# Lecture #19
# Fast Spin Echo, CPMG, and J coupling

- Spin echo vs Fast Spin Echo imaging
- Spin locking
- Decoupling
- References
  - Stables, et al, Analysis of J Coupling-Induced Fat Suppression in DIET Imaging, JMR, **136,** 143–151 (1999)
  - van de Ven, Chp 3.9, 4.9.


Journal of Magnetic Resonance **136,** 143–151 (1999)
Article ID jmre.1998.1628, available online at http://www.idealibrary.com on IDEAL®

# Analysis of *J* Coupling-Induced Fat Suppression in DIET Imaging

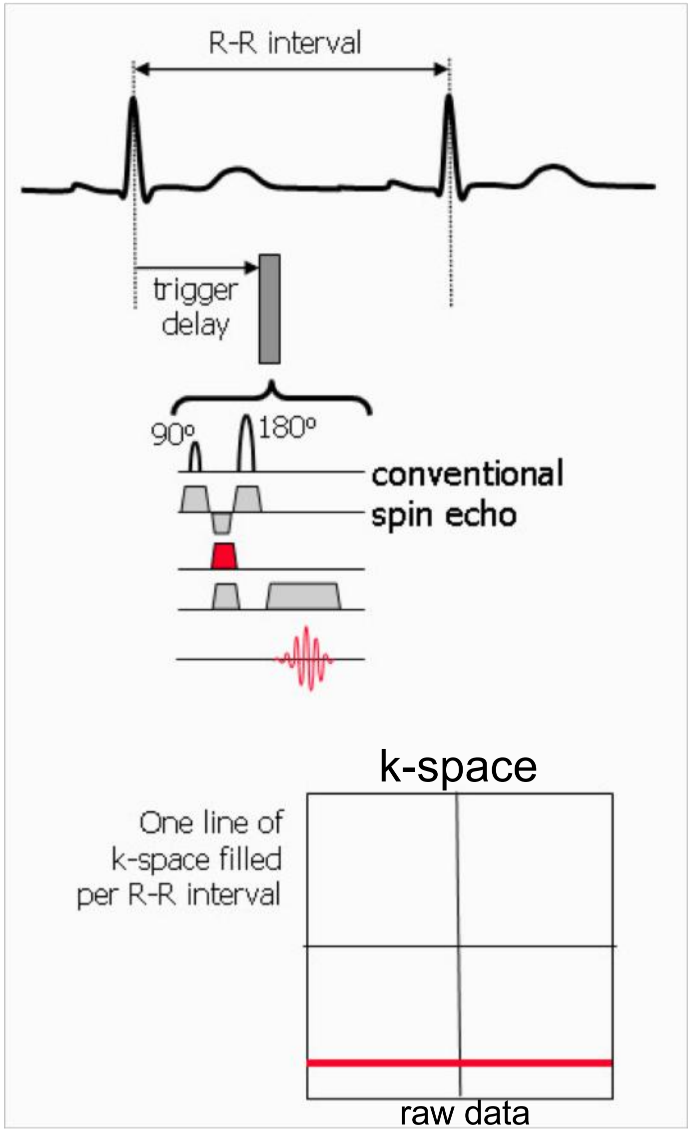
L. A. Stables,* R. P. Kennan,† A. W. Anderson,*·† R. T. Constable,† and J. C. Gore*·†

**Department of Applied Physics and †Department of Diagnostic Radiology, Yale University School of Medicine, New Haven, Connecticut 06520-8042*

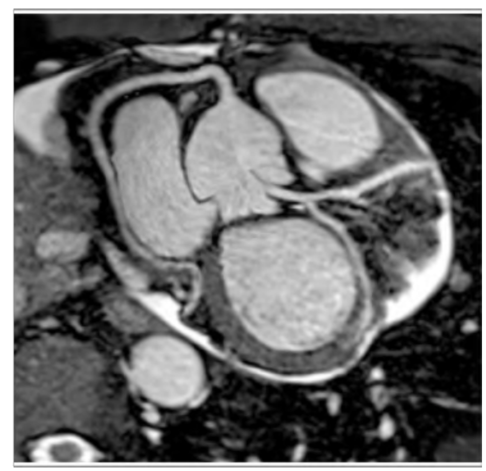
# Spin Echo Imaging

- One k-space line collected each TR

Cardiac MRI example:



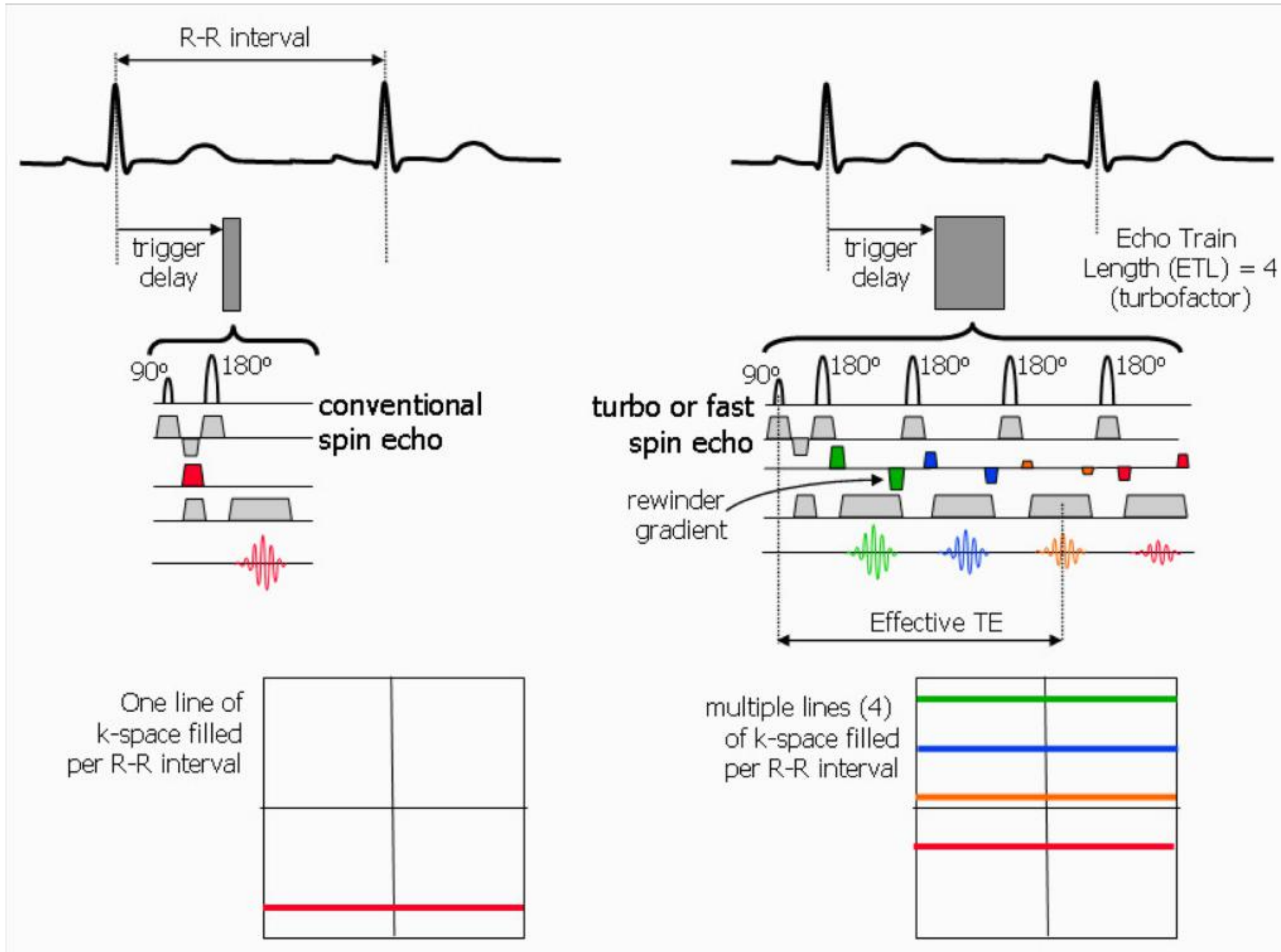

After n lines of data (e.g. 256) are acquired, a FT creates the image



# Fast Spin Echo

- A train of 180s is used to acquire multiple k-space lines each TR

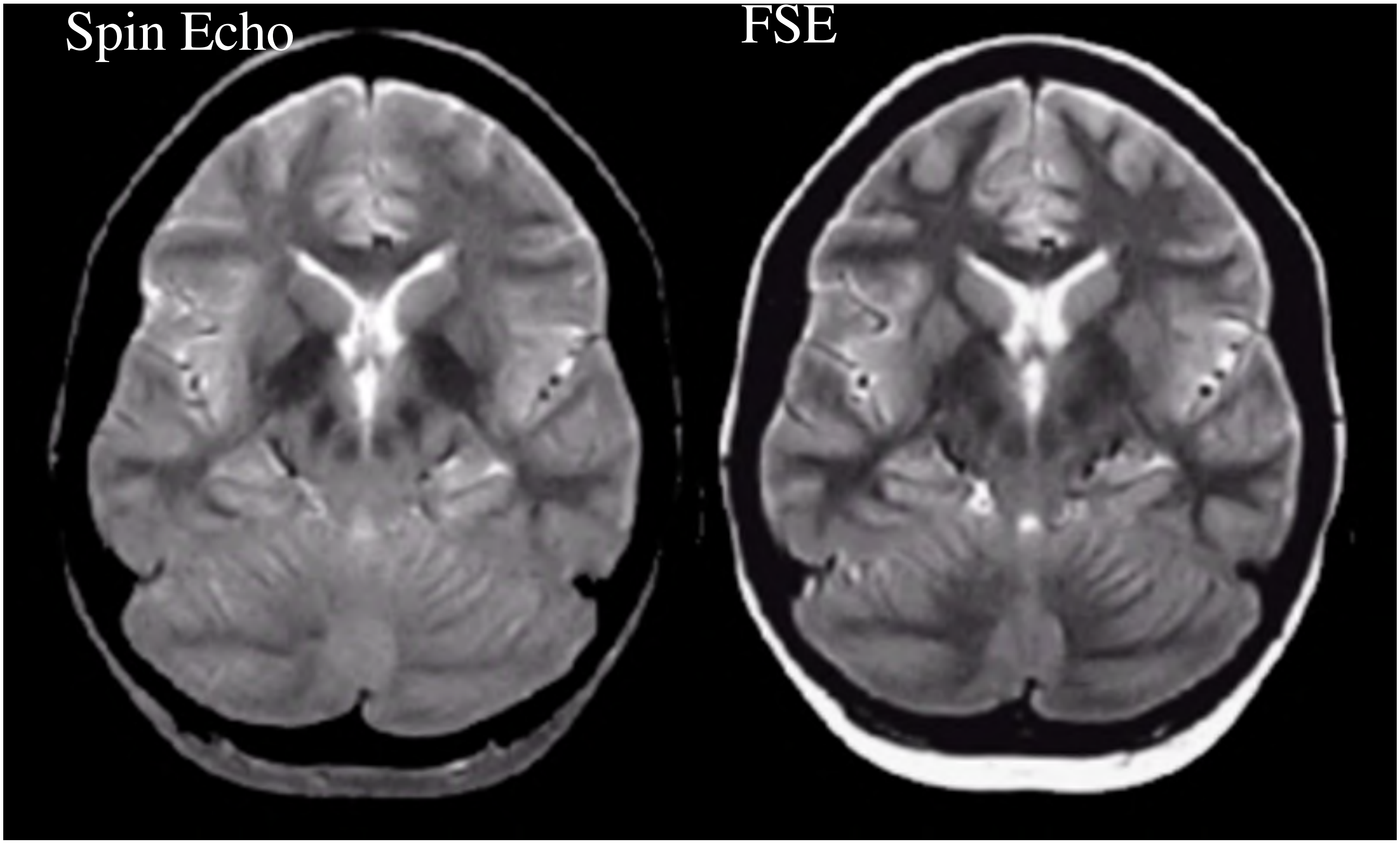
Cardiac MRI example:




- Provides the ability to acquire images much faster, while retaining $T_2$ weighting if desired.

# FSE Neuro Example

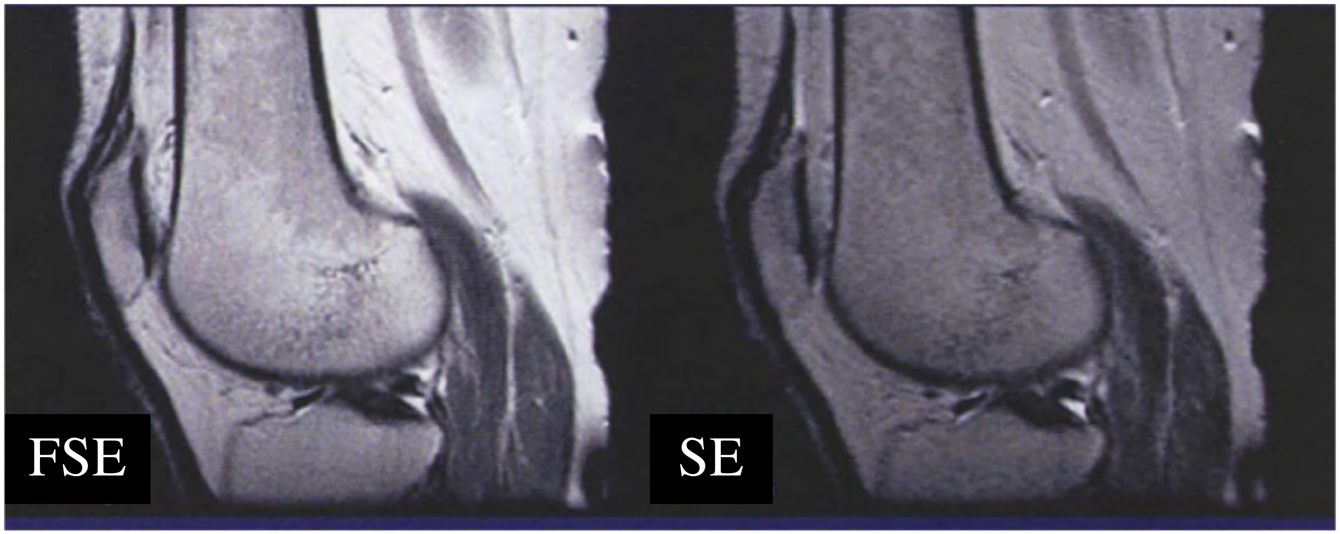
$T_2$-weighted images in much less time (3T, TE/TR = 80/2000 ms)




Acq. time ≈16 min

Acq. time ≈ 1 min
echo train length (ETL) = 16

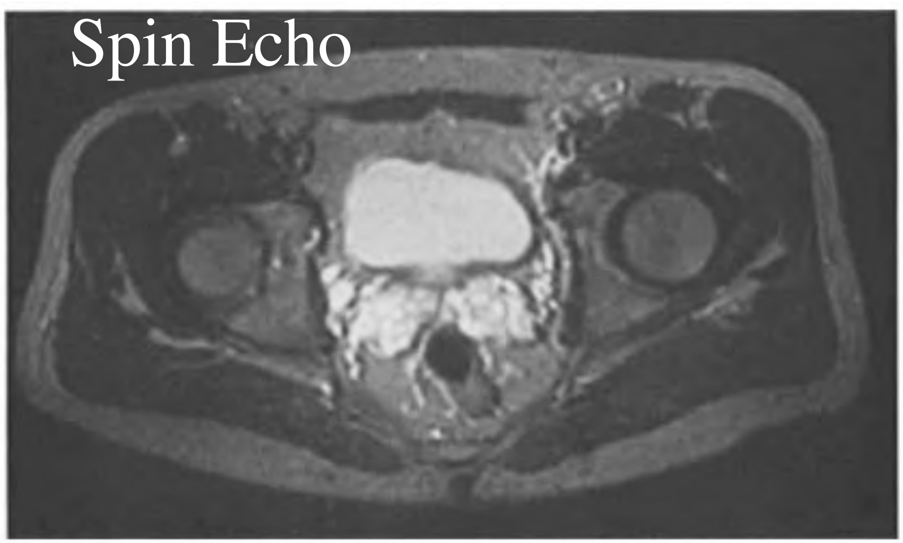
# Why is fat bright in FSE images?




Lipid $T_2$ = ?

SE: lipid $T_2 \sim 35$ ms

FSE: lipid $T_2 \sim 135$ ms




# Multi-Spin Systems

- Ignoring relaxation, the Hamiltonian has the following general form:

$$\hat{H} = \underbrace{\hat{H}^0}_{\text{Static field + J couplings}} + \hat{H}_{Rf}$$

- For a system with multiple spins, the total x, y, and z coherences are

$$\hat{F}_x = \sum_j \hat{I}_{xj} \qquad \hat{F}_y = \sum_j \hat{I}_{yj} \qquad \hat{F}_z = \sum_j \hat{I}_{zj}$$

and the first term of the Hamiltonian can be written as:

$$\hat{H}^0 = \sum_{j=1} \delta_j \hat{I}_{zj} + \sum_{j<k} 2\pi J_{jk} \vec{\hat{I}}_j \cdot \vec{\hat{I}}_k$$

- We wish to consider two cases.
  1. Magnetically equivalent spins, i.e. all $\delta_j$s are equal.
  2. Non-equivalent spins, i.e. unequal $\delta_j$s.

# Equivalent Spins

- Multi-spin system for equivalent spins

$$\hat{H}^0 = \sum_j \hat{I}_{zj} + \sum_{j<k} 2\pi J_{jk} \vec{\hat{I}}_j \cdot \vec{\hat{I}}_k = \hat{H}_1 + \hat{H}_2 \quad \text{One can show} \quad \left[\sum_j \hat{I}_{pj}, \vec{\hat{I}}_j \cdot \vec{\hat{I}}_k\right] = 0, \; p = x, y, z$$

2-spin example

$$\left[\hat{I}_z + \hat{S}_z, \hat{I}_x\hat{S}_x + \hat{I}_y\hat{S}_y + \hat{I}_z\hat{S}_z\right] = \hat{I}_y\hat{S}_x - \hat{I}_x\hat{S}_y + \hat{I}_x\hat{S}_y - \hat{I}_y\hat{S}_x = 0$$

$$\left[\hat{I}_x + \hat{S}_x, \hat{I}_x\hat{S}_x + \hat{I}_y\hat{S}_y + \hat{I}_z\hat{S}_z\right] = \hat{I}_z\hat{S}_y - \hat{I}_y\hat{S}_z + \hat{I}_y\hat{S}_z - \hat{I}_z\hat{S}_y = 0$$

$$\left[\hat{I}_y + \hat{S}_y, \hat{I}_x\hat{S}_x + \hat{I}_y\hat{S}_y + \hat{I}_z\hat{S}_z\right] = \hat{I}_z\hat{S}_x - \hat{I}_x\hat{S}_z + \hat{I}_x\hat{S}_z - \hat{I}_z\hat{S}_x = 0$$

- Theorem: Let $\hat{H} = \hat{H}_1 + \hat{H}_2$ and $\left[\hat{H}_1, \hat{H}_2\right] = \left[\hat{F}_x, \hat{H}_2\right] = \left[\hat{F}_y, \hat{H}_2\right] = \left[\hat{F}_z, \hat{H}_2\right] = 0$

  then the observed signal is independent of $\hat{H}_2$

Proof:

$$\mathrm{Tr}\left[\hat{F}_p e^{-i\hat{H}t}\sigma(0)e^{i\hat{H}t}\right] = \mathrm{Tr}\left[\hat{F}_p e^{-i\left(\hat{H}_1 t + \hat{H}_2 t\right)}\sigma(0)e^{i\left(\hat{H}_1 t + \hat{H}_2 t\right)}\right]$$

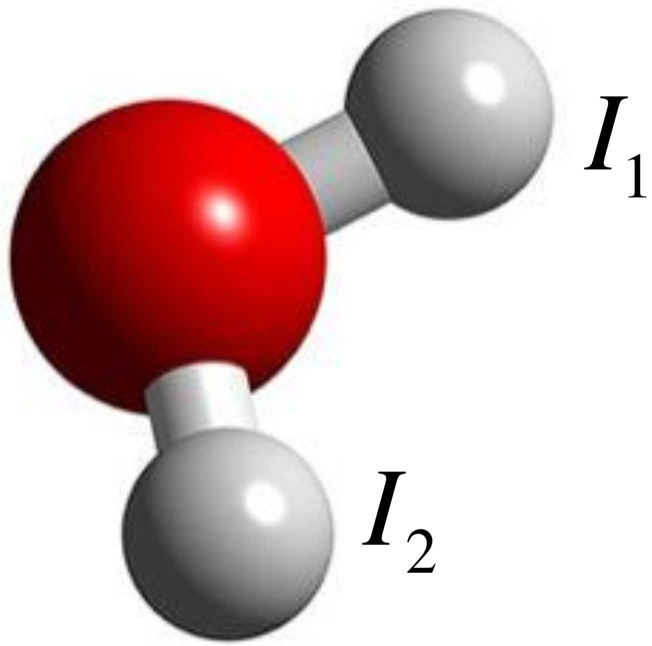
$$= \mathrm{Tr}\left[\hat{F}_p e^{-i\hat{H}_2 t}e^{-i\hat{H}_1 t}\sigma(0)e^{i\hat{H}_1 t}e^{i\hat{H}_2 t}\right]$$

$$= \mathrm{Tr}\left[e^{i\hat{H}_2 t}\hat{F}_p e^{-i\hat{H}_2 t}e^{-i\hat{H}_1 t}\sigma(0)e^{i\hat{H}_1 t}\right]$$ Why is this step legitimate?

$$= \mathrm{Tr}\left[\hat{F}_p e^{i\hat{H}_2 t}e^{-i\hat{H}_2 t}e^{-i\hat{H}_1 t}\sigma(0)e^{i\hat{H}_1 t}\right]$$

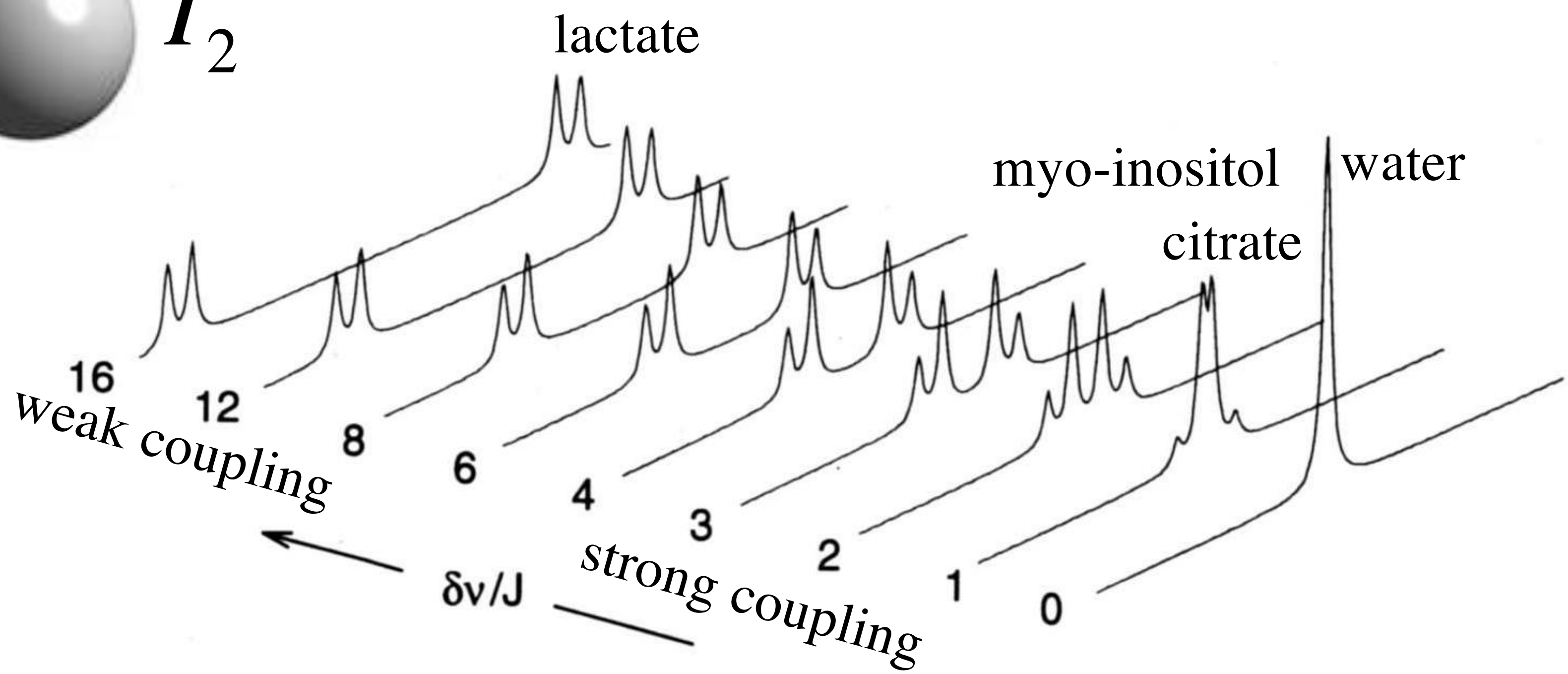
$$= \mathrm{Tr}\left[\hat{F}_p e^{-i\hat{H}_1 t}\sigma(0)e^{i\hat{H}_1 t}\right]$$ Independent of $\hat{H}_2$!

# Water and J-coupling




$$\hat{H}^0 = \Omega \hat{I}_{z1} + \Omega \hat{I}_{z2} + 2\pi J \vec{\hat{I}}_1 \cdot \vec{\hat{I}}_2$$

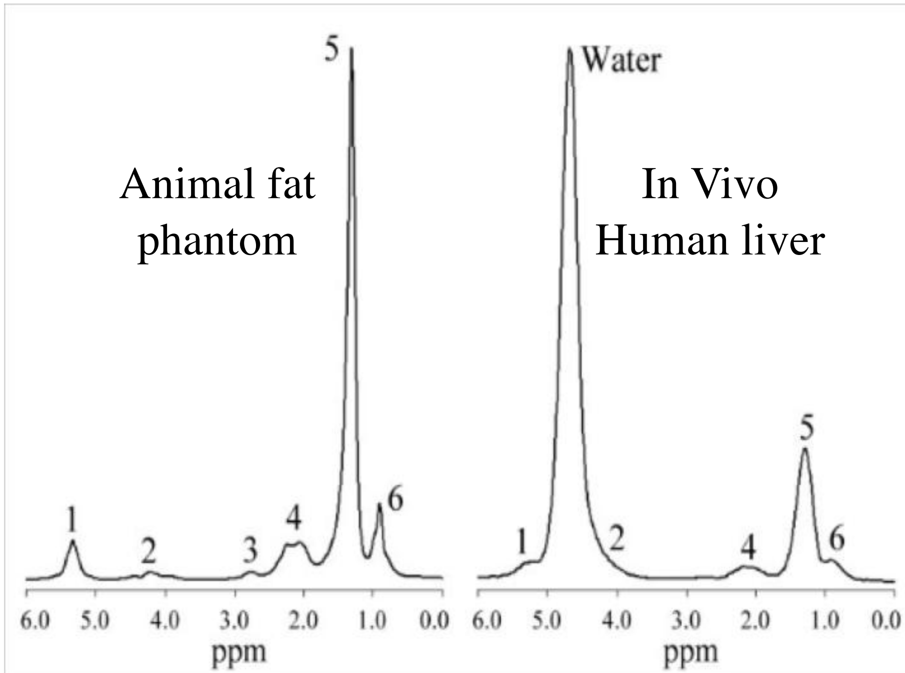
$$\left[\hat{I}_{z1} + \hat{I}_{z2}, \vec{\hat{I}}_1 \cdot \vec{\hat{I}}_2\right] = 0$$




The two water $^1H$ spins are equivalent, hence show no effects due to J-coupling.

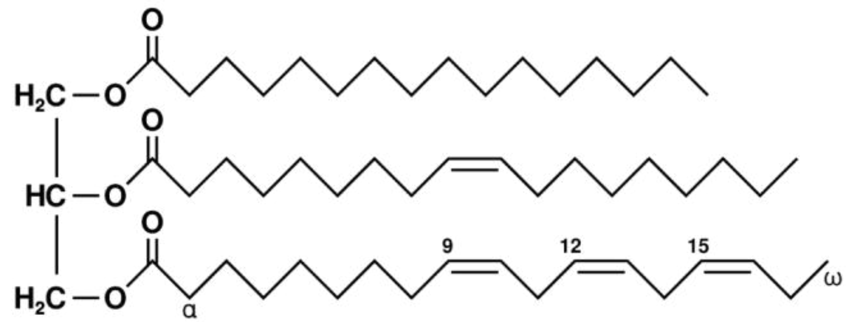
# Lipids

- Lipids consist of multiple J-coupled resonances, and lipid [1]Hs are not equivalent!




| Peak | Location | Assignment | Peak type |
|---|---|---|---|
| 1 | 5.30 ppm | —**CH**═**CH**— | Multiplet |
| | 5.19 ppm | —**CH**—O—CO—R | Multiplet |
| 2 | 4.20 ppm | —**CH**$_2$—O—CO—R | Multiplet |
| 3 | 2.75 ppm | —CH═CH—**CH**$_2$—CH═CH— | Multiplet |
| 4 | 2.20 ppm | —CO—**CH**$_2$—CH$_2$— | Multiplet |
| | 2.02 ppm | —**CH**$_2$—CH═CH—**CH**$_2$— | Multiplet |
| 5 | 1.60 ppm | —CO—CH$_2$—**CH**$_2$— | Multiplet |
| | 1.30 ppm | —(**CH**$_2$)$_n$— | Multiplet |
| 6 | 0.90 ppm | —(CH$_2$)$_n$—**CH**$_3$ | Triplet |

Hamilton, et al, JMRI, 30:145–152 (2009)




$$\hat{H}^0 = \sum_j \delta_j \hat{I}_{zj} + \sum_{j<k} 2\pi J_{jk} \vec{\hat{I}}_j \cdot \vec{\hat{I}}_k$$

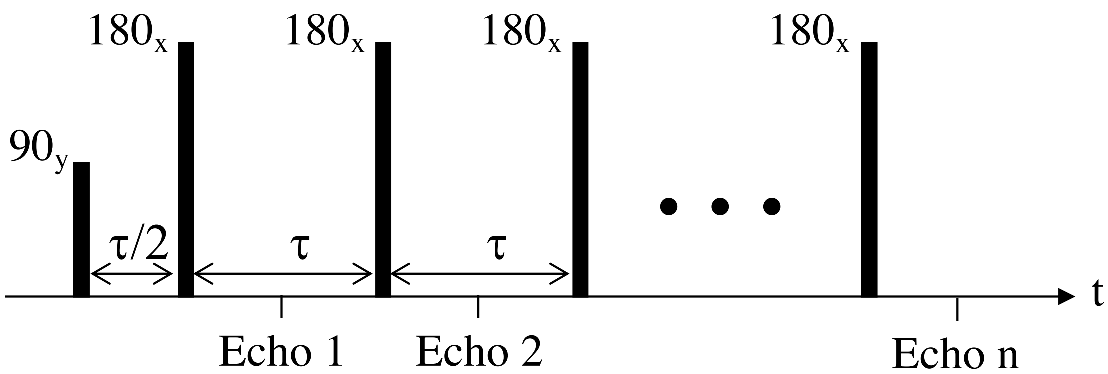
$\delta_j$s not all equal, thus

$$\left[\sum_j \delta_j \hat{I}_{pj}, \vec{\hat{I}}_j \cdot \vec{\hat{I}}_k\right] \neq 0$$

Hence J-coupling can NOT be ignored.

# Carr-Purcell-Meiboom-Gill (CPMG)

- Consider a multi-spin system with j spins, coupling constants $J_{jk}$, rotating frame resonance frequencies $\delta_j$, and the following pulse sequence:



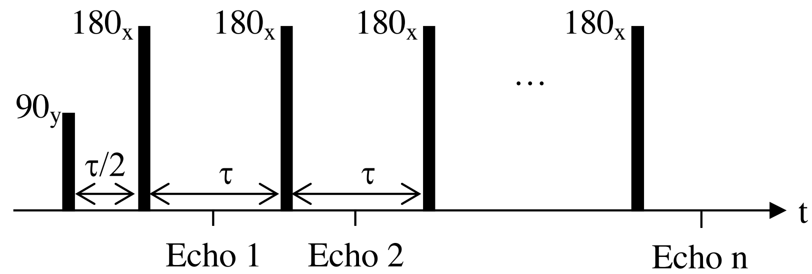

$$\hat{F}_x = \sum_j \hat{I}_{xj}$$

$$\hat{F}_y = \sum_j \hat{I}_{yj}$$

$$\hat{F}_z = \sum_j \hat{I}_{zj}$$

- Assuming hard RF pulses with $\omega_1 >> \delta_j, J_{jk}$ for all $j$, find the Hamiltonian for each time interval.
  - $90_y$ Rf pulse: $\hat{H}_1 = \omega_1 \sum_j \hat{I}_{yj} = \omega_1 \hat{F}_y$ where $\omega_1 = -\gamma B_1 >> \delta_j, J_{jk}$ for all $j,k$.
  - $180_x$ Rf pulses: $\hat{H}_2 = \omega_1 \hat{F}_x$
  - between Rf pulses: $\hat{H}_3 = \sum_j \delta_j \hat{I}_{zj} + \sum_{j<k} 2\pi J_{jk} \vec{\hat{I}}_j \cdot \vec{\hat{I}}_k$

# CPMG Product Operator Analysis




- Ignoring relaxation...

At thermal equilibrium: $\hat{\sigma}_0 \propto \hat{F}_z$

After the $90_y$: $\hat{\sigma} \propto \hat{F}_x$

Before the first $180_x$: $\hat{\sigma} \propto e^{-\frac{i}{2}\hat{H}_3\tau}\hat{F}_x e^{\frac{i}{2}\hat{H}_3\tau}$

After the first $180_x$: $\hat{\sigma} \propto e^{-i\pi\hat{F}_x} e^{-\frac{i}{2}\hat{H}_3\tau}\hat{F}_x e^{\frac{i}{2}\hat{H}_3\tau} e^{i\pi\hat{F}_x}$

$\hat{\sigma} \propto \hat{B}\hat{A}\hat{F}_x\hat{A}^{-1}\hat{B}^{-1}$ where $\hat{A} = e^{-\frac{i}{2}\hat{H}_3\tau}$ and $\hat{B} = e^{-i\pi\hat{F}_x}$

At Echo 1: $\hat{\sigma}_1 \propto \left(\hat{A}\hat{B}\hat{A}\right)\hat{F}_x\left(\hat{A}\hat{B}\hat{A}\right)^{-1}$

Continuing, the spin density at the *n*th echo will be:

$$\hat{\sigma}_n \propto \left(\hat{A}\hat{B}\hat{A}\right)^n \hat{F}_x \left(\hat{A}\hat{B}\hat{A}\right)^{-n}$$

# CPMG Product Operator Analysis

- Let examine $\hat{\sigma}_n \propto \left(\hat{A}\hat{B}\hat{A}\right)^n \hat{F}_x \left(\hat{A}\hat{B}\hat{A}\right)^{-n}$ more closely for the case where τ is short, i.e. $\left|J_{jk}\tau\right|, \left|\delta_j\tau\right| << 1$ for all spin groups $j,k$.

- Expanding $\hat{A}$ to first order in a Taylor series yields

$$\hat{A} = e^{-\frac{i}{2}\hat{H}_3\tau} \approx 1 - \tfrac{i}{2}\hat{H}_3\tau = 1 - \tfrac{i}{2}\sum_j \delta_j \hat{I}_{zj}\tau - \tfrac{i}{2}\sum_{j<k} 2\pi J_{jk}\vec{\hat{I}}_j \cdot \vec{\hat{I}}_k\tau$$

Substituting… $\hat{A}\hat{B}\hat{A} = e^{-\frac{i}{2}\hat{H}_3\tau} e^{-i\pi\hat{F}_x} e^{-\frac{i}{2}\hat{H}_3\tau} \xrightarrow{\text{short } \tau} \left(1 + i\sum_{j<k} 2\pi J_{jk}\vec{\hat{I}}_j \cdot \vec{\hat{I}}_k\tau\right) e^{-i\pi\hat{F}_x}$

However, $\hat{F}_x$ and $e^{-i\pi\hat{F}_x}$ commute…

Therefore, in a CPMG sequence with $\left|J_{jk}\tau\right|, \left|\delta_j\tau\right| << 1$, $\quad \hat{\sigma}_n \xrightarrow{\text{short } \tau} \hat{F}_x$

Independent of $n$,
$\delta_j$, and $J_{jk}$

- Therefore, for this rapidly refocused CPMG sequence, lipids will decay with their true $T_2$s free from the additional dephasing due to multiple J-couplings.

# FSE Simulations

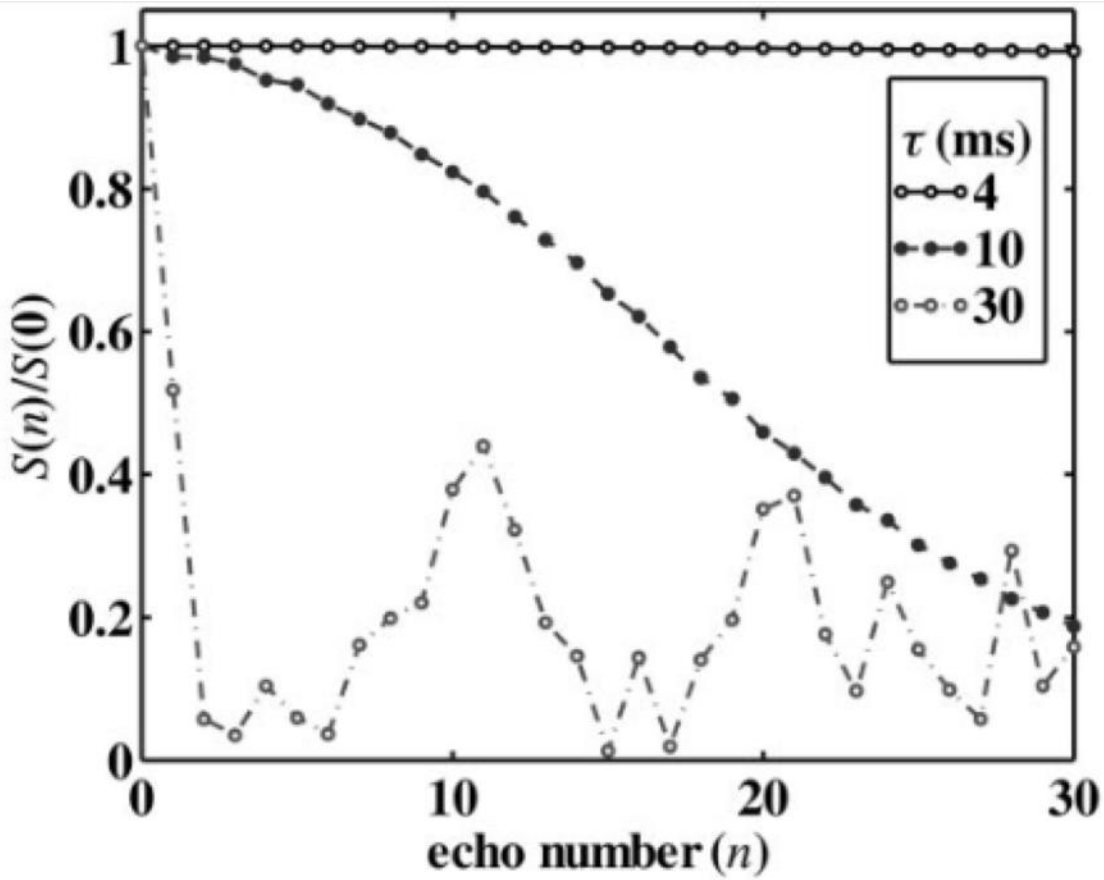
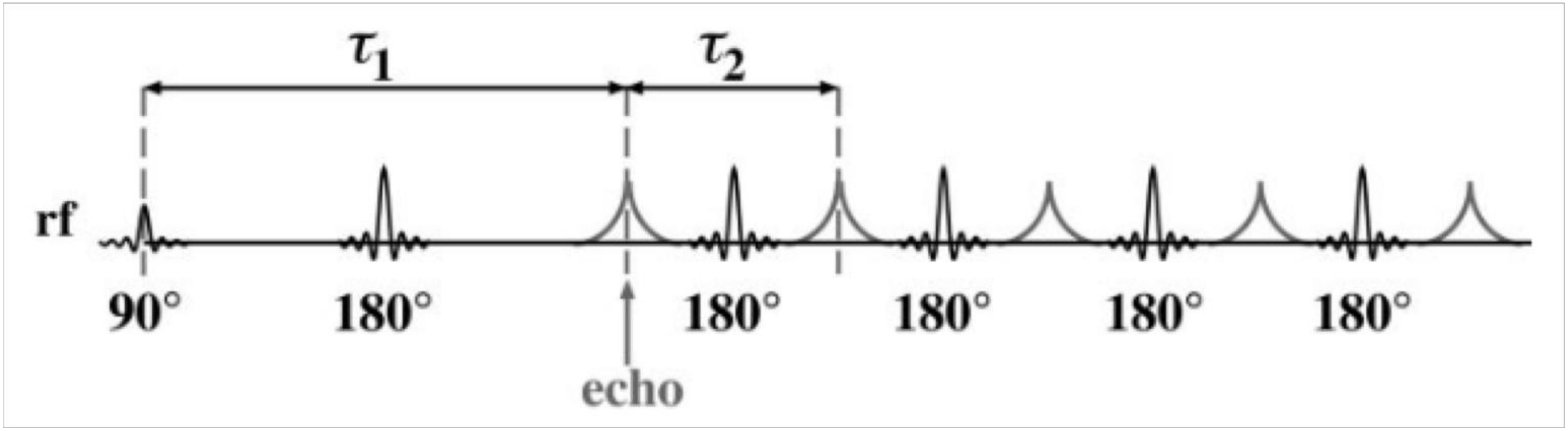



**FIG. 1.** The effect of $J$ coupling on a strongly coupled $A_3B_2$ spin system. The plot shows signal vs echo number for CPMG sequences where $\tau$, the spacing between echoes, is 4, 10, or 30 ms. $J_{AB} = 6$ Hz, $\delta_{AB} = 40$ Hz. Intrinsic $T_2$ relaxation is neglected. Note that as $\tau$ increases, $J$ coupling becomes more effective at suppressing the NMR signal.

Stables, et al, JMR, **136,** 143–151 (1999)

# Dual Interval Echo Train (DIET) FSE

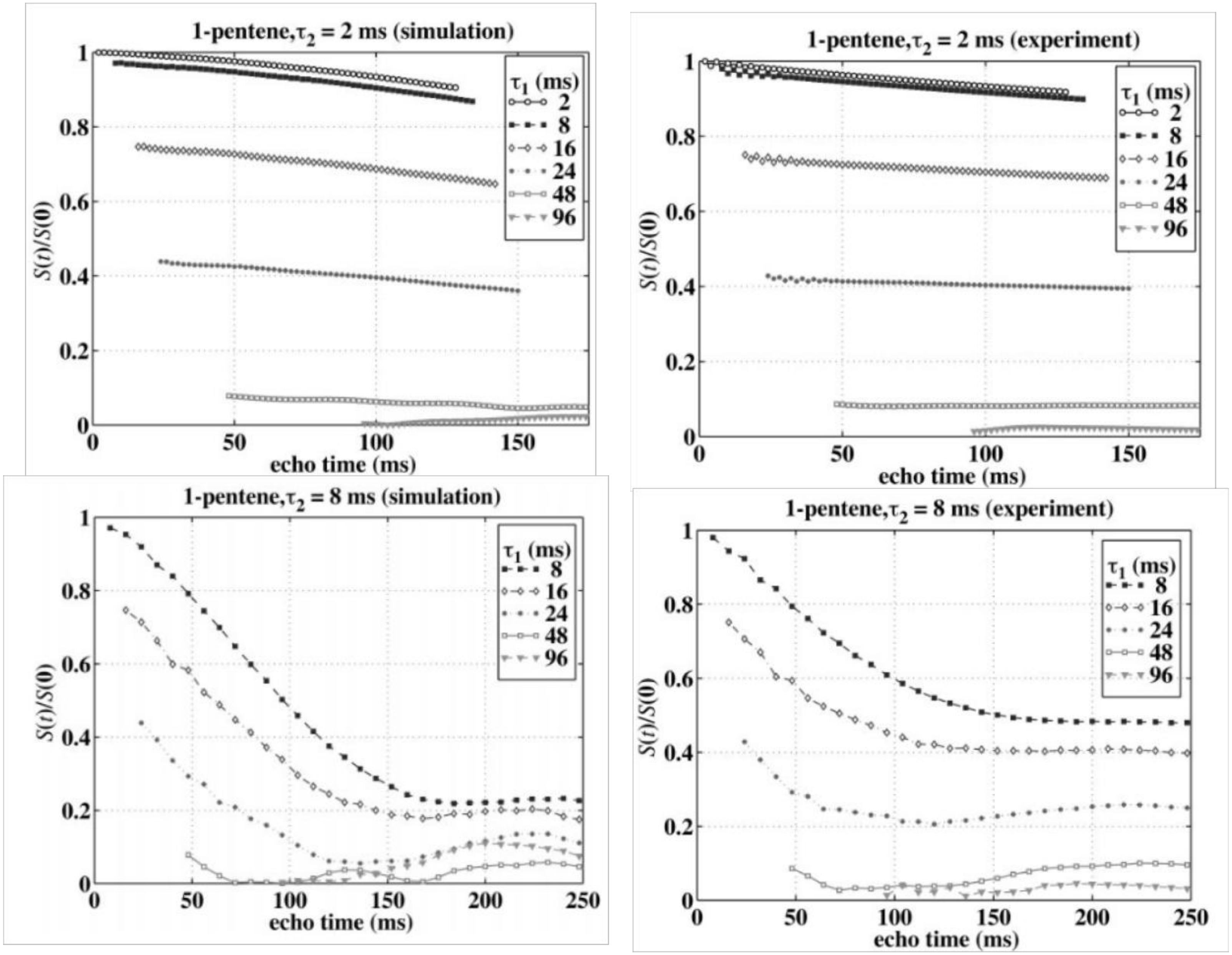
- Goal: lipid suppression




- The relatively long first echo allows for J-coupling-induced dephasing to occur.
- This signal loss is not recovered in the remaining echo train.

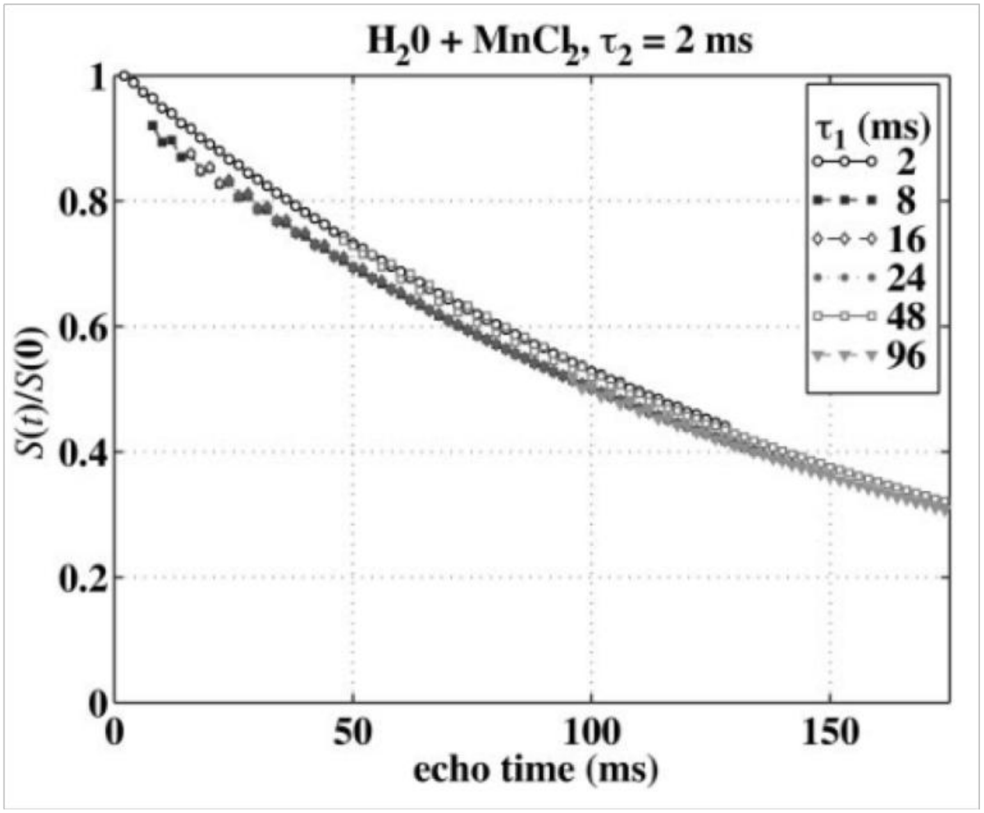
Stables, et al, JMR, **136,** 143–151 (1999)

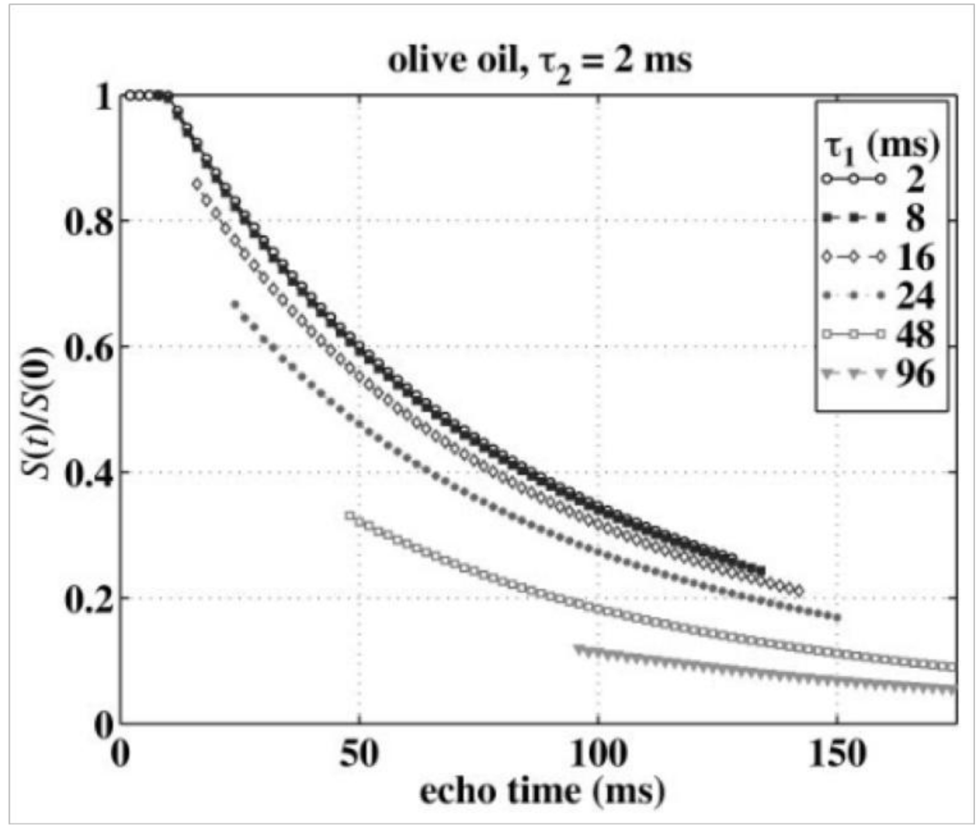
# DIET Results




Stables, et al, JMR, **136,** 143–151 (1999)

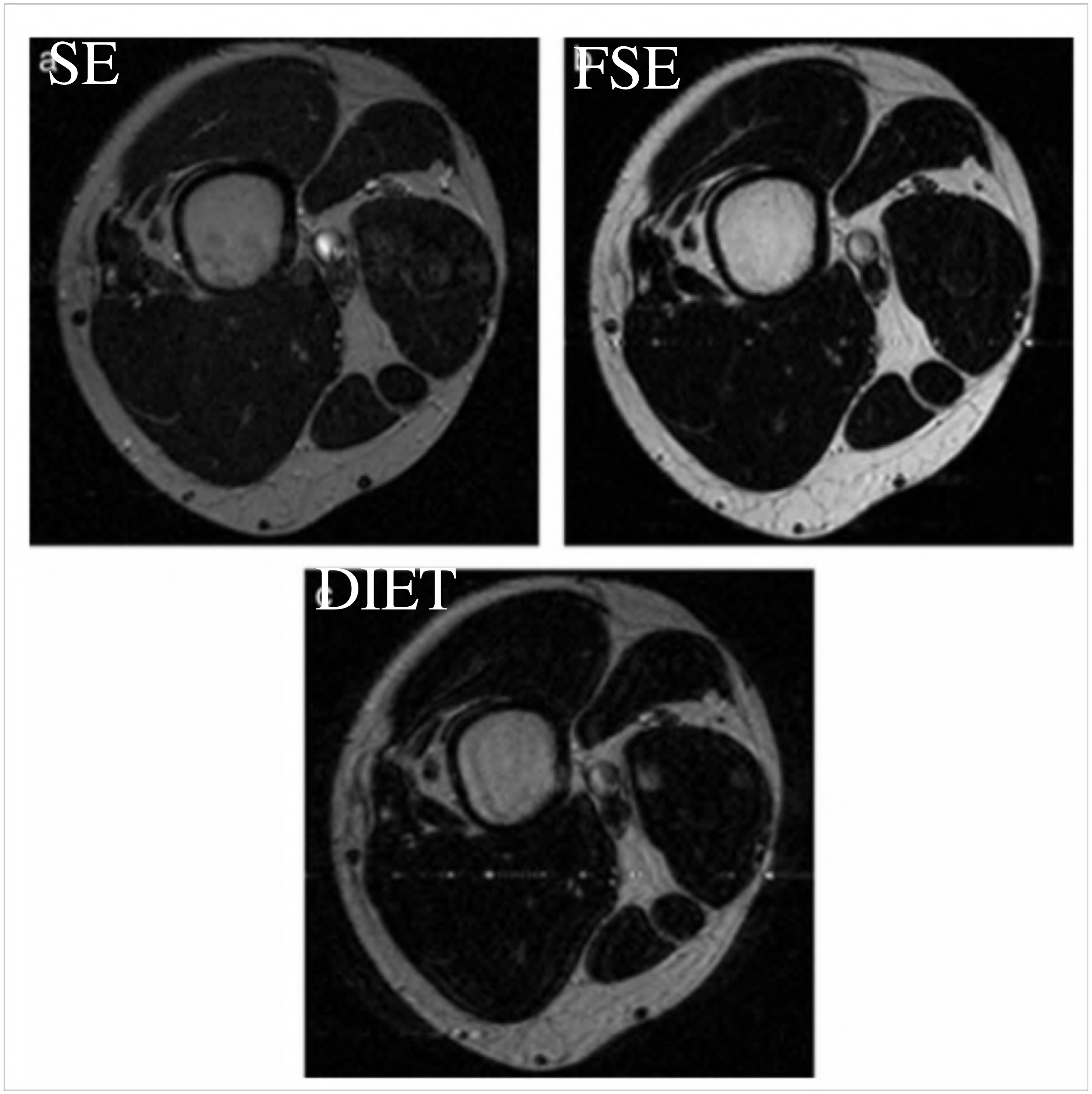
# DIET Results




Stables, et al, JMR, **136,** 143–151 (1999)

# Results

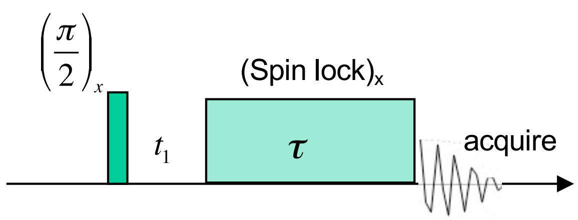


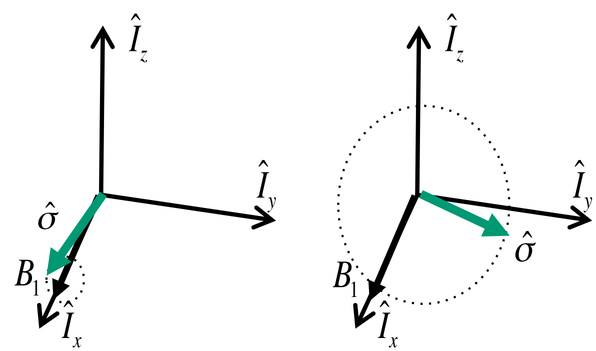

Stables, et al, JMR, **136,** 143–151 (1999)

# Spin Locking

- The suppression of J-coupling during a FSE sequence is considered a nuisance. However, the effect can also have advantages.
- Spin locking: the application of a long, strong continuous Rf pulse along a specified axis, e.g. x, in the rotating frame.
  - Chemical shift is suppressed, and spins are rendered effectively equivalent
  - Coherences along x are retained, those along y are dephased due to Rf inhomogeneity

TOCY
HOHAHA
ROESY



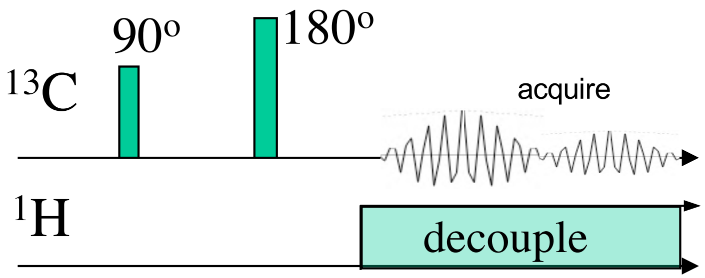

$$\hat{I}_x + \hat{S}_x \xrightarrow{spin\ lock} \hat{I}_x + \hat{S}_x \qquad \hat{I}_x - \hat{S}_x \xrightarrow{spin\ lock} \left(\hat{I}_x - \hat{S}_x\right)\cos 2\pi J\tau + \left(2\hat{I}_y\hat{S}_z - 2\hat{I}_z\hat{S}_y\right)\sin 2\pi J\tau$$

- The observable magnetization for truly equivalent spins does not evolve under J coupling, however spins rendered temporarily equivalent can show much more complex behavior, as they an enter the spin-lock period in a variety of initial states.

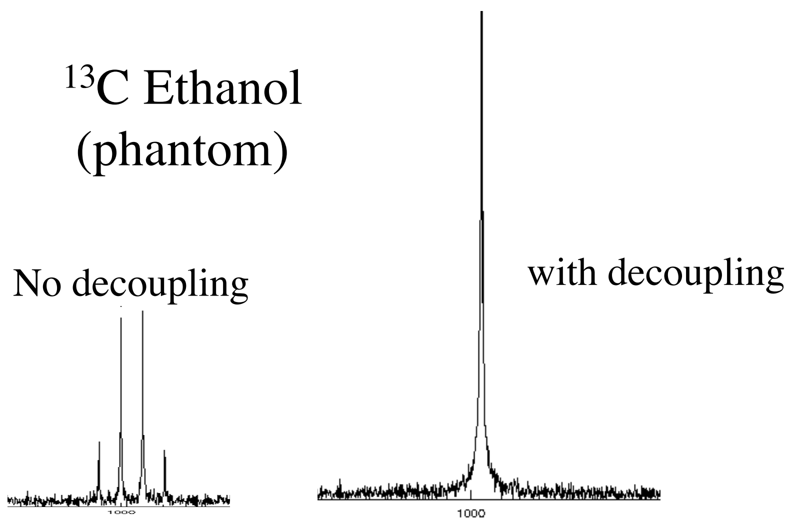
# Decoupling

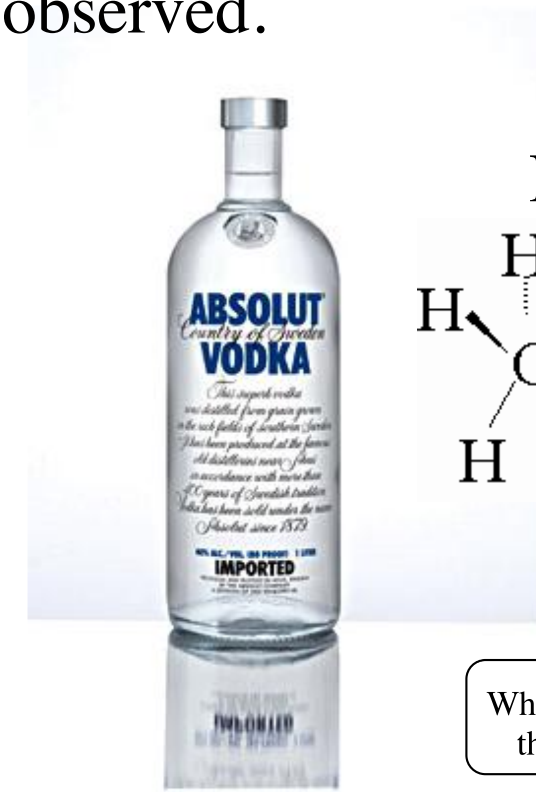
- Line splitting reduces the ability to detect and quantify in vivo peaks.
- Decoupling involves the use of a long strong Rf pulse on the coupled partner of the spin being observed.

Example

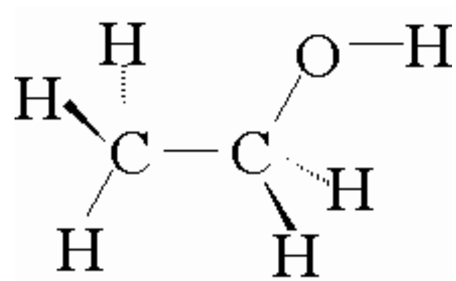


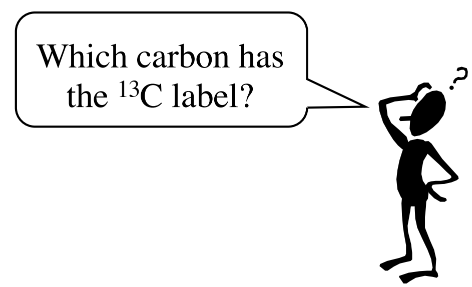

$^{13}C$ Ethanol (phantom)




Ethanol





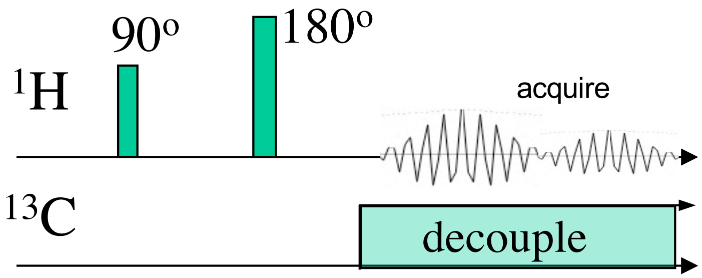
# Decoupling

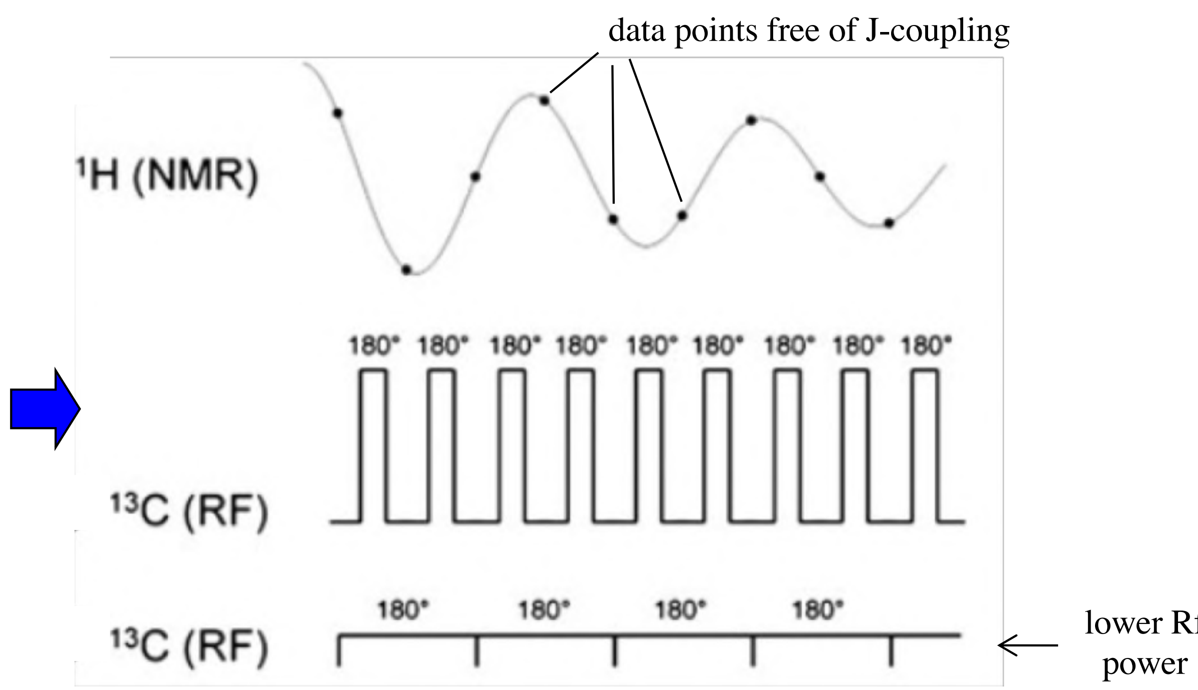
- Hamiltonian: $\hat{H} = \hat{H}_{Zeeman} + \cancel{\hat{H}_{dipole}} + 2\pi J \underbrace{\vec{\hat{I}} \cdot \vec{\hat{S}}}$

time average=0 with rapid molecular tumbling

"decouple" using RF pulses to rapidly flip the S spin

Example

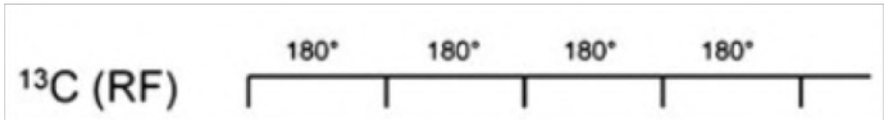


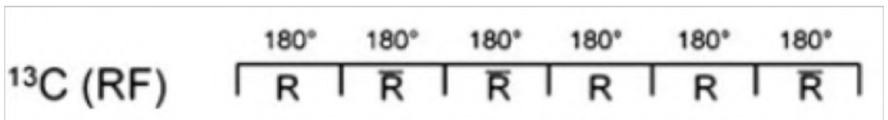

We don't need to synchronize if refocusing on time scale << 1/J

# Broadband Decoupling

- Problem: long Rf pulses have narrow bandwidths.




- Phase cycling the 180$^o$ pulses improves off-resonance behavior.




- Composite 180 pulses are even better:

$$R = 90^\circ_x 180^\circ_{-x} 270^\circ_x = 1\bar{2}3$$

…and are typically used in supercycles.

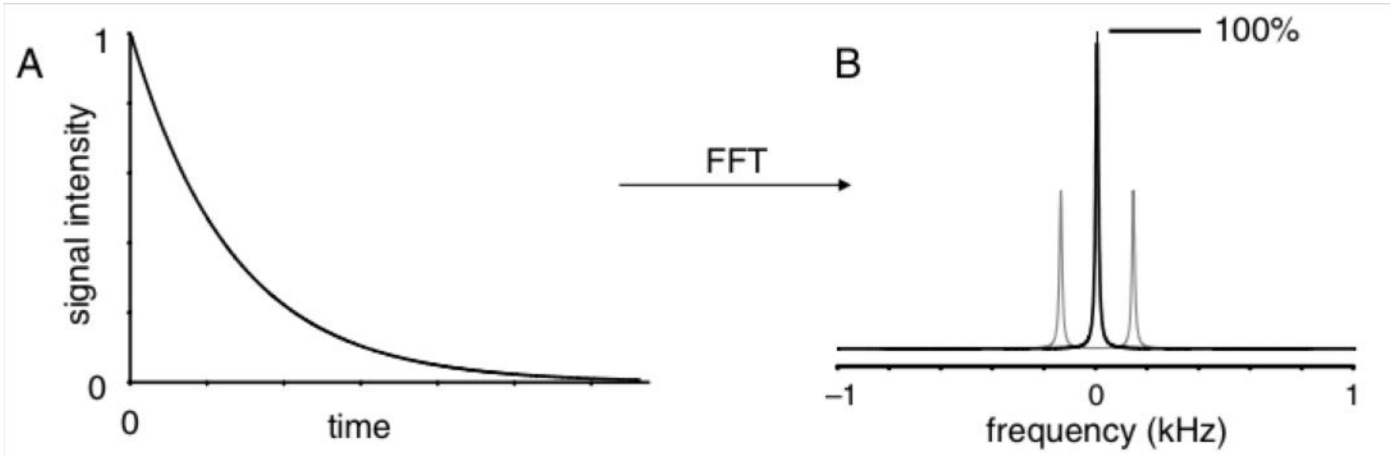
$$1\bar{2}3 \; 1\bar{2}3 \; \bar{1}2\bar{3} \; \bar{1}2\bar{3}$$

<u>W</u>ideband <u>A</u>lternating Phase <u>L</u>ow-power
<u>T</u>echnique for <u>Z</u>ero-residue Splitting

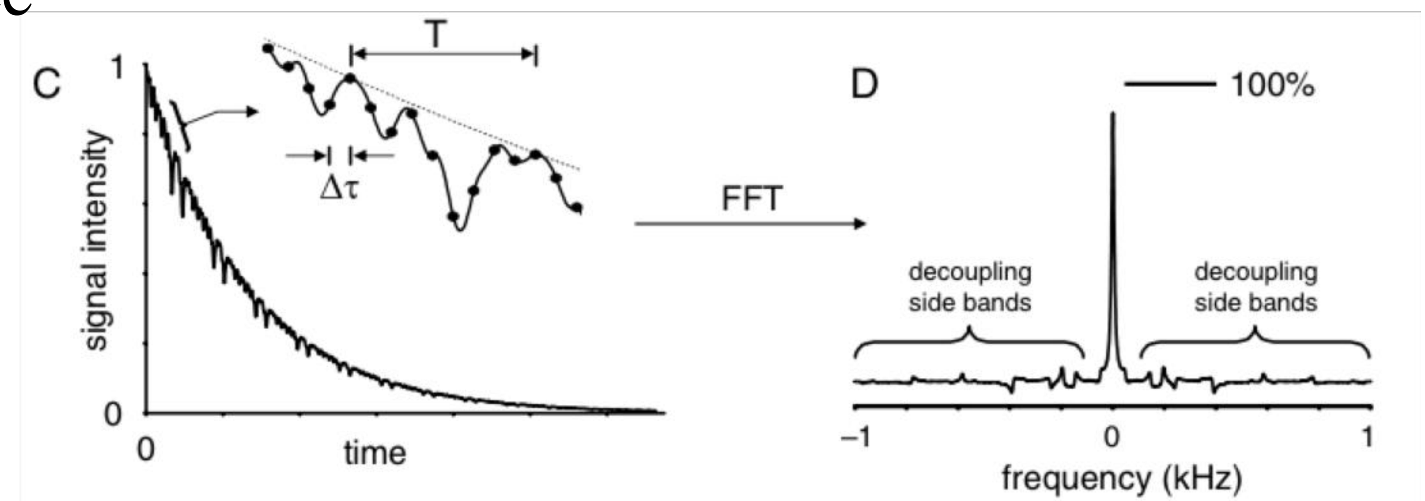
**WALTZ**

# Decoupling in practice

- Theory



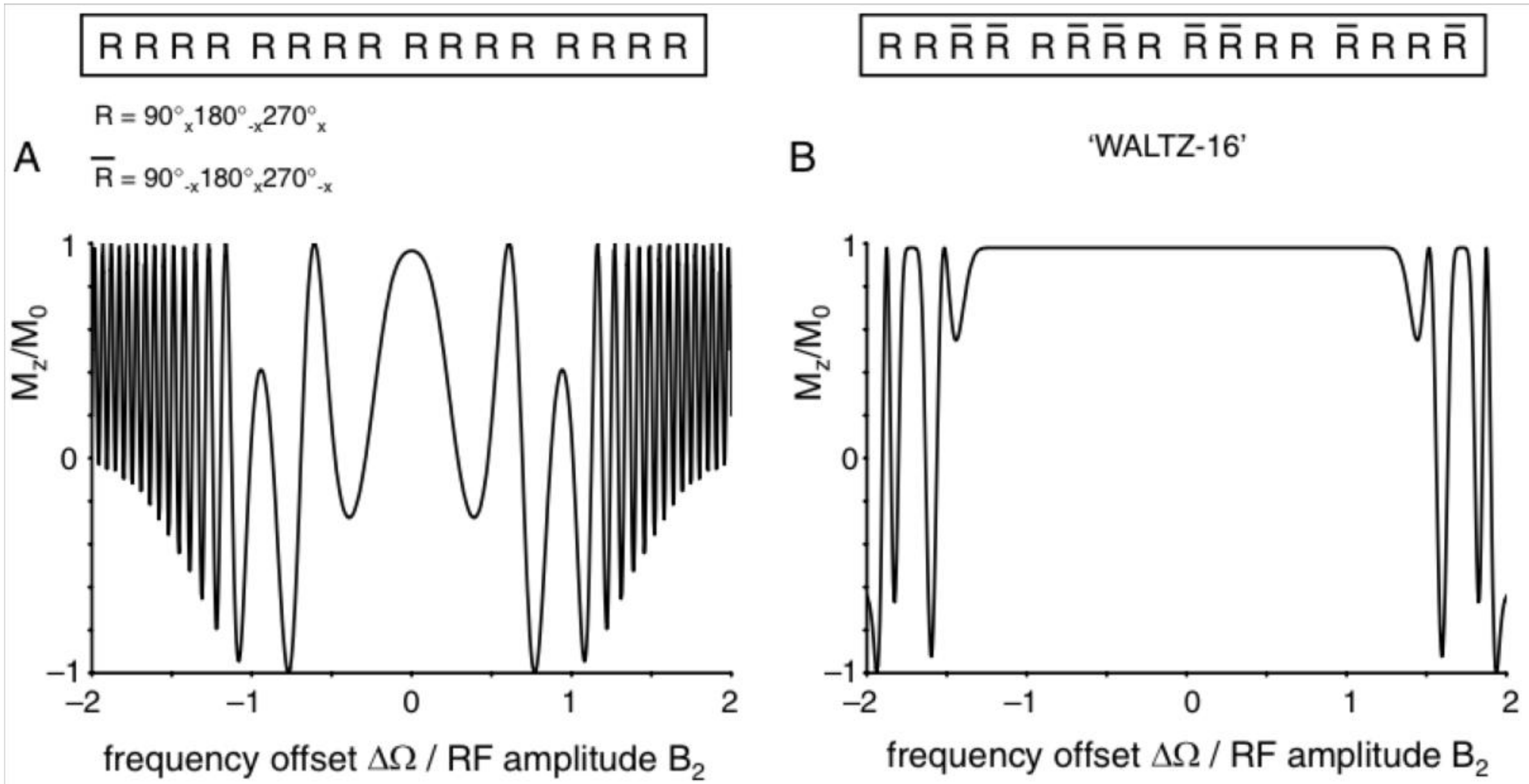

- In practice



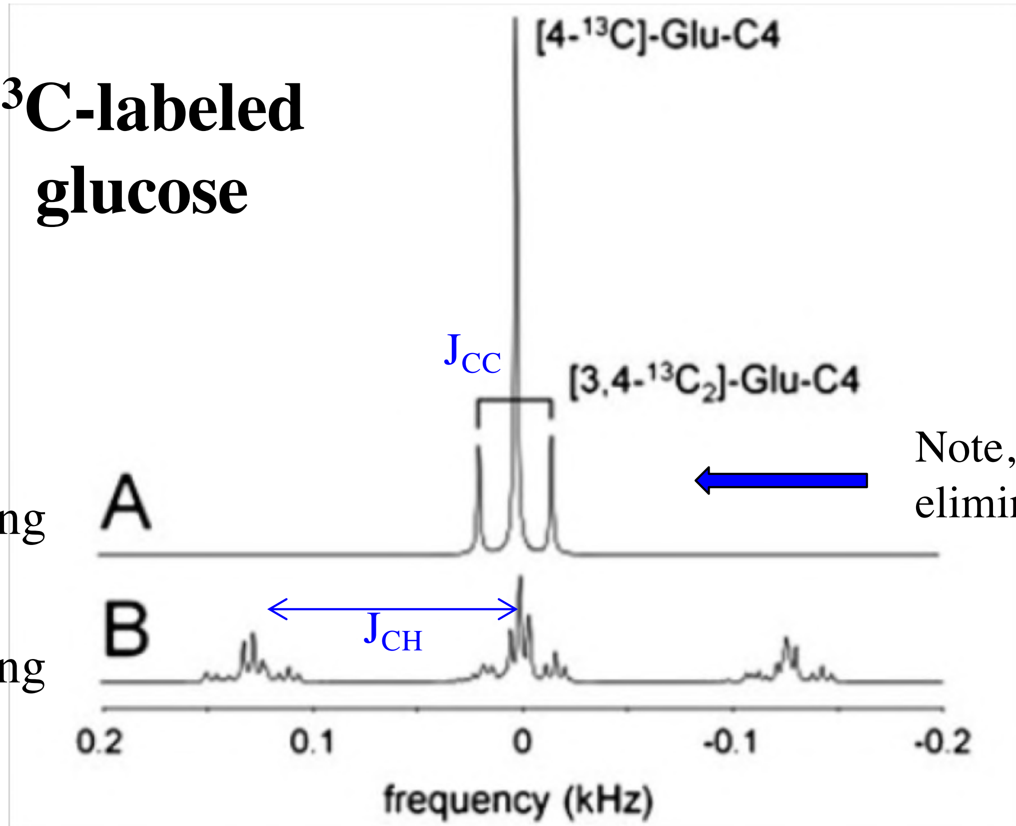

- Rf power deposition, typically measured as Specific Absorption Rate (SAR), is usually the limiting factor.

de Graaf, Chpt 8

# WALTZ Decoupling



# In Vitro Example: $^{13}$C MRS




with 1H decoupling

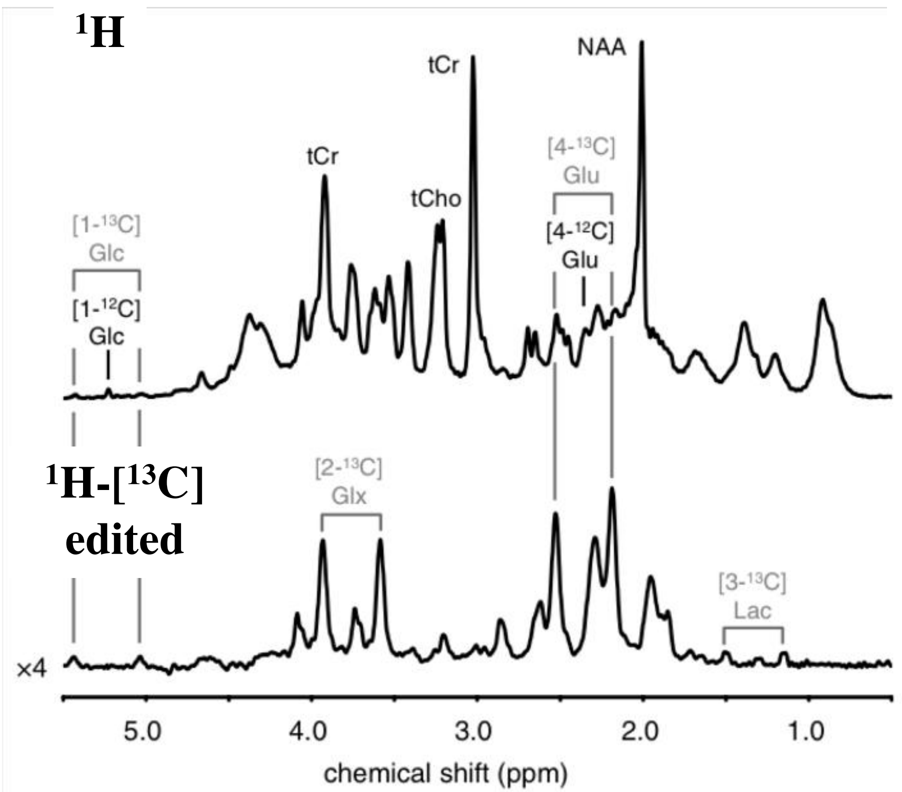
without 1H decoupling

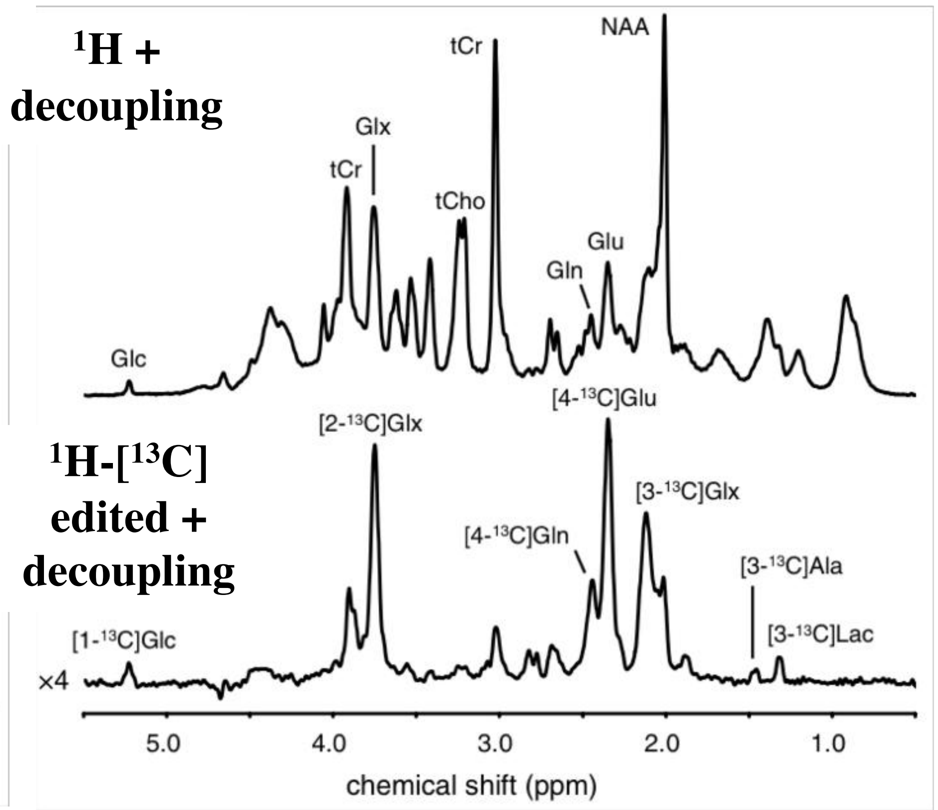
Note, $^{1}$H decoupling does not eliminate $^{13}$C-$^{13}$C J-coupling!

de Graaf, NMR Biomedicine (2011)

# $^{13}$C-Glucose Infusion: $^{1}$H MRS

Rat brain: 180 μl, TR/TE=4000/8.5 ms, 9.4 T
2 hrs post [1,6-$^{13}C_2$]glucose infusion

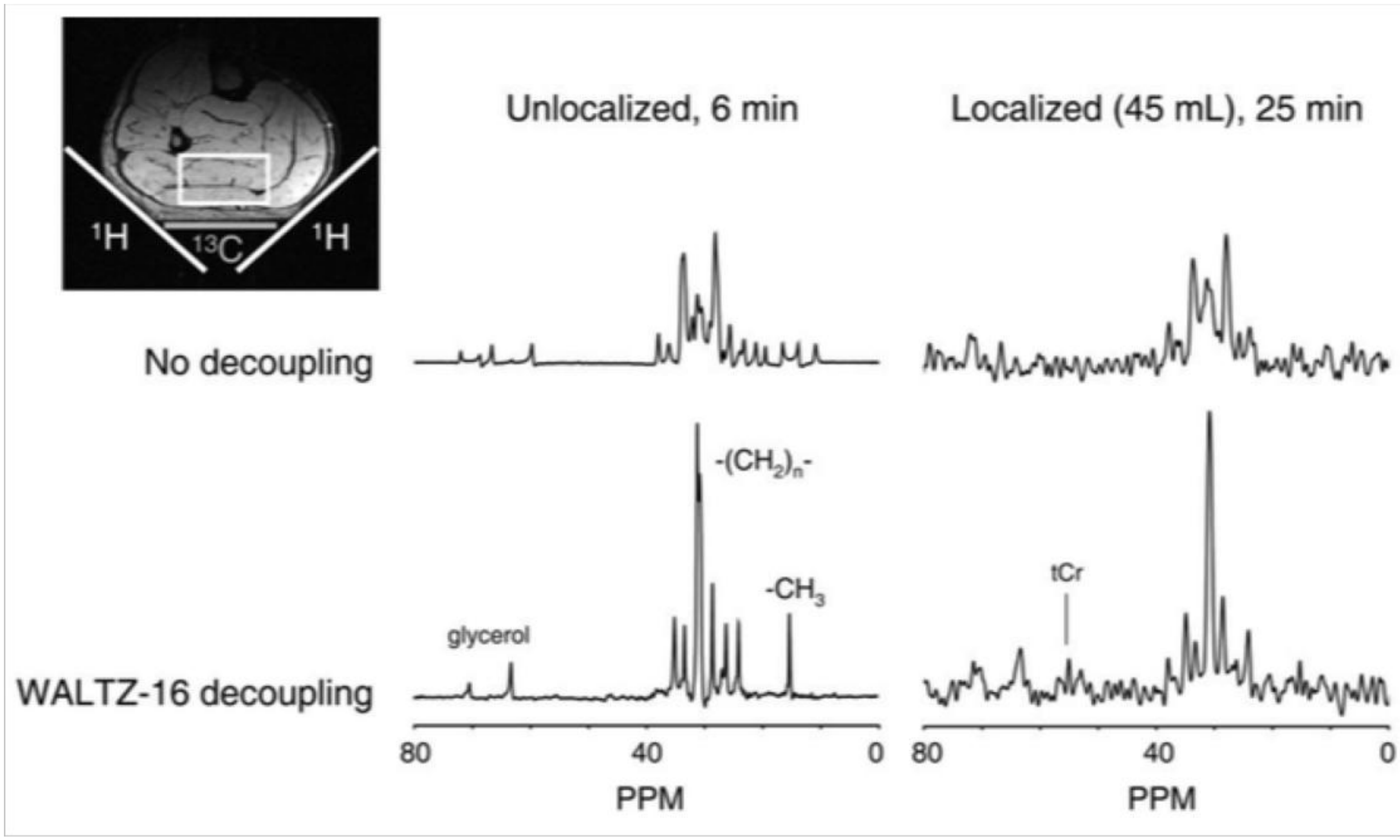



de Graaf, Chpt 8

# In Vivo Human: $^{13}C$ MRS

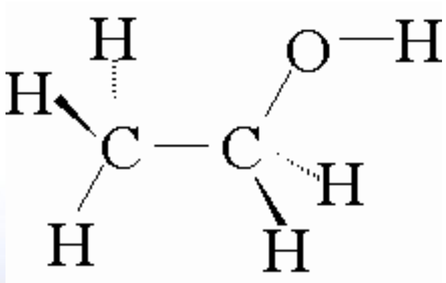
Calf muscle, 4 T, polarization transfer acquisition, natural abundance $^{13}C$




de Graaf, Magnetic Resonance in Medicine 53:1297–1306 (2005)

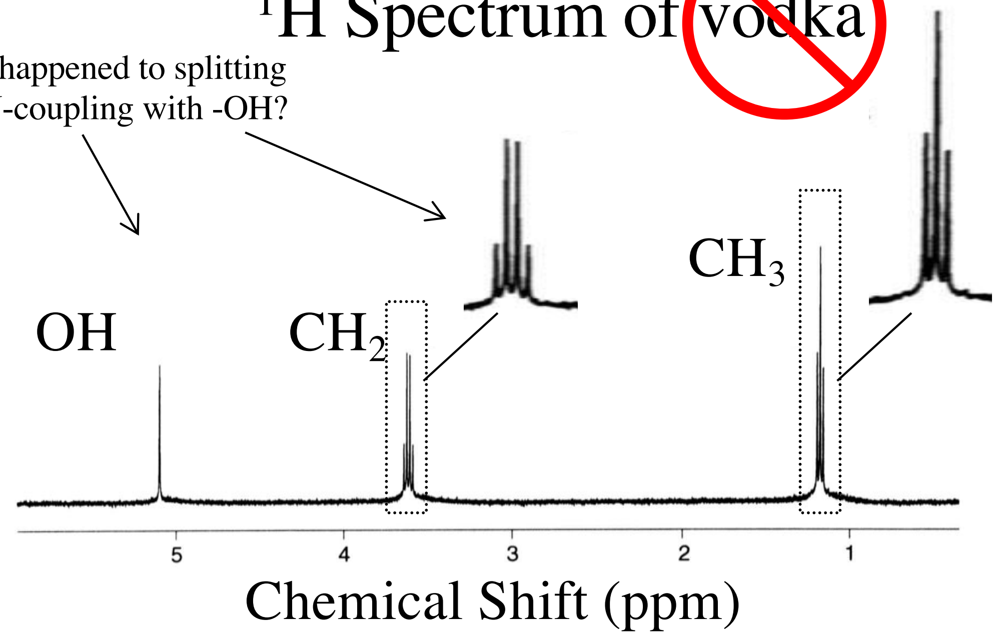
# Decoupling without Rf?


Ethanol



$^1H$ Spectrum of vodka

What happened to splitting
from J-coupling with -OH?



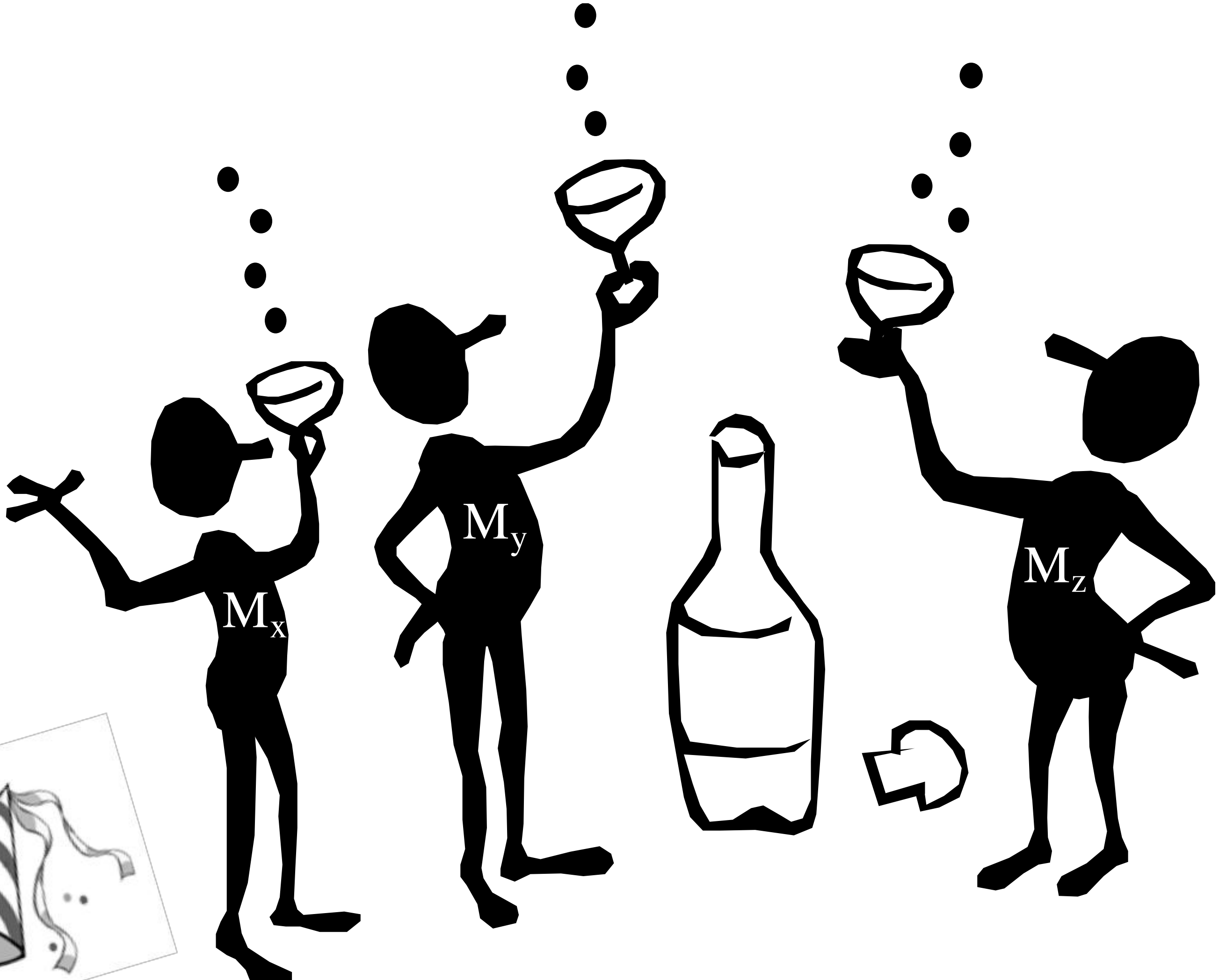

100% Ethanol ➡ $^1H$ Spectrum = ?

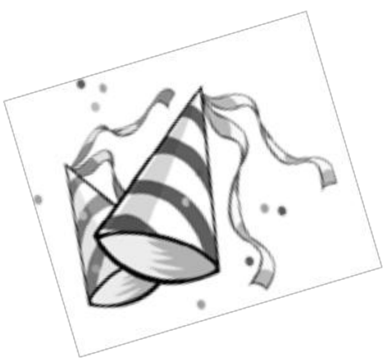
Ethanol + 5% $H_2O$ ➡ $^1H$ Spectrum = ?

Hint: Consider the effects of chemical exchange

$\boxed{\text{Ethanol + 5\% } D_2O \Rightarrow {}^1H \text{ Spectrum = ?}}$

# The End.



(Please fill out the course evaluations forms!)