Particle Size Distributions:
Theory and Application to Aerosols, Clouds, and Soils

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1 Introduction

This document describes mathematical and computational considerations pertaining to size distributions. The application of statistical theory to define meaningful and measurable parameters for

defining generic size distributions is presented in §2. The remaining sections apply these defini-
tions to the size distributions most commonly used to describe clouds and aerosol size distributions
in the meteorological literature. Currently, only the lognormal distribution is presented.

1.1   Modal vs. Sectional Represenatation

mdlsxn *Lu and Bowman* (2004) designed and optimal non-linear least squares-based procedure for
converting from sectional to modal representations.

1.2   Nomenclature

nomenclature There is a bewildering variety of nomenclature associated with size distributions,
probability density functions, and statistics thereof. The nomenclature in this article generally fol-
lows the standard references, (see, e.g., *Hansen and Travis*, 1974; *Patterson and Gillette*, 1977;
Press et al., 1988; *Flatau et al.*, 1989; *Seinfeld and Pandis*, 1997), at least where those references
are in agreement. Quantities whose nomenclature is often confusing, unclear, or simply not stan-
dardized are discussed in the text.

1.3   Distribution Function

This section follows the carefully presented discussion of *Flatau et al.* (1989). The *size distribution*
function $n_{\rm n}(r)$ is defined such that $n_{\rm n}(r)\,{\rm d}r$ is the total concentration (number per unit volume of
air, or $\#\,{\rm m}^{-3}$) of particles with sizes in the domain $[r, r + {\rm d}r]$. The total number concentration of
particles N_0 is obtained by integrating $n_{\rm n}(r)$ over all sizes

$$N_0 = \int_0^\infty n_{\rm n}(r)\,{\rm d}r \tag{1}$$

The size distribution function is also called the *spectral density function*. The dimensions of $n_{\rm n}(r)$
and N_0 are $\#\,{\rm m}^{-3}\,{\rm m}^{-1}$ and $\#\,{\rm m}^{-3}$, respectively. Note that $n_{\rm n}(r)$ is only normalized if $N_0 = 1.0$ (cf.
Section 3.4.2).

  Often N_0 is not an observable quantity. A variety of functional forms, some of which are over-
loaded for clarity, describe the number concentrations actually measured by instruments. Typically
an instrument has a lower detection limit $r_{\rm min}$ and an upper detection limit $r_{\rm max}$ of particle sizes
which it can measure.

$$N(r < r_{\rm max}) = \int_0^{r_{\rm max}} n_{\rm n}(r)\,{\rm d}r \tag{2}$$

$$N(r > r_{\rm max}) = \int_{r_{\rm max}}^\infty n_{\rm n}(r)\,{\rm d}r \tag{3}$$

$$N(r_{\rm min}, r_{\rm max}) = N(r_{\rm min} < r < r_{\rm max}) = \int_{r_{\rm min}}^{r_{\rm max}} n_{\rm n}(r)\,{\rm d}r \tag{4}$$

Equations (2)–(4) define the *cumulative concentration*, *lower bound concentration*, and *truncated
concentration*, respectively. The cumulative concentration is used to define the *median radius* $\tilde{r}_{\rm n}$.

Half the particles are larger and half smaller than $\tilde{r}_\mathrm{n}$

$$N(r < \tilde{r}_\mathrm{n}) = N(r > \tilde{r}_\mathrm{n}) = \frac{N_0}{2} \tag{5}$$

These functions are often used to define $n_\mathrm{n}(r)$ via

$$n_\mathrm{n}(r) = \frac{\mathrm{d}N}{\mathrm{d}r} \tag{6}$$

Note that the concentration nomenclature in (6) is N not $N(r)$. Using $N(r)$ would indicate that the concentration has not been completely integrated over all sizes. By definition, the total concentration N_0 is integrated over all sizes, as defined by (1). A concentration denoted $N(r)$ makes no sense without an associated size bin width Δr, or truncation convention, as in (2)–(4). We try to use N and N_0 for normalized ($N = 1$) and non-normalized ($N_0 \neq 1$, i.e., absolute concentrations). However this convention is not absolute and (1) defines both N and N_0.

1.4   Probability Density Function

Describing size distributions is easier when they are normalized into *probability density functions*, or PDFs. In this context, a PDF is a size distribution function normalized to unity over the domain of interest, i.e., $p(r) = C_n n_\mathrm{n}(r)$ where the normalization constant C_n is defined such that

$$\int_0^\infty p(r)\,\mathrm{d}r = 1 \tag{7}$$

In the following sections we usually work with PDFs because this normalization property is very convenient mathematically. Comparing (7) and (1), it is clear that the normalization constant C_n which transforms a size distribution function (1) into a PDF $p(r)$ is N_0^{-1}

$$p(r) = \frac{1}{N_0}\,n_\mathrm{n}(r) \tag{8}$$

1.4.1   Choice of Independent Variable

The merits of using radius r, diameter D, or some other dimension L, as the independent variable of a size distribution depend on the application. In radiative transfer applications, r prevails in the literature probably because it is favored in electromagnetic and Mie theory. There is, however, a growing recognition of the importance of aspherical particles in planetary atmospheres. Defining an *equivalent radius* or *equivalent diameter* for these complex shapes is not straighforward (consider, e.g., a bullet rosette ice crystal). Important differences exist among the competing definitions, such as *equivalent area spherical radius*, *equivalent volume spherical radius*, (e.g., *Ebert and Curry*, 1992; *McFarquhar and Heymsfield*, 1997).

A direct property of aspherical particles which can often be measured is its maximum dimension, i.e., the greatest distance between any two surface points of the particle. This maximum dimension, usually called L, has proven to be useful for characterizing size distributions of aspherical particles. For a sphere, L is also the diameter. Analyses of mineral dust sediments in ice core deposits or sediment traps, for example, are usually presented in terms of L. The surface area

and volume of ice crystals have been computed in terms of power laws of L (e.g., *Heymsfield and Platt*, 1984; *Takano and Liou*, 1995). Since models usually lack information regarding the shape of particles (early exceptions include *Zender and Kiehl*, 1994; *Chen and Lamb*, 1994), most modelers assume spherical particles, especially for aerosols. Thus, the advantages of using the diameter D as the independent variable in size distribution studies include: D is the dimension often reported in measurements; D is more analogous than r to L.

The remainder of this manuscript assumes spherical particles where r and D are equally useful independent variables. Unless explicitly noted, our convention will be to use D as the independent variable. Thus, it is useful to understand the rules governing conversion of PDFs from D to r and the reverse.

Consider two distinct analytic representations of the same underlying size distribution. The first, $n_{\mathrm{n}}^{D}(D)$, expresses the differential number concentration per unit diameter. The second, $n_{\mathrm{n}}^{r}(r)$, expresses the differential number concentration per unit radius. Both $n_{\mathrm{n}}^{D}(D)$ and $n_{\mathrm{n}}^{r}(r)$ share the same dimensions, $\# \, \mathrm{m}^{-3} \, \mathrm{m}^{-1}$.

$$D = 2r \tag{9}$$

$$\mathrm{d}D = 2\,\mathrm{d}r \tag{10}$$

$$n_{\mathrm{n}}^{D}(D)\,\mathrm{d}D = n_{\mathrm{n}}^{r}(r)\,\mathrm{d}r \tag{11}$$

$$n_{\mathrm{n}}^{D}(D) = \frac{1}{2}n_{\mathrm{n}}^{r}(r) \tag{12}$$

2   Statistics of Size Distributions

2.1   Generic

Consider an arbitrary function $g(x)$ which applies over the domain of the size distribution $p(x)$. For now the exact definition of g is irrelevant, but imagine that $g(x)$ describes the variation of some physically meaningful quantity (e.g., area) with size. The *mean value* of g is the integral of g over the domain of the size distribution, weighted at each point by the concentration of particles

$$\overline{g} = \int_{0}^{\infty} g(x)\,p(x)\,\mathrm{d}x \tag{13}$$

Once $p(x)$ is known, it is always possible to compute $\overline{g}$ for any desired quantity g. Typical quantities represented by $g(x)$ are size, $g(x) = x$; area, $g(x) = A(x) \propto x^{2}$; and volume $g(x) = V(x) \propto x^{3}$. More complicated statistics represented by $g(x)$ include variance, $g(x) = (x - \bar{x})^{2}$. The remainder of this section considers some of these examples in more detail.

2.2   Mean Size

The *number mean size* $\bar{x}$ of a size distribution $p(x)$ is defined as

$$\bar{x} = \int_{0}^{\infty} p(x)\,x\,\mathrm{d}x \tag{14}$$

Synonyms for number mean size include *mean size*, *average size*, *arithmetic mean size*, and *number-weighted mean size* (*Hansen and Travis*, 1974). *Flatau et al.* (1989) define $\bar{D}_{\mathrm{n}} \equiv \bar{D}$, a convention we adopt in the following.

2.3 Variance

The *variance* σ_x^2 of a size distribution $p(x)$ is defined in accord with the statistical variance of a continuous mathematical distribution.

$$\sigma_x^2 = \int_0^\infty p(x)(x - \bar{x})^2 \, \mathrm{d}x \tag{15}$$

The variance measures the mean squared-deviation of the distribution from its mean value. The units of σ_x^2 are [m^2]. Because σ_x^2 is a complicated function for standard aerosol and cloud size distributions, many prefer to work with an alternate definition of variance, called the *effective variance*.

The *effective variance* $\sigma_{x,\mathrm{eff}}^2$ of a size distribution $p(x)$ is the variance about the effective size of the distribution, normalized by x_eff (e.g., *Hansen and Travis*, 1974)

$$\sigma_{x,\mathrm{eff}}^2 = \frac{1}{x_\mathrm{eff}^2} \int_0^\infty p(x)(x - x_\mathrm{eff})^2 \, x^2 \, \mathrm{d}x \tag{16}$$

Because of the x_eff^{-2} normalization, $\sigma_{x,\mathrm{eff}}^2$ is non-dimensional in contrast to typical variances, e.g., (15). In the terminology of *Hansen and Travis* (1974), $\sigma_{x,\mathrm{eff}}^2 = v$.

2.4 Standard Deviation

The *standard deviation* σ_x of a size distribution $p(x)$ is the square root of the variance (15),

$$\sigma_x = \sqrt{\sigma_x^2} \tag{17}$$

σ_x has units of [m]. For standard aerosol and cloud size distributions, σ_x is an ugly expression. Therefore many authors prefer to work with alternate definitions of standard deviation. Unfortunately, nomenclature for these alternate definitions is not standardized.

3 Cloud and Aerosol Size Distributions

3.1 Gamma Distribution

Statistics of the *gamma distribution* are presented in `http://asd-www.larc.nasa.gov/~yhu/paper/thesisall/node8.html`. Currently, the aerosol property program `mie` implements gamma distributions in a limited sense.

3.2 Normal Distribution

The *normal distribution* is the most common statistical distribution. The normal distribution $n(x)$ is expressed in terms of its mean $\bar{x}$ (14) and standard deviation σ_x (17)

$$n(x) = \frac{1}{\sqrt{2\pi}\,\sigma_x} \exp\left[-\frac{1}{2}\left(\frac{x - \bar{x}}{\sigma_x}\right)^2\right] \tag{18}$$

With our standard nomenclature for number distribution n_n and particles diameter D, (18) appears as

$$n_\mathrm{n}(D) \equiv \frac{\mathrm{d}N}{\mathrm{d}D} = \frac{1}{\sqrt{2\pi}\,\sigma_D} \exp\left[-\frac{1}{2}\left(\frac{D - \bar{D}_\mathrm{n}}{\sigma_D}\right)^2\right] \tag{19}$$

The cumulative normal distribution is called the error function and is discussed in Section (5.1). Integration of the error function shows that 68.3% of the values of (19) are in $\bar{D}_\mathrm{n} \pm \sigma_D$, 95.4% are in $\bar{D}_\mathrm{n} \pm 2\sigma_D$, and 99.7% are in $\bar{D}_\mathrm{n} \pm 3\sigma_D$.

3.3   Lognormal Distribution

The *lognormal distribution* is perhaps the most commonly used analytic expression in aerosol studies.

3.3.1   Distribution Function

In a lognormal distribution, the logarithm of abscissa is normally distributed (Section 3.2). Substituting $x = \ln D$ into (18) yields

$$n_\mathrm{n}(\ln D) \equiv \frac{\mathrm{d}N}{\mathrm{d}\ln D} = \frac{1}{\sqrt{2\pi}\,\ln\sigma_\mathrm{g}} \exp\left[-\frac{1}{2}\left(\frac{\ln D - \ln\tilde{D}_\mathrm{n}}{\ln\sigma_\mathrm{g}}\right)^2\right] \tag{20}$$

where σ_g and $\tilde{D}_\mathrm{n}$ are parameters whose physical significance is to be defined. In particular, there is no closed-form algebraic relationship between σ_D (19) and σ_g (20). The former is a true standard deviation and the properties of the latter are as yet unknown.

Substituting $\mathrm{d}\ln D = D^{-1}\,\mathrm{d}D$ in (20) leads to the most commonly used form the *lognormal distribution function*

$$n_\mathrm{n}(D) \equiv \frac{\mathrm{d}N}{\mathrm{d}D} = \frac{1}{\sqrt{2\pi}\,D\ln\sigma_\mathrm{g}} \exp\left[-\frac{1}{2}\left(\frac{\ln(D/\tilde{D}_\mathrm{n})}{\ln\sigma_\mathrm{g}}\right)^2\right] \tag{21}$$

One of the most confusing aspects of size distributions in the meteorological literature is in the usage of σ_g, the *geometric standard deviation*. Some researchers (e.g., *Flatau et al.*, 1989) prefer a different formulation (21) which is equivalent to

$$n_\mathrm{n}(D) = \frac{1}{\sqrt{2\pi}\,\tilde{\sigma}_\mathrm{g}D} \exp\left[-\frac{1}{2}\left(\frac{\ln(D/\tilde{D}_\mathrm{n})}{\tilde{\sigma}_\mathrm{g}}\right)^2\right] \tag{22}$$

where

$$\tilde{\sigma}_\mathrm{g} \equiv \ln\sigma_\mathrm{g} \tag{23}$$

In practice, (21) is used more widely than (22) and we adopt (21) in the following.

The definition of $\tilde{\sigma}_{\mathrm{g}}$ in (22) may be more satisfactory from a mathematical point of view (*Flatau et al.*, 1989), and it subsumes an "ln" which reduces typing. This is seen by transforming x in (18) with

$$
\begin{aligned}
x &= \frac{1}{\tilde{\sigma}_{\mathrm{g}}} \ln\left(\frac{D}{\tilde{D}_{\mathrm{n}}}\right) \\
D &= \tilde{D}_{\mathrm{n}} \exp(\tilde{\sigma}_{\mathrm{g}} x) \\
\mathrm{d}D &= \tilde{\sigma}_{\mathrm{g}} \tilde{D}_{\mathrm{n}} \exp(\tilde{\sigma}_{\mathrm{g}} x) \, \mathrm{d}x \\
\mathrm{d}x &= (\tilde{\sigma}_{\mathrm{g}} \tilde{D}_{\mathrm{n}})^{-1} \, \mathrm{d}D
\end{aligned}
$$

This maps $x \in (-\infty, +\infty)$ into $D \in [0, \infty)$.

One is occasionally given a "standard deviation" or "geometric standard deviation" parameter without clear specification whether it represents σ_{g} (or $\ln \sigma_{\mathrm{g}}$, or $\exp \sigma_{\mathrm{g}}$, or σ_x) in (17), (21), or (22). As a true standard deviation, σ_x has dimensions of x, whereas both σ_{g} and $\tilde{\sigma}_{\mathrm{g}}$ are dimensionless so units cannot disambiguate them. A useful rule of thumb is that σ_{g} in (21) and $\mathrm{e}^{\tilde{\sigma}_{\mathrm{g}}}$ in (22) are usually between 1.5–2.5 for realistic aerosol populations. Since we adopted (21), physically realistic values are $\sigma_{\mathrm{g}} \in (1.5, 2.5)$.

Seinfeld and Pandis (1997) p. 423 describe the physical meaning of the geometric standard deviation σ_{g}. Define the special particle sizes

$$
D_{\sigma_{\mathrm{g}}}^{+} \equiv \tilde{D}_{\mathrm{n}} \sigma_{\mathrm{g}} \tag{24a}
$$

$$
D_{\sigma_{\mathrm{g}}}^{-} \equiv \tilde{D}_{\mathrm{n}} / \sigma_{\mathrm{g}} \tag{24b}
$$

The cumulative concentration smaller than $D_{\sigma_{\mathrm{g}}}^{+}$, simplifies from (38) to

$$
N(D < D_{\sigma_{\mathrm{g}}}^{+}) = \frac{N_0}{2} + \frac{N_0}{2} \operatorname{erf}\left(\frac{1}{\sqrt{2}}\right) = 0.841344746069 N_0 \tag{25}
$$

Numerical integration must be used to obtain the final result, $0.841 N_0$, as erf() has no closed-form solution here. Using (25) to invert (24), we may define σ_{g} as the ratio of the diameter $D_{\sigma_{\mathrm{g}}}^{+}$ (larger than 84.1% of all particles) to the median diameter $\tilde{D}_{\mathrm{n}}$. Monodisperse populations have $\sigma_{\mathrm{g}} \equiv 1$.

Similarly the cumulative concentration smaller than $D_{\sigma_{\mathrm{g}}}^{-}$, simplifies from (38) to

$$
N(D < D_{\sigma_{\mathrm{g}}}^{-}) = \frac{N_0}{2} + \frac{N_0}{2} \operatorname{erf}\left(-\frac{1}{\sqrt{2}}\right) = 0.158655253931 N_0 \tag{26}
$$

where we have used the numerical result in (26) with the error function's anti-symmetric property, $\operatorname{erf}(-x) = -\operatorname{erf}(x)$. Subtracting (26) from (25) shows that 68.3% of all particles in a lognormal distribution lie in $D \in [D_{\sigma_{\mathrm{g}}}^{-}, D_{\sigma_{\mathrm{g}}}^{+}]$.

By raising σ_{g} to any power x in (24), it is straightforward to verify that the number of particles within $D \in [\tilde{D}_{\mathrm{n}} \sigma_{\mathrm{g}}^{-x}, \tilde{D}_{\mathrm{n}} \sigma_{\mathrm{g}}^{x}]$ is

$$
N(\tilde{D}_{\mathrm{n}} \sigma_{\mathrm{g}}^{-x} < D < \tilde{D}_{\mathrm{n}} \sigma_{\mathrm{g}}^{x}) = N_0 \operatorname{erf}(x/\sqrt{2}) \tag{27}
$$

Application of (27) for small integer x shows that 68.3% of all particles lie within $\tilde{D}_{\mathrm{n}}/\sigma_{\mathrm{g}} < D < \tilde{D}_{\mathrm{n}} \sigma_{\mathrm{g}}$, that 95.4% of all particles[1] lie within $\tilde{D}_{\mathrm{n}} \sigma_{\mathrm{g}}^{-2} < D < \tilde{D}_{\mathrm{n}} \sigma_{\mathrm{g}}^{2}$, and that 99.7% of all particles

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[1] *Seinfeld and Pandis* (1997) p. 423 has a typo on this point. That page erroneously states that the bounds bracketing 95% of a lognormal distribution are $\tilde{D}_{\mathrm{n}}/(2\sigma_{\mathrm{g}}) < D < 2\tilde{D}_{\mathrm{n}} \sigma_{\mathrm{g}}$.

lie within $\tilde{D}_\mathrm{n}\sigma_\mathrm{g}^{-3} < D < \tilde{D}_\mathrm{n}\sigma_\mathrm{g}^{3}$. These bounds are identical to the fraction of particles enclosed within one, two, and three standard deviations of the mean of a normal distribution (Section 3.2).

3.3.2   Lognormal Relations

Table 1 summarizes the standard lognormal distribution parameters. Note that $\tilde{\sigma}_\mathrm{g} \equiv \ln \sigma_\mathrm{g}$. The statistics in Table 1 are easy to misunderstand because of the plethora of subtly different definitions. A common mistake is to assume that patterns which seems to apply to one distribution, e.g., the number distribution $n_\mathrm{n}(D)$, apply to distributions of all other moments. For example, the number distribution $n_\mathrm{n}(D)$ is the *only* distribution for which the moment mean size (i.e., number mean size $\bar{D}_\mathrm{n}$) equals the moment-weighted size (i.e., number-weighted size D_n). Also, the number mean size $\bar{D}_\mathrm{n}$ differs from the number median size $\tilde{D}_\mathrm{n}$ by a factor $\exp(\tilde{\sigma}_\mathrm{g}^2/2)$. But this factor is not constant and depends on the moment of the distribution. For instance, $\bar{D}_\mathrm{s}$ differs from $\tilde{D}_\mathrm{s}$ by $\exp(\tilde{\sigma}_\mathrm{g}^2)$, while $\bar{D}_\mathrm{s}$ differs from $\tilde{D}_\mathrm{s}$ by $\exp(3\tilde{\sigma}_\mathrm{g}^2/2)$. Thus converting from mean diameter to median diameter is not the same for number as for mass distributions.

Table 1: **Lognormal Distribution Relations**[234]

Symbol	Value	Units	Description	Defining Relation
N_0	N_0	$\text{\# m}^{-3}$	Total number concentration	$N_0 = \int_0^\infty n_n(D)\,\mathrm{d}D$
D_0	$N_0 \tilde{D}_n \exp(\tilde{\sigma}_g^2/2)$	m m^{-3}	Total diameter	$D_0 = \int_0^\infty D n_n(D)\,\mathrm{d}D$
A_0	$\frac{\pi}{4} N_0 \tilde{D}_n^2 \exp(2\tilde{\sigma}_g^2)$	$\text{m}^2\,\text{m}^{-3}$	Total cross-sectional area	$A_0 = \int_0^\infty \frac{\pi}{4} D^2 n_n(D)\,\mathrm{d}D$
S_0	$\pi N_0 \tilde{D}_n^2 \exp(2\tilde{\sigma}_g^2)$	$\text{m}^2\,\text{m}^{-3}$	Total surface area	$S_0 = \int_0^\infty \pi D^2 n_n(D)\,\mathrm{d}D$
V_0	$\frac{\pi}{6} N_0 \tilde{D}_n^3 \exp(9\tilde{\sigma}_g^2/2)$	$\text{m}^3\,\text{m}^{-3}$	Total volume	$V_0 = \int_0^\infty \frac{\pi}{6} D^3 n_n(D)\,\mathrm{d}D$
M_0	$\frac{\pi}{6} N_0 \rho \tilde{D}_n^3 \exp(9\tilde{\sigma}_g^2/2)$	kg m^{-3}	Total mass	$M_0 = \int_0^\infty \frac{\pi}{6} \rho D^3 n_n(D)\,\mathrm{d}D$
$\bar{D}$	$\tilde{D}_n \exp(\tilde{\sigma}_g^2/2)$	$\text{m \#}^{-1}$	Mean diameter	$N_0 \bar{D} = N_0 \tilde{D}_n = D_0$
$\bar{A}$	$\frac{\pi}{4} \tilde{D}_n^2 \exp(2\tilde{\sigma}_g^2)$	$\text{m}^2\,\text{\#}^{-1}$	Mean cross-sectional area	$N_0 \bar{A} = N_0 \frac{\pi}{4} \bar{D}_s^2 = A_0$
$\bar{S}$	$\pi \tilde{D}_n^2 \exp(2\tilde{\sigma}_g^2)$	$\text{m}^2\,\text{\#}^{-1}$	Mean surface area	$N_0 \bar{S} = N_0 \pi \bar{D}_s^2 = S_0$
$\bar{V}$	$\frac{\pi}{6} \tilde{D}_n^3 \exp(9\tilde{\sigma}_g^2/2)$	$\text{m}^3\,\text{\#}^{-1}$	Mean volume	$N_0 \bar{V} = N_0 \frac{\pi}{6} \bar{D}_v^3 = V_0$
$\bar{M}$	$\frac{\pi}{6} \rho \tilde{D}_n^3 \exp(9\tilde{\sigma}_g^2/2)$	$\text{kg \#}^{-1}$	Mean mass	$N_0 \bar{M} = N_0 \frac{\pi}{6} \rho \bar{D}_v^3 = M_0$
N_0	$\frac{6}{\pi\rho} M_0 \tilde{D}_n^{-3} \exp(-9\tilde{\sigma}_g^2/2)$	$\text{\# m}^{-3}$	Number concentration	$N_0 = \int_0^\infty n_n(D)\,\mathrm{d}D$
$\tilde{D}_n$	$\left(\frac{6M_0}{\pi N_0 \rho}\right)^{1/3} \exp(-3\tilde{\sigma}_g^2/2)$	m	Median diameter	$\int_0^{\tilde{D}_n} n_n(D)\,\mathrm{d}D = \frac{N_0}{2}$
D_{eff}	$\frac{6M_0}{\rho S_0}$	m	Effective diameter	$D_{\text{eff}} = \frac{1}{A_0} \int_0^\infty D \frac{\pi}{4} D^2 n_n(D)\,\mathrm{d}D$
$\hat{S}$	$\frac{6}{\rho D_{\text{eff}}}$	$\text{m}^2\,\text{kg}^{-1}$	Specific surface area	$\hat{S} = S_0/M_0$

Table 1: (continued)

Symbol	Value	Units	Description	Defining Relation
$\tilde{D}_n$	$\bar{D}_n \exp(-\tilde{\sigma}_g^2/2)$	m	Median diameter, Scaling diameter, Number median diameter. Half of particles are larger than, and half smaller than, $\tilde{D}_n$	$\int_0^{\tilde{D}_n} n_n(D)\,\mathrm{d}D = \dfrac{N_0}{2}$
$\bar{D}_n, \bar{D}, \bar{D}_n$	$\tilde{D}_n \exp(\tilde{\sigma}_g^2/2)$	m	Mean diameter, Average diameter, Number-weighted mean diameter	$\bar{D}_n = \dfrac{1}{N_0}\int_0^\infty D n_n(D)\,\mathrm{d}D$
$\bar{D}_s$	$\tilde{D}_n \exp(\tilde{\sigma}_g^2)$	m	Surface mean diameter	$N_0 \pi \bar{D}_s^2 = N_0 \bar{S} = S_0$
$\bar{D}_v$	$\tilde{D}_n \exp(3\tilde{\sigma}_g^2/2)$	m	Volume mean diameter, Mass mean diameter	$N_0 \dfrac{\pi}{6}\bar{D}_v^3 = N_0 \bar{V} = V_0$
$\tilde{D}_s$	$\tilde{D}_n \exp(2\tilde{\sigma}_g^2)$	m	Surface median diameter	$\int_0^{\tilde{D}_s} \pi D^2 n_n(D)\,\mathrm{d}D = \dfrac{S_0}{2}$
D_s, D_{eff}	$\tilde{D}_n \exp(5\tilde{\sigma}_g^2/2)$	m	Area-weighted mean diameter, effective diameter	$D_s = \dfrac{1}{A_0}\int_0^\infty \dfrac{\pi}{4}D^2 n_n(D)\,\mathrm{d}D$
$\tilde{D}_v$	$\tilde{D}_n \exp(3\tilde{\sigma}_g^2)$	m	Volume median diameter, Mass median diameter	$\int_0^{\tilde{D}_v} \dfrac{\pi}{6}D^3 n_n(D)\,\mathrm{d}D = \dfrac{V_0}{2}$
D_v	$\tilde{D}_n \exp(7\tilde{\sigma}_g^2/2)$	m	Mass-weighted mean diameter, Volume-weighted mean diameter	$D_v = \dfrac{1}{V_0}\int_0^\infty \dfrac{\pi}{6}D^3 n_n(D)\,\mathrm{d}D$

For brevity Table 1 presents the lognormal relations in terms of diamter D. Change the relations to befunctions of radius r is straightforward. For example, direct substitution of $D = 2r$ into (21) yields

$$
\begin{aligned}
n_{\mathrm{n}}(D) &= \frac{1}{\sqrt{2\pi}\, 2r \ln \sigma_{\mathrm{g}}} \exp\left[-\frac{1}{2}\left(\frac{\ln(2r/2\tilde{r}_{\mathrm{n}})}{\ln \sigma_{\mathrm{g}}}\right)^2\right] \\
&= \frac{1}{2}\frac{1}{\sqrt{2\pi}\, r \ln \sigma_{\mathrm{g}}} \exp\left[-\frac{1}{2}\left(\frac{\ln(r/\tilde{r}_{\mathrm{n}})}{\ln \sigma_{\mathrm{g}}}\right)^2\right] \\
&= \frac{1}{2}n_{\mathrm{n}}^{r}(r)
\end{aligned}
\tag{28}
$$

in agreement with (12).

Table 2 lists applies the relations in Table 1 to specific size distributions typical of tropospheric aerosols.

Table 2: **Measured Lognormal Dust Size Distributions**[5]

$\tilde{D}_{\mathrm{n}}$	$\tilde{D}_{\mathrm{v}}$	σ_{g}	M	Ref.
µm	µm			
Patterson and Gillette (1977) [6]				
0.08169	0.27	1.88		**??**
0.8674	5.6	2.2		**??**
28.65	57.6	1.62		**??**
Shettle (1984)[7]				
0.003291	0.0111	1.89	2.6×10^{-4}	**??**
0.5972	2.524	2.0^8	0.781	**??, ??**
7.575	42.1	2.13	0.219	**??**
Balkanski et al. (1996)[9]				
0.1600	0.832	2.1	0.036	**??**
1.401	4.82	1.90	0.957	**??**
9.989	19.38	1.60	0.007	**??**

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[6]Detailed fits to dust sampled over Colorado and Texas in *Patterson and Gillette* (1977), p. 2080 Table 1. Original values have been converted from radius to diameter. M was not given. *Patterson and Gillette* (1977) showed soil aerosol could be represented with three modes which they dubbed, in order of increasing size, modes C, A, and B. Mode A is the mineral dust transport mode, seen in source regions and downwind. Mode B is seen in the source soil itself, and in the atmosphere during dust events. Mode C is seen most everywhere, but does not usually correlate with local dust amount. Mode C is usually a global, aged, background, anthropogenic aerosol, typically rich in sulfate and black carbon. Sometimes, however, Mode C has a mineral dust component. Modes C and B are averages from *Patterson and Gillette* (1977) Table 1 p. 2080. Mode B is based on the summary recommendation that $\tilde{r}_{\mathrm{s}} = 1.5$ and $\sigma_{\mathrm{g}} = 2.2$.

[7]Background Desert Model from *Shettle* (1984), p. 75 Table 1.

[9]*Balkanski et al.* (1996), p. 73 Table 2. These are the "background" modes of D'Almeida (1987).

Table 2: (continued)

$\tilde{D}_\mathrm{n}$	$\tilde{D}_\mathrm{v}$	σ_g	M	Ref.
µm	µm			
Alfaro et al. (1998) [10]				
0.6445	1.5	1.7	$(0.22, 0.15)$	??
3.454	6.7	1.6	$(0.69, 0.76)$	??
8.671	14.2	1.5	$(0.09, 0.09)$	??
Dubovik et al. (2002a), Bahrain (1998–2000) [11][12]				
0.1768	0.30 ± 0.08	0.42 ± 0.04		??
1.664	5.08 ± 0.08	0.61 ± 0.02		??
Dubovik et al. (2002a), Solar Village Saudi Arabia (1998–2000) [13]				
0.1485	0.24 ± 0.10	0.40 ± 0.05		??
1.576	4.64 ± 0.06	0.60 ± 0.03		??
Dubovik et al. (2002a), Cape Verde (1993–2000) [14]				
0.1134	0.24 ± 0.06	$0.49 + 0.10\tau \pm 0.04$		??
1.199	3.80 ± 0.06	$0.63 - 0.10\tau \pm 0.03$		??
Maring et al. (2003) [15]				
	3.6 ± 0.3	none		??
	4.1	none		??
Arimoto et al. (2006) [16]				
0.0	1.1	0.0		??
0.0	5.5	0.0		??
0.0	14	0.0		??
Mokhtari et al. (2012) [17]				
0.0	0.2	1.75	0.0008	??
0.0	1.67	1.76	0.0092	??
0.0	11.6	1.70	0.99	??

[10] Mass fractions are reported as (a,b) for measurements and model, respectively, of Spanish soil sample at $u_* = 66\,\mathrm{cm\,s^{-1}}$

[11] All *Dubovik et al.* (2002a) measurements follow certain conventions. Standard deviation of measurements follows $\pm$ sign. Reported σ_g is *not* the geometric standard deviation. Their σ_g is defined as the standard deviation of the logarithm of the size distribution (*Dubovik et al.*, 2002a, p. 606, Equation A2).

[12] Bahrain is an island in the Red Sea.

[13] The Solar Village AERONET station is located in empty land a few kilometers west of Riyadh's international airport.

[14] Values of $\tilde{D}_\mathrm{n}$ were computed using σ_g based on $\tau = 0.1$.

[15] Measurements during PRIDE, July 2000, from Izaña and Puerto Rico. Only $\tilde{D}_\mathrm{v}$ reported as measurements did not fit lognormal distributions.

[16] Original manuscript does not contain σ_g.

[17] AMMA size distribution used in DEAD coupled with SURFEX.

Table 3: **Analytic Lognormal Size Distribution Statistics** [a][b]

$\tilde{D}_{\mathrm{n}}$ $\tilde{r}_{\mathrm{n}}$ µm	$\bar{D}_{\mathrm{n}}, D_{\mathrm{n}}$ $\bar{r}_{\mathrm{n}}, r_{\mathrm{n}}$ µm	$\bar{D}_{\mathrm{s}}$ $\bar{r}_{\mathrm{s}}$ µm	$\bar{D}_{\mathrm{v}}$ $\bar{r}_{\mathrm{v}}$ µm	$\tilde{D}_{\mathrm{s}}$ $\tilde{r}_{\mathrm{s}}$ µm	D_{s} r_{s} µm	$\tilde{D}_{\mathrm{v}}$ $\tilde{r}_{\mathrm{v}}$ µm	D_{v} r_{v} µm	σ_{g}
0.1	0.1272	0.1619	0.2056	0.2614	0.3323	0.4227	0.5373	2.0
0.1861	0.2366	0.3009	0.3825	0.4864	0.6185	0.7864	1.0	2.0
0.2366	0.3008	0.3825	0.4864	0.6185	0.7864	1.0	1.272	2.0
0.3009	0.3825	0.4864	0.6185	0.7864	1.0	1.272	1.617	2.0
0.3825	0.4864	0.6185	0.7864	1.0	1.272	1.617	2.056	2.0
0.4864	0.6185	0.7864	1.0	1.272	1.617	2.056	2.614	2.0
0.5915	0.7521	0.9563	1.216	1.546	1.966	2.5	3.179	2.0
0.6185	0.7864	1.0	1.272	1.617	2.056	2.614	3.324	2.0
0.7864	1.0	1.272	1.617	2.056	2.614	3.324	4.225	2.0
0.8281	1.053	1.339	1.702	2.165	2.753	3.5	4.450	2.0
1.0	1.272	1.617	2.056	2.614	3.324	4.227	5.373	2.0
1.183	1.504	1.913	2.432	3.092	3.932	5.0	6.356	2.0
2.366	3.008	3.825	4.864	6.184	7.864	10.0	12.72	2.0

[a]Shown are statistics for each moment equalling 1 µm, and for $\tilde{D}_{\mathrm{v}} = 0.1, 2.5, 3.5, 5.0, 10.0$ µm.

[b] $\bar{D}_{\mathrm{n}}$, $\bar{D}_{\mathrm{s}}$, and $\bar{D}_{\mathrm{v}}$ are number, surface, and volume-mean diameters, respectively. $\tilde{D}_{\mathrm{n}}$, $\tilde{D}_{\mathrm{s}}$, and $\tilde{D}_{\mathrm{v}}$ are number, surface, and volume median diameters, respectively. D_{n}, D_{s}, and D_{v} are number, surface, and volume-weighted diameters, respectively.

Perry et al. (1997) and *Perry and Cahill* (1999) describe measurements and transport of dust across the Atlantic and Pacific, respectively. *Reid et al.* (2003) summarize historical measurements of dust size distributions, and analyze the influence of measurement technique on the derived size distribution. They show the derived size distribution is strongly sensitive to the measurment technique. During PRIDE, measured $\tilde{D}_{\mathrm{v}}$ varied from 2.5–9 µm depending on the instrument employed. *Maring et al.* (2003) show that the change in mineral dust size distribution across the sub-tropical Atlantic is consistent with a slight updraft of $\sim 0.33\,\mathrm{cm\,s^{-1}}$ during transport. *Ginoux* (2003) and *Colarco et al.* (2003) show that the effects of asphericity on particle settling velocity play an important role in maintaining the large particle tail of the size distribution during long range transport.

Table 3 applies the relations in Table 1 to specific size distributions typical of tropospheric aerosols. Values in Table 3 are valid for radius and diameter distributions. Table 1 shows that all moments of the size distribution depend linearly on $\tilde{D}_{\mathrm{n}}$ (or $\tilde{r}_{\mathrm{n}}$). Therefore all rows in Table 3 scale linearly (for a constant geometric standard deviation). For example, values in the row with $\tilde{D}_{\mathrm{n}} = 1.0$ µm are ten times the corresponding values for the row $\tilde{D}_{\mathrm{n}} = 0.1$ µm. Hence it suffices for Table 3 to show a decade range in $\tilde{D}_{\mathrm{n}}$.

3.3.3   Related Forms

Many important applications make available size distribution information in a form similar to, but hard to recognize as, the analytic lognormal PDF (21). The Aerosol Robotic Network, *AERONET*, for example, retrieves size distributions from solar almucantar radiances[18] (*Dubovik and King*, 2000; *Dubovik et al.*, 2000, 2002b). AERONET labels the retrieved size distribution $\mathrm{d}V(r)/\mathrm{d}\ln r$ and reports the values in [$\mu m^3 \, \mu m^{-2}$] units. The correspondence between the AERONET retrievals and $\mathrm{d}N/\mathrm{d}\ln r$ (21) in [$\# \, m^{-3} \, m^{-1}$] units is not exactly clear. Unfortunately, Table 1 does not help much here. Let us now show how to bridge the gap between theory and measurement.

First, total distributions contain N_0 particles per unit volume and thus N_0 applies as a multiplicative factor to (21)

$$n_{\mathrm{n}}(D) = \frac{N_0}{\sqrt{2\pi}\, D \ln \sigma_{\mathrm{g}}} \exp\left[-\frac{1}{2}\left(\frac{\ln(D/\tilde{D}_{\mathrm{n}})}{\ln \sigma_{\mathrm{g}}}\right)^2\right] \tag{29}$$

Note that (29) is only normalized if $N_0 = 1.0$ (cf. Section 3.4.2).

Applying (6) to (29) yields

$$\frac{\mathrm{d}N}{\mathrm{d}D} = \frac{N_0}{\sqrt{2\pi}\, D \ln \sigma_{\mathrm{g}}} \exp\left[-\frac{1}{2}\left(\frac{\ln(D/\tilde{D}_{\mathrm{n}})}{\ln \sigma_{\mathrm{g}}}\right)^2\right] \tag{30}$$

Multiplying each side of (30) by D and substituting $\mathrm{d}\ln D = D^{-1}\,\mathrm{d}D$ leads to

$$\frac{\mathrm{d}N}{\mathrm{d}\ln D} = \frac{N_0}{\sqrt{2\pi}\, \ln \sigma_{\mathrm{g}}} \exp\left[-\frac{1}{2}\left(\frac{\ln(D/\tilde{D}_{\mathrm{n}})}{\ln \sigma_{\mathrm{g}}}\right)^2\right] \tag{31}$$

The derivative in (31) is with respect to the logarithm of the diameter. The change in the independent variable of differentiation defines a new distribution which could be written $n_{\mathrm{n}}(\ln D)$ to distinguish it from the normal linear distribution $n_{\mathrm{n}}(D)$ (6). However, the nomenclature $n_{\mathrm{n}}(\ln D)$ could be misinterpreted. We follow *Seinfeld and Pandis* (1997) and denote logarithmically-defined distributions with a superscript e on the distribution that re-inforces the use of $\ln D$ as the independent variable

$$n_{\mathrm{n}}^{\mathrm{e}}(\ln D) \equiv n_{\mathrm{n}}(\ln D) \equiv \frac{\mathrm{d}N}{\mathrm{d}\ln D} \tag{32}$$

The SI units of $n_{\mathrm{n}}(D)$ (6) and $n_{\mathrm{n}}^{\mathrm{e}}(\ln D)$ (32) are [$\# \, m^{-3} \, m^{-1}$] and [$\# \, m^{-3}$], respectively.

Remote sensing applications often retrieve columnar distributions rather than volumetric distributions. The columnar number distribution $n_{\mathrm{n}}^{\mathrm{c}}(D)$, for example, is simply the vertical integral

---

[18] The almucantar radiances are radiance measurements in a circle of equal scattering angle centered in a plane about the Sun, i.e., radiance measurements at known forward scattering phase function angles.

of the particle number distribution $n_{\rm n}(D)$,

$$n_{\rm n}^{\rm c}(D) \equiv \frac{{\rm d}N_0^{\rm c}}{{\rm d}D} = \int_{z=0}^{z=\infty} n_{\rm n}(D,z)\,{\rm d}z \; = {\rm same} \tag{33a}$$

$$n_{\rm x}^{\rm c}(D) \equiv \frac{{\rm d}A_0^{\rm c}}{{\rm d}D} = \int_{z=0}^{z=\infty} n_{\rm x}(D,z)\,{\rm d}z \; = \int_{z=0}^{z=\infty} \frac{\pi}{4}D^2 n_{\rm n}(D,z)\,{\rm d}z \tag{33b}$$

$$n_{\rm s}^{\rm c}(D) \equiv \frac{{\rm d}S_0^{\rm c}}{{\rm d}D} = \int_{z=0}^{z=\infty} n_{\rm s}(D,z)\,{\rm d}z \; = \int_{z=0}^{z=\infty} \pi D^2 n_{\rm n}(D,z)\,{\rm d}z \tag{33c}$$

$$n_{\rm v}^{\rm c}(D) \equiv \frac{{\rm d}V_0^{\rm c}}{{\rm d}D} = \int_{z=0}^{z=\infty} n_{\rm v}(D,z)\,{\rm d}z \; = \int_{z=0}^{z=\infty} \frac{\pi}{6}D^3 n_{\rm n}(D,z)\,{\rm d}z \tag{33d}$$

$$n_{\rm m}^{\rm c}(D) \equiv \frac{{\rm d}M_0^{\rm c}}{{\rm d}D} = \int_{z=0}^{z=\infty} n_{\rm m}(D,z)\,{\rm d}z = \int_{z=0}^{z=\infty} \frac{\pi}{6}\rho D^3 n_{\rm n}(D,z)\,{\rm d}z \tag{33e}$$

SI units of the columnar distributions $n_x^{\rm c}$ for $x = {\rm n, x, s, v, m}$ (33) are one less "per meter" than the corresponding volumetric distributions, e.g., $n_{\rm v}$ and $n_{\rm v}^{\rm c}$ are in $[{\rm m}^3\,{\rm m}^{-3}\,{\rm m}^{-1}]$ and $[{\rm m}^3\,{\rm m}^{-2}\,{\rm m}^{-1}]$, respectively. This is because of integration over the vertical coordinate.

Combining (33) with (31) leads to

$$n_{\rm n}^{\rm e,c}(\ln D) \equiv \frac{{\rm d}N_0^{\rm c}}{{\rm d}\ln D} = \frac{N_0^{\rm c}}{\sqrt{2\pi}\,\ln \sigma_{\rm g}} \exp\left[-\frac{1}{2}\left(\frac{\ln(D/\tilde{D}_{\rm n})}{\ln \sigma_{\rm g}}\right)^2\right] \tag{34a}$$

$$n_{\rm x}^{\rm e,c}(\ln D) \equiv \frac{{\rm d}A_0^{\rm c}}{{\rm d}\ln D} = \sqrt{\frac{\pi}{2}}\frac{N_0^{\rm c}D^2}{4\ln \sigma_{\rm g}} \exp\left[-\frac{1}{2}\left(\frac{\ln(D/\tilde{D}_{\rm n})}{\ln \sigma_{\rm g}}\right)^2\right] \tag{34b}$$

$$n_{\rm s}^{\rm e,c}(\ln D) \equiv \frac{{\rm d}S_0^{\rm c}}{{\rm d}\ln D} = \sqrt{\frac{\pi}{2}}\frac{N_0^{\rm c}D^2}{\ln \sigma_{\rm g}} \exp\left[-\frac{1}{2}\left(\frac{\ln(D/\tilde{D}_{\rm n})}{\ln \sigma_{\rm g}}\right)^2\right] \tag{34c}$$

$$n_{\rm v}^{\rm e,c}(\ln D) \equiv \frac{{\rm d}V_0^{\rm c}}{{\rm d}\ln D} = \sqrt{\frac{\pi}{2}}\frac{N_0^{\rm c}D^3}{6\ln \sigma_{\rm g}} \exp\left[-\frac{1}{2}\left(\frac{\ln(D/\tilde{D}_{\rm n})}{\ln \sigma_{\rm g}}\right)^2\right] \tag{34d}$$

$$n_{\rm m}^{\rm e,c}(\ln D) \equiv \frac{{\rm d}M_0^{\rm c}}{{\rm d}\ln D} = \sqrt{\frac{\pi}{2}}\frac{\rho N_0^{\rm c}D^3}{6\ln \sigma_{\rm g}} \exp\left[-\frac{1}{2}\left(\frac{\ln(D/\tilde{D}_{\rm n})}{\ln \sigma_{\rm g}}\right)^2\right] \tag{34e}$$

These logarithmic columnar (vertically integrated) distributions (34) are one less "per meter" than the corresponding linear columnar distributions (33), e.g., $n_{\rm v}^{\rm c}$ and $n_{\rm v}^{\rm e,c}$ are in $[{\rm m}^3\,{\rm m}^{-2}\,{\rm m}^{-1}]$ and $[{\rm m}^3\,{\rm m}^{-2}]$, respectively. In order for the area under the curve to be proportional to the integrated distributions, logarithmic distributions should be plotted on semi-log axes, e.g., horizontal axis with logarithmic size D and vertical axis with linearly spaced values of $n_{\rm v}^{\rm e}(\ln D)$ (*Seinfeld and Pandis*, 1997, p. 415).

Measurements (or retrievals such as AERONET) are usually reported in historical units that can be counted rather than in pure SI. SI units for $n_{\rm v}(D) = {\rm d}V(D)/{\rm d}D$ are $[{\rm m}^3\,{\rm m}^{-3}\,{\rm m}^{-1}]$, i.e., particle volume per unit air volume per unit particle diameter. These units condense to $[{\rm m}^3\,{\rm m}^{-2}]$, or, multiplying by 10^6, $[\mu{\rm m}^3\,\mu{\rm m}^{-2}]$. These condensed units may be confused with particle volume per unit particle surface area ($V(D)/S(D)$), or with columnar particle volume per unit horizontal

surface (e.g., ground or ocean) area ($\int V(z)\,\mathrm{d}z$). AERONET most definitely does *not* report any of these three quantities $\mathrm{d}V/\mathrm{d}r$, $V(D)/S(D)$, or $\int V(z)\,\mathrm{d}z$. AERONET reports $n_{\mathrm{v}}^{\mathrm{e,c}}(\ln D)$ the vertically integrated logarithmic volume distribution (34d), the logarithmic derivative of the columnar volume V_0^{c}.

3.3.4   Variance

According to (15), the variance σ_D^2 of the lognormal distribution (21) is

$$\sigma_D^2 = \frac{1}{\sqrt{2\pi}\,\ln\sigma_{\mathrm{g}}} \int_0^\infty \frac{1}{D}\exp\left[-\frac{1}{2}\left(\frac{\ln(D/\tilde{D}_{\mathrm{n}})}{\ln\sigma_{\mathrm{g}}}\right)^2\right](D-\bar{D})^2\,\mathrm{d}D \qquad (35)$$

3.3.5   Non-standard terminology

Non-standard terminology leads to much confusion in the literature. For example, *Dubovik et al.* (2002a) provide precise analytic definitions of their supposedly lognormal size distribution parameters. However, their terminology is inconsistent with their definitions. Distributions computed according to their definitions are not lognormal distributions. *Dubovik et al.* (2002a) Equation A1 (their p. 606) defines the *mean* logarithmic radius $\bar{r}_{\mathrm{v}}$ of the volume distribution which they confusingly name the volume *median* radius $\tilde{r}_{\mathrm{v}}$. *Dubovik et al.* (2002a) Equation A2 (their p. 606) defines the standard deviation of the logarithm of the volume distribution. This differs from the geometric standard deviation σ_{g} of a lognormal distribution. The correct parameters of a lognormal distribution (21) are $\tilde{r}_{\mathrm{n}}$ and σ_{g} (or $\tilde{\sigma}_{\mathrm{g}} \equiv \ln\sigma_{\mathrm{g}}$) For a lognormal volume path distribution $n_{\mathrm{v}}^{\mathrm{e,c}}(\ln D)$ (34d) the appropriate parameters are $\tilde{r}_{\mathrm{v}}$ and σ_{g} (or $\tilde{\sigma}_{\mathrm{g}} \equiv \ln\sigma_{\mathrm{g}}$), not $\bar{r}_{\mathrm{v}}$ and $\sqrt{\sigma_r^2}$ (35). *Dubovik et al.* (2002a) Equation 1 (their p. 593) is the correct form for $n_{\mathrm{v}}^{\mathrm{e,c}}(\ln D)$ (34d).

3.3.6   Bounded Distribution

The statistical properties of a bounded lognormal distribution are expressed in terms of the error function (§5.2). The cumulative concentration bounded by $D_{\max}$ is given by applying (2) to (21)

$$N(D < D_{\max}) = \frac{N_0}{\sqrt{2\pi}\,\ln\sigma_{\mathrm{g}}} \int_0^{D_{\max}} \frac{1}{D}\exp\left[-\frac{1}{2}\left(\frac{\ln(D/\tilde{D}_{\mathrm{n}})}{\ln\sigma_{\mathrm{g}}}\right)^2\right]\mathrm{d}D \qquad (36)$$

We make the change of variable $z = (\ln D - \ln\tilde{D}_{\mathrm{n}})/\sqrt{2}\,\ln\sigma_{\mathrm{g}}$

$$
\begin{aligned}
z &= (\ln D - \ln\tilde{D}_{\mathrm{n}})/\sqrt{2}\,\ln\sigma_{\mathrm{g}} \\
D &= \tilde{D}_{\mathrm{n}}\mathrm{e}^{\sqrt{2}\,z\ln\sigma_{\mathrm{g}}} \\
&= \tilde{D}_{\mathrm{n}}\sigma_{\mathrm{g}}^{\sqrt{2}\,z} \\
\mathrm{d}z &= (\sqrt{2}\,D\ln\sigma_{\mathrm{g}})^{-1}\,\mathrm{d}D \\
\mathrm{d}D &= \sqrt{2}\,\ln\sigma_{\mathrm{g}}\tilde{D}_{\mathrm{n}}\mathrm{e}^{\sqrt{2}\,z\ln\sigma_{\mathrm{g}}}\,\mathrm{d}z \\
&= \sqrt{2}\,\ln\sigma_{\mathrm{g}}\tilde{D}_{\mathrm{n}}\sigma_{\mathrm{g}}^{\sqrt{2}\,z}\,\mathrm{d}z
\end{aligned}
\qquad (37)
$$

which maps $D \in (0, D_{\max})$ into $z \in (-\infty, \ln D_{\max} - \ln \tilde{D}_n)/\sqrt{2} \ln \sigma_g)$. In terms of z we obtain

$$
\begin{aligned}
N(D < D_{\max}) &= \frac{N_0}{\sqrt{2\pi} \ln \sigma_g} \int_{-\infty}^{(\ln D_{\max} - \ln \tilde{D}_n)/\sqrt{2} \ln \sigma_g} \frac{1}{\tilde{D}_n e^{\sqrt{2} z \ln \sigma_g}} \, e^{-z^2} \sqrt{2} \ln \sigma_g \tilde{D}_n e^{\sqrt{2} z \ln \sigma_g} \, dz \\
&= \frac{N_0}{\sqrt{\pi}} \int_{-\infty}^{(\ln D_{\max} - \ln \tilde{D}_n)/\sqrt{2} \ln \sigma_g} e^{-z^2} \, dz \\
&= \frac{N_0}{\sqrt{\pi}} \left(\int_{-\infty}^{0} e^{-z^2} \, dz + \int_{0}^{(\ln D_{\max} - \ln \tilde{D}_n)/\sqrt{2} \ln \sigma_g} e^{-z^2} \, dz \right) \\
&= \frac{N_0}{2} \left(\frac{2}{\sqrt{\pi}} \int_{0}^{+\infty} e^{-z^2} \, dz + \frac{2}{\sqrt{\pi}} \int_{0}^{(\ln D_{\max} - \ln \tilde{D}_n)/\sqrt{2} \ln \sigma_g} e^{-z^2} \, dz \right) \\
&= \frac{N_0}{2} \left[\mathrm{erf}(\infty) + \mathrm{erf}\left(\frac{\ln(D_{\max}/\tilde{D}_n)}{\sqrt{2} \ln \sigma_g} \right) \right] \\
&= \frac{N_0}{2} + \frac{N_0}{2} \mathrm{erf}\left(\frac{\ln(D_{\max}/\tilde{D}_n)}{\sqrt{2} \ln \sigma_g} \right)
\end{aligned}
\tag{38}
$$

where we have used the properties of the error function (§5.2). The same procedure can be performed to compute the cumulative concentration of particles smaller than $D_{\min}$. When $N(D < D_{\min})$ is subtracted from (38) we obtain the truncated concentration (4)

$$
N(D_{\min}, D_{\max}) = \frac{N_0}{2} \left[\mathrm{erf}\left(\frac{\ln(D_{\max}/\tilde{D}_n)}{\sqrt{2} \ln \sigma_g} \right) - \mathrm{erf}\left(\frac{\ln(D_{\min}/\tilde{D}_n)}{\sqrt{2} \ln \sigma_g} \right) \right]
\tag{39}
$$

We are also interested in the bounded distributions of higher moments, e.g., the mass of particles lying between $D_{\min}$ and $D_{\max}$. The cross-sectional area, surface area, volume, and mass distributions of spherical particles are related to their number distribution by

$$
n_x(D) = \frac{\pi}{4} D^2 n_n(D) \tag{40a}
$$

$$
n_s(D) = \pi D^2 n_n(D) \tag{40b}
$$

$$
n_v(D) = \frac{\pi}{6} D^3 n_n(D) \tag{40c}
$$

$$
n_m(D) = \frac{\pi}{6} \rho D^3 n_n(D) \tag{40d}
$$

so that we may simply substitute $\tilde{D}_n = \tilde{D}_v$, for example, in (39) and we obtain

$$
V(D_{\min}, D_{\max}) = \frac{N_0}{2} \left[\mathrm{erf}\left(\frac{\ln(D_{\max}/\tilde{D}_v)}{\sqrt{2} \ln \sigma_g} \right) - \mathrm{erf}\left(\frac{\ln(D_{\min}/\tilde{D}_v)}{\sqrt{2} \ln \sigma_g} \right) \right]
\tag{41}
$$

3.3.7   Statistics of Bounded Distributions

All of the relations given in Table 1 may be re-expressed in terms of truncated lognormal distributions, but doing so is tedious, and requires new terminology. Instead we derive the expression for a

typical size distribution statistic, and allow the reader to generalize. We generalize (13) to consider

$$\overline{g}^* = \int_{D_{\min}}^{D_{\max}} D\, p^*(D)\, \mathrm{d}D \tag{42}$$

Note the domain of integration, $D \in (D_{\min}, D_{\max})$, reflects the fact that we are considering a bounded distribution. The superscript * indicates that the average statistic refers to a truncated distribution and reminds us that $\overline{g}^* \neq \overline{g}$. Defining a closed form expression for $p^*(D)$ requires some consideration. This truncated distribution has N_0^* defined by (39), and is completely specified on $D \in (0, \infty)$ by

$$p^*(D) = \begin{cases} 0 & , & 0 < D < D_{\min} \\ N(D_{\min}, D_{\max})\, p(D)/N_0 & , & D_{\min} \leq D \leq D_{\max} \\ 0 & , & D_{\max} < D < \infty \end{cases} \tag{43}$$

The difficulty is that the three parameters of the lognormal distribution, $\tilde{D}_{\mathrm{n}}$, σ_{g}, and N_0 are defined in terms of an untruncated distribution. Using (39) we can write

$$p^*(D) = \frac{1}{N_0^*}\, n_{\mathrm{n}}(D) N_0^* = N(D_{\min}, D_{\max}) \tag{44}$$

If we think of p^* order to be properly normalized to unity, note that (fxm) Thus when we speak of truncated distributions it is important to keep in mind that the parameters $\tilde{D}_{\mathrm{n}}$, σ_{g}, and N_0 refer to the untruncated distribution.

The properties of the truncated distribution will be expressed in terms of $\tilde{D}_{\mathrm{n}}^*$, σ_{g}^*, and N_0^*, respectively.

Consider the mean size, D. In terms of (13) we have $g(D) = D$ so that

$$\bar{D} = \int_{D_{\min}}^{D_{\max}} D\, p(D)\, \mathrm{d}x \tag{45}$$

3.3.8   Overlapping Distributions

Consider the problem of distributing I independent and possibly *overlapping distributions* of particles into J independent and possibly overlapping distributions of particles. To reify the problem we call the I bins the source bins (these bins represent the parent size distributions in mineral dust source areas) and the J bins as sink bins (which represent sizes transported in the atmosphere). Typically we know the total mass M_0 or number N_0 of source particles to distribute into the sink bins and we know the fraction of the total mass to distribute which resides in each source distribution, M_i. The problem is to determine matrices of overlap factors $N_{i,j}$ and $M_{i,j}$ which determine what number and mass fraction, respectively, of each source bin i is blown into each sink bin j.

The mass and number fractions contained by the source distributions are normalized such that

$$\sum_{i=1}^{I} M_i = \sum_{i=1}^{I} N_i = 1 \tag{46}$$

In the case of dust emissions, M_i and N_i may vary with spatial location.

The overlap factors $N_{i,j}$ and $M_{i,j}$ are defined by the relations

$$N_j = \sum_{i=1}^{I} N_{i,j} N_i$$

$$= N_0 \sum_{i=1}^{I} N_{i,j} N_i \tag{47}$$

$$M_j = \sum_{i=1}^{I} M_{i,j} M_i$$

$$= M_0 \sum_{i=1}^{I} M_{i,j} M_i \tag{48}$$

Using (39) and (46) we find

$$N_{i,j} = \frac{1}{2} \left[\mathrm{erf}\left(\frac{\ln(D_{\mathrm{max},j}/\tilde{D}_{\mathrm{n},i})}{\sqrt{2} \ln \sigma_{\mathrm{g},i}} \right) - \mathrm{erf}\left(\frac{\ln(D_{\mathrm{min},j}/\tilde{D}_{\mathrm{n},i})}{\sqrt{2} \ln \sigma_{\mathrm{g},i}} \right) \right] \tag{49}$$

$$M_{i,j} = \frac{1}{2} \left[\mathrm{erf}\left(\frac{\ln(D_{\mathrm{max},j}/\tilde{D}_{\mathrm{v},i})}{\sqrt{2} \ln \sigma_{\mathrm{g},i}} \right) - \mathrm{erf}\left(\frac{\ln(D_{\mathrm{min},j}/\tilde{D}_{\mathrm{v},i})}{\sqrt{2} \ln \sigma_{\mathrm{g},i}} \right) \right] \tag{50}$$

fxm: The mathematical derivation appears correct but the overlap factor appears to asymptote to 0.5 rather than to 1.0 for $D_{\mathrm{max}} \gg \tilde{D}_{\mathrm{n}} \gg D_{\mathrm{min}}$.

A mass distribution has the same form as a lognormal number distribution but has a different median diameter. Thus the overlap matrix elements apply equally to mass and number distributions depending on the median diameter used in the following formulae. For the case where both source and sink distributions are complete lognormal distributions,

$$M(D) = \sum_{i=1}^{i=I} M_i(D)$$

3.3.9 Median Diameter

Substituting $D = \tilde{D}_{\mathrm{n}}$ into (38) we obtain

$$N(D < \tilde{D}_{\mathrm{n}}) = \frac{N_0}{2} \tag{51}$$

This proves that $\tilde{D}_{\mathrm{n}}$ is the median diameter (5). The lognormal distribution is the only distribution known (to us) which is most naturally expressed in terms of its median diameter.

3.3.10 Mode Diameter

The *mode* is the most frequently occuring value of a distribution. The *mode diameter* or *modal diameter* of the number distribution $n_{\mathrm{n}}(D)$ is the diameter $\hat{D}_{\mathrm{n}}$ that satisfies

$$\left. \frac{\mathrm{d} n_{\mathrm{n}}(D)}{\mathrm{d} D} \right|_{D=\hat{D}_{\mathrm{n}}} = 0 \tag{52}$$

Applying condition (52) to (21) proves that the median and modal diameters are identical for lognormal distributions

$$\hat{D}_{\mathrm{n}} = \tilde{D}_{\mathrm{n}} \tag{53}$$

The number, surface, volume, and mass distributions are all lognormal if any one is. Therefore (53) implies $\hat{D}_{\mathrm{s}} = \tilde{D}_{\mathrm{s}}$, and $\hat{D}_{\mathrm{v}} = \tilde{D}_{\mathrm{v}}$.

3.3.11   Multimodal Distributions

Realistic particle size distributions may be expressed as an appropriately weighted sum of individual modes.

$$n_{\mathrm{n}}(D) = \sum_{i=1}^{I} n_{\mathrm{n}}^i(D) \tag{54}$$

where $n_{\mathrm{n}}^i(D)$ is the number distribution of the ith component mode[19]. Such particle size distributions are called *multimodal istributions* because they contain one maximum for each component distribution. Generalizing (1), the total number concentration becomes

$$
\begin{aligned}
N_0 &= \sum_{i=1}^{I} \int_0^{\infty} n_{\mathrm{n}}^i(D)\,\mathrm{d}D \\
&= \sum_{i=1}^{I} N_0^i
\end{aligned}
\tag{55}
$$

where N_0^i is the total number concentration of the ith component mode.

The median diameter of a multimodal distribution is obtained by following the logic of (36)–(39). The number of particles smaller than a given size is

$$N(D < D_{\mathrm{max}}) = \sum_{i=1}^{I} \frac{N_0^i}{2} + \frac{N_0^i}{2}\,\mathrm{erf}\left(\frac{\ln(D_{\mathrm{max}}/\tilde{D}_{\mathrm{n}}^i)}{\sqrt{2}\,\ln\sigma_{\mathrm{g}}^i}\right) \tag{56}$$

For the median particle size, $D_{\mathrm{max}} \equiv \tilde{D}_{\mathrm{n}}$, and we can move the unknown $\tilde{D}_{\mathrm{n}}$ to the LHS yielding

$$
\begin{aligned}
\sum_{i=1}^{I} \frac{N_0^i}{2} + \frac{N_0^i}{2}\,\mathrm{erf}\left(\frac{\ln(\tilde{D}_{\mathrm{n}}/\tilde{D}_{\mathrm{n}}^i)}{\sqrt{2}\,\ln\sigma_{\mathrm{g}}^i}\right) &= \frac{N_0}{2} \\
\sum_{i=1}^{I} N_0^i\,\mathrm{erf}\left(\frac{\ln(\tilde{D}_{\mathrm{n}}/\tilde{D}_{\mathrm{n}}^i)}{\sqrt{2}\,\ln\sigma_{\mathrm{g}}^i}\right) &= 0
\end{aligned}
\tag{57}
$$

where we have used $N_0 = \sum_i^I N_0^i$. Obtaining $\tilde{D}_{\mathrm{n}}$ for a multimodal distribution requires numerically solving (57) given the N_0^i, $\tilde{D}_{\mathrm{n}}^i$, and σ_{g}^i.

---

[19]Throughout this section the i superscript represents an index of the component mode, not an exponent.

3.4 Higher Moments

It is often useful to compute higher moments of the number distribution. Each factor of the independent variable weighting the number distribution function $n_\mathrm{n}(D)$ in the integrand of (14) counts as a *moment*. The kth moment of $n_\mathrm{n}(D)$ is

$$F(k) = \int_0^\infty n_\mathrm{n}(D) D^k \, \mathrm{d}D \tag{58}$$

The statistical properties of higher moments of the lognormal size distribution may be obtained by direct integration of (58).

$$
\begin{aligned}
F(k) &= \frac{N_0}{\sqrt{2\pi}\,\ln\sigma_\mathrm{g}} \int_0^\infty \frac{1}{D} \exp\left[-\frac{1}{2}\left(\frac{\ln(D/\tilde{D}_\mathrm{n})}{\ln\sigma_\mathrm{g}}\right)^2\right] D^k \, \mathrm{d}D \\[2mm]
&= \frac{N_0}{\sqrt{2\pi}\,\ln\sigma_\mathrm{g}} \int_0^\infty D^{k-1} \exp\left[-\frac{1}{2}\left(\frac{\ln(D/\tilde{D}_\mathrm{n})}{\ln\sigma_\mathrm{g}}\right)^2\right] \mathrm{d}D
\end{aligned} \tag{59}
$$

We make the same change of variable $z = (\ln D - \ln\tilde{D}_\mathrm{n})/\sqrt{2}\,\ln\sigma_\mathrm{g}$ as in (37). This maps $D \in (0, +\infty)$ into $z \in (-\infty, +\infty)$. In terms of z we obtain

$$
\begin{aligned}
F(k) &= \frac{N_0}{\sqrt{2\pi}\,\ln\sigma_\mathrm{g}} \int_{-\infty}^{+\infty} (\tilde{D}_\mathrm{n}\mathrm{e}^{\sqrt{2}\,z\ln\sigma_\mathrm{g}})^{k-1} \mathrm{e}^{-z^2}\sqrt{2}\,\ln\sigma_\mathrm{g}\tilde{D}_\mathrm{n}\mathrm{e}^{\sqrt{2}\,z\ln\sigma_\mathrm{g}}\,\mathrm{d}z \\[2mm]
&= \frac{N_0}{\sqrt{\pi}} \int_{-\infty}^{+\infty} (\tilde{D}_\mathrm{n}\mathrm{e}^{\sqrt{2}\,z\ln\sigma_\mathrm{g}})^k \mathrm{e}^{-z^2}\,\mathrm{d}z \\[2mm]
&= \frac{N_0\tilde{D}_\mathrm{n}^k}{\sqrt{\pi}} \int_{-\infty}^{+\infty} \mathrm{e}^{\sqrt{2}kz\ln\sigma_\mathrm{g}}\mathrm{e}^{-z^2}\,\mathrm{d}z \\[2mm]
&= \frac{N_0\tilde{D}_\mathrm{n}^k}{\sqrt{\pi}} \int_{-\infty}^{+\infty} \mathrm{e}^{-z^2+\sqrt{2}kz\ln\sigma_\mathrm{g}}\,\mathrm{d}z \\[2mm]
&= \frac{N_0\tilde{D}_\mathrm{n}^k}{\sqrt{\pi}}\sqrt{\pi}\exp\left(\frac{2k^2\ln^2\sigma_\mathrm{g}}{4}\right) \\[2mm]
&= N_0\tilde{D}_\mathrm{n}^k\exp(\tfrac{1}{2}k^2\ln^2\sigma_\mathrm{g})
\end{aligned} \tag{60}
$$

where we have used (73) with $\alpha = 1$ and $\beta = \sqrt{2}k\ln\sigma_\mathrm{g}$.

Applying the formula (60) to the first five moments of the lognormal distribution function we obtain

$$
\begin{aligned}
F(0) &= \int_0^\infty n_\mathrm{n}(D)\,\mathrm{d}D &&= N_0 &&= N_0 &&= N_0 \\[2mm]
F(1) &= \int_0^\infty n_\mathrm{n}(D)D\,\mathrm{d}D &&= N_0\tilde{D}_\mathrm{n}\exp(\tfrac{1}{2}\ln^2\sigma_\mathrm{g}) &&= D_0 &&= N_0\bar{D}_\mathrm{n} \\[2mm]
F(2) &= \int_0^\infty n_\mathrm{n}(D)D^2\,\mathrm{d}D &&= N_0\tilde{D}_\mathrm{n}^2\exp(2\ln^2\sigma_\mathrm{g}) &&= \frac{S_0}{\pi} &&= N_0\bar{D}_\mathrm{s}^2 \\[2mm]
F(3) &= \int_0^\infty n_\mathrm{n}(D)D^3\,\mathrm{d}D &&= N_0\tilde{D}_\mathrm{n}^3\exp(\tfrac{9}{2}\ln^2\sigma_\mathrm{g}) &&= \frac{6V_0}{\pi} &&= N_0\bar{D}_\mathrm{v}^3 \\[2mm]
F(4) &= \int_0^\infty n_\mathrm{n}(D)D^4\,\mathrm{d}D &&= N_0\tilde{D}_\mathrm{n}^4\exp(8\ln^2\sigma_\mathrm{g})
\end{aligned} \tag{61}
$$

Table 1 includes these relations in slightly different forms.

The first few moments of the number distribution are related to measurable properties of the size distribution. In particular, $F(k = 0)$ is the number concentration. Other quantities of merit are ratios of consecutive moments. For example, the volume-weighted diameter D_v is computed by weighted each diameter by the volume of particles at that diameter and then normalizing by the total volume of all particles.

$$
\begin{aligned}
D_\mathrm{v} &= \int_0^\infty D \frac{\pi}{6} D^3 n_\mathrm{n}(D)\,\mathrm{d}D \bigg/ \int_0^\infty \frac{\pi}{6} D^3 n_\mathrm{n}(D)\,\mathrm{d}D \\
&= \int_0^\infty D^4 n_\mathrm{n}(D)\,\mathrm{d}D \bigg/ \int_0^\infty D^3 n_\mathrm{n}(D)\,\mathrm{d}D \\
&= F(4)/F(3) \\
&= \frac{N_0 \tilde{D}_\mathrm{n}^4 \exp(8\ln^2 \sigma_\mathrm{g})}{N_0 \tilde{D}_\mathrm{n}^3 \exp(\frac{9}{2}\ln^2 \sigma_\mathrm{g})} \\
&= \tilde{D}_\mathrm{n} \exp(\tfrac{7}{2}\ln^2 \sigma_\mathrm{g}) \tag{62}
\end{aligned}
$$

The *surface-weighted diameter* D_s is defined analogously to D_v. D_s is more often known by its other name, the effective diameter (twice the effective radius). The term "effective" refers to the light extinction properties of the distribution. Light impinging on a particle distribution is, in the limit of geometric optics, extinguished in proportion to the cross-sectional area of the particles. Hence the effective diameter (or radius) characterizes the extinction properties of the distribution. Following (62), the effective diameter of a lognormal distribution is

$$
\begin{aligned}
D_\mathrm{s} &= F(3)/F(2) \\
&= \frac{N_0 \tilde{D}_\mathrm{n}^3 \exp(\frac{9}{2}\ln^2 \sigma_\mathrm{g})}{N_0 \tilde{D}_\mathrm{n}^2 \exp(2\ln^2 \sigma_\mathrm{g})} \\
&= \tilde{D}_\mathrm{n} \exp(\tfrac{5}{2}\ln^2 \sigma_\mathrm{g}) \tag{63}
\end{aligned}
$$

Moment-weighted diameters, such as the volume-weighted diameter D_v (62), characterize disperse distributions. A disperse mass distribution $n_\mathrm{m}(D)$ behaves most like a monodisperse distribution with all mass residing at $D = D_\mathrm{v}$. Due to approximations, physical operators may be constrained to act on a single, representative diameter rather than an entire distribution. The "least-wrong" diameter to pick is the moment-weighted diameter most relevant to the process being modeled. For example, D_v best represents the gravitational sedimentation of a distribution of particles. On the other hand, D_s (63) best represents the scattering cross-section of a distribution of particles.

3.4.1   Aspherical Particles

The useful relation (60) is a property of the lognormal distribution itself, rather than the particle shape. A lognormal distribution of aspherical particles also obeys (60). Important measurable properties of most convex aspherical habits may be represented by a constant times the kth moment $F(k)$ of the distribution. For example, the surface area S_h [m^2] and volume V_h [m^3] of hexagonal

prisms are given by (81)–(83)[20].

To be consistent with the diameter-centric expressions in Table 1, we introduce $D_{\rm h}$, the hexagonal prism diameter. Adopting the convention that $D_{\rm h} \equiv 2a$, the full-width of the basal face, we obtain

$$S_{\rm h} = \left(\frac{3\sqrt{3}}{4} + 3\Gamma\right) D_{\rm h}^2 \tag{64}$$

$$V_{\rm h} = \frac{3\sqrt{3}}{8}\Gamma D_{\rm h}^3 \tag{65}$$

The functional forms for $S_{\rm h}$ and $V_{\rm h}$ consist of constants multiplying the diameter's second and third moments, respectively. The surface area (πD^2) and volume ($\pi D^3/6$) of spheres have the same form. Therefore the higher moments of aspherical particle distributions must be the same as spherical particle distributions modulo the leading constant expressions. Inserting $S_{\rm h}$ and $V_{\rm h}$ into (64), (65), and (60) leads to analytic expressions for the total surface area $S_{0,\rm h}$ [m^2 m^{-3}] and volume $V_{0,\rm h}$ [m^3 m^{-3}] of a lognormal distribution of hexagonal prisms:

$$S_{0,\rm h} = \left(3\Gamma + \frac{3\sqrt{3}}{4}\right) \tilde{D}_{\rm n,h}^2 \exp(2\tilde{\sigma}_{\rm g}^2) \tag{66}$$

$$V_{0,\rm h} = \frac{3\sqrt{3}}{8}\Gamma \tilde{D}_{\rm n,h}^3 \exp(9\tilde{\sigma}_{\rm g}^2/2) \tag{67}$$

The total concentration $N_{0,\rm V/S}$ of equivalent V/S-spheres corresponding to a known distribution of hexagonal prisms must be computed numerically unless the size dependence of the aspect ratio $\Gamma(D)$ takes an analytic form. In the simplest case, one can imagine or assume distributions of hexagonal prisms with constant aspect ratio, i.e, $\Gamma \neq \Gamma(D)$. In this idealized case, the ratio $N_{\rm V/S}/N_{\rm h}$ (**??**) is constant throughout the distribution. Then the analytic number concentration of equivalent V/S-spheres is simply $N_{\rm V/S}/N_{\rm h}$ times the analytic number concentration of hexagonal prisms which is presumably known directly from the lognormal size distribution parameters (cf. Table 1).

3.4.2 Normalization

We show that (21) is normalized by considering

$$n_{\rm n}(D) = \frac{C_n}{D} \exp\left[-\frac{1}{2}\left(\frac{\ln(D/\tilde{D}_{\rm n})}{\ln\sigma_{\rm g}}\right)^2\right] \tag{68}$$

---

[20] My LaTeX skills are too poor to reference equations in other documents. All "undefined" analytic formula below are in the aerosol FACT, aer.pdf, and that's why their equation numbers appear as question marks in this FACT. 20150303: Some equations are now in a new appendix of this document. Look in aer.pdf to see the other equations.

where C_n is the normalization constant determined by (7). First we change variables to $y = \ln(D/\tilde{D}_n)$

$$
\begin{aligned}
y &= \ln D - \ln \tilde{D}_n \\
D &= \tilde{D}_n e^y \\
\mathrm{d}y &= D^{-1}\,\mathrm{d}D \\
\mathrm{d}D &= \tilde{D}_n e^y\,\mathrm{d}y
\end{aligned}
\tag{69}
$$

This transformation maps $D \in (0, +\infty)$ into $y \in (-\infty, +\infty)$. In terms of y, the normalization condition (7) becomes

$$
\int_{-\infty}^{+\infty} \frac{C_n}{\tilde{D}_n \exp y} \exp\left[-\frac{1}{2}\left(\frac{y}{\ln \sigma_g}\right)^2\right] \tilde{D}_n \exp^y\,\mathrm{d}y = 1
$$

$$
\int_{-\infty}^{+\infty} C_n \exp\left[-\frac{1}{2}\left(\frac{y}{\ln \sigma_g}\right)^2\right]\,\mathrm{d}y = 1
$$

Next we change variables to $z = y/\ln \sigma_g$

$$
\begin{aligned}
z &= y/\ln \sigma_g \\
y &= z \ln \sigma_g \\
\mathrm{d}z &= (\ln \sigma_g)^{-1}\,\mathrm{d}y \\
\mathrm{d}y &= \ln \sigma_g\,\mathrm{d}z
\end{aligned}
\tag{70}
$$

This transformation does not change the limits of integration and we obtain

$$
\begin{aligned}
\int_{-\infty}^{+\infty} C_n \exp\left(\frac{-z^2}{2}\right) \ln \sigma_g\,\mathrm{d}z &= 1 \\
C_n \sqrt{2\pi}\, \ln \sigma_g &= 1 \\
C_n &= \frac{1}{\sqrt{2\pi}\, \ln \sigma_g}
\end{aligned}
\tag{71}
$$

In the above we used the well-known normalization property of the Gaussian distribution function, $\int_{-\infty}^{+\infty} e^{-x^2/2}\,\mathrm{d}x = \sqrt{2\pi}$ (72).

4   Implementation in NCAR models

The discussion thus far has centered on the theoretical considerations of size distributions. In practice, these ideas must be implemented in computer codes which model, e.g., Mie scattering parameters or thermodynamic growth of aerosol populations. This section describes how these ideas have been implemented in the NCAR-Dust and Mie models.

Soil Texture	$\tilde{D}_{\mathrm{n}}$	σ_{g}	Description
Sand			Sand
Silt			Silt
Clay			Clay
Soil Texture	$\tilde{D}_{\mathrm{n}}$	σ_{g}	Description
Sand			Sand
Silt			Silt
Clay			Clay

Table 4: Source size distribution associated with surface soil texture data of *Webb et al.* (1993) and of *Foley* (1998).

4.1   NCAR-Dust Model

The NCAR-Dust model uses as input a time invariant dataset of surface soil size distribution. The two such datasets currently used are from *Webb et al.* (1993) and from IBIS (*Foley*, 1998). The *Webb et al.* dataset provides global information for three soil texture types: sand, clay and silt. At each gridpoint, the mass flux of dust is partitioned into mass contributions from each of these soil types. To accomplish this, the partitioning scheme assumes a size distribution for the source soil of the deflated particles. Table 4 lists the lognormal distribution parameters associated with the surface soil texture data of *Webb et al.* (1993) and of *Foley* (1998). The dust model is a size resolving aerosol model. Thus, overlap factors are computed to determine the fraction of each parent size type which is mobilized into each atmospheric dust size bin during a deflation event.

4.2   Mie Scattering Model

This section documents the Mie scattering code `mie`. `mie` is box model intended to provide exact simulations of microphysical processes for the purpose of parameterization into larger scale models. `mie` provides instantaneous and equilibrium decriptions of many processes ranging from surface flux exchange, dust production, reflection of polarized radiation, and, as its name suggests, the interaction of particles and radiation. Thus the inputs to `mie` are the instantaneous state (boundary and initial conditions) of the environment. Given these, the program solves for the associated rates of change and unknown variables.

There is no time-stepping loop primarily because `mie` generates an extraordinary amount of information about the instantaneous state. Time-stepping this environment in a box-model-like format would be prohibitive if all quantities were allowed to evolve.

4.2.1   Input switches

The flexibility and power of `mie` can only be exercised by actively using the hundreds of input switches which control its behavior. This section describes how some of these switches are commonly used to control fundamental properties of the microphysical environment. A complete reference table for these switches, there default values, and dimensional units, is presented in Appendix 5.5.

The heart of `mie` is an aerosol size distribution. Most users will wish to initialize this size distribution to a particular type of aerosol, and to a particular shape. This is accomplished with the `cmp_aer` and `psd_typ` keywords. The linearity, range, and resolution of the grid on which the analytic size distribution is discretized are controlled by the `sz_grd`, `sz_mnm`, `sz_mxm`, `sz_nbr` switches, respectively. Compute size distribution characteristics of a lognormal distribution

```
mie -dbg -no_mie --psd_typ=lognormal --sz_grd=log --sz_mnm=0.01 \
--sz_mxm=10.0 --sz_nbr=300 --rds_nma=0.4 --gsd_anl=2.2
mie -dbg -no_mie --psd_typ=lognormal --sz_grd=log --sz_mnm=1.0 \
--sz_mxm=10.0 --sz_nbr=25 --rds_nma=2.0 --gsd_anl=2.2
```

4.2.2   Moments of Size Distribution

Determine the analytic (or resolved) moments of an arbitrary size distribution.

1. Generate the size distribution. (It may have more than one moment)

2. Select the statistics of interest

```
# 1. Lognormal distribution with mass median diameter 3.5 um, GSD = 2.0
mie -no_mie --psd_typ=lognormal --sz_grd=log --sz_nbr=1000 \
--sz_mnm=0.005 --sz_mxm=50.0 --dmt_vma=3.5 --gsd_anl=2.0
# 2. Extract median and weighted analytic moments of diameter
ncks -H -v dmt_vwa,dmt_vma,dmt_swa,dmt_sma,dmt_nwa,dmt_nma ${DATA}/mie/mie.r
# 3. Extract median and weighted resolved moments of diameter
ncks -H -v dmt_vwr,dmt_vmr,dmt_swr,dmt_smr,dmt_nwr,dmt_nmr ${DATA}/mie/mie.r
# 4. Extract median and weighted analytic moments of diameter
```

```
ncks -H -v rds_vwa,rds_vma,rds_swa,rds_sma,rds_nwa,rds_nma ${DATA}/mie/mie.
# 5. Extract median and weighted resolved moments of diameter
ncks -H -v rds_vwr,rds_vmr,rds_swr,rds_smr,rds_nwr,rds_nmr ${DATA}/mie/mie.
# 6. Extract number, surface area, and volume distributions at specific size
ncks -H -C -F -u -v dst,dst_rds,dst_sfc,dst_vlm -d sz,1.0e-6 ${DATA}/mie/mie
```

4.2.3 Generating Properties for Multi-Bin

On occasion, a seriouly masochistic scientist will decide to create the ultimate hybrid bin-spectral aerosol method by discretizing the size distribution into a finite number of bins each with an independently configurable analytic sub-bin distribution. Generating properties for all the bins in such a scheme requires enormous amounts of bookkeeping, or, if a computer is available, a relatively simple Perl batch script named psd.pl.

The psd.pl batch script calls mie repeatedly in a loop over particle bin. As input, psd.pl accepts concise array representations of each property of a bin. For example, --sz_nbr={200,25,25,25} specifies that bin 1 is discretized into 200 sub-bins, and the remaining three bins are each discretized into only 25 sub-bins.

```
~/dst/psd.pl --dbg=1 --CCM_SW --ftn_fxd --psd_nbr=4 --spc_idx_sng={01,02,03,
--sz_mnm={0.05,0.5,1.25,2.5} --sz_mxm={0.5,1.25,2.5,5.0} --sz_nbr={200,25,2
--dmt_vma_dfl=3.5 > ${DATA}/dst/mie/psd_CCM_SW.txt.v3 2>&1 &
```

5 Appendix

5.1 Properties of Gaussians

The area under a Gaussian distribution may be expressed in closed form for infinite domains. This result can be obtained (IIRC) by transforming to polar coordinates in the complex plane $x = r(\cos\theta + \mathrm{i}\sin\theta)$.

$$\int_{-\infty}^{+\infty} \mathrm{e}^{-x^2/2}\,\mathrm{d}x = \sqrt{2\pi} \tag{72}$$

This is a special case of a more general result

$$\int_{-\infty}^{+\infty} \exp(-\alpha x^2 - \beta x)\,\mathrm{d}x = \sqrt{\frac{\pi}{\alpha}}\exp\left(\frac{\beta^2}{4\alpha}\right) \qquad \text{where } \alpha > 0 \tag{73}$$

To obtain this result, complete the square under the integrand, change variables to $y = x + \beta/2\alpha$, and then apply (72). Substituting $\alpha = 1/2$ and $\beta = 0$ into (73) yields (72).

5.2 Error Function

The *error function* $\mathrm{erf}(x)$ may be defined as the partial integral of a Gaussian curve

$$\mathrm{erf}(z) = \frac{2}{\sqrt{\pi}}\int_0^z \mathrm{e}^{-x^2}\,\mathrm{d}x \tag{74}$$

Using (72) and the symmetry of a Gaussian curve, it is simple to show that the error function is bounded by the limits $\mathrm{erf}(0) = 0$ and $\mathrm{erf}(\infty) = 1$. Thus $\mathrm{erf}(z)$ is the cumulative probability function for a normally distributed variable z (fxm: True??). Most compilers implement $\mathrm{erf}(x)$ as an intrinsic function. Thus $\mathrm{erf}(x)$ is used to compute areas bounded by finite lognormal distributions (§3.3.6).

5.3   Properties of Binomials

The number N of trials that result in a given outcome k times in n total trials is

$$N(k,n) = \binom{n}{k} = \frac{n!}{k!(n-k)!} \tag{75}$$

$N(k,n)$ is known as the *binomial coefficient*.

If the probability of the outcome of a single independent trial (e.g., flipping a coin) is p, then the probability P of that outcome occurring k times in n trials is

$$
\begin{aligned}
P(k,n) &= N(k,n)p^k(1-k)^{(n-k)} \\
&= \binom{n}{k}p^k(1-k)^{(n-k)} \\
&= \frac{n!}{k!(n-k)!}p^k(1-k)^{(n-k)}
\end{aligned}
\tag{76}
$$

$P(k,n)$ is known as the *binomial distribution*.

The outcome of the binomial distribution is difficult to compute exactly for large n because evaluating the factorial function for large numbers is numerically unwieldy. Instead we make use of *Stirling's Approximation*:

$$n! = \sqrt{2\pi n}\, n^n \mathrm{e}^{-n}\left(1 + \mathcal{O}\left(\frac{1}{n}\right)\right) \tag{77}$$

Substituting (77) in (76) we obtain

$$
\begin{aligned}
P(k,n) &= \frac{\sqrt{2\pi n}\, n^n \mathrm{e}^{-n}}{\sqrt{2\pi k}\, k^k \mathrm{e}^{-k}\sqrt{2\pi(n-k)}\,(n-k)^{(n-k)}\mathrm{e}^{-(n-k)}}p^k(1-k)^{(n-k)} \\
&= \sqrt{\frac{n}{2\pi k(n-k)}}\frac{n^n \mathrm{e}^{-n}}{k^k \mathrm{e}^{-k}\,(n-k)^{(n-k)}\mathrm{e}^{-(n-k)}}p^k(1-k)^{(n-k)}
\end{aligned}
\tag{78}
$$

$$\tag{79}$$

5.4   Formulae from other FACTs

The surface area S_h [m^2] and volume V_h [m^3] of a hexagonal prism are

$$
\begin{aligned}
S_\mathrm{h} &= 6ac + 3\sqrt{3}a^2 = 3a(2c + \sqrt{3}a) \tag{80} \\
V_\mathrm{h} &= \frac{3\sqrt{3}a^2 c}{2} \tag{81}
\end{aligned}
$$

These definitions may be re-cast in terms of a and Γ by using $c = 2a\Gamma$ from (**??**)

$$
\begin{aligned}
S_\mathrm{h} &= 12a^2\Gamma + 3\sqrt{3}a^2 = 3a^2(4\Gamma + \sqrt{3}) & (82) \\
V_\mathrm{h} &= 3\sqrt{3}a^3\Gamma & (83)
\end{aligned}
$$

It is more convenient to describe hexagons in terms of (a,Γ) than (a,c). This is because natural hexagonal prisms may share similar aspect ratios over a large range of sizes.

5.5   Command Line Switches for `mie` Code

Table 5 summarizes all of the *command line arguments* available to control the behavior of the `mie` program. This is a summary only—it is impractical to think that written documentation could ever convey the exact meaning of all the switches[21]. The most frequently used switches are described in Section 4.2.1. The only way to learn the full meaning of the more obscure switches is to read the source code itself.

---

[21]Perhaps the most useful way to begin to contribute to this FACT would be to systematize and extend the documentation of command line switches

Table 5: **Command Line Switches for `mie` code**[22][23]

Switch	Purpose	Default	Units
	Boolean flags		
`--abc_flg`	Alphabetize output with `ncks`	true	Flag
`--abs_ncl_wk_mdm_flg`	Absorbing inclusion in weakly-absorbing sphere	false	Flag
`--bch_flg`	Batch behavior	false	Flag
`--coat_flg`	Assume coated spheres	false	Flag
`--drv_rds_nma_flg`	Derive `rds_nma` from bin boundaries	false	Flag
`--fdg_flg`	Tune the extinction of a particular band	false	Flag
`--hxg_flg`	Aspherical particles are hexagonal prisms	true	Flag
`--vts_flg`	Apply equal-V/S approximation for aspherical optical properties	false	Flag
`--ftn_fxd_flg`	Fortran fixed format	false	Flag
`--hrz_flg`	Print size-resolved optical properties at debug wavelength	false	Flag
`--mca_flg`	Multi-component aerosol with effective medium approximation	false	Flag
`--mie_flg`	Perform Mie scattering calculation	true	Flag
`--no_abc_flg`	Set `abc_flg` to `false`		Flag
`--no_bch_flg`	Set `bch_flg` to `false`		Flag
`--no_hrz_flg`	Set `hrz_flg` to `false`		Flag
`--no_mie_flg`	Set `mie_flg` to `false`		Flag
`--no_wrn_ntp_flg`	Set `wrn_ntp_flg` to `false`		Flag
`--wrn_ntp_flg`	Print WARNINGs from `ntp_vec()`	true	Flag
	Variables		
`--RH_lqd`	Relative humidity w/r/t liquid water	0.8	Fraction
`--asp_rat_hxg_dfl`	Hexagonal prism aspect ratio	1.0	Fraction

Table 5: (continued)

Switch	Purpose	Default	Units
--asp_rat_lps_dfl	Ellipsoidal aspect ratio	1.0	Fraction
--bnd_SW_LW	Boundary between SW and LW weighting	5.0×10^{-6}	m
--bnd_nbr	Number of sub-bands per output band	1	Number
--cmp_cor	Composition of core	"air"	String
--cmp_mdm	Composition of medium	"air"	String
--cmp_mnt	Composition of mantle	"air"	String
--cmp_mtx	Composition of matrix	"air"	String
--cmp_ncl	Composition of inclusion	"air"	String
--cmp_prt	Composition of particle	"saharan_dust"	String
--cnc_nbr_anl_dfl	Number concentration analytic, default	1.0	$\#\,m^{-3}$
--cnc_nbr_pcp_anl	Number concentration analytic, raindrop	1.0	$\#\,m^{-3}$
--cpv_foo	Intrinsic computational precision temporary variable	0.0	Fraction
--dbg_lvl	Debugging level	0	Index
--dmn_nbr_max	Maximum number of dimensions allowed in single variable in output file	2	Number
--dmn_frc	Fractal dimensionality of inclusions	3.0	Fraction
--dmn_rcd	Record dimension name	""	String
--dmt_dtc	Diameter of detector	0.001	m
--dmt_nma_mcr	Number median analytic diameter	cmd_ln_dfl	μm
--dmt_pcp_nma_mcr	Diameter number median analytic, raindrop, microns	1000.0	μm
--dmt_swa_mcr	Surface area weighted mean diameter analytic	cmd_ln_dfl	μm
--dmt_vma_mcr	Volume median diameter analytic	cmd_ln_dfl	μm
--dns_cor	Density of core	0.0	$kg\,m^{-3}$

Table 5: (continued)

Switch	Purpose	Default	Units
--dns_mdm	Density of medium	0.0	$\mathrm{kg\,m^{-3}}$
--dns_mnt	Density of mantle	0.0	$\mathrm{kg\,m^{-3}}$
--dns_mtx	Density of matrix	0.0	$\mathrm{kg\,m^{-3}}$
--dns_ncl	Density of inclusion	0.0	$\mathrm{kg\,m^{-3}}$
--dns_prt	Density of particle	0.0	$\mathrm{kg\,m^{-3}}$
--doy	Day of year [1.0..367.0]	135.0	day
--drc_dat	Data directory	/data/zender/aca	String
--drc_in	Input directory	${HOME}/nco/data	String
--drc_out	Output directory	${HOME}/c++	String
--dsd_dbg_mcr	Debugging size for raindrops	1000.0	μm
--dsd_mnm_mcr	Minimum diameter in raindrop distribution	999.0	μm
--dsd_mxm_mcr	Maximum diameter in raindrop distribution	1001.0	μm
--dsd_nbr	Number of raindrop size bins	1	Number
--fdg_idx	Band to tune by fdg_val	0	Index
--fdg_val	Tuning factor for all bands	1.0	Fraction
--fl_err	File for error messages	"cerr"	String
--fl_idx_rfr_cor	File or function for refractive indices of core	""	String
--fl_idx_rfr_mdm	File or function for refractive indices of medium	""	String
--fl_idx_rfr_mnt	File or function for refractive indices of mantle	""	String
--fl_idx_rfr_mtx	File or function for refractive indices of matrix	""	String
--fl_idx_rfr_ncl	File or function for refractive indices of inclusion	""	String
--fl_idx_rfr_prt	File or function for refractive indices of particle	""	String
--fl_slr_spc	File or function for solar spectrum	""	String
--flt_foo	Intrinsic float temporary variable	0.0	Fraction

Table 5: (continued)

Switch	Purpose	Default	Units
--flx_LW_dwn_sfc	Longwave downwelling flux at surface	0.0	$\mathrm{W\,m^{-2}}$
--flx_SW_net_gnd	Solar flux absorbed by ground	450.0	$\mathrm{W\,m^{-2}}$
--flx_SW_net_vgt	Solar flux absorbed by vegetation	0.0	$\mathrm{W\,m^{-2}}$
--flx_frc_drc_sfc_cmd_ln	Surface insolation fraction in direct beam	0.85	Fraction
--flx_vlm_pcp_rsl	Precipitation volume flux, resolved	-1.0	$\mathrm{m^3\,m^{-2}\,s^{-1}}$
--gsd_anl_dfl	Geometric standard deviation, default	2.0	Fraction
--gsd_pcp_anl	Geometric standard deviation, raindrop	1.86	Fraction
--hgt_mdp	Midlayer height above surface	95.0	m
--hgt_rfr	Reference height (i.e., 10 m) at which surface winds are evaluated for dust mobilization	10.0	m
--hgt_zpd_dps_cmd_ln	Zero plane displacement height	cmd_ln_dfl	m
--hgt_zpd_mbl	Zero plane displacement height for erodible surfaces	0.0	m
--idx_rfr_cor_usr	Refractive index of core	$1.0 + 0.0i$	Complex
--idx_rfr_mdm_usr	Refractive index of medium	$1.0 + 0.0i$	Complex
--idx_rfr_mnt_usr	Refractive index of mantle	$1.33 + 0.0i$	Complex
--idx_rfr_mtx_usr	Refractive index of matrix	$1.0 + 0.0i$	Complex
--idx_rfr_ncl_usr	Refractive index of inclusion	$1.0 + 0.0i$	Complex
--idx_rfr_prt_usr	Refractive index of particle	$1.33 + 0.0i$	Complex
--lat_dgr	Latitude	40.0	°
--lbl_sng	Line-by-line test	"CO2"	String
--lgn_nbr	Number of terms in Legendre expansion of phase function	8	Number
--lnd_frc_dry	Dry land fraction	1.0	Fraction
--mmw_prt	Mean molecular weight	0.0	$\mathrm{kg\,mol^{-1}}$

Table 5: (continued)

Switch	Purpose	Default	Units
--mno_lng_dps_cmd_ln	Monin-Obukhov length	cmd_ln_dfl	m
--mss_frc_cly	Mass fraction clay	0.19	Fraction
--mss_frc_snd	Mass fraction sand	0.777	Fraction
--ngl_nbr	Number of angles in Mie computation	11	Number
--oro	Orography: ocean=0.0, land=1.0, sea ice=2.0	1.0	Fraction
--pnt_typ_idx	Plant type index	14	Index
--prs_mdp	Environmental pressure	100825.0	Pa
--prs_ntf	Environmental surface pressure	prs_STP	Pa
--psd_typ	Particle size distribution type	"lognormal"	String
--q_H2O_vpr	Specific humidity	cmd_ln_dfl	$kg\,kg^{-1}$
--rds_ffc_gmm_mcr	Effective radius of Gamma distribution	50.0	µm
--rds_nma_mcr	Number median analytic radius	0.2986	µm
--rds_swa_mcr	Surface area weighted mean radius analytic	cmd_ln_dfl	µm
--rds_vma_mcr	Volume median radius analytic	cmd_ln_dfl	µm
--rgh_mmn_dps_cmd_ln	Roughness length momentum	cmd_ln_dfl	m
--rgh_mmn_ice_std	Roughness length over sea ice	0.0005	m
--rgh_mmn_mbl	Roughness length momentum for erodible surfaces	100.0×10^{-6}	m
--rgh_mmn_smt	Smooth roughness length	10.0×10^{-6}	m
--rfl_gnd_dff	Diffuse reflectance of ground (beneath snow)	0.20	Fraction
--sfc_typ	LSM surface type [0..28]	2	Index
--slf_tst_typ	Self-test type	"BoH83"	String
--slr_cst	Solar constant	1367.0	$W\,m^{-2}$
--slr_spc_key	Solar spectrum string	"LaN68"	String

Table 5: (continued)

Switch	Purpose	Default	Units
`--slr_zen_ngl_cos`	Cosine solar zenith angle	1.0	Fraction
`--slv_sng`	Mie solver to use	"Wis79"	String
`--snw_hgt_lqd`	Equivalent liquid water snow depth	0.0	m
`--soi_typ`	LSM soil type [1..5]	1	Index
`--spc_heat_prt`	Specific heat capacity	0.0	$\mathrm{J\,kg^{-1}\,K^{-1}}$
`--spc_abb_sng`	Species abbreviation for Fortran data	"foo"	String
`--spc_idx_sng`	Species index for Fortran data	"foo"	String
`--ss_alb_cmd_ln`	Single scattering albedo	1.0	Fraction
`--sz_dbg_mcr`	Debugging size	1.0	μm
`--sz_grd_sng`	Type of size grid	"logarithmic"	String
`--sz_mnm_mcr`	Minimum size in distribution	0.9	μm
`--sz_mxm_mcr`	Maximum size in distribution	1.1	μm
`--sz_nbr`	Number of particle size bins	1	Number
`--sz_prm_rsn`	Size parameter resolution	0.1	Fraction
`--thr_nbr`	Thread number	0	Number
`--tm_dlt`	Timestep	1200.0	s
`--tpt_bbd_wgt`	Blackbody temperature of radiation	273.15	K
`--tpt_gnd`	Ground temperature	300.0	K
`--tpt_ice`	Ice temperature	`tpt_frz_pnt`	K
`--tpt_mdp`	Environmental temperature	300.0	K
`--tpt_prt`	Particle temperature	273.15	K
`--tpt_soi`	Soil temperature	297.0	K
`--tpt_sst`	Sea surface temperature	300.0	K
`--tpt_vgt`	Vegetation temperature	300.0	K

Table 5: (continued)

Switch	Purpose	Default	Units
--tst_sng	Name of test to perform (htg, lbl, nc, nsz, psd_ntg_dgn)	""	String
--var_ffc_gmm	Effective variance of Gamma distribution	1.0	Fraction
--vlm_frc_mntl	Fraction of volume in mantle	0.5	Fraction
--vmr_CO2	Volume mixing ratio of CO_2	355.0×10^{-6}	molecule molecule^{-1}
--vmr_HNO3_gas	Volume mixing ratio of gaseous HNO_3	0.05×10^{-9}	molecule molecule^{-1}
--vwc_sfc	Volumetric water content	0.03	$m^3\,m^{-3}$
--wbl_shp	Weibull distribution shape parameter	2.4	Fraction
--wnd_frc_dps_cmd_ln	Friction speed	cmd_ln_dfl	$m\,s^{-1}$
--wnd_mrd_mdp	Surface layer meridional wind speed	0.0	$m\,s^{-1}$
--wnd_znl_mdp	Surface layer zonal wind speed	10.0	$m\,s^{-1}$
--wvl_dbg_mcr	Debugging wavelength	0.50	μm
--wvl_grd_sng	Type of wavelength grid	"regular"	String
--wvl_dlt_mcr	Bandwidth	0.1	μm
--wvl_mdp_mcr	Midpoint wavelength	cmd_ln_dfl	μm
--wvl_mnm_mcr	Minimum wavelength	0.45	μm
--wvl_mxm_mcr	Maximum wavelength	0.55	μm
--wvl_nbr	Number of output wavelength bands	1	Number
--wvn_dlt_xcm	Bandwidth	1.0	cm^{-1}
--wvn_mdp_xcm	Midpoint wavenumber	cmd_ln_dfl	cm^{-1}
--wvn_mnm_xcm	Minimum wavenumber	18182	cm^{-1}
--wvn_mxm_xcm	Maximum wavenumber	22222	cm^{-1}
--wvn_nbr	Number of output wavenumber bands	1	Number
--xpt_dsc	Experiment description	""	String

Table 6 summarizes the *fields* output by SWNB.

Table 6: **SWNB Output Fields**[24]

Name(s)	Purpose	Units
`abs_bb_SAS`	Broadband absorptance of surface-atmosphere system	fraction
`abs_bb_atm`	Broadband absorptance of surface	fraction
`abs_bb_sfc`	Broadband absorptance of atmosphere	fraction
`abs_nst_SAS`	FSBR absorptance of surface-atmosphere system	fraction
`abs_nst_atm`	FSBR absorptance of surface	fraction
`abs_nst_sfc`	FSBR absorptance of atmosphere	fraction
`abs_spc_SAS`	Spectral absorptance of surface-atmosphere system	fraction
`abs_spc_atm`	Spectral absorptance of atmosphere	fraction
`abs_spc_sfc`	Spectral absorptance of surface	fraction
`alb_sfc`	Specified Lambertian surface albedo	fraction
`alt_cld_btm`	Highest interface beneath all clouds in column	meter
`alt_cld_thick`	Thickness of region containing all clouds	meter
`alt_ntf`	Interface altitude	meter
`alt`	Altitude	meter
`azi_dgr`	Azimuthal angle (degrees)	degree
`azi`	Azimuthal angle (radians)	radian
`bnd`	Midpoint wavelength	meter
`flx_abs_atm_rdr`	Flux absorbed in atmosphere at longer wavelengths	$\mathrm{W\,m^{-2}}$
`flx_bb_abs_atm`	Broadband flux absorbed by atmospheric column only	$\mathrm{W\,m^{-2}}$
`flx_bb_abs_sfc`	Broadband flux absorbed by surface only	$\mathrm{W\,m^{-2}}$

Table 6: (continued)

Name(s)	Purpose	Units
flx_bb_abs_ttl	Broadband flux absorbed by surface-atmosphere system	$\mathrm{W\,m^{-2}}$
flx_bb_abs	Broadband flux absorbed by layer	$\mathrm{W\,m^{-2}}$
flx_bb_dwn_TOA	Broadband incoming flux at TOA (total insolation)	$\mathrm{W\,m^{-2}}$
flx_bb_dwn_dff	Diffuse downwelling broadband flux	$\mathrm{W\,m^{-2}}$
flx_bb_dwn_drc	Direct downwelling broadband flux	$\mathrm{W\,m^{-2}}$
flx_bb_dwn_sfc	Broadband downwelling flux at surface	$\mathrm{W\,m^{-2}}$
flx_bb_dwn	Total downwelling broadband flux (direct+diffuse)	$\mathrm{W\,m^{-2}}$
flx_bb_net	Net broadband flux (downwelling−upwelling)	$\mathrm{W\,m^{-2}}$
flx_bb_up	Upwelling broadband flux	$\mathrm{W\,m^{-2}}$
flx_nst_abs_atm	FSBR flux absorbed by atmospheric column only	$\mathrm{W\,m^{-2}}$
flx_nst_abs_sfc	FSBR flux absorbed by surface only	$\mathrm{W\,m^{-2}}$
flx_nst_abs_ttl	FSBR flux absorbed by surface-atmosphere system	$\mathrm{W\,m^{-2}}$
flx_nst_abs	FSBR flux absorbed by layer	$\mathrm{W\,m^{-2}}$
flx_nst_dwn_TOA	FSBR incoming flux at TOA (total insolation)	$\mathrm{W\,m^{-2}}$
flx_nst_dwn_sfc	FSBR downwelling flux at surface	$\mathrm{W\,m^{-2}}$
flx_nst_dwn	Total downwelling FSBR flux (direct+diffuse)	$\mathrm{W\,m^{-2}}$
flx_nst_net	Net FSBR flux (downwelling−upwelling)	$\mathrm{W\,m^{-2}}$
flx_nst_up	Upwelling FSBR flux	$\mathrm{W\,m^{-2}}$
flx_slr_frc	Fraction of solar flux	fraction
flx_spc_abs_SAS	Spectral flux absorbed by surface-atmosphere system	$\mathrm{W\,m^{-2}\,m^{-1}}$

Table 6: (continued)

Name(s)	Purpose	Units
`flx_spc_abs_atm`	Spectral flux absorbed by atmospheric column only	$\mathrm{W\,m^{-2}\,m^{-1}}$
`flx_spc_abs_sfc`	Spectral flux absorbed by surface only	$\mathrm{W\,m^{-2}\,m^{-1}}$
`flx_spc_abs`	Spectral flux absorbed by layer	$\mathrm{W\,m^{-2}\,m^{-1}}$
`flx_spc_act_pht_TOA`	Spectral actinic photon flux at TOA	$\mathrm{\#\,m^{-2}\,s^{-1}\,m^{-1}}$
`flx_spc_act_pht_sfc`	Spectral actinic photon flux at surface	$\mathrm{\#\,m^{-2}\,s^{-1}\,m^{-1}}$
`flx_spc_dwn_TOA`	Spectral solar insolation at TOA	$\mathrm{W\,m^{-2}\,m^{-1}}$
`flx_spc_dwn_dff`	Spectral diffuse downwelling flux	$\mathrm{W\,m^{-2}\,m^{-1}}$
`flx_spc_dwn_drc`	Spectral direct downwelling flux	$\mathrm{W\,m^{-2}\,m^{-1}}$
`flx_spc_dwn_sfc`	Spectral solar insolation at surface	$\mathrm{W\,m^{-2}\,m^{-1}}$
`flx_spc_dwn`	Spectral downwelling flux	$\mathrm{W\,m^{-2}\,m^{-1}}$
`flx_spc_pht_dwn_sfc`	Spectral photon flux downwelling at surface	$\mathrm{\#\,m^{-2}\,s^{-1}\,m^{-1}}$
`flx_spc_up`	Spectral upwelling flux	$\mathrm{W\,m^{-2}\,m^{-1}}$
`frc_ice_ttl`	Fraction of column condensate that is ice	fraction
`htg_rate_bb`	Broadband heating rate	$\mathrm{K\,s^{-1}}$
`j_NO2`	Photolysis rate for $NO_2 + h\nu \longrightarrow O(^3P) + NO$	$\mathrm{s^{-1}}$
`j_spc_NO2_sfc`	Spectral photolysis rate at sfc for $NO_2 + h\nu \longrightarrow O(^3P) + NO$	$\mathrm{s^{-1}\,m^{-1}}$
`lat_dgr`	Latitude (degrees)	degree
`lcl_time_hr`	Local day hour	hour
`lcl_yr_day`	Day of year in local time	day
`levp`	Interface pressure	pascal
`lev`	Layer pressure	pascal
`mpc_CWP`	Total column Condensed Water Path	$\mathrm{kg\,m^{-2}}$

Table 6: (continued)

Name(s)	Purpose	Units
`nrg_pht`	Energy of photon at band center	joule photon-1
`ntn_bb_aa`	Broadband azimuthally averaged intensity	$W\,m^{-2}\,sr^{-1}$
`ntn_bb_mean`	Broadband mean intensity	$W\,m^{-2}\,sr^{-1}$
`ntn_spc_aa_ndr_sfc`	Spectral intensity of nadir radiation at surface	$W\,m^{-2}\,m^{-1}\,sr^{-1}$
`ntn_spc_aa_ndr`	Spectral intensity of nadir radiation	$W\,m^{-2}\,m^{-1}\,sr^{-1}$
`ntn_spc_aa_sfc`	Spectral intensity of radiation at surface	$W\,m^{-2}\,m^{-1}\,sr^{-1}$
`ntn_spc_aa_zen_sfc`	Spectral intensity of zenith radiation at surface	$W\,m^{-2}\,m^{-1}\,sr^{-1}$
`ntn_spc_aa_zen`	Spectral intensity of zenith radiation	$W\,m^{-2}\,m^{-1}\,sr^{-1}$
`ntn_spc_chn`	Full spectral intensity of particular band	$W\,m^{-2}\,m^{-1}\,sr^{-1}$
`ntn_spc_mean`	Spectral mean intensity	$W\,m^{-2}\,m^{-1}\,sr^{-1}$
`odac_spc_aer`	Aerosol absorption optical depth to surface	fraction
`odac_spc_bga`	Background aerosol absorption optical depth to surface	fraction
`odac_spc_ice`	Liquid water absorption optical depth to surface	fraction
`odac_spc_lqd`	Ice water absorption optical depth to surface	fraction
`odal_obs_aer`	Layer aerosol absorption optical depth	fraction
`odal_obs_bga`	Layer background aerosol absorption optical depth	fraction
`odsl_obs_aer`	Layer aerosol scattering optical depth	fraction
`odsl_obs_bga`	Layer background aerosol scattering optical depth	fraction
`odxc_obs_aer`	Column aerosol extinction optical depth	fraction
`odxc_obs_bga`	Column background aerosol extinction optical depth	fraction
`odxc_spc_CO2`	CO2 optical depth to surface	fraction
`odxc_spc_H2OH2O`	H2O dimer optical depth to surface	fraction

Table 6: (continued)

Name(s)	Purpose	Units
odxc_spc_H2O	H2O optical depth to surface	fraction
odxc_spc_NO2	NO$_2$optical depth to surface	fraction
odxc_spc_O2N2	O2N2 optical depth to surface	fraction
odxc_spc_O2O2	O2O2 optical depth to surface	fraction
odxc_spc_O2	O2 optical depth to surface	fraction
odxc_spc_O3	O3 optical depth to surface	fraction
odxc_spc_OH	OH optical depth to surface	fraction
odxc_spc_Ray	Rayleigh scattering optical depth to surface	fraction
odxc_spc_aer	Aerosol extinction optical depth to surface	fraction
odxc_spc_bga	Background aerosol extinction optical depth to surface	fraction
odxc_spc_ice	Ice water extinction optical depth to surface	fraction
odxc_spc_lqd	Liquid water extinction optical depth to surface	fraction
odxc_spc_ttl	Total extinction optical depth to surface	fraction
odxl_obs_aer	Layer aerosol extinction optical depth	fraction
odxl_obs_bga	Layer background aerosol extinction optical depth	fraction
plr_cos	Cosine polar angle (degrees)	fraction
plr_dgr	Polar angle (degrees)	degree
plr	Polar angle (radians)	radian
rfl_bb_SAS	Broadband albedo of entire surface-atmosphere system	fraction
rfl_bb_sfc	Broadband albedo of surface	fraction
rfl_nst_SAS	FSBR albedo of entire surface-atmosphere system	fraction
rfl_nst_sfc	FSBR albedo of surface	fraction

Table 6: (continued)

Name(s)	Purpose	Units
`rfl_spc_SAS`	Spectral planetary flux reflectance	fraction
`slr_zen_ngl_cos`	Cosine solar zenith angle	fraction
`tau_prs`	Optical level (pressure)	pascal
`tau`	Optical level (optical depth)	fraction
`tpt_ntf`	Interface temperature	kelvin
`tpt`	Layer Temperature	kelvin
`trn_bb_atm`	Broadband transmission of atmospheric column	fraction
`trn_nst_atm`	FSBR transmission of atmospheric column	fraction
`trn_spc_atm_CO2`	Column transmission due to CO2 absorption	fraction
`trn_spc_atm_H2OH2O`	Column transmission due to H2O dimer absorption	fraction
`trn_spc_atm_H2O`	Column transmission due to H2O absorption	fraction
`trn_spc_atm_NO2`	Column transmission due to NO_2absorption	fraction
`trn_spc_atm_O2N2`	Column transmission due to O2-N2 absorption	fraction
`trn_spc_atm_O2O2`	Column transmission due to O2-O2 absorption	fraction
`trn_spc_atm_O2`	Column transmission due to O2 absorption	fraction
`trn_spc_atm_O3`	Column transmission due to O3 absorption	fraction
`trn_spc_atm_OH`	Column transmission due to OH absorption	fraction
`trn_spc_atm_Ray`	Column transmission due to Rayleigh scattering	fraction
`trn_spc_atm_aer`	Column transmission due to aerosol extinction	fraction
`trn_spc_atm_bga`	Column transmission due to background aerosol extinction	fraction
`trn_spc_atm_ice`	Column transmission due to ice extinction	fraction
`trn_spc_atm_lqd`	Column transmission due to liquid extinction	fraction

Table 6: (continued)

Name(s)	Purpose	Units
trn_spc_atm_ttl	Spectral flux transmission of entire column	fraction
wvl_ctr	Midpoint wavelength in band	meter
wvl_dlt	Width of band	meter
wvl_grd	Wavelength grid	meter
wvl_max	Maximum wavelength in band	meter
wvl_min	Minimum wavelength in band	meter
wvl_obs_aer	Wavelength of aerosol optical depth specification	meter
wvl_obs_bga	Wavelength of background aerosol optical depth specification	meter
wvn_ctr	Midpoint wavenumber in band	centimeter-1
wvn_dlt	Bandwidth in wavenumbers	centimeter-1
wvn_max	Maximum wavenumber in band	centimeter-1
wvn_min	Minimum wavenumber in band	centimeter-1

Table 7 summarizes the *fields* output by CLM.

Table 7: **CLM Output Fields**[25]

Name(s)	Purpose	Units
CO2_vmr_clm	Carbon Dioxide volume mixing ratio	fraction
N2O_vmr_clm	Nitrous Oxide volume mixing ratio	fraction
CH4_vmr_clm	Methane volume mixing ratio	fraction
CFC11_vmr_clm	CFC11 volume mixing ratio	fraction
CFC12_vmr_clm	CFC12 volume mixing ratio	fraction
RH_ice	Relative humidity w/r/t ice	fraction
RH	Relative humidity	fraction
RH_lqd	Relative humidity w/r/t liquid	fraction
alb_sfc_NIR_drc	Albedo for NIR radiation at strong zenith angles	fraction
alb_sfc_NIR_dff	Albedo for NIR radiation at weak zenith angles	fraction
alb_sfc	Prescribed surface albedo	fraction
alb_sfc_vsb_drc	Albedo for visible radiation at strong zenith angles	fraction
alb_sfc_vsb_dff	Albedo for visible radiation at weak zenith angles	fraction
alt_cld_btm	Highest interface beneath all clouds in column	meter
alt_cld_mid	Altitude at midpoint of all clouds in column	meter
alt_cld_thick	Thickness of region containing all clouds	meter
alt_cld_top	Lowest interface above all clouds in column	meter
alt_dlt	Layer altitude thickness	meter
alt	Altitude	meter
alt_ntf	Interface altitude	meter
cld_frc	Cloud fraction	fraction
cnc_CO2	CO2 concentration	molecule m^{-3}
cnc_CH4	CH4 concentration	molecule m^{-3}

Table 7: (continued)

Name(s)	Purpose	Units
cnc_N2O	N2O concentration	molecule m^{-3}
cnc_CFC11	CFC11 concentration	molecule m^{-3}
cnc_CFC12	CFC12 concentration	molecule m^{-3}
cnc_H2OH2O	H2O dimer concentration	molecule m^{-3}
cnc_H2O	H2O concentration	molecule m^{-3}
cnc_N2	N2 concentration	molecule m^{-3}
cnc_NO2	NO$_2$concentration	molecule m^{-3}
cnc_O2O2	O2O2 concentration	molecule m^{-3}
cnc_O2_cnc_N2	O2 number concentration times N2 number concentration	molecule^2 m^{-6}
cnc_O2_cnc_O2	O2 number concentration squared	molecule^2 m^{-6}
cnc_O2	O2 concentration	molecule m^{-3}
cnc_O2_npl_N2_clm	Column total O2 number concentration times N2 number path	molecule^2 m^{-5}
cnc_O2_npl_N2	O2 number concentration times N2 number path	molecule^2 m^{-5}
cnc_O2_npl_O2_clm	Column total O2 number concentration times O2 number path	molecule^2 m^{-5}
cnc_O2_npl_O2_clm_frc	Fraction of column total O2-O2 at or above each layer	fraction
cnc_O2_npl_O2	O2 number concentration times O2 number path	molecule^2 m^{-5}
cnc_O3	O3 concentration	# m^{-3}
cnc_OH	OH concentration	# m^{-3}
cnc_dry_air	Dry concentration	# m^{-3}
cnc_mst_air	Moist air concentration	# m^{-3}
dns_CO2	Density of CO2	kg m^{-3}

Table 7: (continued)

Name(s)	Purpose	Units
dns_CH4	Density of CH4	$\mathrm{kg\,m^{-3}}$
dns_N2O	Density of N2O	$\mathrm{kg\,m^{-3}}$
dns_CFC11	Density of CFC11	$\mathrm{kg\,m^{-3}}$
dns_CFC12	Density of CFC12	$\mathrm{kg\,m^{-3}}$
dns_H2OH2O	Density of H2OH2O	$\mathrm{kg\,m^{-3}}$
dns_H2O	Density of H2O	$\mathrm{kg\,m^{-3}}$
dns_N2	Density of N2	$\mathrm{kg\,m^{-3}}$
dns_NO2	Density of $\mathrm{NO_2}$	$\mathrm{kg\,m^{-3}}$
dns_O2O2	Density of O2-O2	$\mathrm{kg\,m^{-3}}$
dns_O2_dns_N2	O2 mass concentration times N2 mass concentration	$\mathrm{kg^2\,m^{-6}}$
dns_O2_dns_O2	O2 mass concentration squared	$\mathrm{kg^2\,m^{-6}}$
dns_O2	Density of O2	$\mathrm{kg\,m^{-3}}$
dns_O2_mpl_N2_clm	Column total O2 mass concentration times N2 mass path	$\mathrm{kg^2\,m^{-5}}$
dns_O2_mpl_N2	O2 mass concentration times N2 mass path	$\mathrm{kg^2\,m^{-5}}$
dns_O2_mpl_O2_clm	Column total O2 mass concentration times O2 mass path	$\mathrm{kg^2\,m^{-5}}$
dns_O2_mpl_O2	O2 mass concentration times O2 mass path	$\mathrm{kg^2\,m^{-5}}$
dns_O3	Density of O3	$\mathrm{kg\,m^{-3}}$
dns_OH	Density of OH	$\mathrm{kg\,m^{-3}}$
dns_aer	Aerosol density	$\mathrm{kg\,m^{-3}}$
dns_bga	Background aerosol density	$\mathrm{kg\,m^{-3}}$
dns_dry_air	Density of dry air	$\mathrm{kg\,m^{-3}}$
dns_mst_air	Density of moist air	$\mathrm{kg\,m^{-3}}$

Table 7: (continued)

Name(s)	Purpose	Units
eqn_time_sec	foo	second
ext_cff_mss_aer	Aerosol mass extinction coefficient	$m^2\,kg^{-1}$
ext_cff_mss_bga	Background aerosol mass extinction coefficient	$m^2\,kg^{-1}$
frc_ice	Fraction of condensate that is ice	fraction
frc_ice_ttl	Fraction of column condensate that is ice	fraction
frc_str_zen_ngl_sfc	Surface fraction of strong zenith angle dependence	fraction
gas_cst_mst_air	Specific gas constant for moist air	joule kilogram-1 kelvin-1
gmt_day	foo	day
gmt_doy	foo	day
gmt_hr	foo	hour
gmt_mnt	foo	minute
gmt_mth	foo	month
gmt_sec	foo	second
gmt_ydy	foo	day
gmt_yr	foo	year
grv	Gravity	meter second-2
oro	Orography flag	flag
lat_cos	Cosine of latitude	fraction
lat_dgr	Latitude (degrees)	degree
lat	Latitude (radians)	radian
lcl_time_hr	Local day hour	hour
lcl_yr_day	Day of year in local time	day
lev	Layer pressure	pascal

Table 7: (continued)

Name(s)	Purpose	Units
levp	Interface pressure	pascal
lmt_day	foo	day
lmt_doy	foo	day
lmt_hr	foo	hour
lmt_mnt	foo	minute
lmt_mth	foo	month
lmt_sec	foo	second
lmt_ydy	foo	day
lmt_yr	foo	year
lon_dgr	foo	degree
lon	foo	radian
lon_sec	foo	second
ltst_day	foo	day
ltst_doy	foo	day
ltst_hr	foo	hour
ltst_mnt	foo	minute
ltst_mth	foo	month
ltst_sec	foo	second
ltst_ydy	foo	day
ltst_yr	foo	year
mmw_mst_air	Mean molecular weight of moist air	kilogram mole-1
mpc_CO2	Mass path of CO2 in column	$kg\,m^{-2}$
mpc_CH4	Mass path of CH4 in column	$kg\,m^{-2}$
mpc_N2O	Mass path of N2O in column	$kg\,m^{-2}$

Table 7: (continued)

Name(s)	Purpose	Units
`mpc_CFC11`	Mass path of CFC11 in column	$\text{kg}\,\text{m}^{-2}$
`mpc_CFC12`	Mass path of CFC12 in column	$\text{kg}\,\text{m}^{-2}$
`mpc_CWP`	Total column Condensed Water Path	$\text{kg}\,\text{m}^{-2}$
`mpc_H2OH2O`	Mass path of H2O dimer in column	$\text{kg}\,\text{m}^{-2}$
`mpc_H2O`	Mass path of H2O in column	$\text{kg}\,\text{m}^{-2}$
`mpc_IWP`	Total column Ice Water Path	$\text{kg}\,\text{m}^{-2}$
`mpc_LWP`	Total column Liquid Water Path	$\text{kg}\,\text{m}^{-2}$
`mpc_N2`	Mass path of N2 in column	$\text{kg}\,\text{m}^{-2}$
`mpc_NO2`	Mass path of NO_2in column	$\text{kg}\,\text{m}^{-2}$
`mpc_O2O2`	Mass path of O2-O2 in column	$\text{kg}\,\text{m}^{-2}$
`mpc_O2`	Mass path of O2 in column	$\text{kg}\,\text{m}^{-2}$
`mpc_O3_DU`	Mass path of O3 in column	Dobson
`mpc_O3`	Mass path of O3 in column	$\text{kg}\,\text{m}^{-2}$
`mpc_OH`	Mass path of OH in column	$\text{kg}\,\text{m}^{-2}$
`mpc_aer`	Total column mass path of aerosol	$\text{kg}\,\text{m}^{-2}$
`mpc_bga`	Total column mass path of background aerosol	$\text{kg}\,\text{m}^{-2}$
`mpc_dry-air`	Mass path of dry air in column	$\text{kg}\,\text{m}^{-2}$
`mpc_mst-air`	Mass path of moist air in column	$\text{kg}\,\text{m}^{-2}$
`mpl_CO2`	Mass path of CO2 in layer	$\text{kg}\,\text{m}^{-2}$
`mpl_CH4`	Mass path of CH4 in layer	$\text{kg}\,\text{m}^{-2}$
`mpl_N2O`	Mass path of N2O in layer	$\text{kg}\,\text{m}^{-2}$
`mpl_CFC11`	Mass path of CFC11 in layer	$\text{kg}\,\text{m}^{-2}$
`mpl_CFC12`	Mass path of CFC12 in layer	$\text{kg}\,\text{m}^{-2}$
`mpl_CWP`	Layer Condensed Water Path	$\text{kg}\,\text{m}^{-2}$

Table 7: (continued)

Name(s)	Purpose	Units
mpl_H2OH2O	Mass path of H2O dimer in layer	$kg\,m^{-2}$
mpl_H2O	Mass path of H2O in layer	$kg\,m^{-2}$
mpl_IWP	Layer Ice Water Path	$kg\,m^{-2}$
mpl_LWP	Layer Liquid Water Path	$kg\,m^{-2}$
mpl_N2	Mass path of N2 in layer	$kg^2\,m^{-5}$
mpl_NO2	Mass path of NO_2 in layer	$kg\,m^{-2}$
mpl_O2O2	Mass path of O2-O2 in layer	$kg\,m^{-2}$
mpl_O2	Mass path of O2 in layer	$kg^2\,m^{-5}$
mpl_O3	Mass path of O3 in layer	$kg\,m^{-2}$
mpl_OH	Mass path of OH in layer	$kg\,m^{-2}$
mpl_aer	Layer mass path of aerosol	$kg\,m^{-2}$
mpl_bga	Layer mass path of aerosol	$kg\,m^{-2}$
mpl_dry_air	Mass path of dry air in layer	$kg\,m^{-2}$
mpl_mst_air	Mass path of moist air in layer	$kg\,m^{-2}$
npc_CO2	Column number path of CO2	$molecule\,m^{-2}$
npc_CH4	Column number path of CH4	$molecule\,m^{-2}$
npc_N2O	Column number path of N2O	$molecule\,m^{-2}$
npc_CFC11	Column number path of CFC11	$molecule\,m^{-2}$
npc_CFC12	Column number path of CFC12	$molecule\,m^{-2}$
npc_H2OH2O	Column number path of H2O dimer	$molecule\,m^{-2}$
npc_H2O	Column number path of H2O	$molecule\,m^{-2}$
npc_N2	Column number path of O2	$molecule\,m^{-2}$
npc_NO2	Column number path of NO_2	$molecule\,m^{-2}$
npc_O2O2	Column number path of O2O2	$molecule\,m^{-2}$

Table 7: (continued)

Name(s)	Purpose	Units
`npc_O2`	Column number path of O2	molecule m^{-2}
`npc_O3`	Column number path of O3	molecule m^{-2}
`npc_OH`	Column number path of OH	molecule m^{-2}
`npc_dry-air`	Column number path of dry air	molecule m^{-2}
`npc_mst-air`	Column number path of moist air	molecule m^{-2}
`npl_CO2`	Number path of CO2 in layer	molecule m^{-2}
`npl_CH4`	Number path of CH4 in layer	molecule m^{-2}
`npl_N2O`	Number path of N2O in layer	molecule m^{-2}
`npl_CFC11`	Number path of CFC11 in layer	molecule m^{-2}
`npl_CFC12`	Number path of CFC12 in layer	molecule m^{-2}
`npl_H2OH2O`	Number path of H2O dimer in layer	molecule m^{-2}
`npl_H2O`	Number path of H2O in layer	molecule m^{-2}
`npl_N2`	Number path of N2 in layer	molecule^2 m^{-5}
`npl_NO2`	Number path of NO2 in layer	molecule m^{-2}
`npl_O2O2`	Number path of O2-O2 in layer	molecule m^{-2}
`npl_O2`	Number path of O2 in layer	molecule^2 m^{-5}
`npl_O3`	Number path of O3 in layer	molecule m^{-2}
`npl_OH`	Number path of OH in layer	molecule m^{-2}
`npl_dry-air`	Number path of dry air in layer	molecule m^{-2}
`npl_mst-air`	Number path of moist air in layer	molecule m^{-2}
`odxc_obs_aer`	Column aerosol extinction optical depth	fraction
`odxc_obs_bga`	Column background aerosol extinction optical depth	fraction
`odxl_obs_aer`	Layer aerosol extinction optical depth	fraction

Table 7: (continued)

Name(s)	Purpose	Units
odxl_obs_bga	Layer background aerosol extinction optical depth	fraction
oneD_foo		
ppr_CO2	Partial pressure of CO2	pascal
ppr_CH4	Partial pressure of CH4	pascal
ppr_N2O	Partial pressure of N2O	pascal
ppr_CFC11	Partial pressure of CFC11	pascal
ppr_CFC12	Partial pressure of CFC12	pascal
ppr_H2OH2O	Partial pressure of H2O dimer	pascal
ppr_H2O	Partial pressure of H2O	pascal
ppr_N2	Partial pressure of N2	pascal
ppr_NO2	Partial pressure of NO_2	pascal
ppr_O2O2	Partial pressure of O2O2	pascal
ppr_O2	Partial pressure of O2	pascal
ppr_O3	Partial pressure of O3	pascal
ppr_OH	Partial pressure of OH	pascal
ppr_dry_air	Partial pressure of dry air	pascal
prs_cld_btm	Highest interface beneath all clouds in column	pascal
prs_cld_mid	Pressure at midpoint of all clouds in column	pascal
prs_cld_thick	Thickness of region containing all clouds	meter
prs_cld_top	Lowest interface above all clouds in column	pascal
prs_dlt	Layer pressure thickness	pascal
prs	Pressure	pascal
prs_ntf	Interface pressure	pascal

Table 7: (continued)

Name(s)	Purpose	Units
`prs_sfc`	Surface pressure	pascal
`q_CO2`	Mass mixing ratio of CO2	$\mathrm{kg\,kg^{-1}}$
`q_CH4`	Mass mixing ratio of CH4	$\mathrm{kg\,kg^{-1}}$
`q_N2O`	Mass mixing ratio of N2O	$\mathrm{kg\,kg^{-1}}$
`q_CFC11`	Mass mixing ratio of CFC11	$\mathrm{kg\,kg^{-1}}$
`q_CFC12`	Mass mixing ratio of CFC12	$\mathrm{kg\,kg^{-1}}$
`q_H2OH2O`	Water vapor dimer mass mixing ratio	$\mathrm{kg\,kg^{-1}}$
`q_H2OH2O_rcp-q_H2O`	Ratio of dimer mmr to monomer mmr	fraction
`q_H2O`	Water vapor mass mixing ratio	fraction
`q_N2`	Mass mixing ratio of N2	$\mathrm{kg\,kg^{-1}}$
`q_NO2`	Mass mixing ratio of NO_2	$\mathrm{kg\,kg^{-1}}$
`q_O2O2`	Ozone mass mixing ratio	$\mathrm{kg\,kg^{-1}}$
`q_O2`	Mass mixing ratio of O2	$\mathrm{kg\,kg^{-1}}$
`q_O3`	Ozone mass mixing ratio	$\mathrm{kg\,kg^{-1}}$
`q_OH`	Mass mixing ratio of OH	$\mathrm{kg\,kg^{-1}}$
`qst_H2O_ice`	Saturation mixing ratio w/r/t ice	$\mathrm{kg\,kg^{-1}}$
`qst_H2O_lqd`	Saturation mixing ratio w/r/t liquid	$\mathrm{kg\,kg^{-1}}$
`r_CO2`	Dry-mass mixing ratio (r) of CO2	$\mathrm{kg\,kg^{-1}}$
`r_CH4`	Dry-mass mixing ratio (r) of CH4	$\mathrm{kg\,kg^{-1}}$
`r_N2O`	Dry-mass mixing ratio (r) of N2O	$\mathrm{kg\,kg^{-1}}$
`r_CFC11`	Dry-mass mixing ratio (r) of CFC11	$\mathrm{kg\,kg^{-1}}$
`r_CFC12`	Dry-mass mixing ratio (r) of CFC12	$\mathrm{kg\,kg^{-1}}$
`r_H2OH2O`	Dry-mass mixing ratio (r) of H2O dimer	$\mathrm{kg\,kg^{-1}}$
`r_H2O`	Dry-mass mixing ratio (r) of H2O	$\mathrm{kg\,kg^{-1}}$

Table 7: (continued)

Name(s)	Purpose	Units
r_N2	Dry-mass mixing ratio (r) of N2	$kg\,kg^{-1}$
r_NO2	Dry-mass mixing ratio (r) of NO_2	$kg\,kg^{-1}$
r_O2O2	Dry-mass mixing ratio (r) of O2O2	$kg\,kg^{-1}$
r_O2	Dry-mass mixing ratio (r) of O2	$kg\,kg^{-1}$
r_O3	Dry-mass mixing ratio (r) of O3	$kg\,kg^{-1}$
r_OH	Dry-mass mixing ratio (r) of OH	$kg\,kg^{-1}$
rds_fct_ice	Effective radius of ice crystals	micron
rds_fct_lqd	Effective radius of liquid droplets	micron
rgh_len	Aerodynamic roughness length	meter
scl_hgt	Local scale height	meter
sfc_ems	Surface emissivity	fraction
slr_azi_dgr	Solar azimuth angle	degree
slr_crd_gmm_dgr	foo	degree
slr_cst	Solar constant	$W\,m^{-2}$
slr_dcl_dgr	Solar declination	degree
slr_dmt_dgr	Diameter of solar disc	degree
slr_dst_au	Earth-sun distance	astronomical units
slr_elv_dgr	Solar elevation	degree
slr_flx_TOA	Solar flux at TOA	$W\,m^{-2}$
slr_flx_nrm_TOA	Solar constant corrected for orbital position	$W\,m^{-2}$
slr_hr_ngl_dgr	Solar hour angle	degree
slr_rfr_ngl_dgr	Solar refraction angle	degree
slr_rgt_asc_dgr	Solar right ascension	degree
slr_zen_ngl_cos	Cosine solar zenith angle	fraction

Table 7: (continued)

Name(s)	Purpose	Units
slr_zen_ngl_dgr	Solar zenith angle in degrees	degree
slr_zen_ngl	Solar zenith angle	radian
snow_depth	Snow depth	meter
spc_heat_mst_air	Specific heat at constant pressure of moist air	joule kilogram-1 kelvin-1
time_lmt	Seconds between 1969 and LMT of simulation	second
time_ltst	Seconds between 1969 and LTST of simulation	second
time_unix	Seconds between 1969 and GMT of simulation	second
tpt_cls	Layer temperature (Celsius)	celsius
tpt_cls_ntf	Interface temperature (Celsius)	celsius
tpt	Layer Temperature	kelvin
tpt_ntf	Interface temperature	kelvin
tpt_sfc	Temperature of air in contact with surface	kelvin
tpt_skn	Temperature of surface	kelvin
tpt_vrt	Virtual temperature	kelvin
vmr_CO2	Volume mixing ratio of CO2	number number-1
vmr_CH4	Volume mixing ratio of CH4	number number-1
vmr_N2O	Volume mixing ratio of N2O	number number-1
vmr_CFC11	Volume mixing ratio of CFC11	number number-1
vmr_CFC12	Volume mixing ratio of CFC12	number number-1
vmr_H2OH2O	Volume mixing ratio of H2O dimer	number number-1
vmr_H2O	Volume mixing ratio of H2O	number number-1
vmr_N2	Volume mixing ratio of N2	number number-1
vmr_NO2	Volume mixing ratio of NO_2	number number-1
vmr_O2O2	Volume mixing ratio of O2O2	number number-1

Table 7: (continued)

Name(s)	Purpose	Units
vmr_O2	Volume mixing ratio of O2	number number-1
vmr_O3	Volume mixing ratio of O3	number number-1
vmr_OH	Volume mixing ratio of OH	number number-1
wvl_obs_aer	Wavelength of aerosol optical depth specification	meter
wvl_obs_bga	Wavelength of background aerosol optical depth specification	meter
xnt_fac	Eccentricity factor	fraction

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