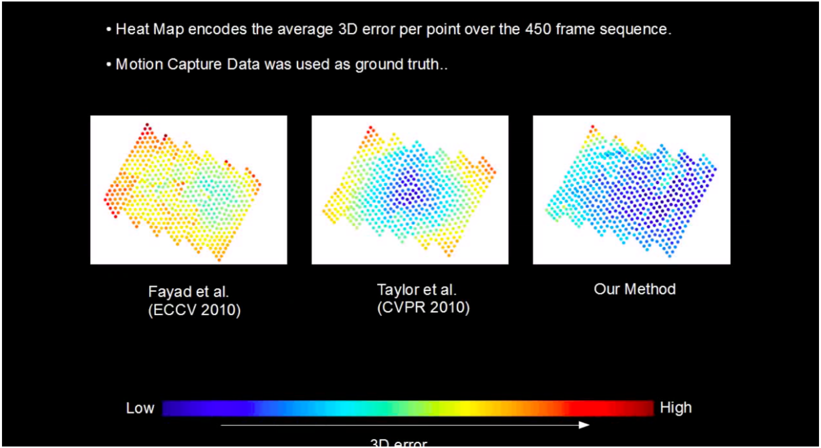
Algorithm Evolution



Practical Issues

- We introduce a robustness term to cope with outliers.
- We introduce a global MDL prior to penalize solutions with a large number of models.
- Our formulation can deal with models of any type: linear, quadratic, planar... or a mixture of models.

Experimental Results



Numeric scores

Data set	Fayad ECCV10	Taylor CVPR10	Our work
Flag	3.25%	2.63%	1.59%
Back	15.20%	-	9.17%

Proportional 3d error

Numeric scores

Data set	Fayad ECCV10	Taylor CVPR10	Our work
Flag	3.25%	2.63%	1.59%
Back	15.20%	-	9.17%

Proportional 3d error
Our time 20 minutes

Numeric scores

Data set	Fayad ECCV10	Taylor CVPR10	Our work
Flag	3.25%	2.63%	1.59%
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Proportional 3d error
Our time 20 minutes
Fayad – 10 minutes

Numeric scores

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Proportional 3d error
Our time 20 minutes
Fayad – 10 minutes (excluding human labelling)

Numeric scores

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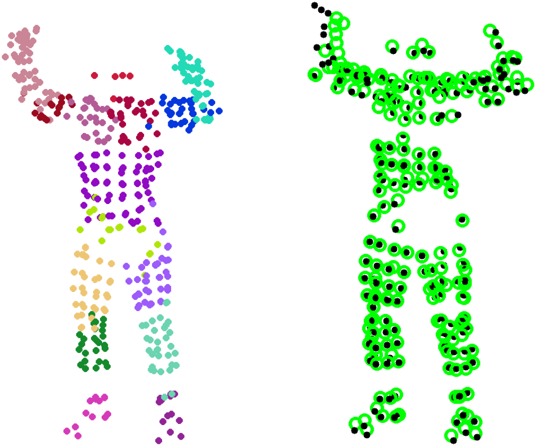
Back	15.20%	-	9.17%

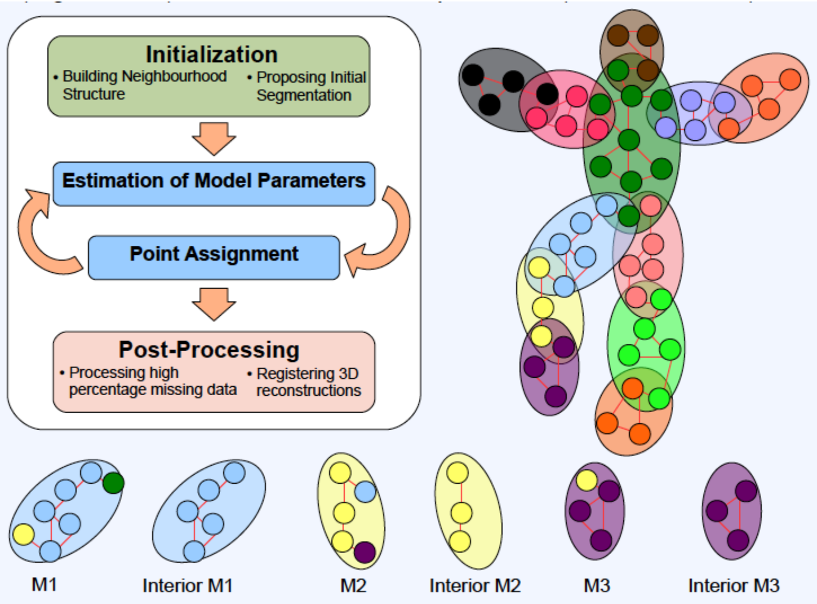
Proportional 3d error
Our time 20 minutes
Fayad – 10 minutes (excluding human labelling)
Taylor – 7 hours

Articulated Motion (ICCV '11)



Articulated motion is piecewise rigid motion.
Areas of overlap are joints.

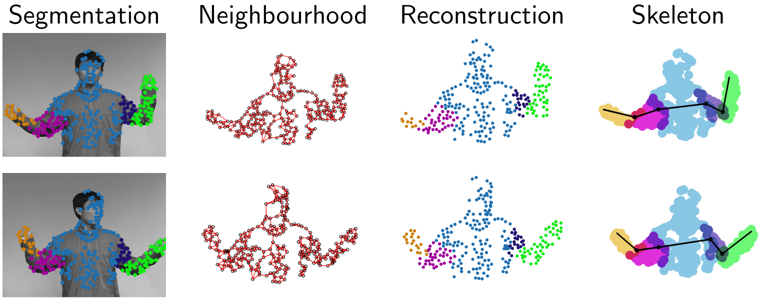
Method


Initialization
• Building Neighbourhood Structure
• Proposing Initial Segmentation
Estimation of Model Parameters
Point Assignment
Post-Processing
• Processing high percentage missing data
• Registering 3D reconstructions
M1
Interior M1
M2
Interior M2
M3
Interior M3

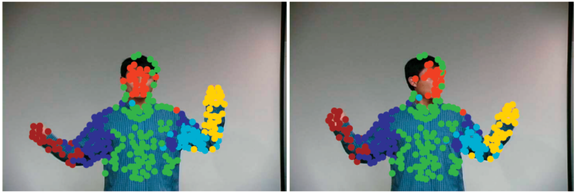
Articulated Motion

- Simultaneous segmentation into body parts and 3D reconstruction.
- Decomposing the points in to rigid pieces, allows us to use pre-existing robust methods.
- (*Marques and Costeira. Estimating 3D shape from degenerate sequences with missing data.* 2008) to reconstruct partial tracks.
- We create a skeleton by ‘joining up’ the joint centres
- Come and see our poster on Thursday!

Dance sequence

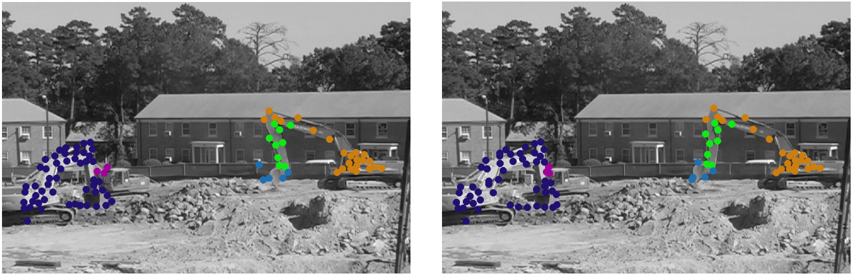


‘Dance’ data-set Yan,Pollefeys (IJCV ’08).



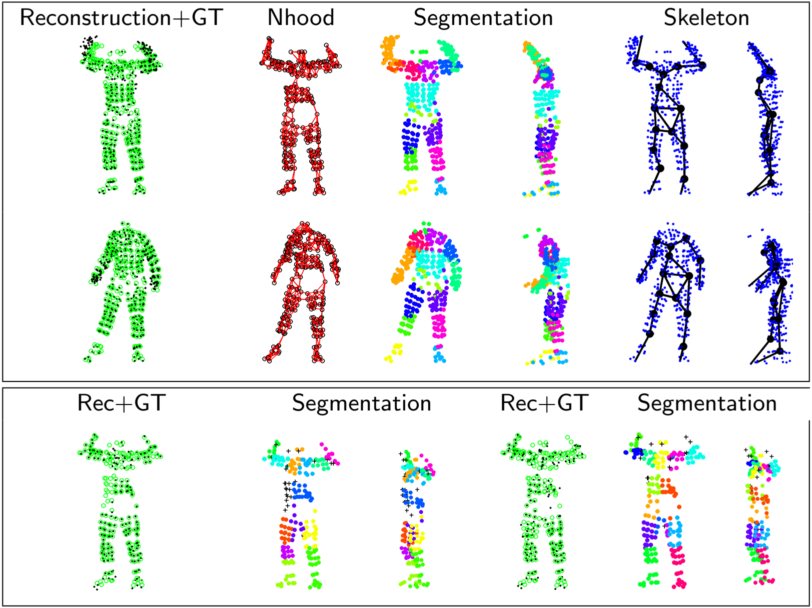
Digger

Our approach



Yan and Pollefeys

Mo-cap


Reconstruction+GT
Nhood
Segmentation
Skeleton
Rec+GT
Segmentation
Rec+GT
Segmentation

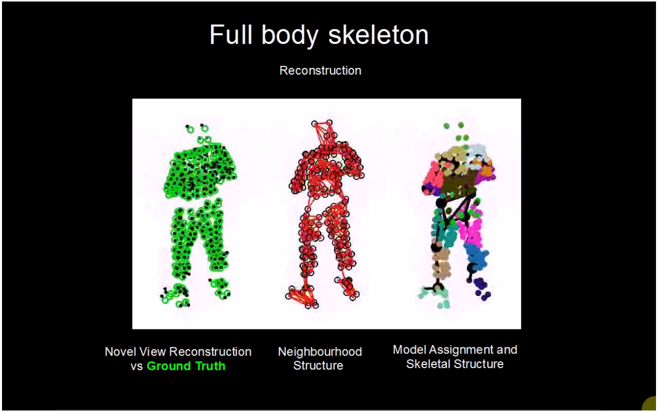
Mo-cap scores

Our approach
Over 461 tracks we have 7.31% 3d error.
Ignoring the hands,
357 tracks 4.87% 3d error.

Mo-cap scores

Our approach
Over 461 tracks we have 7.31% 3d error.
Ignoring the hands,
357 tracks 4.87% 3d error.
Yan and Pollefeys
219 complete tracks
We individually reconstruct and manually aligned each segment with the ground truth
Reconstructions on this has 7.13% error
Ransac to discard outliers leaves 195 tracks
5.09% error.

Video


Full body skeleton
Reconstruction
Novel View Reconstruction
vs Ground Truth
Neighbourhood
Structure
Model Assignment and
Skeletal Structure

Conclusion

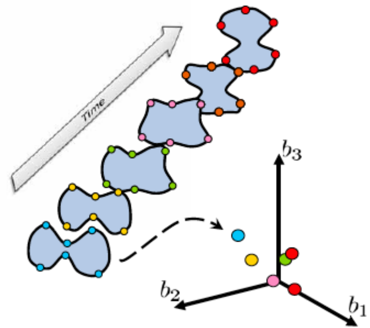
- A novel approach to 3d reconstruction
- State of the art results in articulated motion and NRSfM
- Efficient – Scales to dense reconstruction

Roadmap

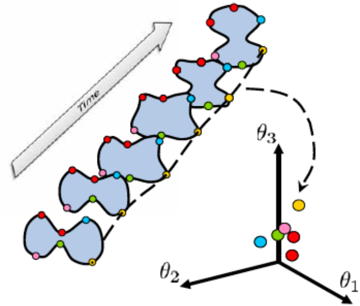
- Piecewise Approaches to NRSfM.
- **Trajectory Space.**
- Smooth Time Shape Trajectory.

Trajectory Space

(Akhter et al. NIPS'08)



Shape Basis

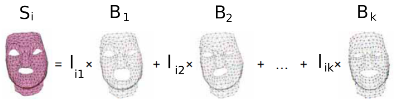


Trajectory basis

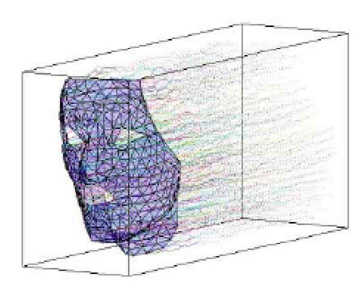
- **Shape Basis**: The 3D shape at each time instant lies in a low dimensional space.
- **Trajectory Basis**: Trajectory of each point over time lies in a low dimensional space.

Images courtesy of Y.Sheikh and S. Khan

Trajectory Space



Shape space

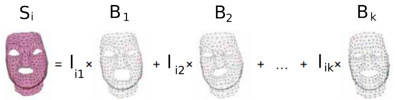


Trajectory space

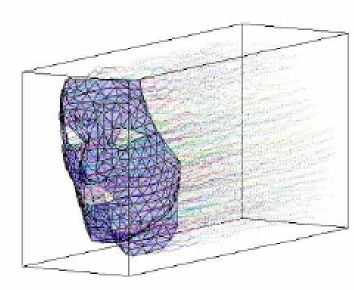
$$S_{3F \times P} = \begin{bmatrix} \mathbf{X}_{11} & \mathbf{X}_{12} & \cdots & \mathbf{X}_{1P} \\ \mathbf{X}_{21} & \mathbf{X}_{22} & \cdots & \mathbf{X}_{2P} \\ & & \vdots & \\ \mathbf{X}_{F1} & \mathbf{X}_{F2} & \cdots & \mathbf{X}_{FP} \end{bmatrix}$$

Images courtesy of Y.Sheikh and S. Khan

Trajectory Space



Shape space

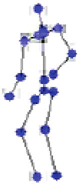


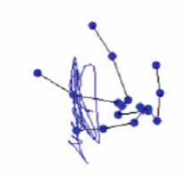
Trajectory space

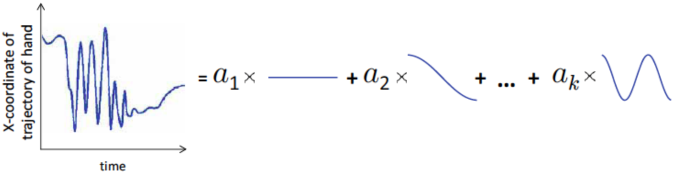
$$S_{3F\times P} = \begin{bmatrix} \mathbf{X}_{11} & \mathbf{X}_{12} & \cdots & \mathbf{X}_{1P} \\ \mathbf{X}_{21} & \mathbf{X}_{22} & \cdots & \mathbf{X}_{2P} \\ & & \vdots & \\ \mathbf{X}_{F1} & \mathbf{X}_{F2} & \cdots & \mathbf{X}_{FP} \end{bmatrix}$$

Images courtesy of Y.Sheikh and S. Khan

Linear Trajectory Model

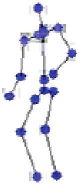


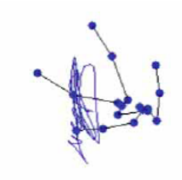


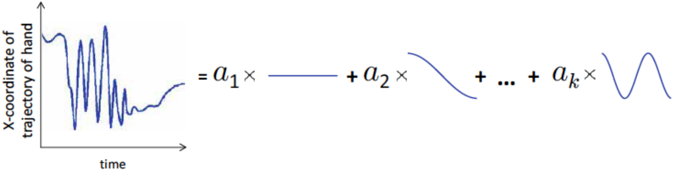


$$\mathrm{T}_j^X = \sum_{k=1}^{K} a_{jk}^X \theta^k$$

Linear Trajectory Model

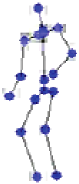


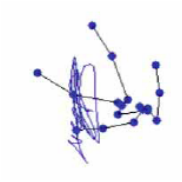


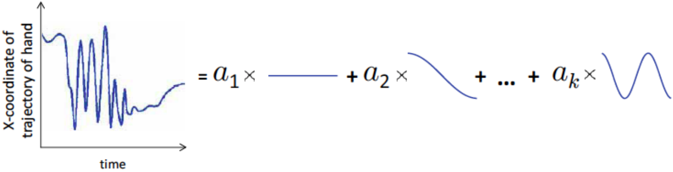


$$\mathrm{T}_j^X = \sum_{k=1}^{K} a_{jk}^X \theta^k \qquad \mathrm{T}_j^Y = \sum_{k=1}^{K} a_{jk}^Y \theta^k$$

Linear Trajectory Model



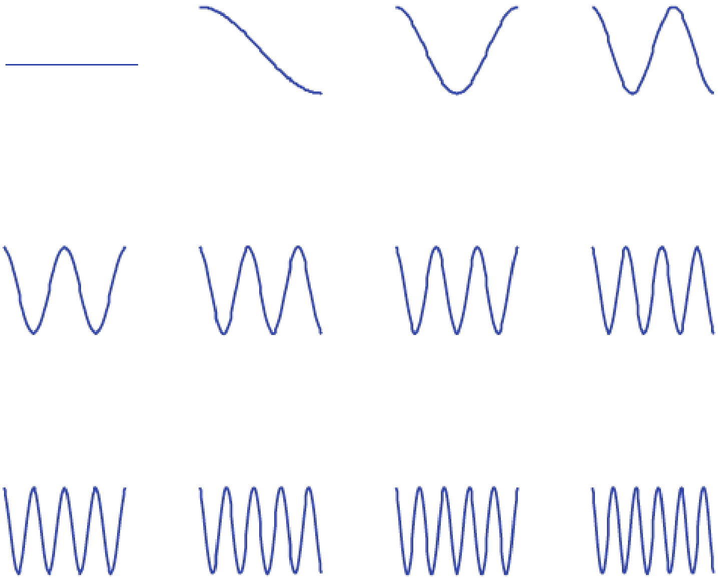




$$\mathrm{T}_j^X = \sum_{k=1}^{K} a_{jk}^X \boldsymbol{\theta}^k \qquad \mathrm{T}_j^Y = \sum_{k=1}^{K} a_{jk}^Y \boldsymbol{\theta}^k \qquad \mathrm{T}_j^Z = \sum_{k=1}^{K} a_{jk}^Z \boldsymbol{\theta}^k$$

Images courtesy of Y.Sheikh and S. Khan

DCT basis



Linear Trajectory Model

DYNAMIC STRUCTURE

$$\begin{bmatrix} X_{1j} \\ X_{2j} \\ \vdots \\ X_{Fj} \end{bmatrix} = a^X_{j1} \begin{bmatrix} \theta^1_1 \\ \theta^1_2 \\ \vdots \\ \theta^1_F \end{bmatrix} + a^X_{j2} \begin{bmatrix} \theta^2_1 \\ \theta^2_2 \\ \vdots \\ \theta^2_F \end{bmatrix} + \ldots + a^X_{jK} \begin{bmatrix} \theta^K_1 \\ \theta^K_2 \\ \vdots \\ \theta^K_F \end{bmatrix}$$

X-component of trajectory of jth point as linear combination of K basis trajectories

X-component of trajectory of **all** point as linear combination of K basis trajectories

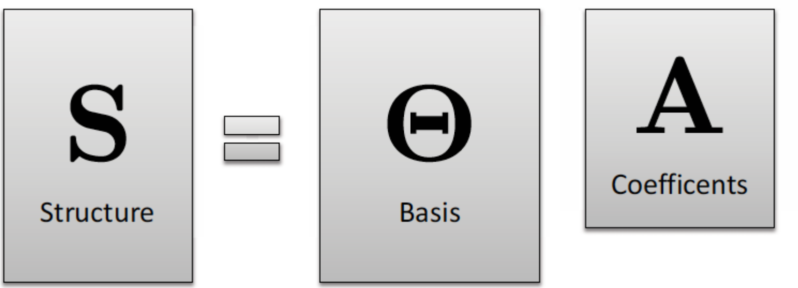
$$\begin{bmatrix} X_{11} & X_{12} & \ldots & X_{1P} \\ X_{21} & X_{22} & \ldots & X_{2P} \\ \vdots & \vdots & \vdots & \vdots \\ X_{F1} & X_{F2} & \ldots & X_{FP} \end{bmatrix} = \begin{bmatrix} \theta^1_1 & \theta^2_1 & \ldots & \theta^K_1 \\ \theta^1_2 & \theta^2_2 & \ldots & \theta^K_2 \\ \vdots & \vdots & \vdots & \vdots \\ \theta^1_F & \theta^2_F & \ldots & \theta^K_F \end{bmatrix} \begin{bmatrix} a^X_{11} & a^X_{21} & \ldots & a^X_{P1} \\ a^X_{12} & a^X_{22} & \ldots & a^X_{P2} \\ \vdots & \vdots & \vdots & \vdots \\ a^X_{1K} & a^X_{2K} & \ldots & a^X_{PK} \end{bmatrix}$$

$$\underset{F \times P}{\mathbf{S}^X} = \underset{F \times K}{\mathbf{\Theta}^X} \times \underset{K \times P}{\mathbf{A}^X}$$

Images courtesy of Y.Sheikh and S. Khan

Linear Trajectory Model

- Advantage: The DCT basis is known in advance.

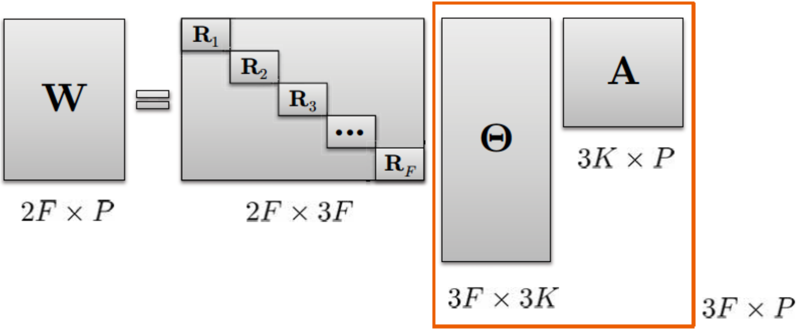


$3F \times P$ $\quad$ $3F \times 3K$ $\quad$ $3K \times P$

Images courtesy of Y.Sheikh and S. Khan

Linear Trajectory Model

- Projection with an orthographic camera.



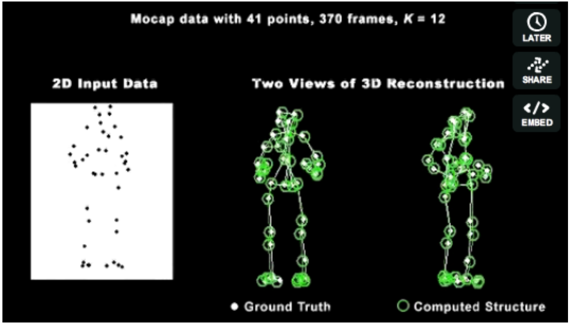
Structure $\mathbf{S}$, in trajectory subspace represented by K trajectory basis

- Two steps: SVD (rank 3K), compute corrective transformation Q.

Images courtesy of Y.Sheikh and S. Khan

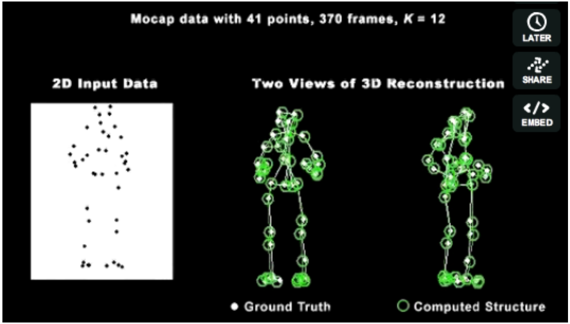
Experimental Results

Yoga Sequence



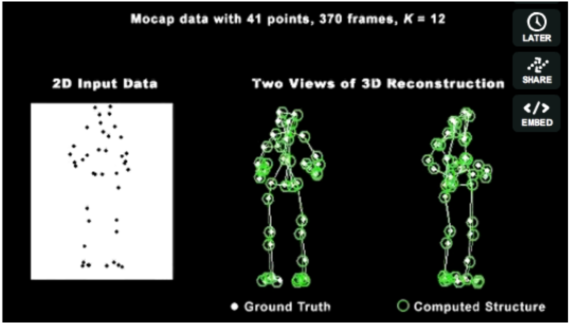
Experimental Results

Dance Sequence



Experimental Results

Stretch Sequence



Conclusions

- Advantages of trajectory space: the DCT basis is pre-determined.
- Metric upgrade is simpler than in the shape basis case.
- Disadvantages: large rotations are needed to achieve reconstructions.
- Spatial smoothness is not imposed.

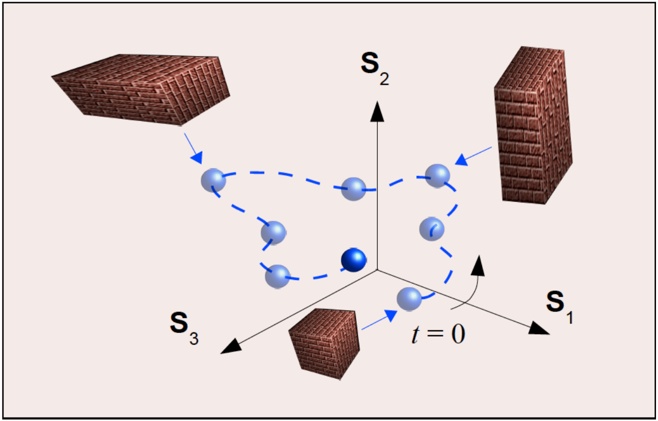
Roadmap

- Piecewise Approaches to NRSfM.
- Trajectory Space.
- **Smooth Shape Time-Trajectories**.

Smooth shape time-trajectory

(Gotardo Martinez PAMI'11, CVPR'11)

- Objects deform smoothly throughout a sequence.
- The shape in each frame is a single K-dim point in shape space.
- The object moves (deforms) along a smooth time-trajectory.



Smooth shape time-trajectory

(Gotardo Martinez PAMI'11, CVPR'11)

Recap: Non-rigid factorization formulation with low-rank shape basis:

$$\underbrace{\begin{bmatrix} x_{1,1} & x_{1,2} & \cdots & x_{1,n} \\ y_{1,1} & y_{1,2} & \cdots & y_{1,n} \\ x_{2,1} & x_{2,2} & \cdots & x_{2,n} \\ y_{2,1} & y_{2,2} & \cdots & y_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{T,1} & x_{T,2} & \cdots & x_{T,n} \\ y_{T,1} & y_{T,2} & \cdots & y_{T,n} \end{bmatrix}}_{\mathbf{W}=\mathbf{MS}+\mathbf{t1}^T} = \underbrace{\begin{bmatrix} \widehat{\mathbf{R}}_1 & & & \\ & \widehat{\mathbf{R}}_2 & & \\ & & \ddots & \\ & & & \widehat{\mathbf{R}}_T \end{bmatrix}}_{\mathbf{D}} \left(\underbrace{\begin{bmatrix} c_{1,1} & c_{1,2} & \cdots & c_{1,K} \\ c_{2,1} & c_{2,2} & \cdots & c_{2,K} \\ \vdots & \vdots & \ddots & \vdots \\ c_{T,1} & c_{T,2} & \cdots & c_{T,K} \end{bmatrix}}_{\mathbf{C}} \otimes \mathbf{I}_3 \right) \underbrace{\begin{bmatrix} \widehat{\mathbf{S}}_1 \\ \widehat{\mathbf{S}}_2 \\ \vdots \\ \widehat{\mathbf{S}}_K \end{bmatrix}}_{\mathbf{S} \in \mathbb{R}^{3K \times n}} + \mathbf{t1}^T$$

$$\mathbf{M} = \mathbf{D}(\mathbf{C} \otimes \mathbf{I}_3) \in \mathbb{R}^{2T \times 3K}$$

Smooth shape time-trajectory

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$$\mathbf{M} = \mathbf{D}(\mathbf{C} \otimes \mathbf{I}_3) \in \mathbb{R}^{2T\times 3K}$$

- Each column of matrix **C** encodes the evolution over time of the coefficient associated with shape mode S_K.

Smooth shape time-trajectory

(Gotardo Martinez PAMI'11, CVPR'11)

Recap: Non-rigid factorization formulation with low-rank shape basis:

$$\underbrace{\begin{bmatrix} x_{1,1} & x_{1,2} & \cdots & x_{1,n} \\ y_{1,1} & y_{1,2} & \cdots & y_{1,n} \\ x_{2,1} & x_{2,2} & \cdots & x_{2,n} \\ y_{2,1} & y_{2,2} & \cdots & y_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{T,1} & x_{T,2} & \cdots & x_{T,n} \\ y_{T,1} & y_{T,2} & \cdots & y_{T,n} \end{bmatrix}}_{\mathbf{W}=\mathbf{MS}+\mathbf{t}\mathbf{1}^T} = \underbrace{\begin{bmatrix} \widehat{\mathbf{R}}_1 & & & \\ & \widehat{\mathbf{R}}_2 & & \\ & & \ddots & \\ & & & \widehat{\mathbf{R}}_T \end{bmatrix}}_{\mathbf{D}} \left(\underbrace{\begin{bmatrix} c_{1,1} & c_{1,2} & \cdots & c_{1,K} \\ c_{2,1} & c_{2,2} & \cdots & c_{2,K} \\ \vdots & \vdots & \ddots & \vdots \\ c_{T,1} & c_{T,2} & \cdots & c_{T,K} \end{bmatrix}}_{\mathbf{C}} \otimes \mathbf{I}_3 \right) \underbrace{\begin{bmatrix} \widehat{\mathbf{S}}_1 \\ \widehat{\mathbf{S}}_2 \\ \vdots \\ \widehat{\mathbf{S}}_K \end{bmatrix}}_{\mathbf{S} \in \mathbb{R}^{3K \times n}} + \mathbf{t}\mathbf{1}^T$$

$$\mathbf{M} = \mathbf{D}(\mathbf{C} \otimes \mathbf{I}_3) \in \mathbb{R}^{2T \times 3K}$$

- Each column of matrix **C** encodes the evolution over time of the coefficient associated with shape mode S_K.
- **C** can be represented **compactly** with a DCT basis.

Smooth shape time-trajectory

(Gotardo Martinez PAMI'11, CVPR'11)

Recap: Non-rigid factorization formulation with low-rank shape basis:

$$\underbrace{\begin{bmatrix} x_{1,1} & x_{1,2} & \cdots & x_{1,n} \\ y_{1,1} & y_{1,2} & \cdots & y_{1,n} \\ x_{2,1} & x_{2,2} & \cdots & x_{2,n} \\ y_{2,1} & y_{2,2} & \cdots & y_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{T,1} & x_{T,2} & \cdots & x_{T,n} \\ y_{T,1} & y_{T,2} & \cdots & y_{T,n} \end{bmatrix}}_{\mathbf{W}=\mathbf{MS}+\mathbf{t1}^T} = \underbrace{\begin{bmatrix} \widehat{\mathbf{R}}_1 & & & \\ & \widehat{\mathbf{R}}_2 & & \\ & & \ddots & \\ & & & \widehat{\mathbf{R}}_T \end{bmatrix}}_{\mathbf{D}} \left(\underbrace{\begin{bmatrix} c_{1,1} & c_{1,2} & \cdots & \boxed{c_{1,K}} \\ c_{2,1} & c_{2,2} & \cdots & c_{2,K} \\ \vdots & \vdots & \ddots & \vdots \\ c_{T,1} & c_{T,2} & \cdots & c_{T,K} \end{bmatrix}}_{\mathbf{C}} \otimes \mathbf{I}_3 \right) \underbrace{\begin{bmatrix} \widehat{\mathbf{S}}_1 \\ \widehat{\mathbf{S}}_2 \\ \vdots \\ \widehat{\mathbf{S}}_K \end{bmatrix}}_{\mathbf{S}\,\in\,\mathbb{R}^{3K\times n}} + \mathbf{t1}^T$$

$$\mathbf{M} = \mathbf{D}(\mathbf{C} \otimes \mathbf{I}_3) \in \mathbb{R}^{2T\times 3K}$$

- Each column of matrix **C** encodes the evolution over time of the coefficient associated with shape mode S_K.
- **C** can be represented **compactly** with a DCT basis.

$$\mathbf{C} = \mathbf{\Omega}_d \mathbf{X}$$

$\mathbf{\Omega}_d = \Big[$  $\ldots \Big]$, d low-frequency basis vectors

Smooth shape time-trajectory

(Gotardo Martinez PAMI'11, CVPR'11)

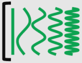
Recap: Non-rigid factorization formulation with low-rank shape basis:

$$\underbrace{\begin{bmatrix} x_{1,1} & x_{1,2} & \cdots & x_{1,n} \\ y_{1,1} & y_{1,2} & \cdots & y_{1,n} \\ x_{2,1} & x_{2,2} & \cdots & x_{2,n} \\ y_{2,1} & y_{2,2} & \cdots & y_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{T,1} & x_{T,2} & \cdots & x_{T,n} \\ y_{T,1} & y_{T,2} & \cdots & y_{T,n} \end{bmatrix}}_{\mathbf{W}=\mathbf{MS}+\mathbf{t1}^T} = \underbrace{\begin{bmatrix} \widehat{\mathbf{R}}_1 & & & \\ & \widehat{\mathbf{R}}_2 & & \\ & & \ddots & \\ & & & \widehat{\mathbf{R}}_T \end{bmatrix}}_{\mathbf{D}} \left(\underbrace{\begin{bmatrix} c_{1,1} & c_{1,2} & \cdots & \boxed{c_{1,K}} \\ c_{2,1} & c_{2,2} & \cdots & c_{2,K} \\ \vdots & \vdots & \ddots & \vdots \\ c_{T,1} & c_{T,2} & \cdots & c_{T,K} \end{bmatrix}}_{\mathbf{C}} \otimes \mathbf{I}_3 \right) \underbrace{\begin{bmatrix} \widehat{\mathbf{S}}_1 \\ \widehat{\mathbf{S}}_2 \\ \vdots \\ \widehat{\mathbf{S}}_K \end{bmatrix}}_{\mathbf{S}\,\in\,\mathbb{R}^{3K\times n}} + \mathbf{t1}^T$$

$$\mathbf{M} = \mathbf{D}(\mathbf{C}\otimes\mathbf{I}_3) \in \mathbb{R}^{2T\times 3K}$$

- Each column of matrix **C** encodes the evolution over time of the coefficient associated with shape mode S_K.
- **C** can be represented **compactly** with a DCT basis.

$$\mathbf{C} = \mathbf{\Omega}_d \mathbf{X}$$

$\mathbf{\Omega}_d = \Big[$  $\ldots \Big]$, d low-frequency basis vectors

Dimensionality reduction:

Number of Unknowns:

$\mathbf{C} \in \mathbb{R}^{T\times K}$ — TK

$\mathbf{X} \in \mathbb{R}^{d\times K}$ — dK $(d \ll T)$

Smooth shape time-trajectory

Approach

- Shape matrix **S** is expressed implicitly in terms of **M** and **W**.

$$\mathbf{S} = S(\mathbf{M},\mathbf{W})$$

- Optimize with respect to **M**.

$$\min_{\mathbf{M}} f(\mathbf{M}) = \|\mathbf{W} - \mathbf{M}\mathbf{S}\|_F^2, \qquad \mathbf{M} = \mathbf{D}(\underbrace{\mathbf{\Omega}_d \mathbf{X}}_{\mathbf{C}} \otimes \mathbf{I}_3)$$

Smooth shape time-trajectory

Approach

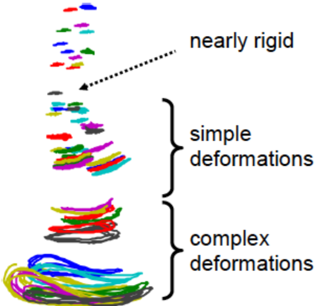
- Shape matrix **S** is expressed implicitly in terms of **M** and **W**.

$$\mathbf{S} = S(\mathbf{M},\mathbf{W})$$

- Optimize with respect to **M**.

$$\min_{\mathbf{M}} f(\mathbf{M}) = \|\mathbf{W} - \mathbf{M}\mathbf{S}\|_F^2 , \qquad \mathbf{M} = \mathbf{D}(\underbrace{\mathbf{\Omega}_d \mathbf{X}}_{\mathbf{C}} \otimes \mathbf{I}_3)$$

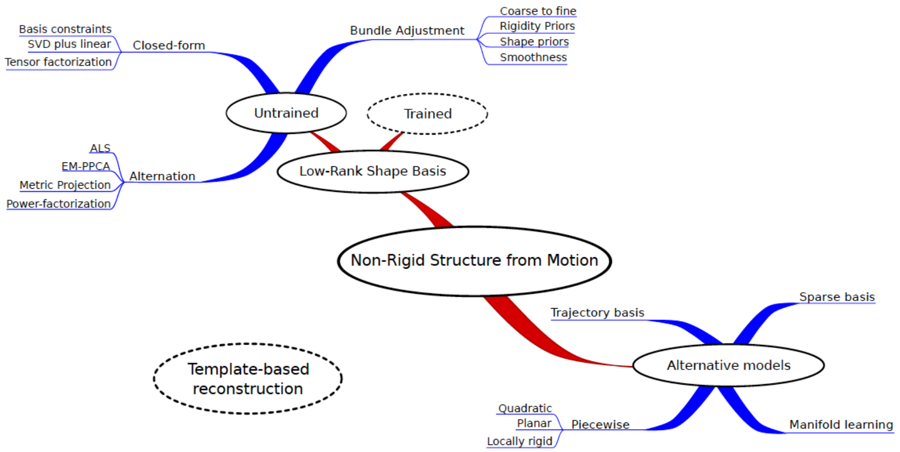
- Variant: Deformation modes S_k are estimated in a coarse-to-fine approach.
- Basis shapes are PCA-like deformations of decreasing importance.



Smooth shape time-trajectory: Results

Dataset	EM-PPCA	MP	PTA	CSF1	CSF2
Drink	0.3393	0.4604	**0.0250**	**0.0223**	**0.0223**
Stretch	1.1111	0.8549	0.1088	**0.0710**	**0.0684**
Pick-up	0.5822	0.4332	**0.2369**	**0.2301**	**0.2277**
Yoga	0.8097	0.8039	0.1625	**0.1467**	**0.1465**
Dance	0.9839	0.2639	0.2958	0.2705	**0.1942**
Walking	0.4917	0.5607	0.3954	0.1863	**0.1041**
Face1	**0.0434**	0.0734	0.1247	0.0637	**0.0526**
Face2	**0.0329**	**0.0357**	0.0444	**0.0363**	**0.0312**
Shark1	0.0501	0.1571	0.1796	**0.0081**	0.0437
Shark2	0.0529	0.1346	0.3120	0.2538	**0.0052**

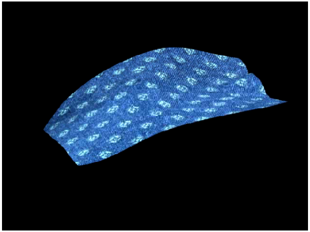
A Taxonomy of Approaches to Non-Rigid 3D Reconstruction

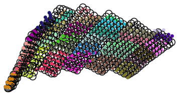


Current Challenges I

Current Challenges I

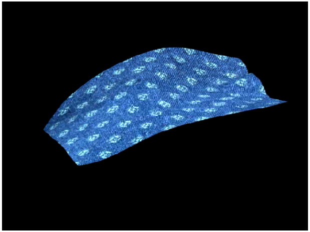
- **Strong Deformations: Piecewise Modelling**

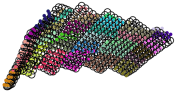




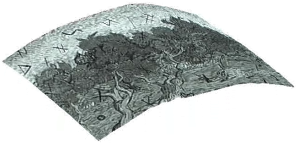
Current Challenges I

- **Strong Deformations: Piecewise Modelling**



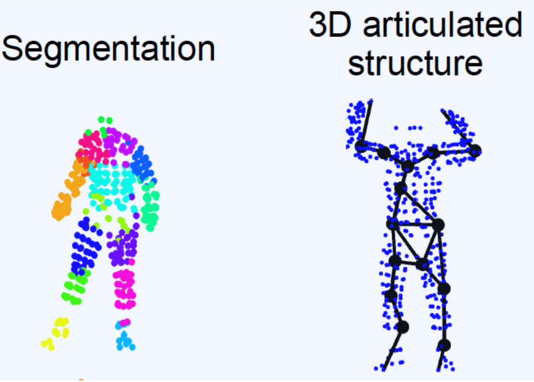


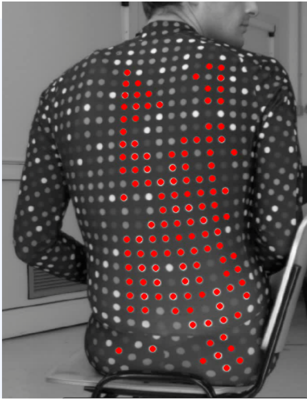
- **Dense: Optical Flow/ 3D Modelling**



Current Challenges II

- **Human Motion Modelling**





Conclusions: My to do list

- Unified approach: 3D shape estimation directly from pixel intensity information. Simultaneous 2D registration + segmentation + 3D reconstruction.

- Robustify approaches: Dealing with more than one object, occlusions, changes in lighting...

- Real time?