ASTR 610
Theory of Galaxy Formation

Lecture 7: The Transfer Function & Cosmic Microwave Background

FRANK VAN DEN BOSCH
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Evolution of the Linear Density Field

So far we have seen how (individual) linear perturbations evolve in an expanding space-time. We will now develop some useful `machinery' to describe how the entire cosmological density field (in the linear regime) evolves as function of time.

Topics that will be covered include:

- Power Spectrum
- Two-Point Correlation Function
- Gaussian Random Fields
- Transfer Function
- Harrison-Zel'dovich spectrum

The Cosmological Density Field

How can we describe the cosmological (over)density field, $\delta(\vec{x}, t)$, without having to specify the actual value of δ at each location in space-time, $(\vec{x}, t)$?

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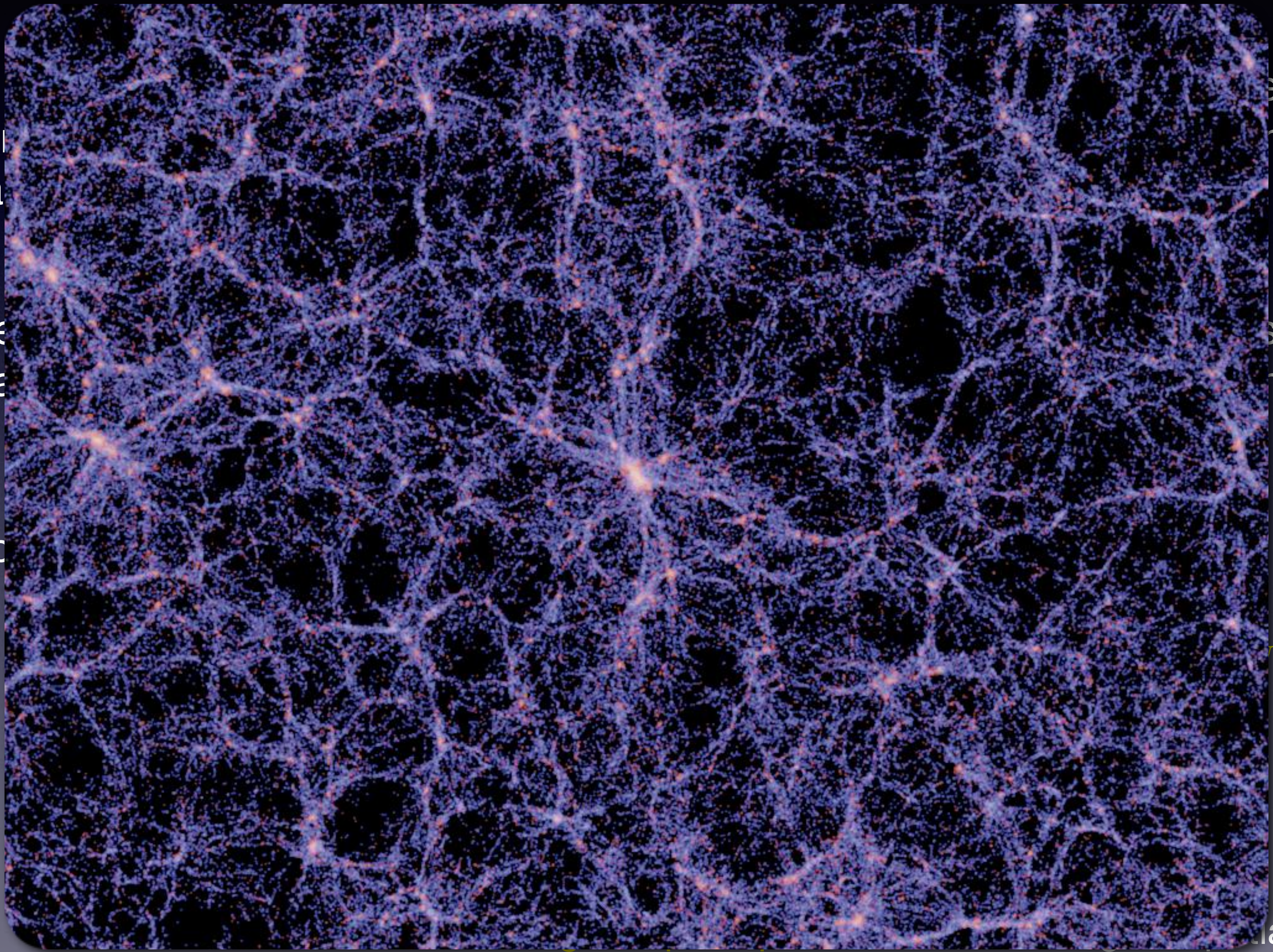

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The Two-Point Correlation Function

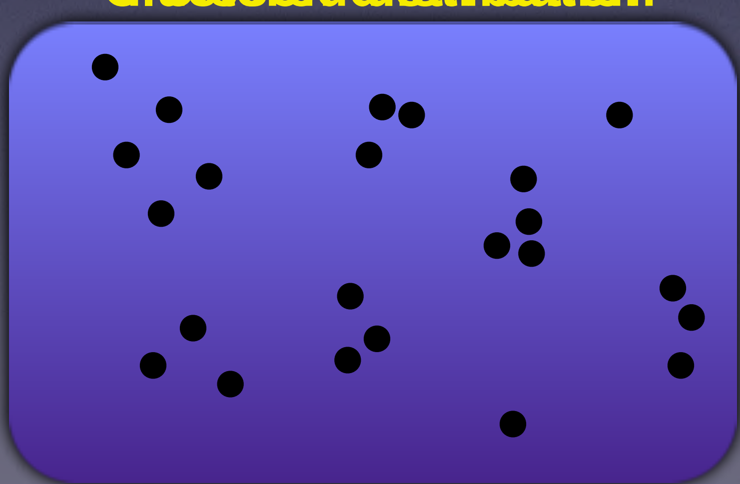
Second Moment $\boxed{\langle \delta_1 \, \delta_2 \rangle \equiv \xi(r_{12}) \qquad r_{12} = |\vec{x}_1 - \vec{x}_2|}$

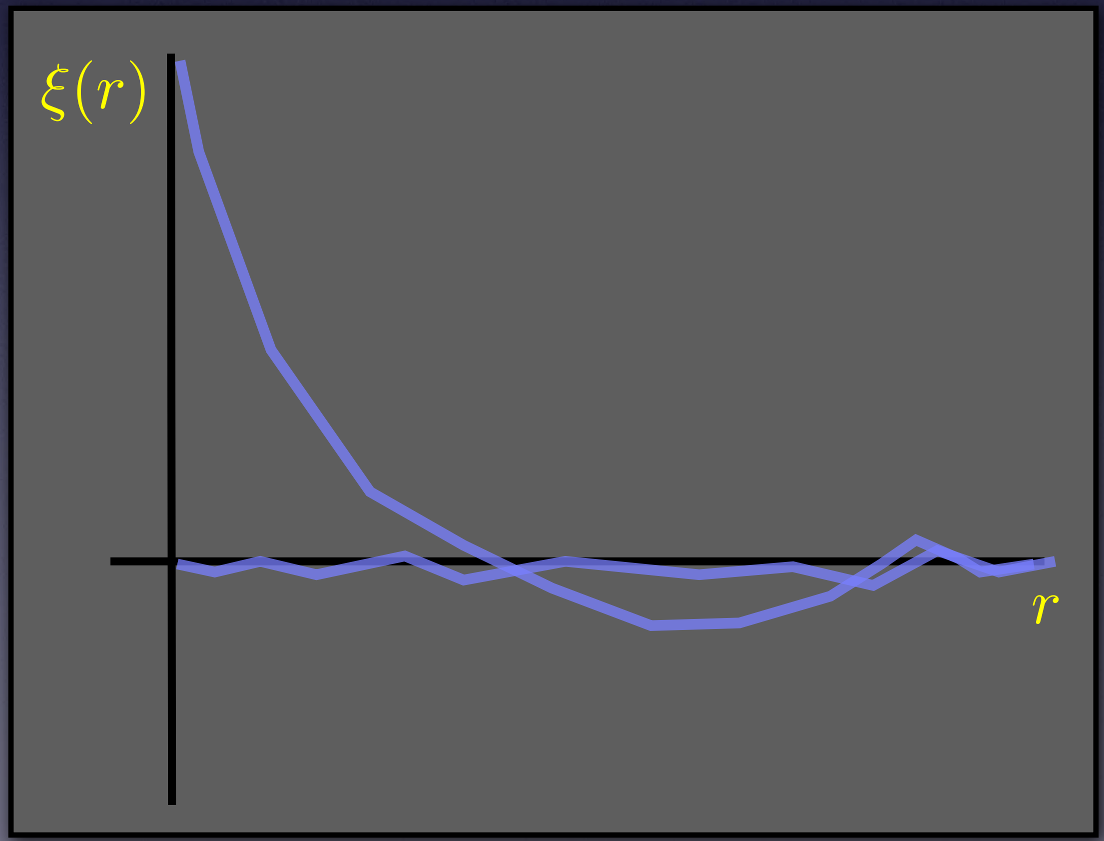
$\xi(r)$ is called the two-point correlation function

Note that this two-point correlation function is defined for a continuous field, $\delta(\vec{x})$. However, one can also define it for a point distribution:

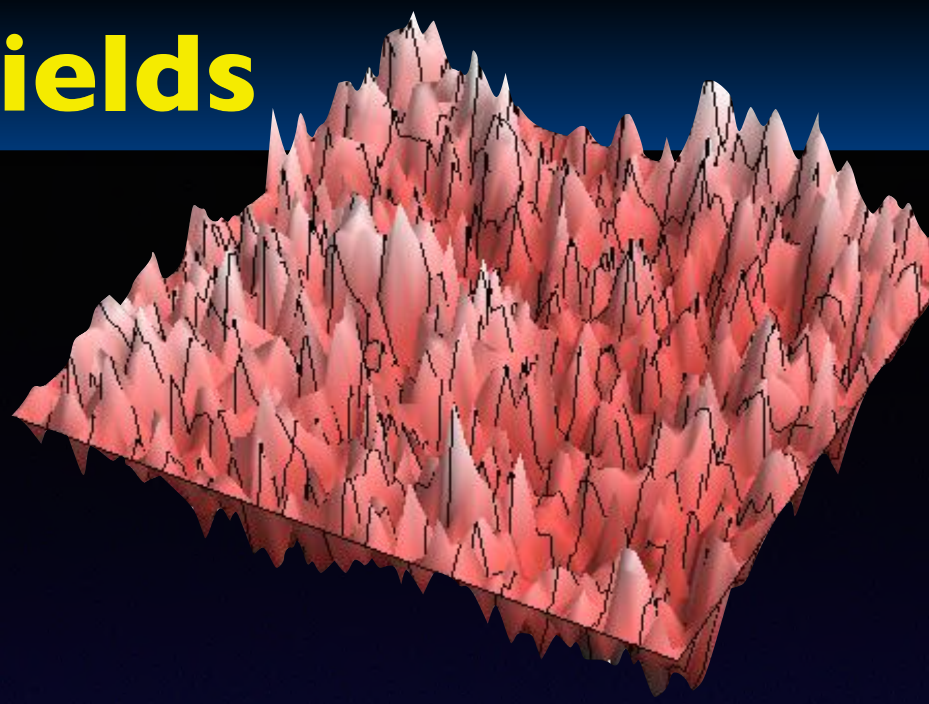
$$\boxed{1 + \xi(r) = \frac{n_{\rm pair}(r \pm {\rm d}r)}{n_{\rm random}(r \pm {\rm d}r)}}$$

Poisson distribution
Clustered distribution





Gaussian Random Fields



Thus far we discussed the first and second moments; how many moments do we need to completely specify the matter distribution?

In principle infinitely many......

However, there are good reasons to believe that the density distribution of the Universe is special, in that it is a Gaussian random field...

A random field $\delta(\vec{x})$ is said to be Gaussian if the distribution of the field values at an arbitrary set of N points is an N-variate Gaussian:

$$\mathcal{P}(\delta_1, \delta_2, ..., \delta_N) = \frac{\exp(-Q)}{[(2\pi)^N \det(\mathcal{C})]^{1/2}}$$

$$Q \equiv \frac{1}{2} \sum_{i,j} \delta_i \, (\mathcal{C}^{-1})_{ij} \delta_j$$

$$\mathcal{C}_{ij} = \langle \delta_i \delta_j \rangle = \xi(r_{12})$$

As you can see, such a Gaussian random field is completely specified by its second moment, the two-point correlation function $\xi(r)$!!!!

The Power Spectrum

Often it is very useful to describe the matter field in Fourier space:

$$\delta(\vec{x}) = \sum_k \delta_{\vec{k}}\, e^{+i\vec{k}\cdot\vec{x}} \qquad \delta_{\vec{k}} = \frac{1}{V}\int \delta(\vec{x})\, e^{-i\vec{k}\cdot\vec{x}}\, \mathrm{d}^3\vec{x}$$

Here V is the volume over which the Universe is assumed to be periodic.

Note: the perturbed density field can be written as a sum of plane waves of different wave numbers k (called `modes')

The Fourier transform (FT) of the two-point correlation function is called the power spectrum and is given by

$$\begin{aligned} P(\vec{k}) &\equiv V\langle|\delta_{\vec{k}}|^2\rangle \\ &= \int \xi(\vec{x})\, e^{-i\vec{k}\cdot\vec{x}}\, \mathrm{d}^3\vec{x} \\ &= 4\pi \int \xi(r)\, \frac{\sin kr}{kr}\, r^2\, \mathrm{d}r \end{aligned}$$

Note: P(k) has units of volume!

A Gaussian random field is completely specified by either the two-point correlation function $\xi(r)$, or, equivalently, the power spectrum $P(k)$

Evolution of the Power Spectrum

Our goal in what follows is to derive the evolution of the Power Spectrum $P(k,t)$

As we have seen, in the linear regime the linearized fluid equations reduce to

$$\boxed{\frac{\mathrm{d}^2\delta_{\vec{k}}}{\mathrm{d}t^2} + 2\frac{\dot{a}}{a}\frac{\mathrm{d}\delta_{\vec{k}}}{\mathrm{d}t} = \left[4\pi G\bar{\rho} - \frac{k^2 c_{\rm s}^2}{a^2}\right]\delta_{\vec{k}} - \frac{2}{3}\frac{\bar{T}}{a^2}k^2\,S_{\vec{k}}}$$

which show that each mode, $\delta_{\vec{k}}(t)$, evolves independently!

Since $P(k,t) = V\langle|\delta_{\vec{k}}(t)|^2\rangle$, we therefore need to solve the above equation for each individual mode. In the previous lecture, we have seen how to do this. All we need is a convenient and concise way to write this down...

As we shall see, we can simply write $\boxed{P(k,t) = P_{\rm i}(k)\,T^2(k)\,D^2(t)}$

$P_{\rm i}(k)$ is the initial power spectrum (i.e., shortly after creation of perturbations)

$T(k)$ is called the transfer function, and will be defined below

$D(t)$ is the linear growth rate, defined in the previous lecture.

The Transfer Function

As we have seen in Lecture 4, during the matter dominated era, sub-horizon perturbations in dark matter and baryons (both are, at that time, pressureless) evolve as

$$\delta_{\vec{k}} \propto D(a) \qquad\qquad \Phi_{\vec{k}} \propto D(a)/a$$

Recall that the density and potential modes are related via the Poisson equation:

$$-k^2 \Phi_{\vec{k}} = 4\pi G a^2 \, \bar{\rho}_{\rm m} \, \delta_{\vec{k}}$$

Note: here and in what follows we use the scale-factor a as our time-variable.

We have also seen that in a EdS cosmology, $D(a) \propto a$, so that $\Phi_{\vec{k}}$ remains constant
This is the same behavior as for super-horizon perturbations.

Growth of sub-horizon perturbations in pressureless fluid (i.e., dark matter or baryons past recombination) in an EdS cosmology is identical to that of super-horizon perturbations

The Transfer Function

As we have seen in Lecture 3, the Friedmann equation implies that every cosmology behaves as an EdS cosmology at early enough times, i.e., has that

$$\boxed{\lim_{a \to 0} \Omega_{\rm m}(a) = 1}$$

At early times, but after recombination, all matter perturbations, both sub- and super-horizon, have $\Phi_{\vec{k}} = \text{constant}$

We will use this fact to define our transfer function:

Define a scale-factor $a_{\rm m} > a_{\rm rec}$ such that $\Omega(a_{\rm m}) \simeq 1$, i.e., Universe in still in EdS phase. Then, all modes with $\lambda < \lambda_{\rm H}(a_{\rm m})$, which applies effectively to all modes of interest to us, are sub-horizon for $a > a_{\rm m}$.

In the linear regime, all these modes evolve independently according to

$$\boxed{\Phi_{\vec{k}}(a) = \Phi_{\vec{k}}(a_{\rm m}) \, \frac{D(a)}{D(a_{\rm m})} \, \frac{a_{\rm m}}{a} = \Phi_{\vec{k}}(a_{\rm m}) \frac{D(a)}{a}} \qquad a > a_{\rm m}$$

The Transfer Function

Next we use the Poisson equation to write $\delta_{\vec{k}}(a) = -\frac{k^2 \Phi_{\vec{k}}(a)}{4\pi G\, a^2\, \bar{\rho}} = -\frac{k^2 \Phi_{\vec{k}}(a_{\rm m})}{4\pi G\, a^3\, \bar{\rho}}\, D(a)$

Using that $a^3 \bar{\rho} = \bar{\rho}_{\rm m,0} = \Omega_{\rm m,0}\rho_{\rm crit,0}$ this yields that $\delta_{\vec{k}}(a) = -\frac{2}{3}\frac{k^2 \Phi_{\vec{k}}(a_{\rm m})}{\Omega_{\rm m,0}\, H_0^2}\, D(a)$

We can thus relate the mode amplitude of potential perturbations in the linear regime to those at some earlier time $a_{\rm m}$(defined as above). However, what we want is to relate them to the initial conditions, i.e., the perturbations shortly after their creation.

Between `creation' and $a_{\rm m}$there are a number of processes that affect the growth of our perturbations:

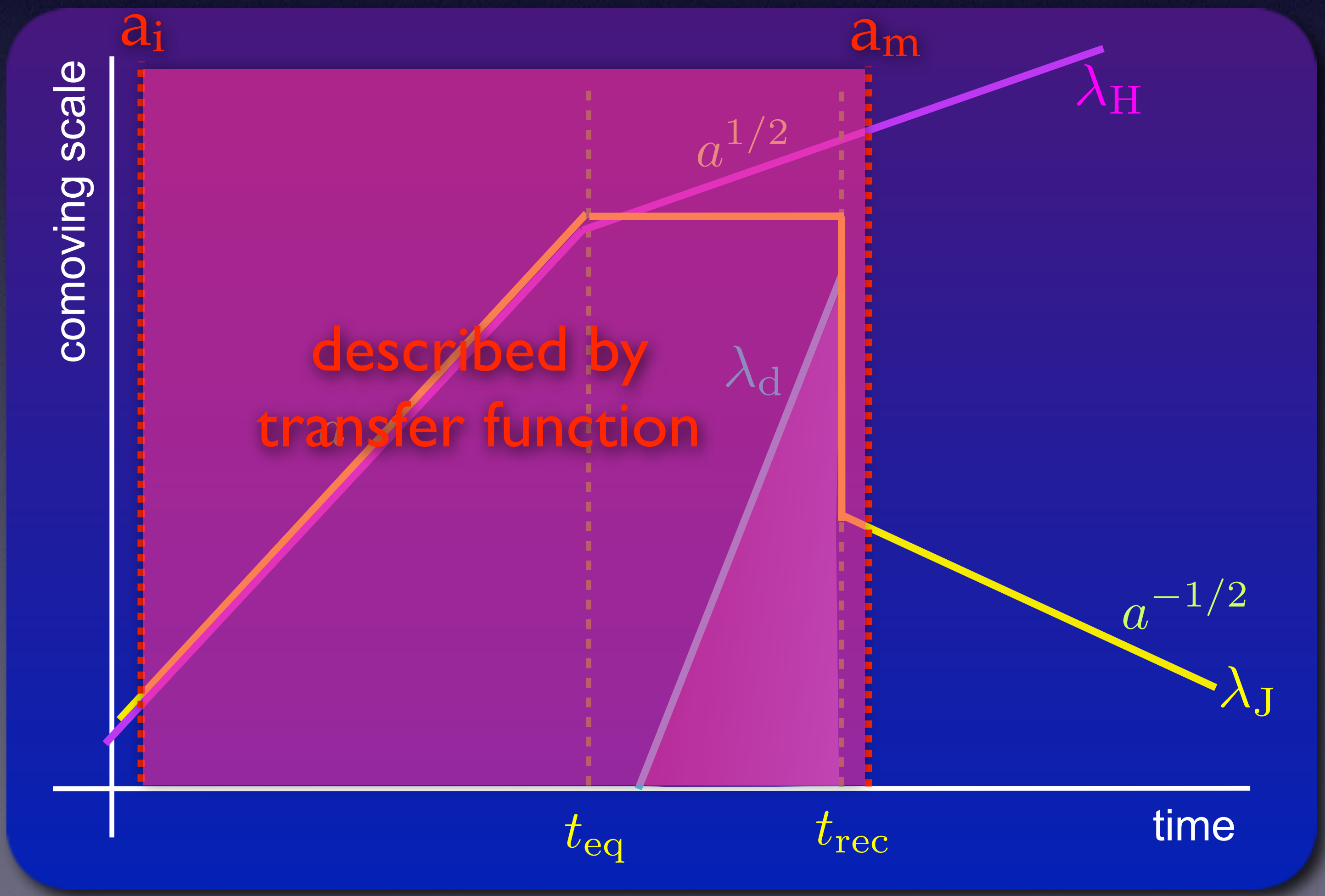
- Meszaros effect (stagnation in pressureless fluid during radiation dominated era)
- acoustic oscillations (no net growth due to pressure; Jeans criterion)
- Silk damping (damping on small scales due to imperfections in photon-baryon fluid)
- free-streaming damping (damping on small scales due to non-zero velocity of dark matter)
- radiation drag (stagnation that effects isothermal baryonic modes prior to equality)

The transfer function is devised to describe the combined effect of all these processes.

The Transfer Function

We define the transfer function as $$T(k) = \frac{\Phi_{\vec{k}}(a_\mathrm{m})}{\Phi_{\vec{k}}(a_\mathrm{i})}$$

Here a_i is the scale factor at our `initial' time. Note that the transfer function is independent of a_m, which follows from the fact that potential modes are frozen during the EdS phase where a_m is defined.



The Transfer Function

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Here $a_{\rm i}$ is the scale factor at our `initial' time. Note that the transfer function is independent of $a_{\rm m}$, which follows from the fact that potential modes are frozen during the EdS phase where $a_{\rm m}$ is defined.

$$\delta_{\vec{k}}(a) = -\frac{2}{3}\frac{k^2\Phi_{\vec{k}}(a_{\rm m})}{\Omega_{\rm m,0}\,H_0^2}\,D(a) \quad \Rightarrow \quad \delta_{\vec{k}}(a) = -\frac{2}{3}\frac{k^2\Phi_{\vec{k}}(a_{\rm i})}{\Omega_{\rm m,0}\,H_0^2}\,T(k)\,D(a)$$

This finally allows us to write the power spectrum as

$$\boxed{P(k,a) = \langle|\delta_{\vec{k}}(a)|^2\rangle = \frac{4}{9}\frac{k^4\langle|\Phi_{\vec{k},\,\rm i}|^2\rangle}{\Omega_{\rm m,0}^2\,H_0^4}\,T^2(k)\,D^2(a) = P_{\rm i}(k)\,T^2(k)\,D^2(a)}$$

Defining the power spectrum of potential perturbation as $P_\Phi(k) = \langle|\Phi_{\vec{k}}|^2\rangle$ we have that $P(k) \propto k^4 P_\Phi(k)$. We will use this at a later stage to get some insight into the nature of the initial power spectrum....

The Transfer Function

$$T(k) = \frac{\Phi_{\vec{k}}(a_{\rm m})}{\Phi_{\vec{k}}(a_{\rm i})}$$

$\Rightarrow$ Thus, in order to compute $T(k)$ we need to evolve different modes from their initial conditions to some fiducial time shortly after recombination (EdS phase).

In Lecture 4 we have seen how this can be done using Newtonian perturbation theory.

$$\frac{{\rm d}^2\delta_{\vec{k}}}{{\rm d}t^2} + 2\frac{\dot{a}}{a}\frac{{\rm d}\delta_{\vec{k}}}{{\rm d}t} = \left[4\pi G\bar{\rho} - \frac{k^2 c_{\rm s}^2}{a^2}\right]\delta_{\vec{k}} - \frac{2}{3}\frac{\bar{T}}{a^2}k^2\, S_{\vec{k}}$$

However, accurate calculations of $T(k)$ requires solving the Boltzmann equation in a perturbed FRW metric. This is a formidable task, that will not be covered in this course. (if interested, see MBW §4.2 or textbook Modern Cosmology by S. Dodelson).

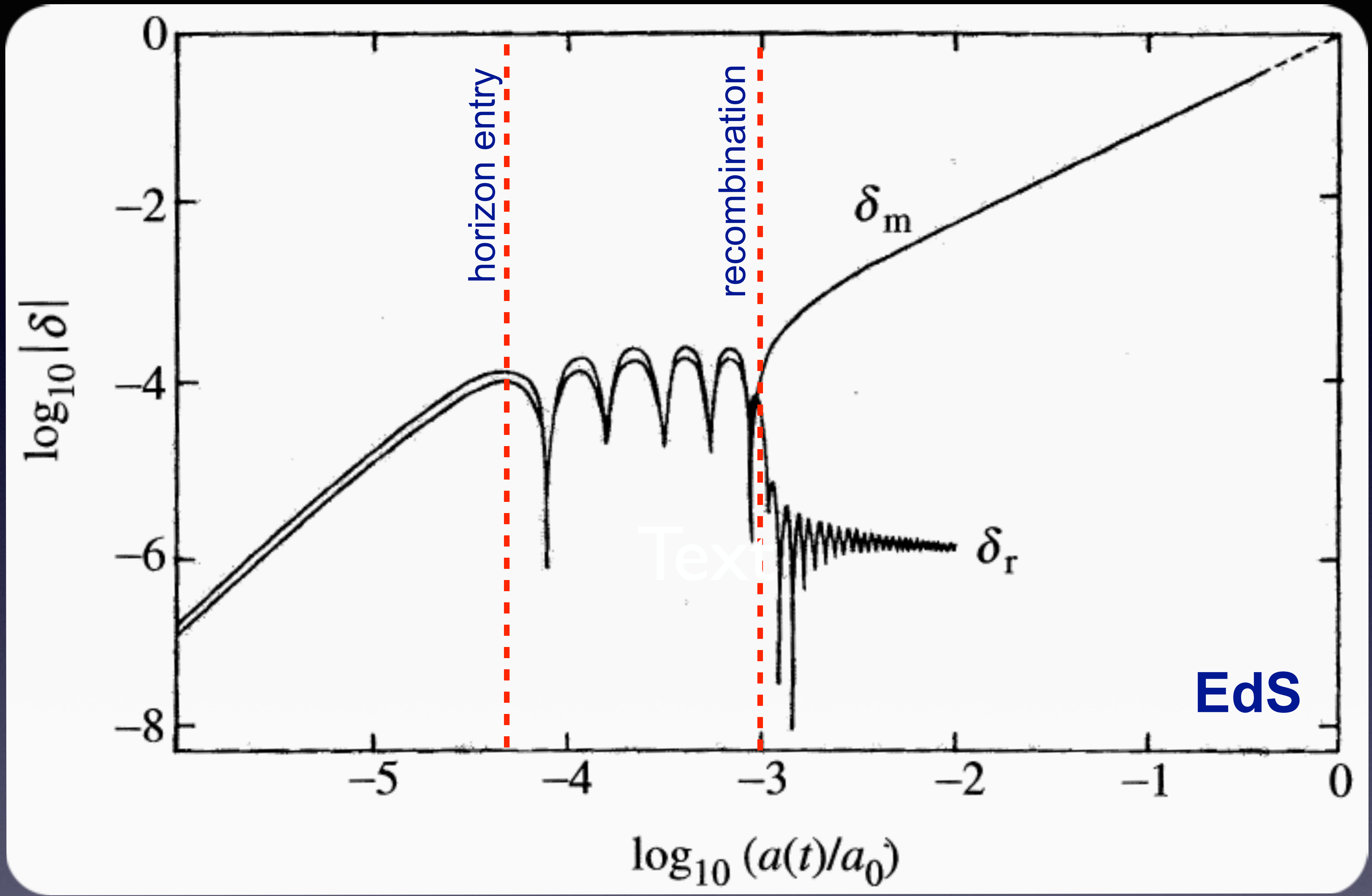
Fortunately, nowadays a number of codes to compute $T(k)$ are publicly available:

Websites:

CMBFAST: http://lambda.gsfc.nasa.gov/toolbox/tb_cmbfast_ov.cfm
CMBEASY: http://www.thphys.uni-heidelberg.de/~robbers/cmbeasy/
CAMB: http://camb.info/
CLASS: http://class-code.net/

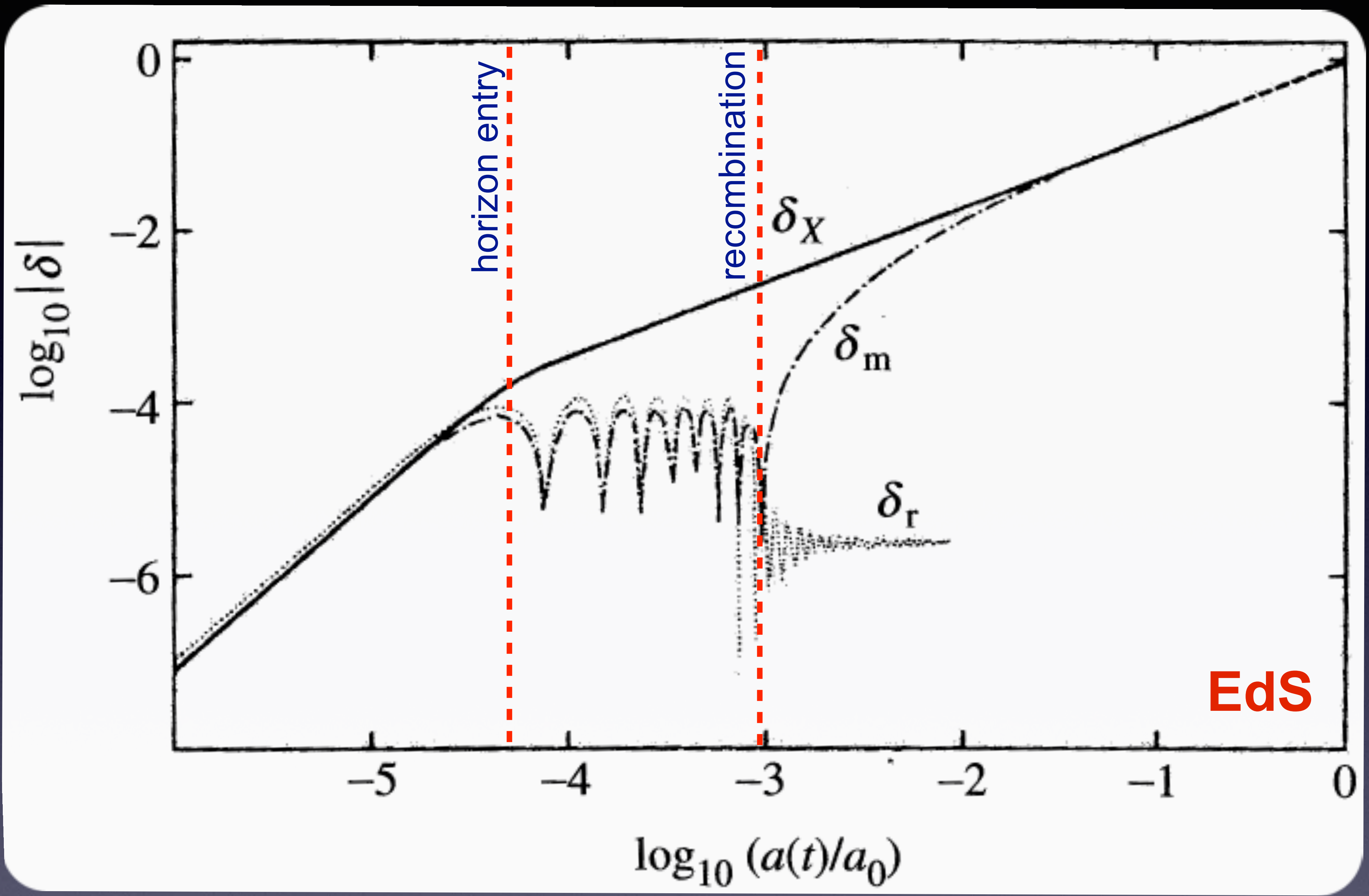
The next two pages show examples of mode-evolution computed using such codes....

Growth of Isentropic, Baryonic Perturbation



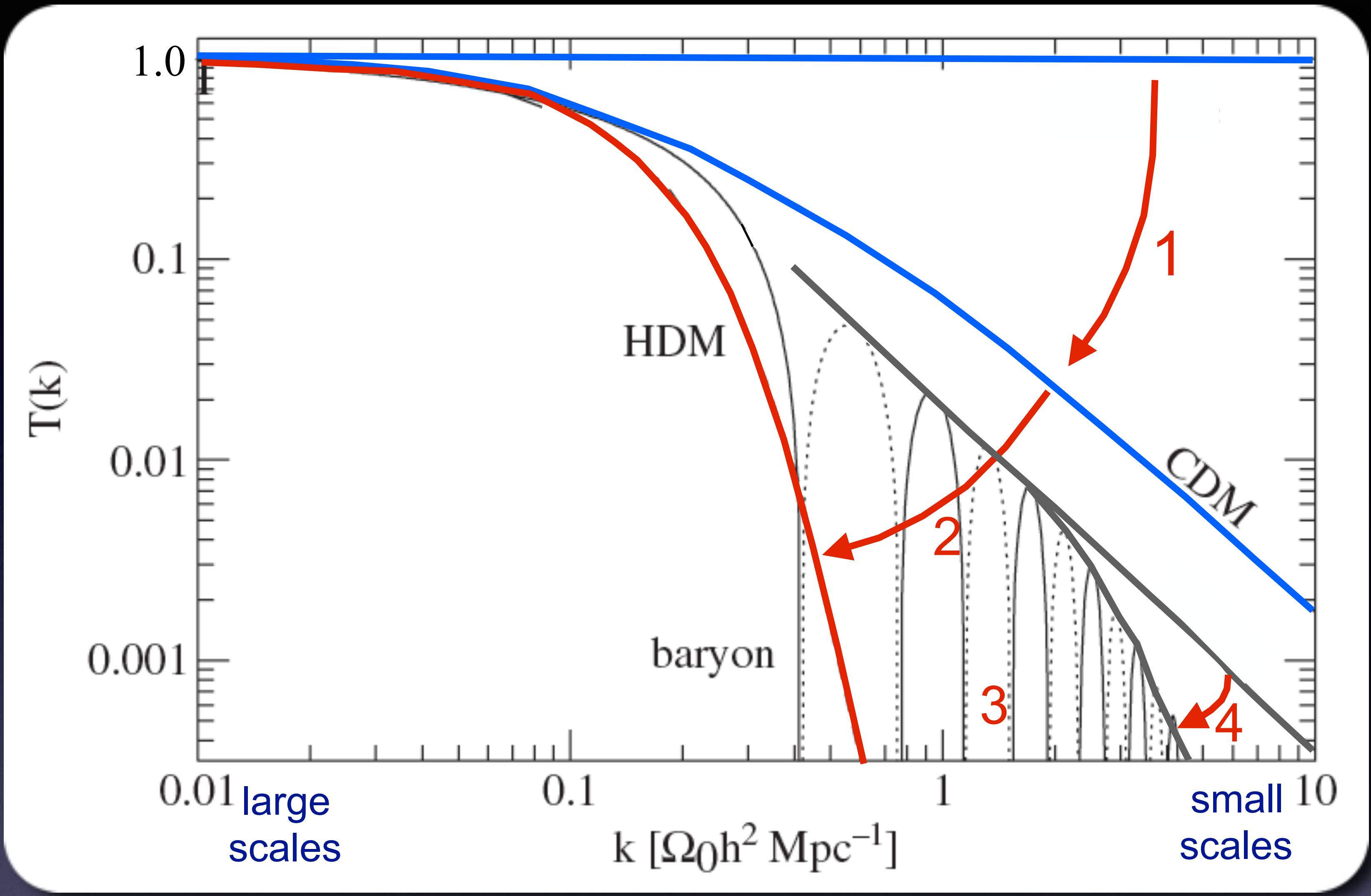
The above example shows the evolution of the amplitude of a mode corresponding to a mass scale of $10^{15} M_{\odot}$ in an EdS cosmology. Note that $M_{\rm d}(z_{\rm rec}) < M < M_{\rm J}(z_{\rm rec})$ so that there is no Silk damping.

Growth of Isentropic Perturbation



Same mode/cosmology as before, except that we have now added dark matter. Since this mode ($M = 10^{15} M_\odot$) enters horizon after matter-radiation equality, there is no Meszaros effect. After recombination, baryons quickly catch-up with dark matter (they fall in the dark matter potential wells)

Examples of Transfer Functions



This figure shows examples of three transfer functions for isentropic perturbations.

CDM = Cold Dark Matter

HDM = Hot Dark Matter

baryon = no Dark Matter

Question: what are the physical processes giving rise to 1, 2, 3, and 4?

1

2

3

4

The Initial Power Spectrum

As we have seen, $P(k,t) = P_{\rm i}(k)\, T^2(k)\, D^2(t)$. It is common practice to assume that the initial power spectrum has a power-law form

$$\boxed{P_{\rm i}(k) \propto k^n}$$

where n is called the spectral index. As described in MBW §4.5, the power spectra predicted by inflation models typically have this form (roughly).

Recall that the power spectrum $P(k)$ has the units of volume. It is often useful to define the dimensionless quantity

$$\boxed{\Delta^2(k) \equiv \frac{1}{2\pi^2}\, k^3 P(k)}$$

which expresses the contribution to the variance by the power in a unit logarithmic interval of k. For the initial power spectrum: $\Delta^2_{\rm i}(k) \propto k^{3+n}$

The corresponding quantity for the gravitational potential is

$$\boxed{\Delta^2_\Phi(k) \equiv \frac{1}{2\pi^2} k^3 P_\Phi(k) \propto k^{-4}\Delta^2(k) \propto k^{n-1}}$$

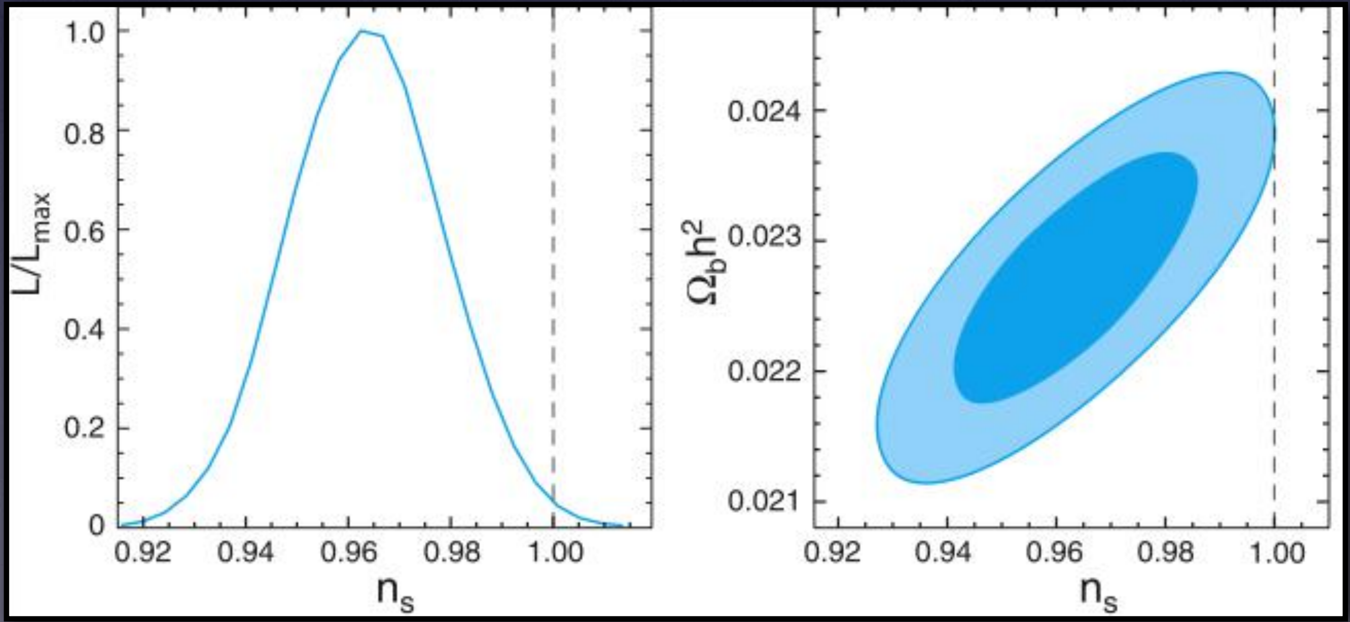
where the second step follows straightforward from the Poisson equation..

The Initial Power Spectrum

$$\boxed{\Delta^2_\Phi(k) \equiv \frac{1}{2\pi^2} k^3 P_\Phi(k) \propto k^{-4}\Delta^2(k) \propto k^{n-1}}$$

Note that $\Delta^2_\Phi(k)$ is independent of k for $n = 1$. This special case is called the Harrison-Zel'dovich spectrum or scale-invariant spectrum, which has the desirable property that the gravitational potential is finite on both small and large scales. Inflation predicts that the `tilt' $|n-1|$ is very small, which is supported by observations of the CMB power spectrum.

The normalization of the initial power spectrum is normally defined via the parameter σ_8, which will be described in detail once we discuss filtering of the cosmological density field.



Komatsu et al. (2009)

The Cosmic Microwave Background

The Cosmic Microwave Background

The Cosmic Microwave Background is one of the three pillars of Big Bang cosmology. Its anisotropy power spectrum has a rich structure that can tell us much about our cosmological world-models. Understanding these structures is a perfect application of what we have learned above regarding perturbation growth.

Topics that will be covered include:

- CMB Power Spectrum
- CMB dipole
- CMB acoustic peaks
- Sachs-Wolfe effect
- Diffusion damping
- Cosmological Parameters

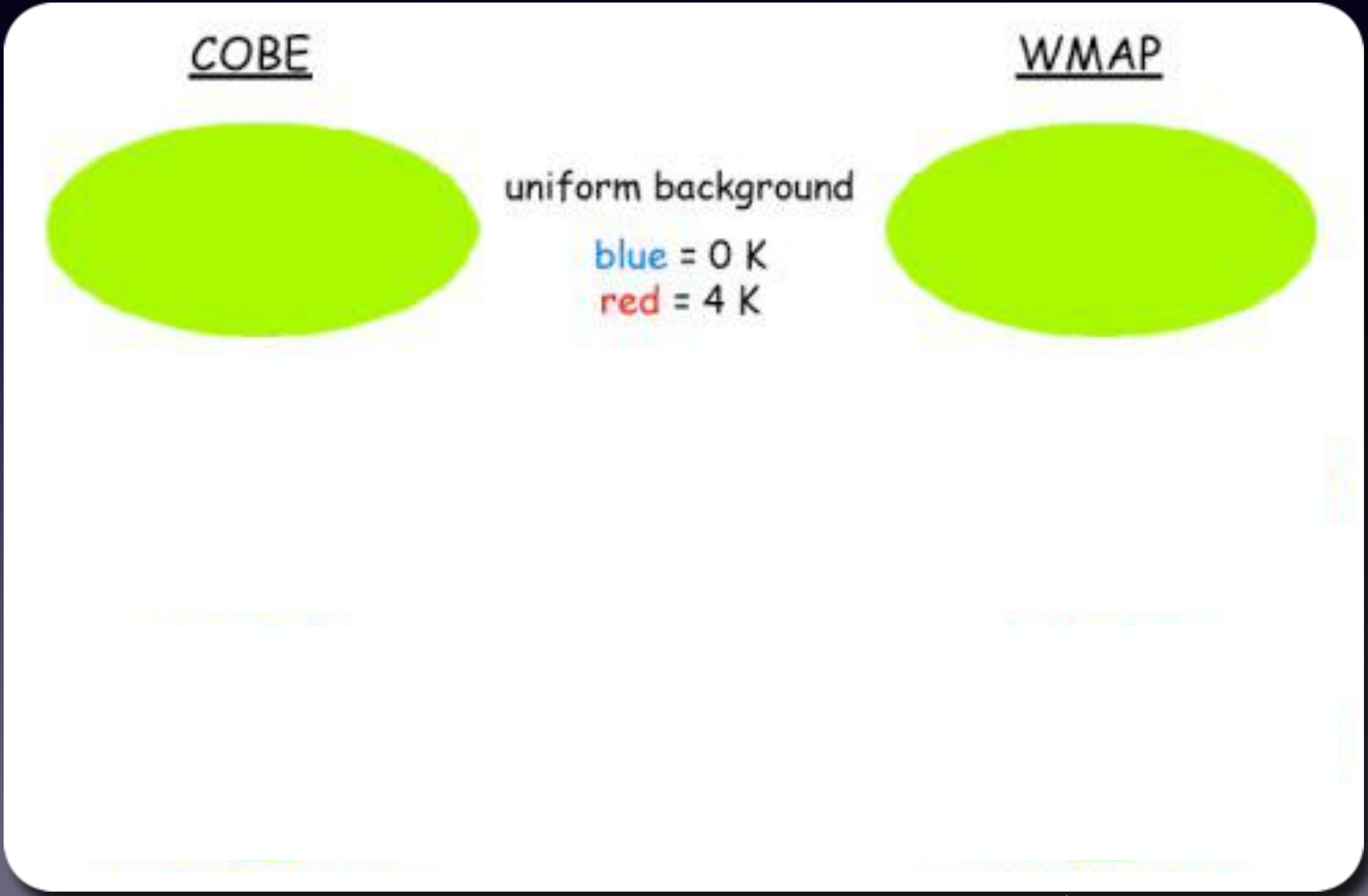
Many of the materials used in this section are taken from Wayne Hu's website (background.uchicago.edu)

CMB Anisotropy

$$\frac{\Delta T}{T} = 1.5$$

$$\frac{\Delta T}{T} = 3 \times 10^{-3}$$

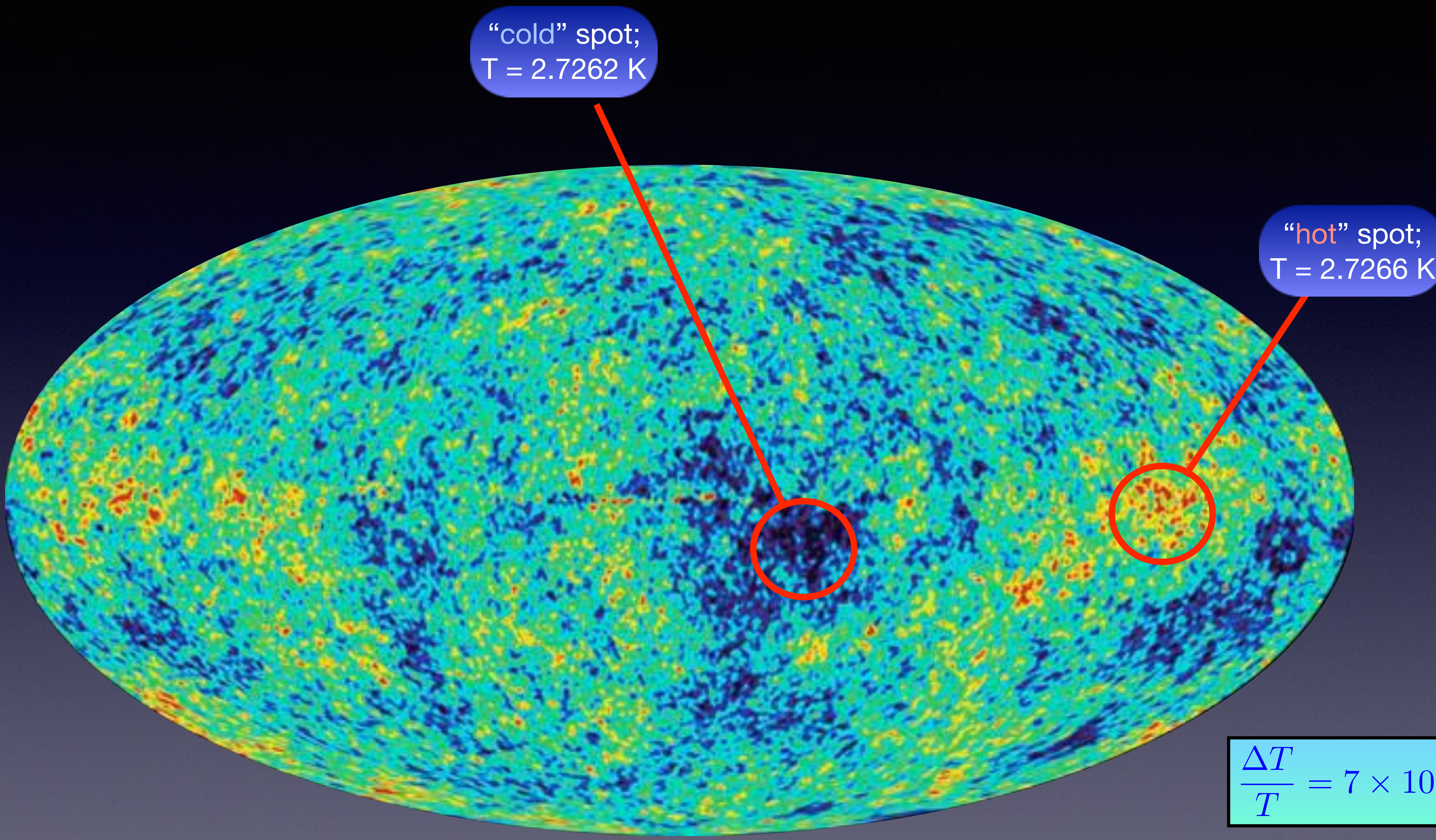
$$\frac{\Delta T}{T} = 7 \times 10^{-5}$$



increasing temperature sensitivity

increasing spatial resolution

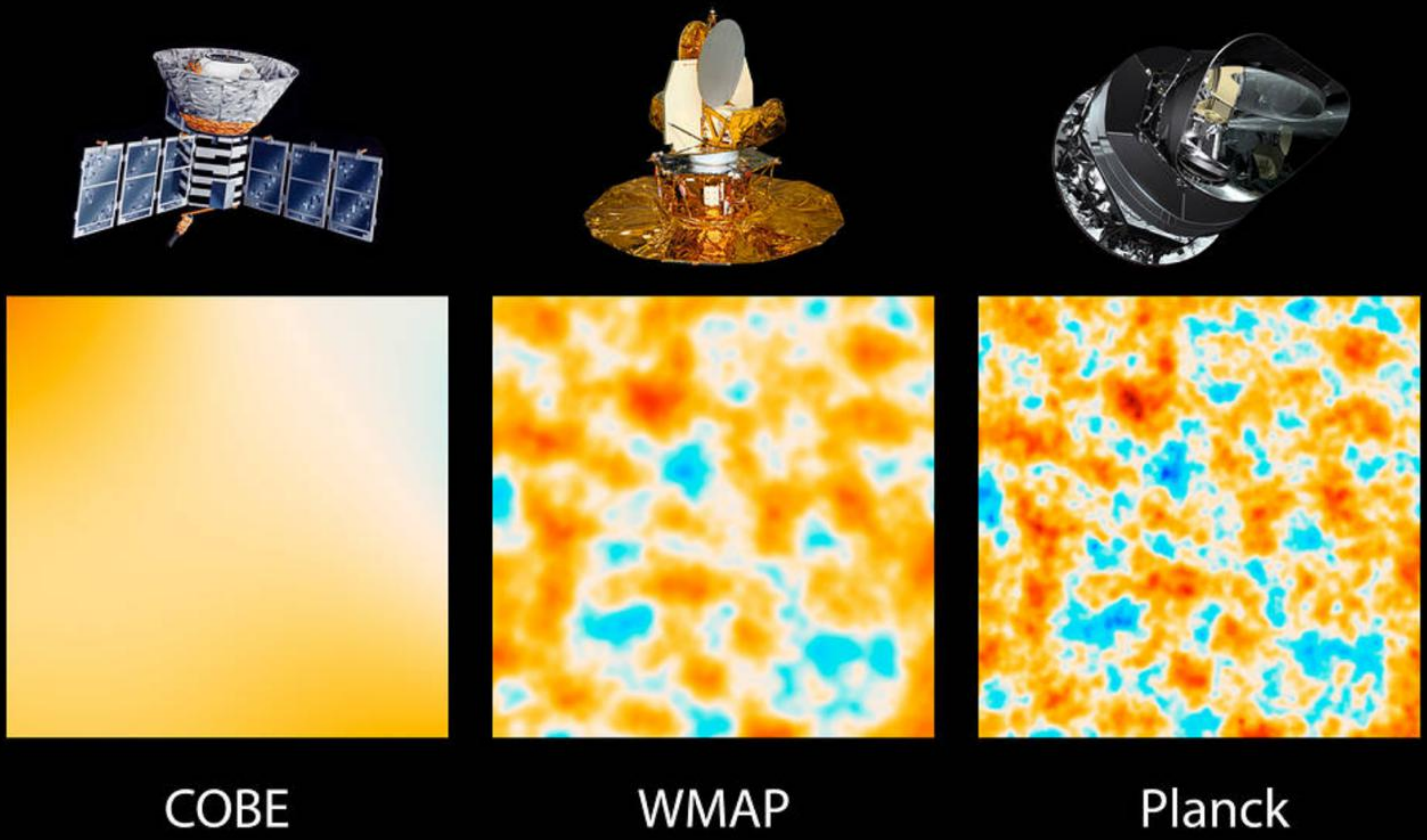
CMB Anisotropy



$$\frac{\Delta T}{T} = 7 \times 10^{-5}$$

The WMAP all sky map, after removal of the radiation coming from the Milky Way disk

...and then there was Planck...



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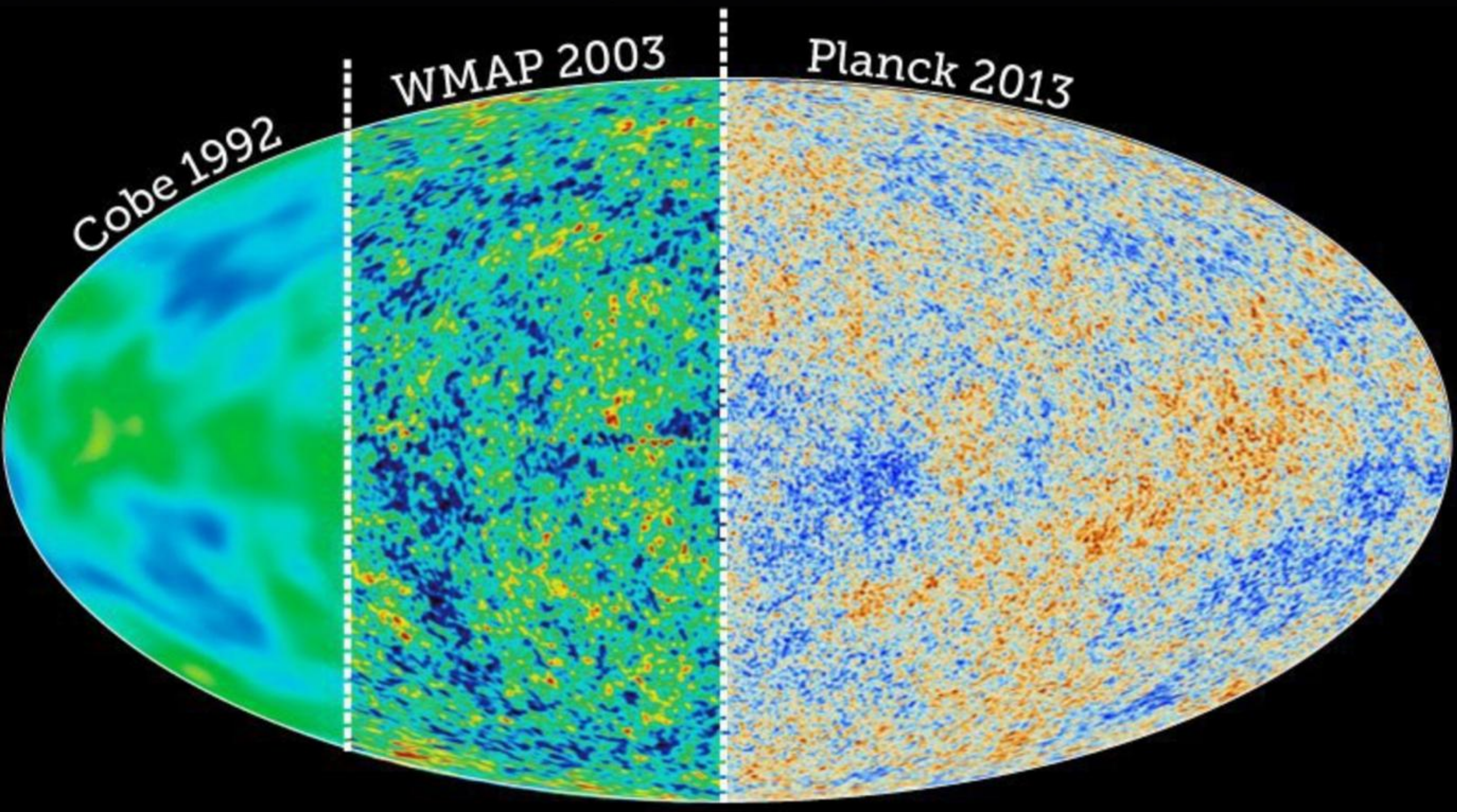

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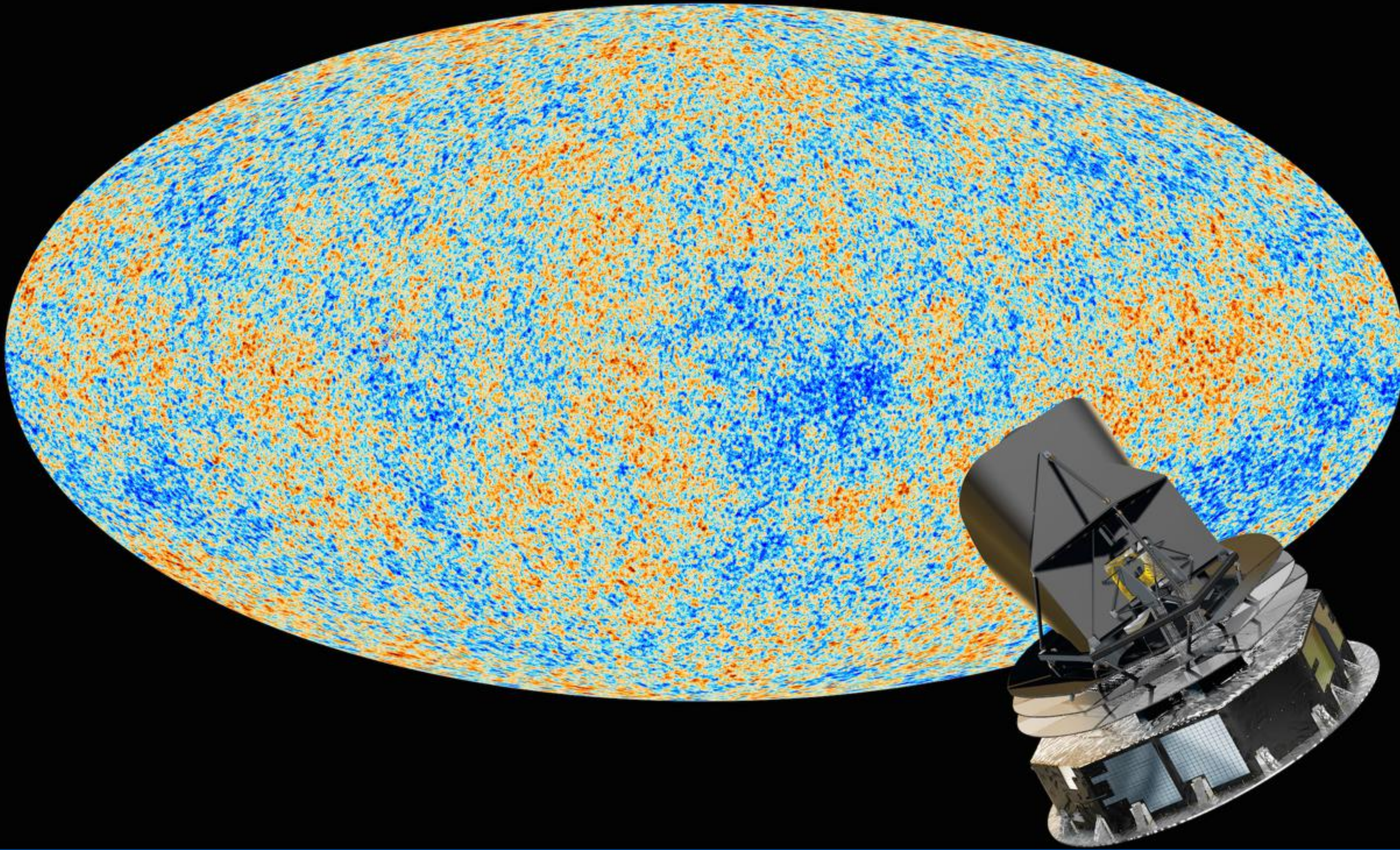
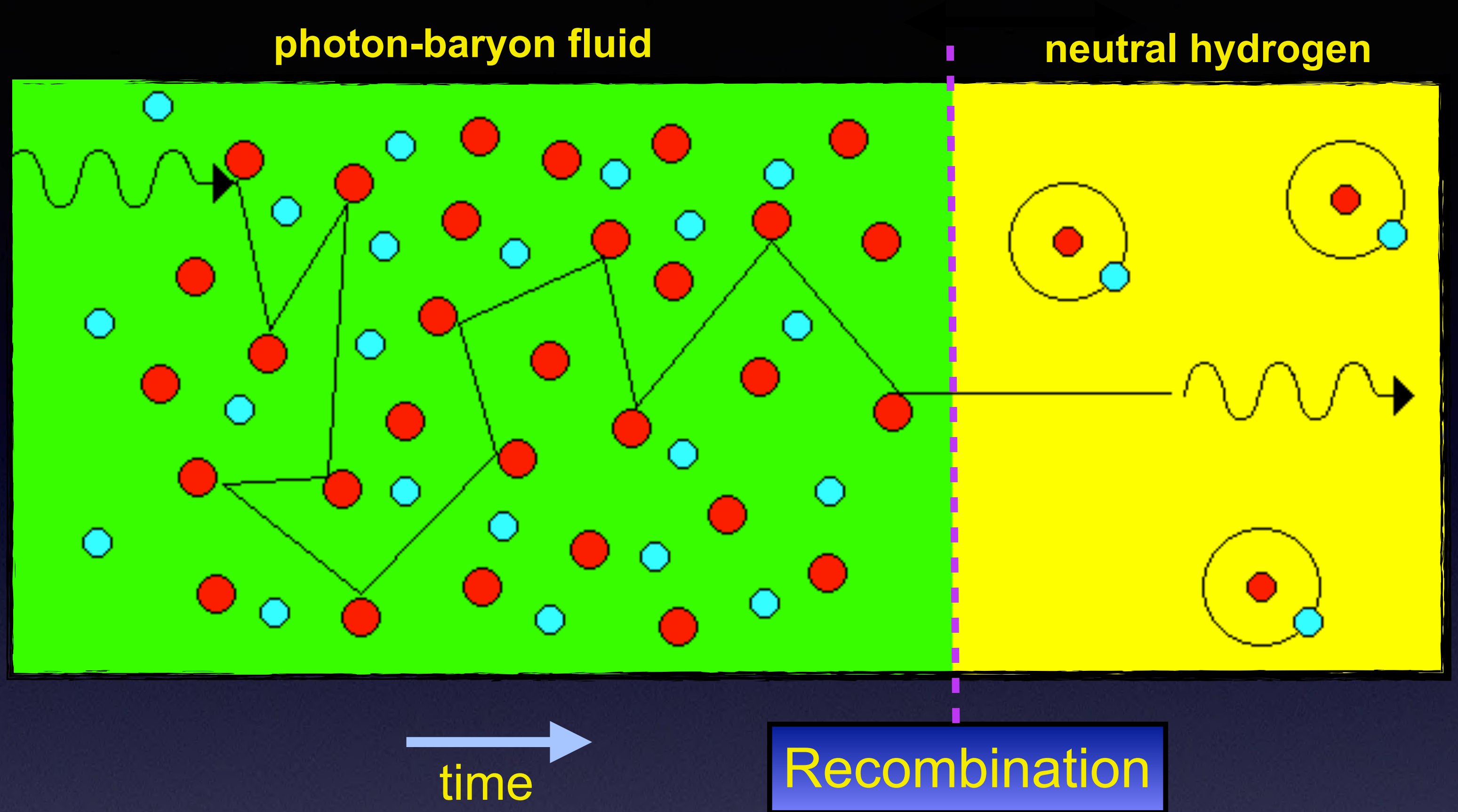


Recombination and Decoupling



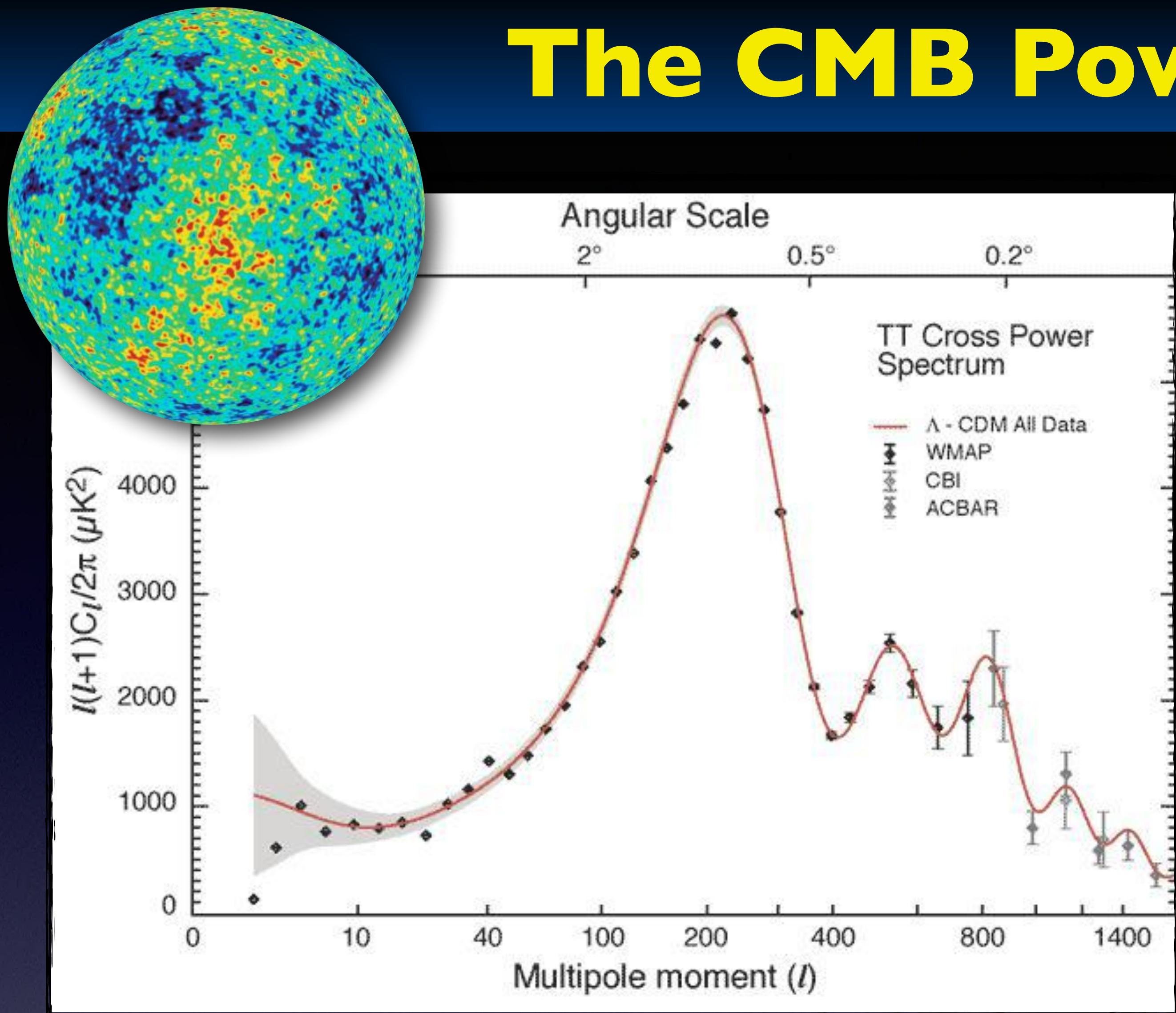
CMB radiation comes to us from last scattering surface (LSS). Since recombination is not instantaneous, in general $z_{\rm LSS} \neq z_{\rm rec}$. Here, the redshift of recombination, $z_{\rm rec}$, is defined as the redshift at which the ionization fraction drops below some value (typically 0.1).

Rather $z_{\rm LSS} = z_{\rm dec}$, where the latter is the redshift of decoupling, defined as the epoch at which the Thomson scattering rate $\Gamma_{\rm T} = n_{\rm e}\,\sigma_{\rm T}\,c$ is equal to the Hubble expansion rate $H(z)$

Detailed calculations, using Boltzmann codes, show that for $\Omega_{\rm b,0}/\Omega_{\rm m,0} \simeq 0.17$, the probability $P(z)$ that a photon had a last scattering at redshift z has a median at $z_{\rm dec} \simeq 1100$ and a width $\Delta z \simeq 80$ (see MBW §3.5.2).

As we shall see, this non-zero width of the LSS causes damping (called diffusion damping) of the CMB anisotropies on smal scales.

The CMB Power Spectrum



Similar to $\delta(\vec{x})$, the CMB has to be considered a particular realization of a random process.

Define the CMB anisotropy distribution

$$\boxed{\Theta(\hat{n}) \equiv \frac{\Delta T}{T}(\hat{n}) = \frac{T(\hat{n}) - \bar{T}}{\bar{T}}}$$

Here $\hat{n} = (\vartheta, \phi)$ is direction on the sky, and $\bar{T}$ is the average CMB temperature.

We expand this in Spherical Harmonics:‡

$$\boxed{\Theta(\hat{n}) = \sum_{l,m} a_{lm}\, Y_{lm}(\vartheta, \phi)}$$

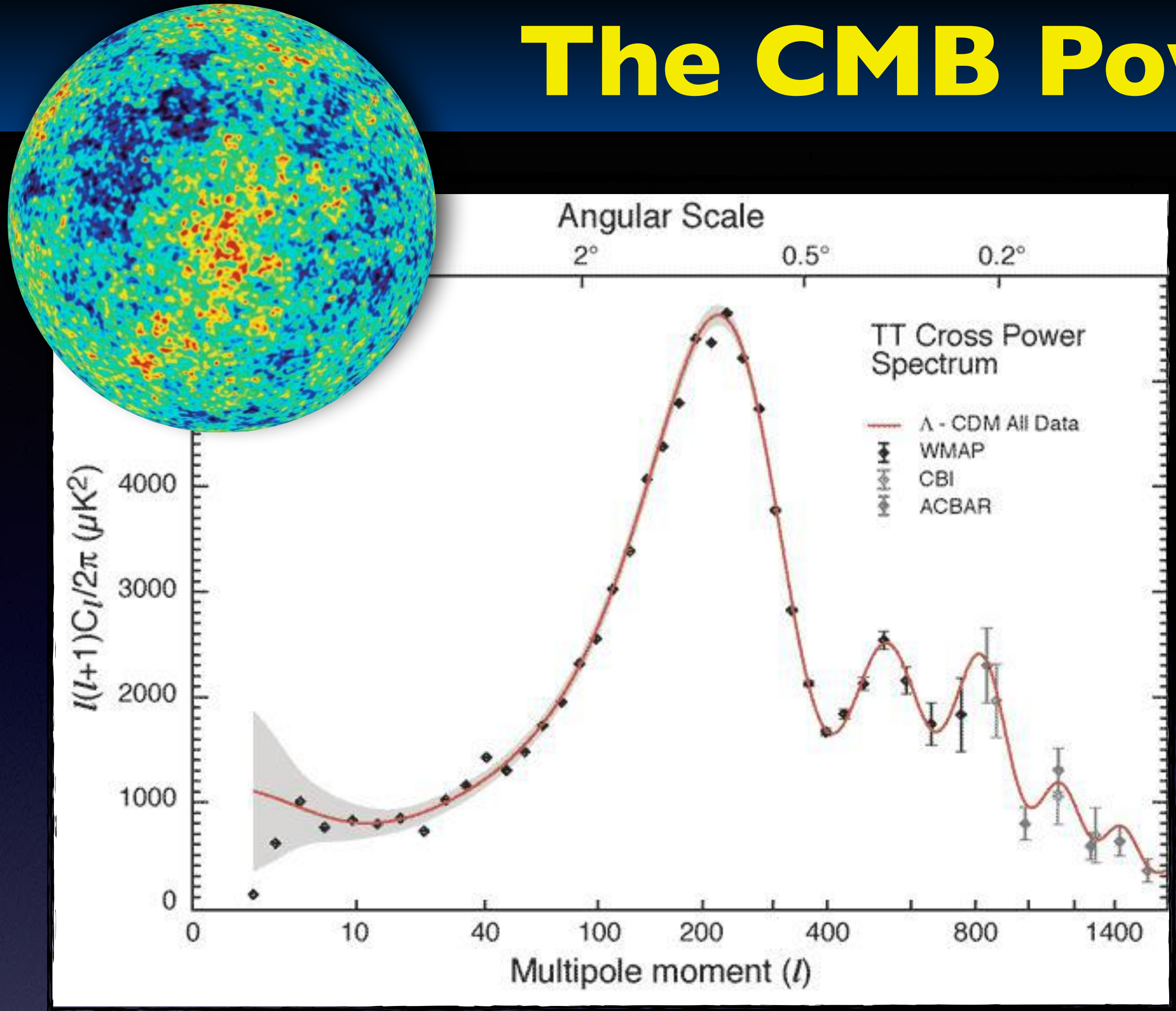
and define the power spectrum as

$$\boxed{\mathcal{C}_l = \langle |a_{lm}|^2 \rangle}$$

Almost always, the power spectrum that people plot is not $\mathcal{C}_l$ but $l(l+1)\mathcal{C}_l$. The reason is that for a Harrison-Zel'dovich spectrum in a EdS cosmology, the latter is independent of l on large scales (= small l). The small upturn at large scales in the WMAP power spectrum therefore indicates that $n_{\rm s} \neq 1$ and/or $\Omega_{\rm m,0} \neq 1$ (due to integrated Sachs-Wolfe effect).

‡ **NOTE:** this is similar to an expansion in plane-waves (i.e., Fourier Transform), except that here a different set of basis-functions is used, optimized to describe a distribution on a spherical surface.

The CMB Power Spectrum



As a rule of thumb, the relation between l and the associated angular scale θ is:

$$\boxed{\theta \sim \frac{\pi}{l}\,\mathrm{rad} \sim \frac{180^\circ}{l}}$$

A comoving length $\lambda^{\rm com}$ at last scattering surface (i.e., at $z = z_{\rm dec}$), subtends an angle

$$\boxed{\theta = \frac{\lambda^{\rm phys}}{d_{\rm A}(z_{\rm dec})} = \frac{\lambda^{\rm com}}{d_{\rm A}(z_{\rm dec})\,(1+z_{\rm dec})}}$$

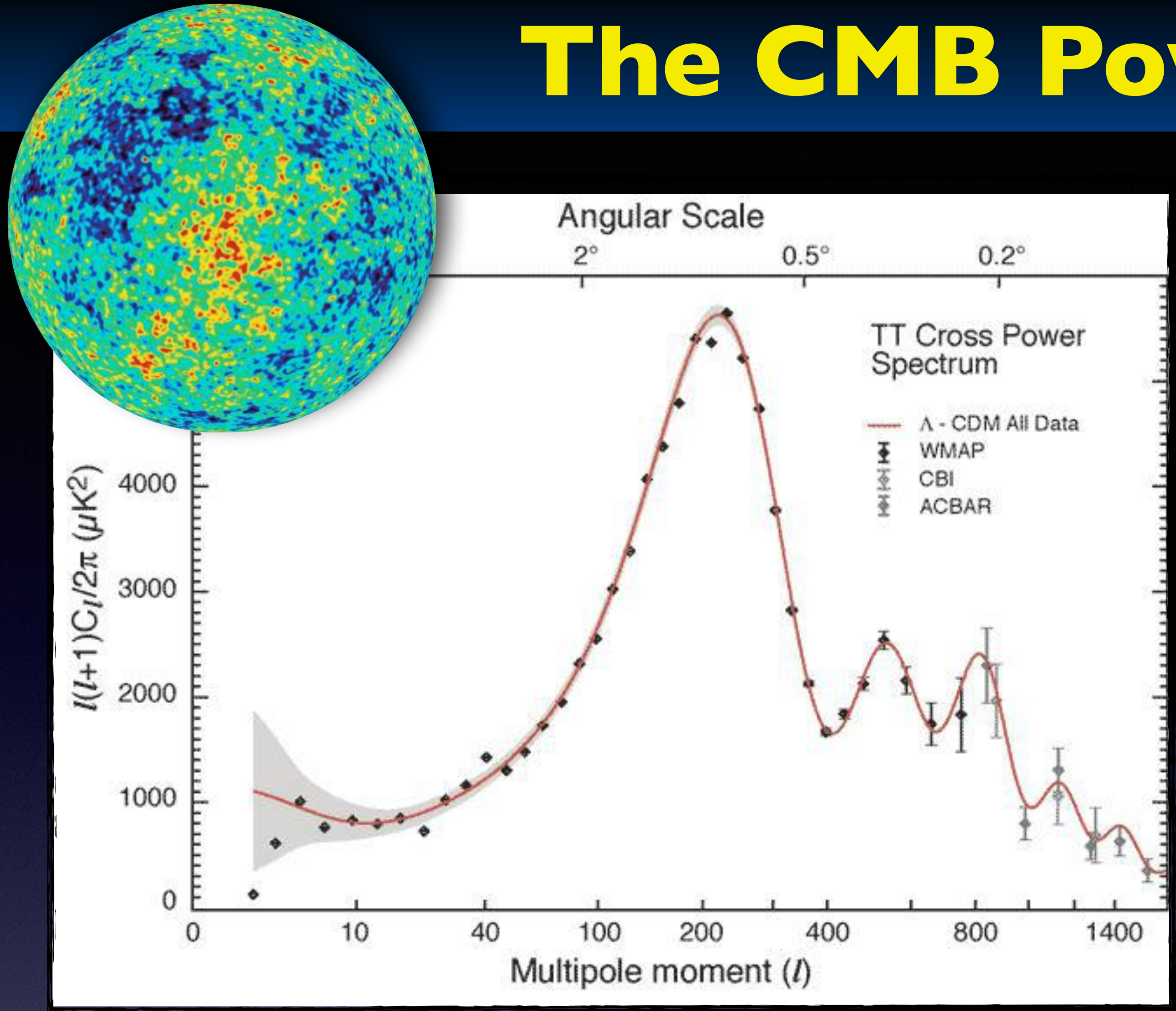
For a flat ΛCDM cosmology, this yields: $\theta \sim 0.3' \left(\frac{\lambda^{\rm com}}{1h^{-1}{\rm Mpc}}\right)\left(\frac{\Omega_{\rm m,0}}{0.3}\right)^{1/2}$

An important scale is the comoving Hubble radius at decoupling, $r_{\rm H} = c/H(z_{\rm dec})$, which is similar to the particle horizon at $z_{\rm dec}$ except for a factor of order unity.

For a flat ΛCDM cosmology $\theta_{\rm H} \sim 0.87^\circ \left(\frac{z_{\rm dec}}{1100}\right)^{-1/2}$, which corresponds to $l \sim 200$.

CMB anisotropies with $l < 200$ correspond to super-horizon scale perturbations.

The CMB Power Spectrum



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$$\boxed{\theta = \frac{\lambda^{\mathrm{phys}}}{d_{\mathrm{A}}(z_{\mathrm{dec}})} = \frac{\lambda^{\mathrm{com}}}{d_{\mathrm{A}}(z_{\mathrm{dec}})\,(1+z_{\mathrm{dec}})}}$$

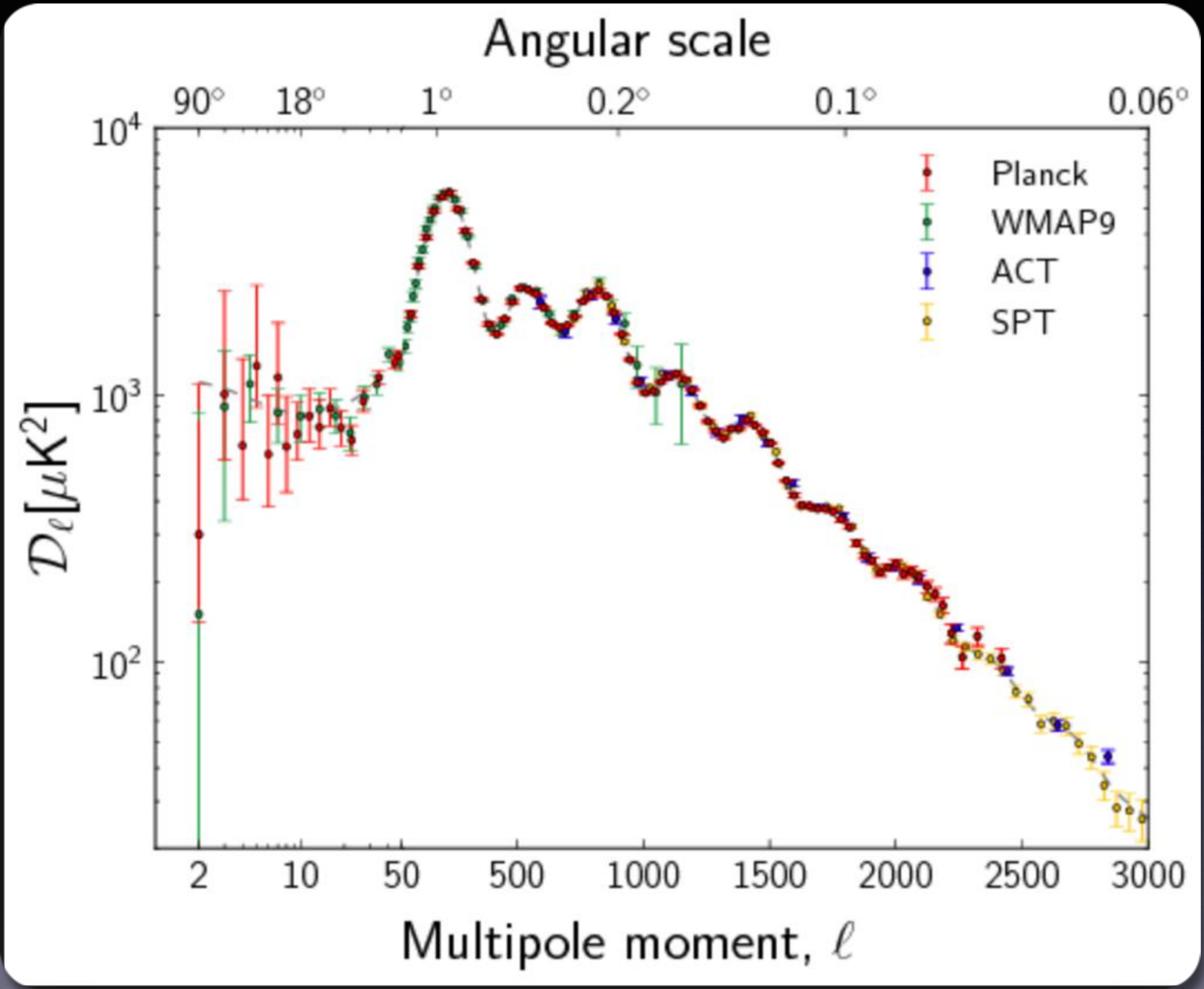
CMB anisotropies with $l < 200$ correspond to super-horizon scale perturbations.

On these super-horizon scales, only two effects can contribute to non-zero $\Delta T/T$

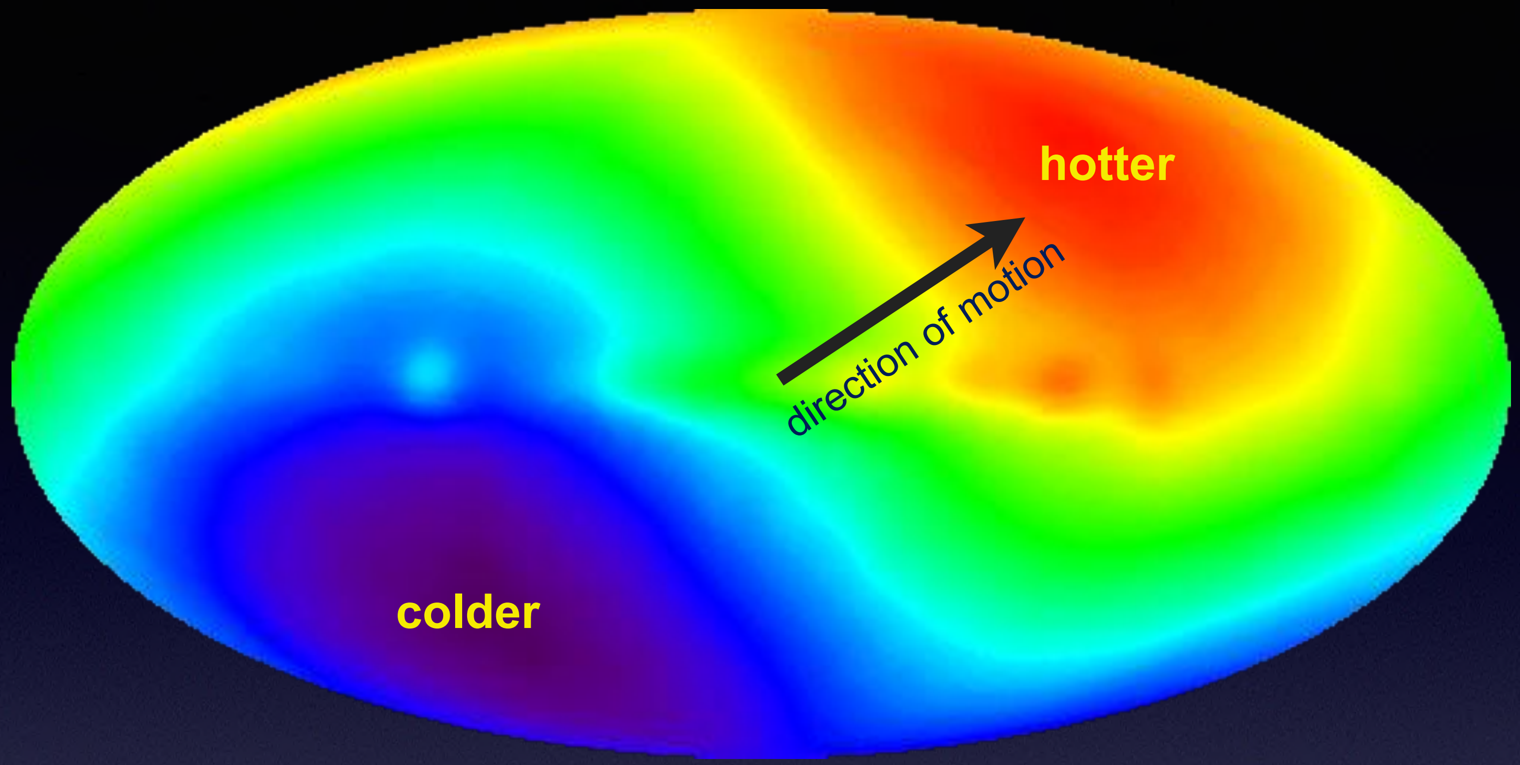
- fluctuations in the energy density of the photons $\delta_\gamma \propto \delta_{\mathrm{r}}$
- fluctuations in the gravitational potential $\Phi_{\vec{k}}$ (photons lose energy when climbing out of a potential well....)

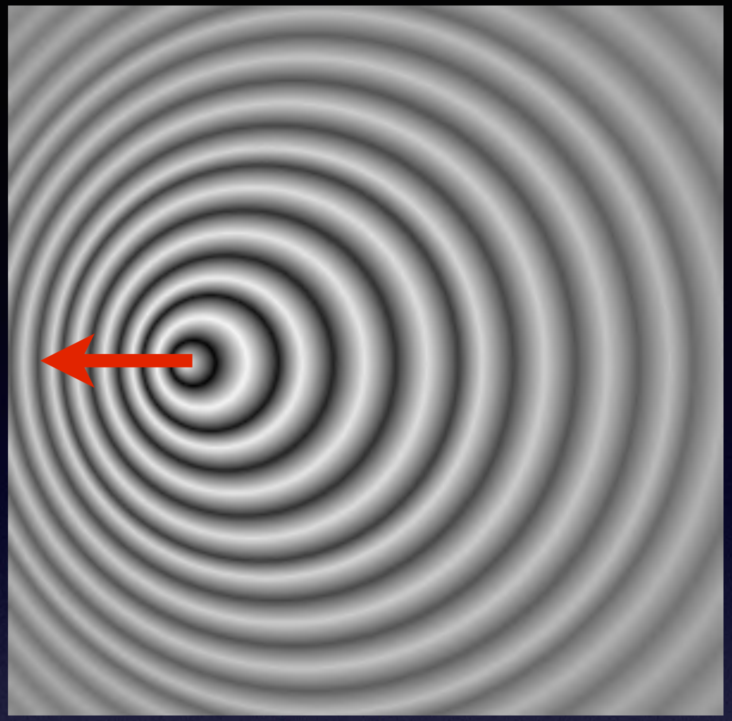
The combination of these two effects is known as the Sachs-Wolfe effect.

Power Spectrum; current status



The CMB Dipole





Origin of CMB dipole is Doppler effect due to our peculiar motion

Our peculiar motion is made up of:

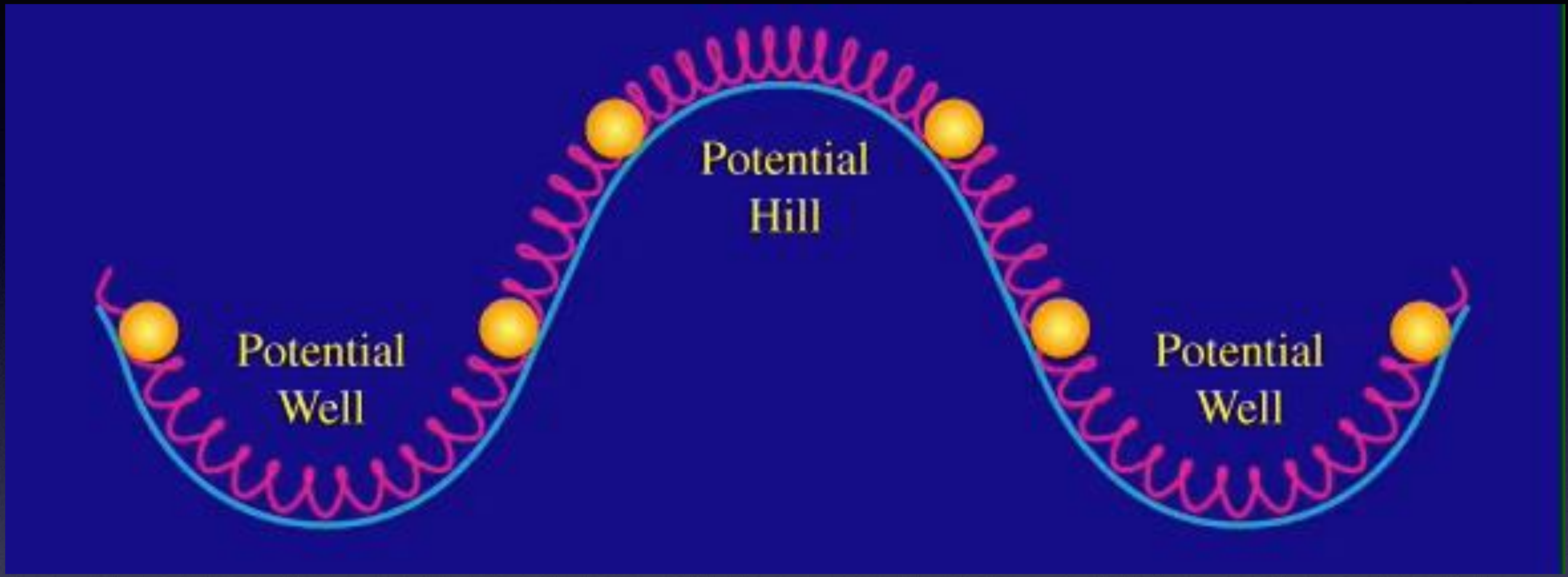
- Motion of Earth around Sun (~30 km/s)
- Motion of Sun around MW center (~220 km/s)
- Motion of MW towards Virgo cluster (~300 km/s)

Total vector sum of 369 km/s

Photons coming from the direction in which we are moving are blueshifted (as if that direction is moving towards us). Photons of a shorter wavelength correspond to photons of a higher temperature (i.e., Wien's law)

Origin of Acoustic Peaks

After entering horizon, baryonic perturbations below Jeans mass start acoustic oscillations. These are driven by the potential perturbations in the dark matter.



Enormous pressure of tightly coupled photon-baryon fluid, due to Thomson scattering of photons off free electrons, resists gravitational compression.

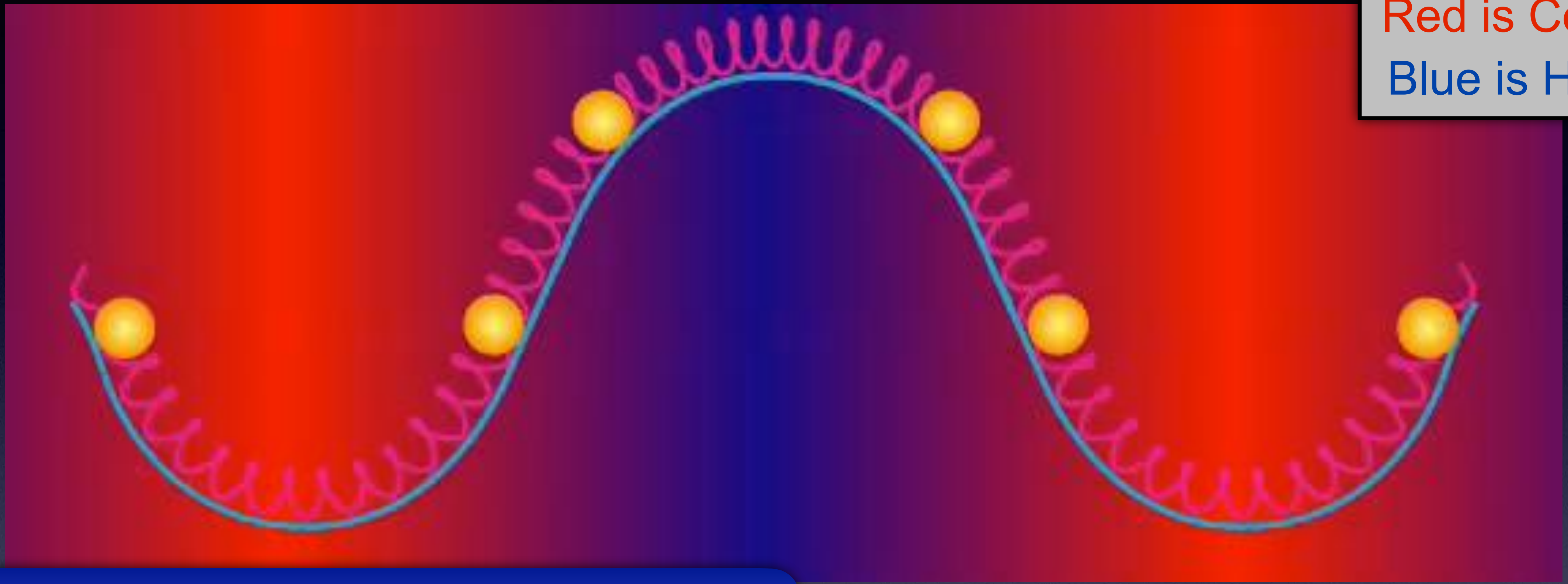
→ acoustic oscillations (compression --> rarefaction --> compression --> rarefaction).

The resulting sound waves in photon-baryon fluid create **temperature fluctuations**

Adiabatic compression of gas heats it up
Adiabatic expansion of gas cools it down
→ Temperature fluctuations

Origin of Acoustic Peaks

After entering horizon, baryonic perturbations below Jeans mass start acoustic oscillations. These are driven by the potential perturbations in the dark matter.

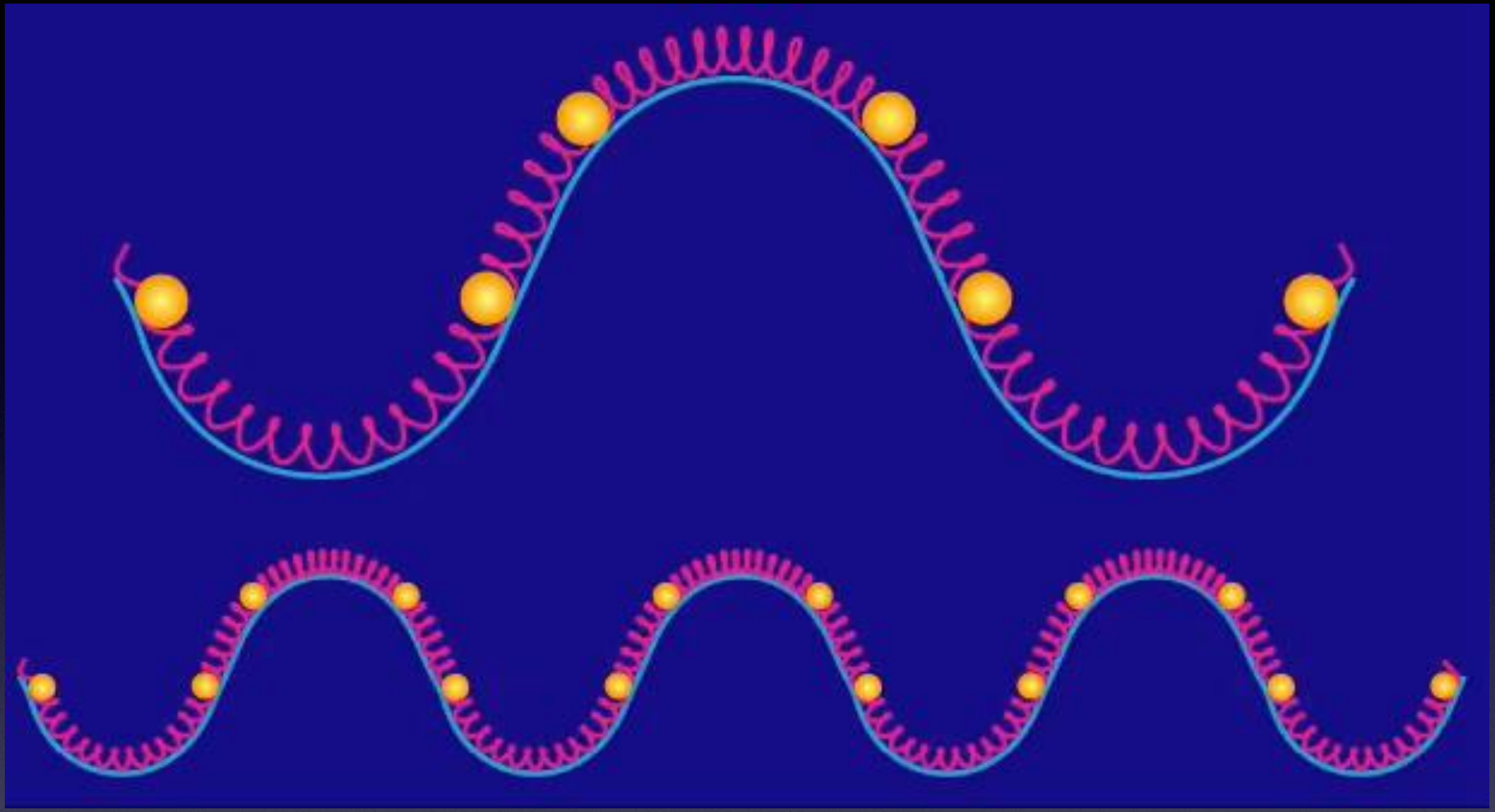


Red is Cold
Blue is Hot

Compression results in higher temperature
Rarefaction results in lower temperature

Oscillations: Compression in valley (hot) & rarefaction at hill (cold)
is followed by rarefaction in valley (cold) & compression at hill (hot)
is followed by compression in valley (hot) & rarefaction at hill (cold), etc

Origin of Acoustic Peaks



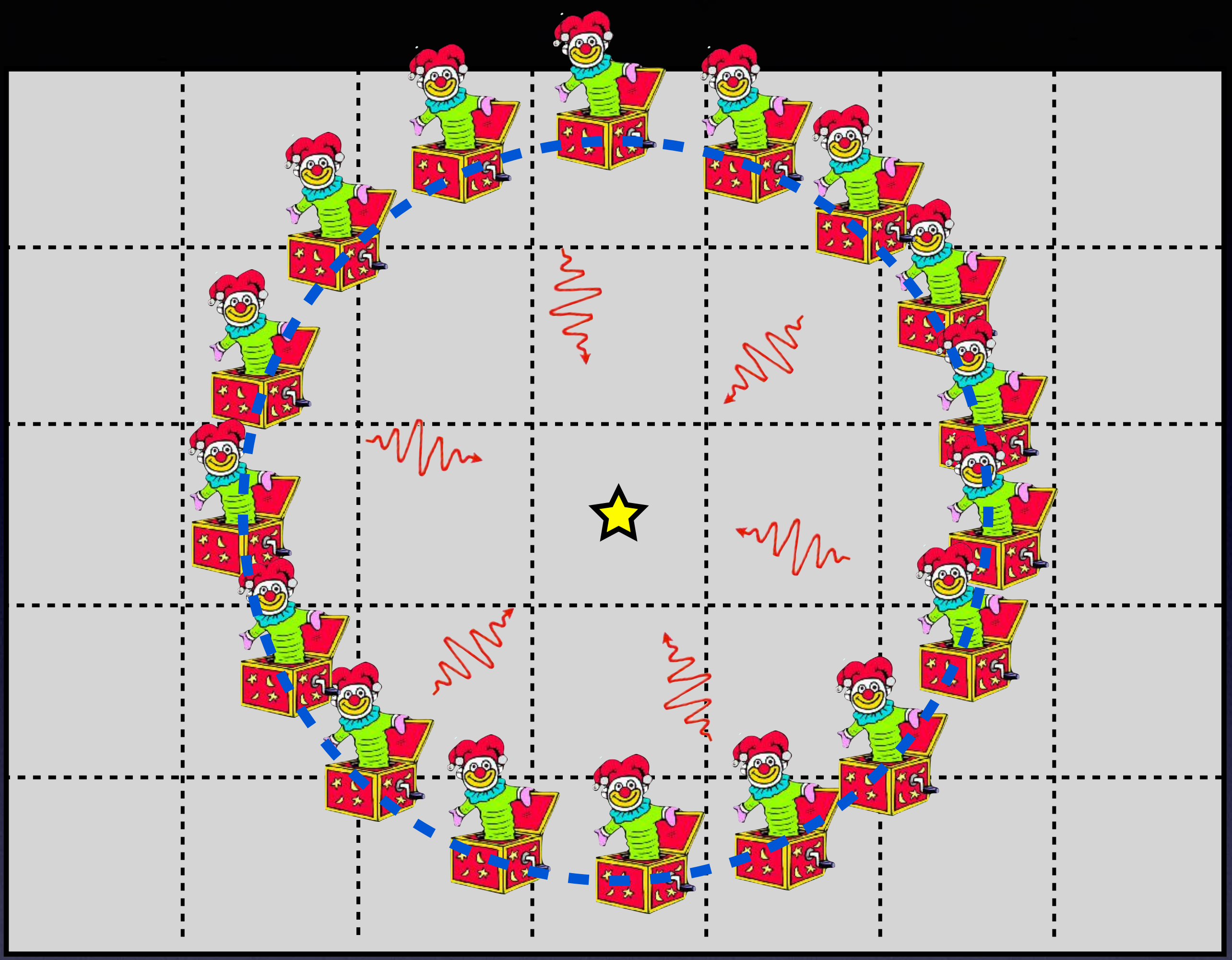
long-wavelength mode

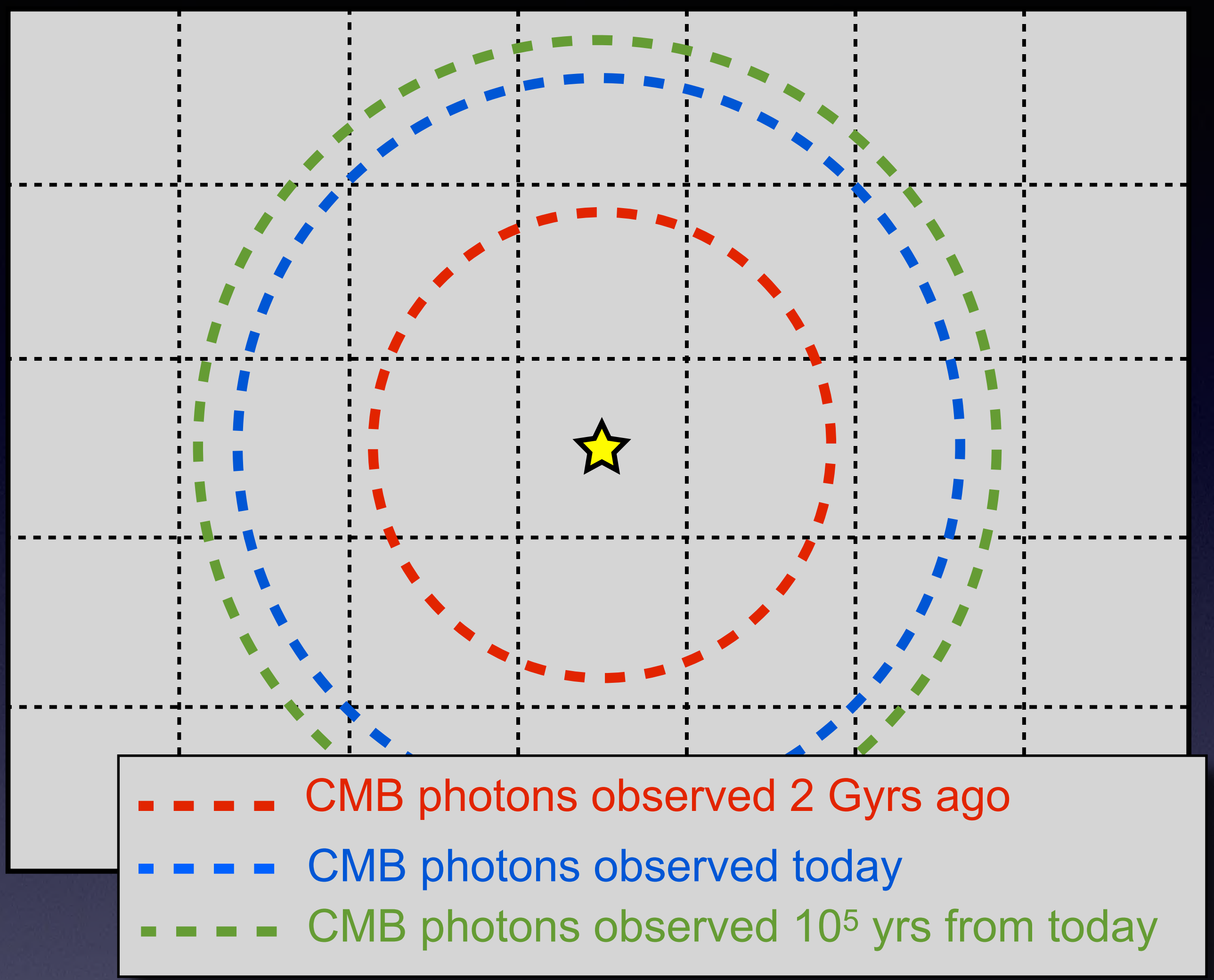
short-wavelength mode

Since sound speed of photon-baryon fluid is the same for all modes, those with a smaller wavelengths oscillate faster....

At recombination, photons are released, and pressure of photon-baryon fluid abruptly drops to (almost) zero. Temperature of photons at release is frozen at that at recombination. Put differently; the last-scattering surface is a snap-shot view of oscillation phases of all different modes.

Observing the CMB



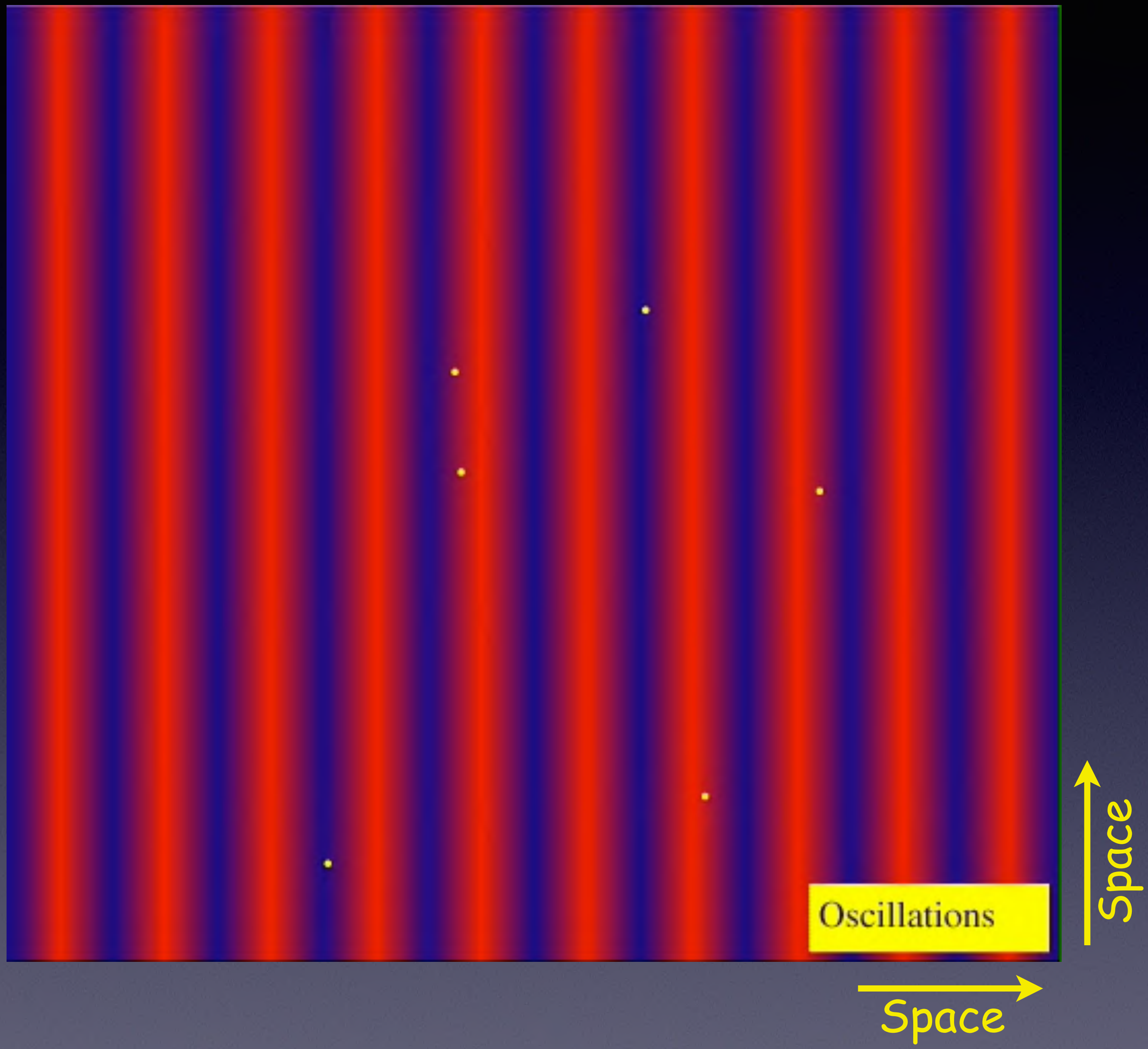


Useful mnemonic:

The CMB photons observed today were all released at decoupling from jack-in-the-boxes that are equi-distant from us (indicated by blue, dashed circle.

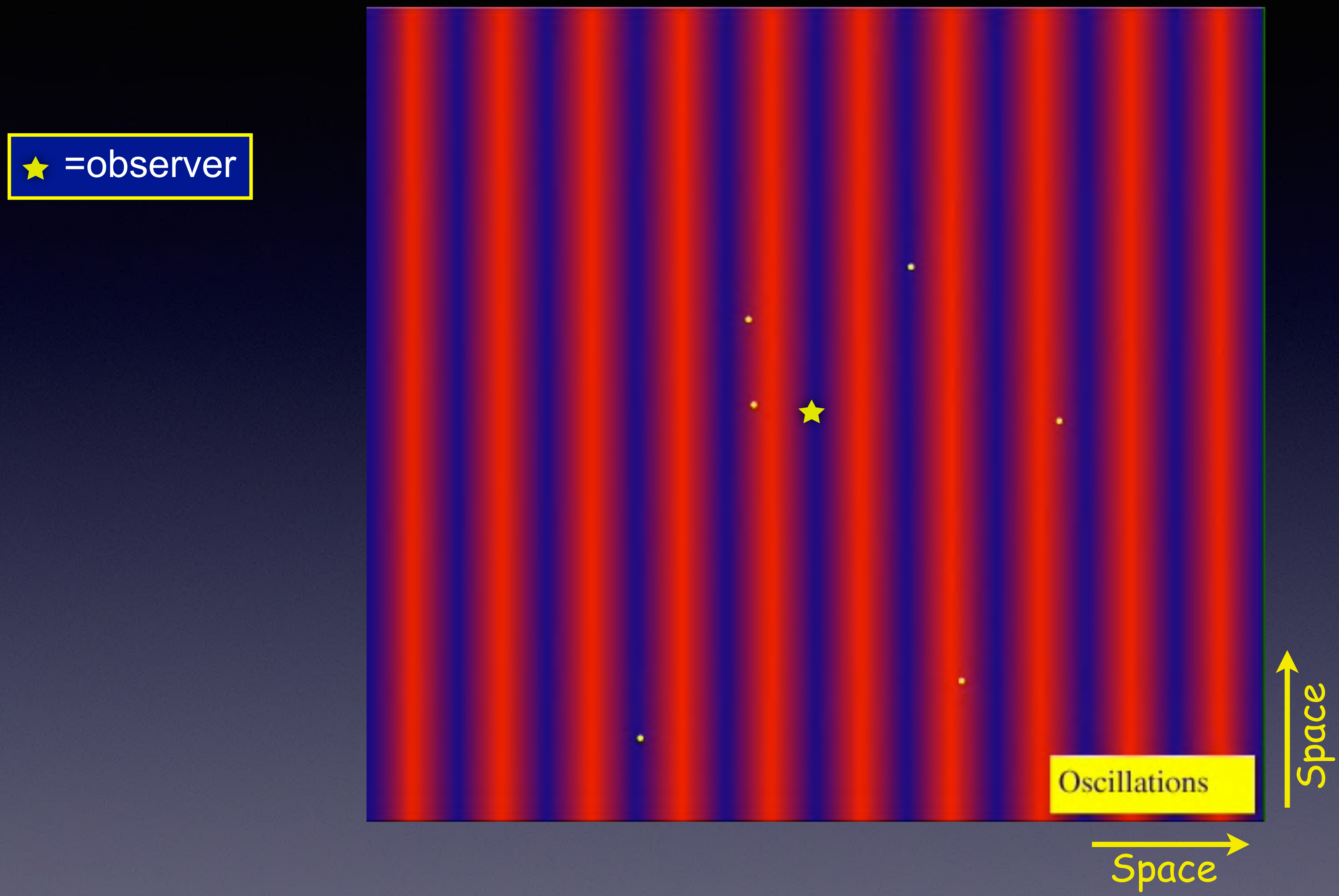
At each point in time, one observes CMB photons coming from jack-in-the-boxes at different locations...

Origin of Acoustic Peaks



Shown is the time-evolution of a single perturbation mode, together with the locations of six `jack-in-the-boxes'.

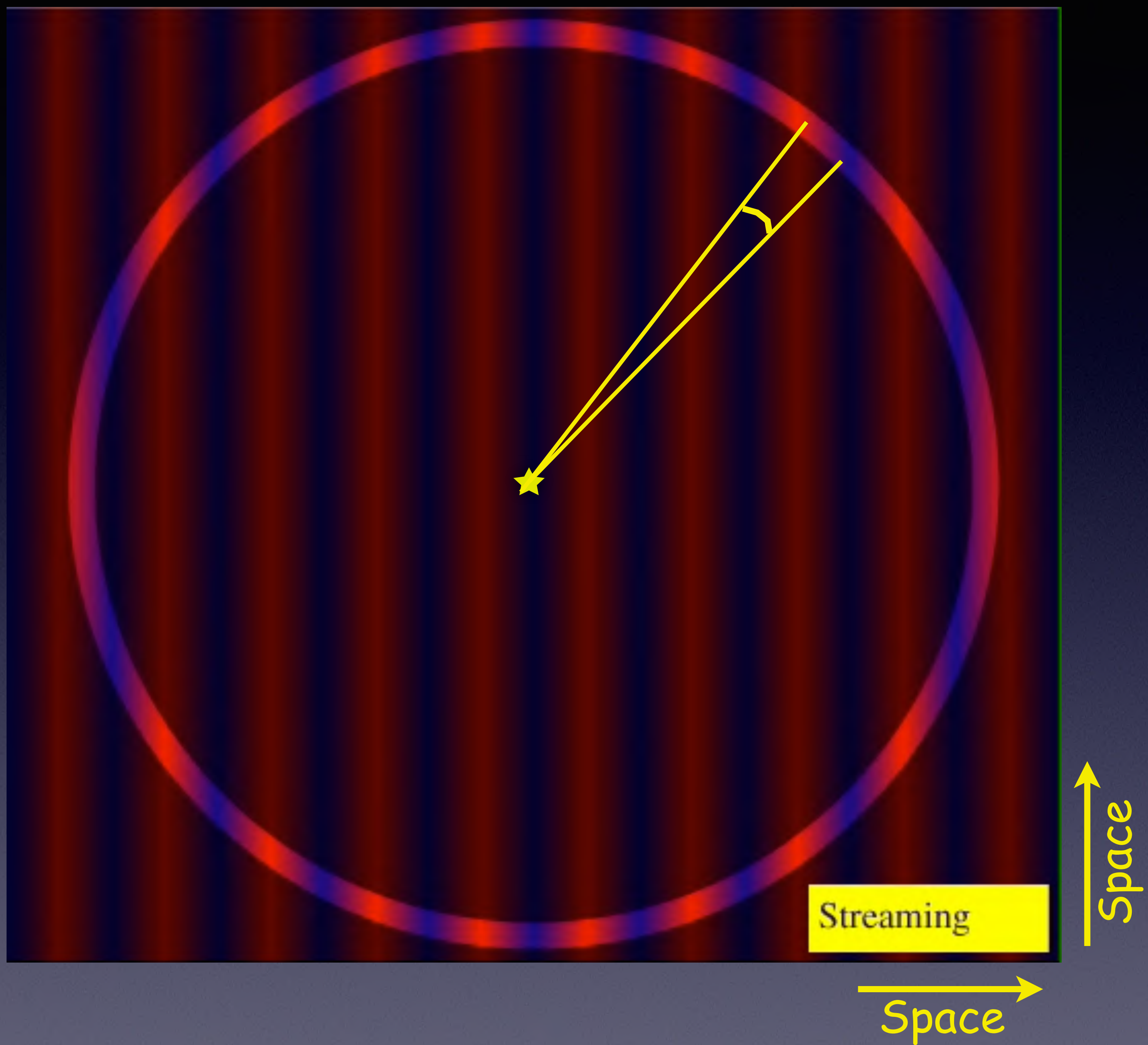
Origin of Acoustic Peaks



At recombination, jack-in-the-boxes open (photons `decouple') and the photons start to free-stream through space.

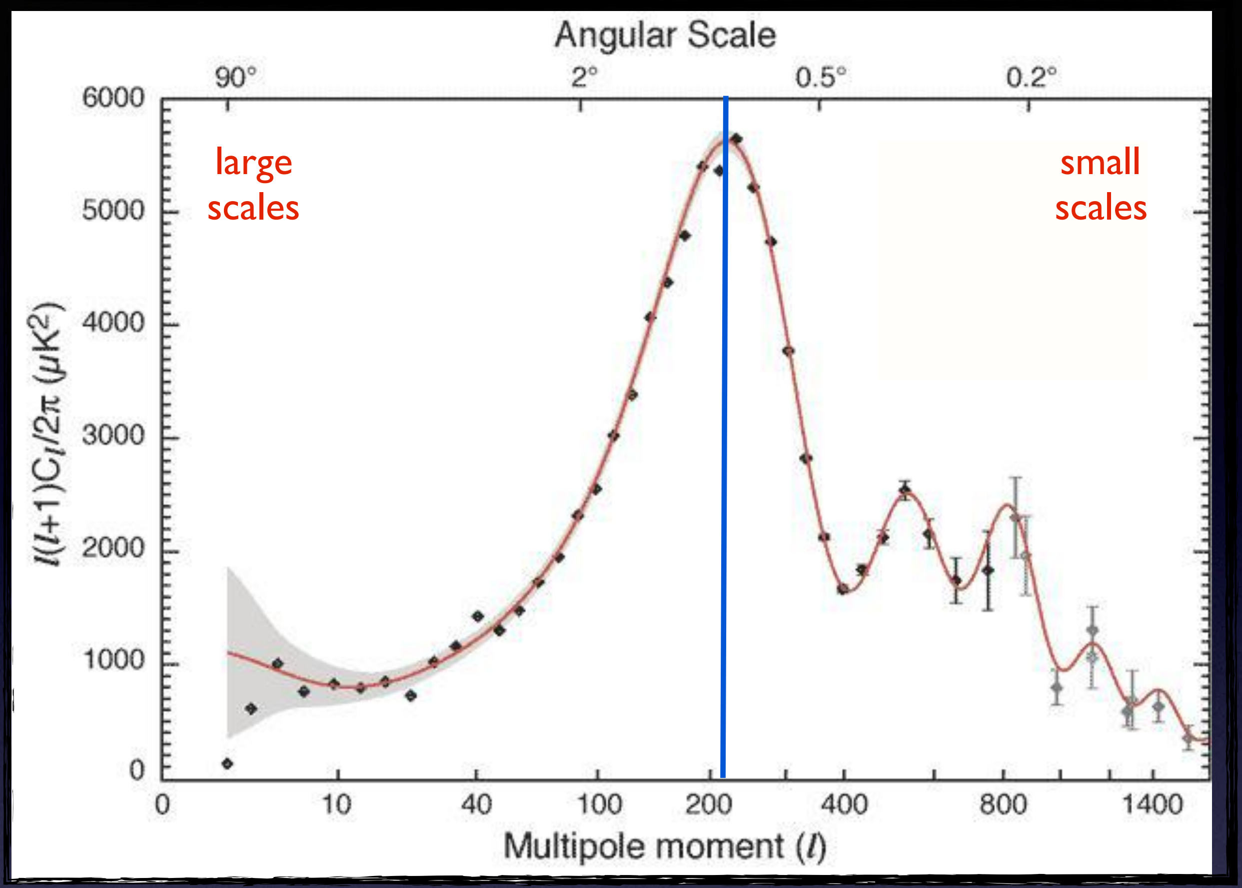
Origin of Acoustic Peaks

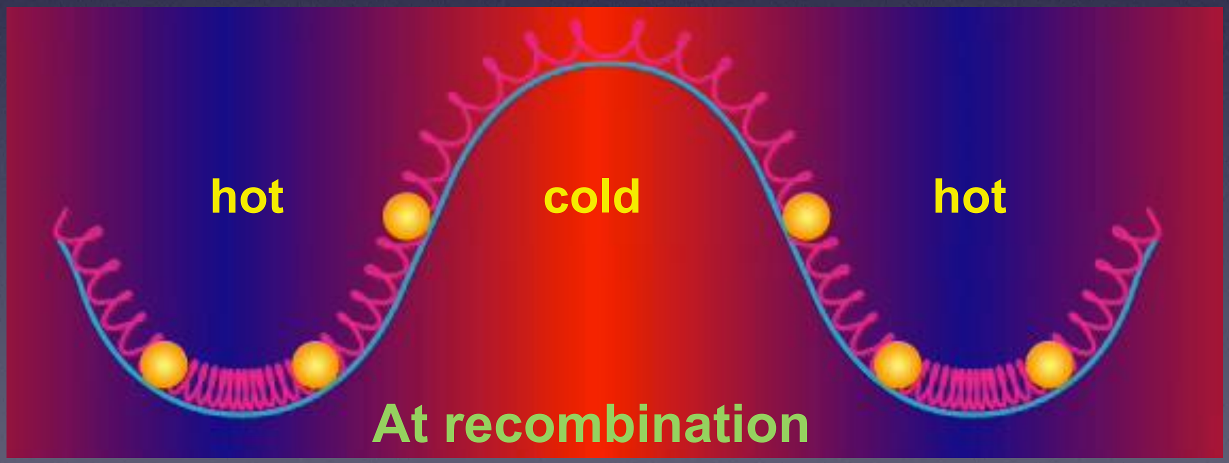
★ =observer

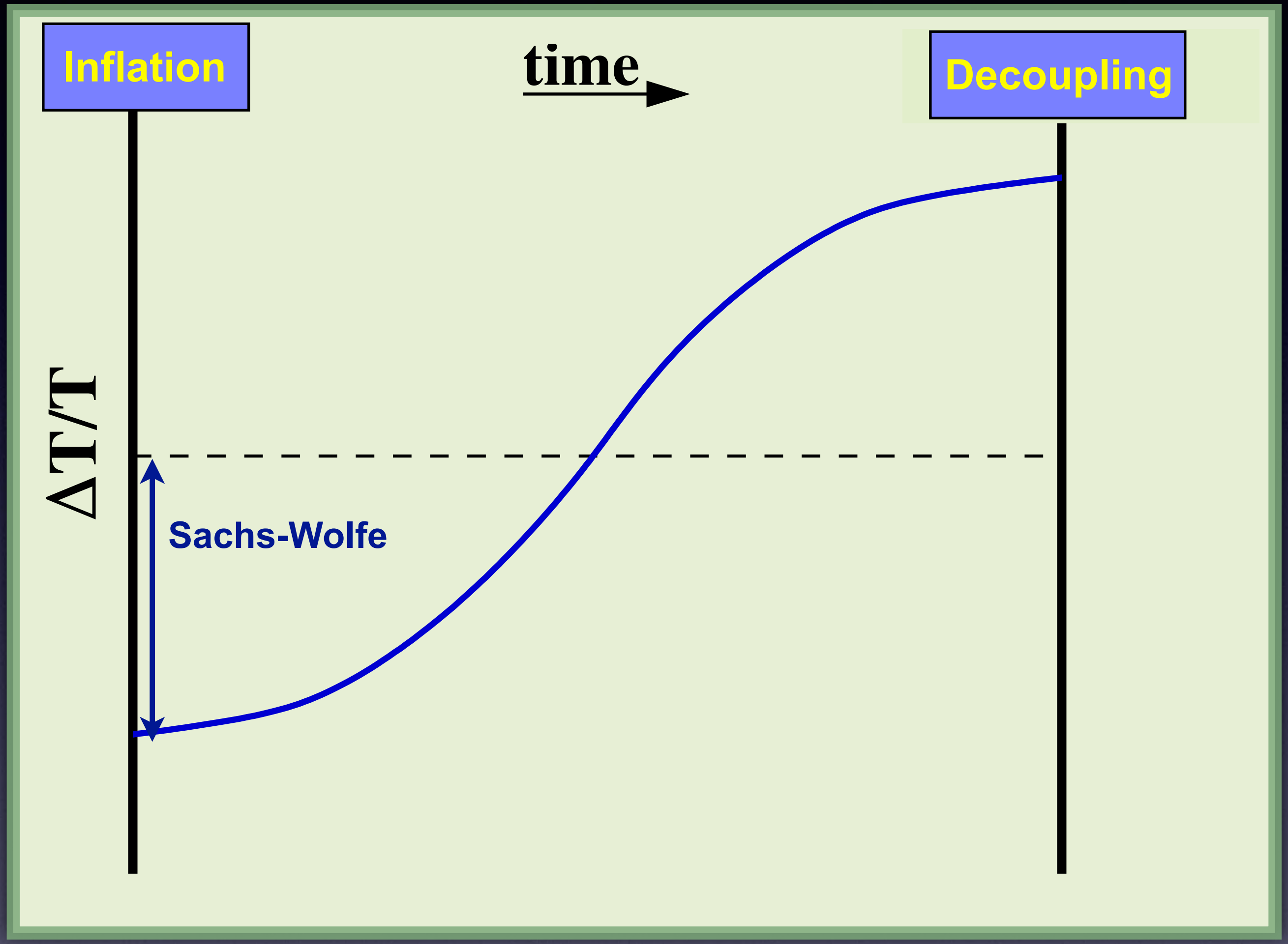


The observer sees this mode as angular temperature fluctuation on the sky, with a characteristic angular scale set by the wavelength of the mode.

The Origin of the first Acoustic Peak

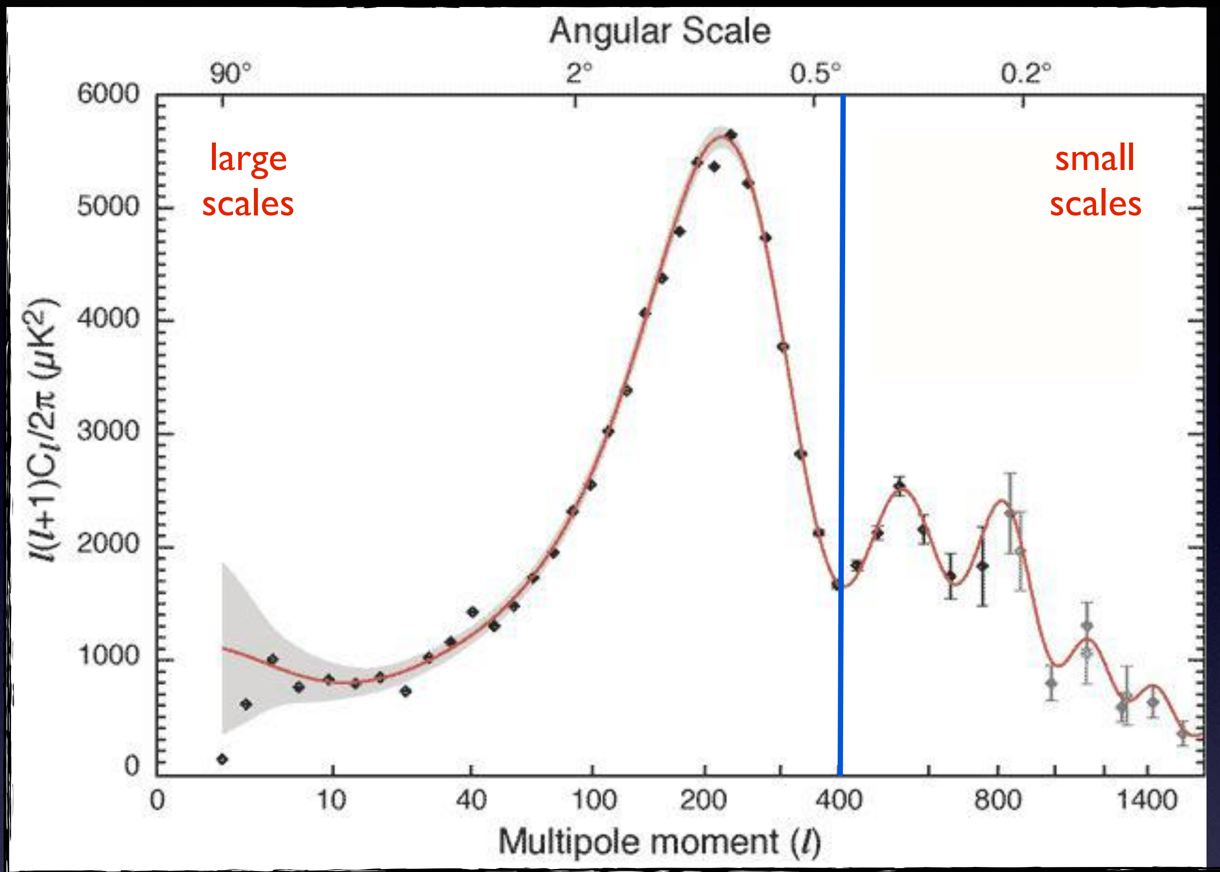


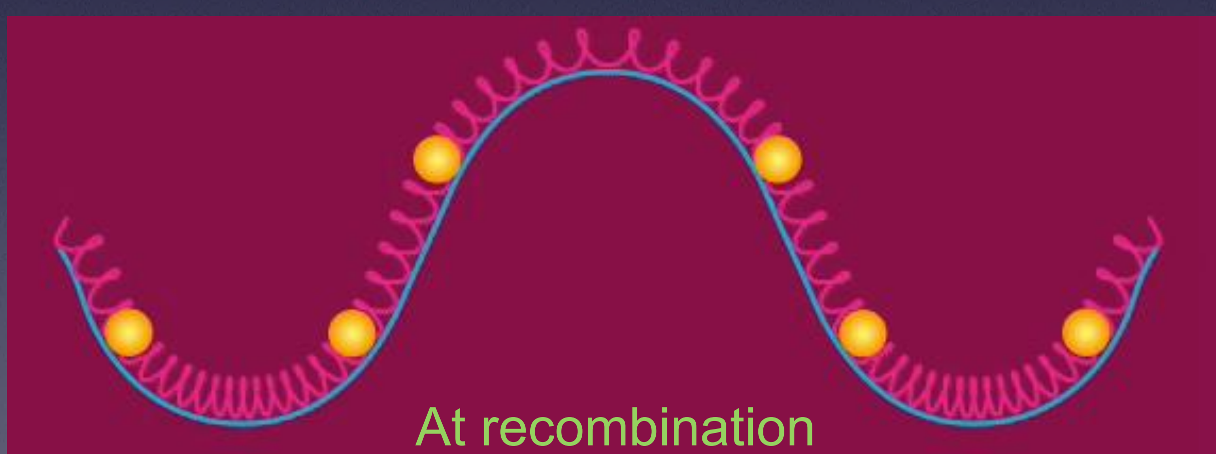


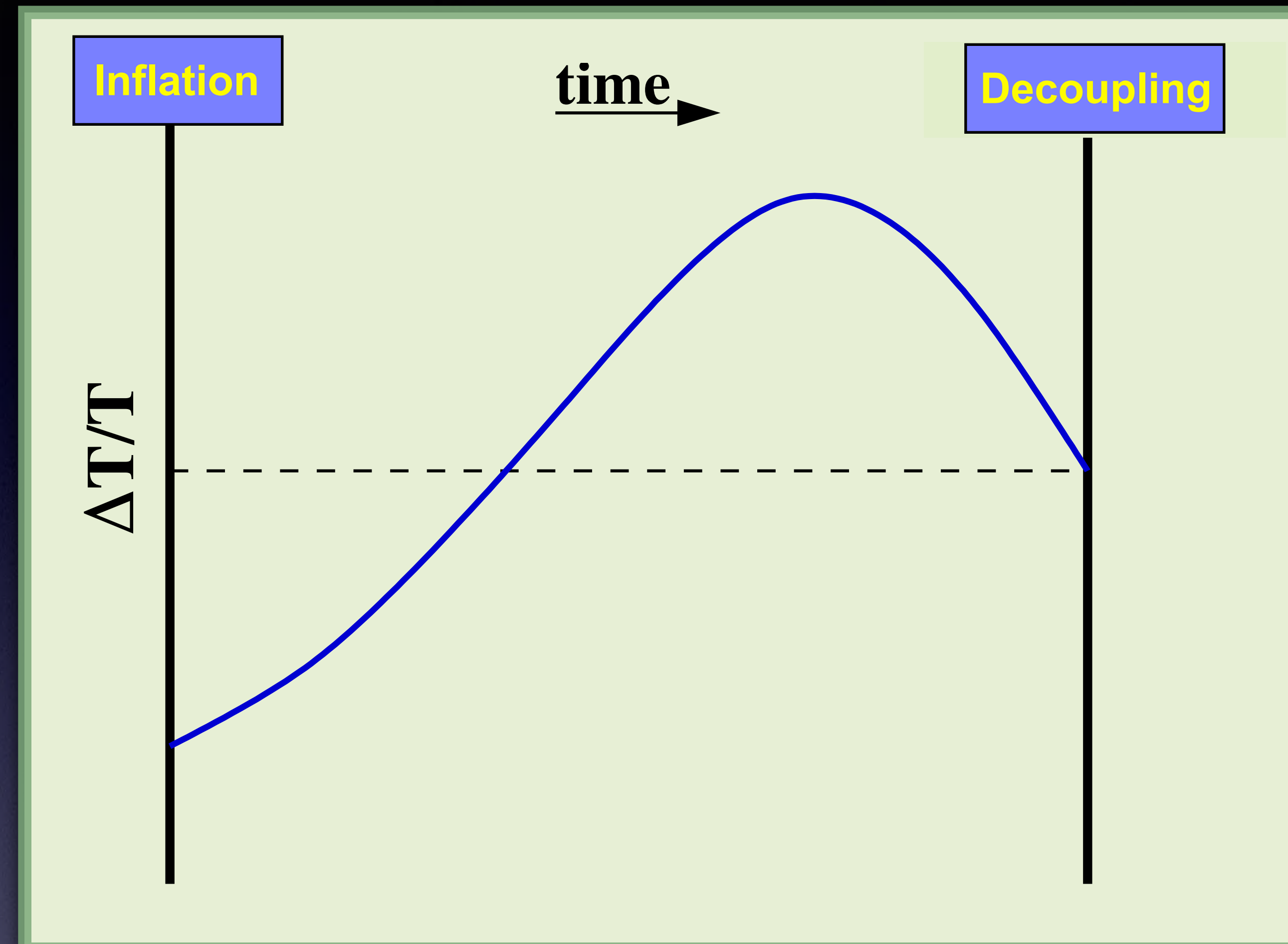


The first acoustic peak is due to the mode that just reaches maximal compression in valley/rarefaction on hill top for first time at recombination

The Origin of the first Acoustic Through

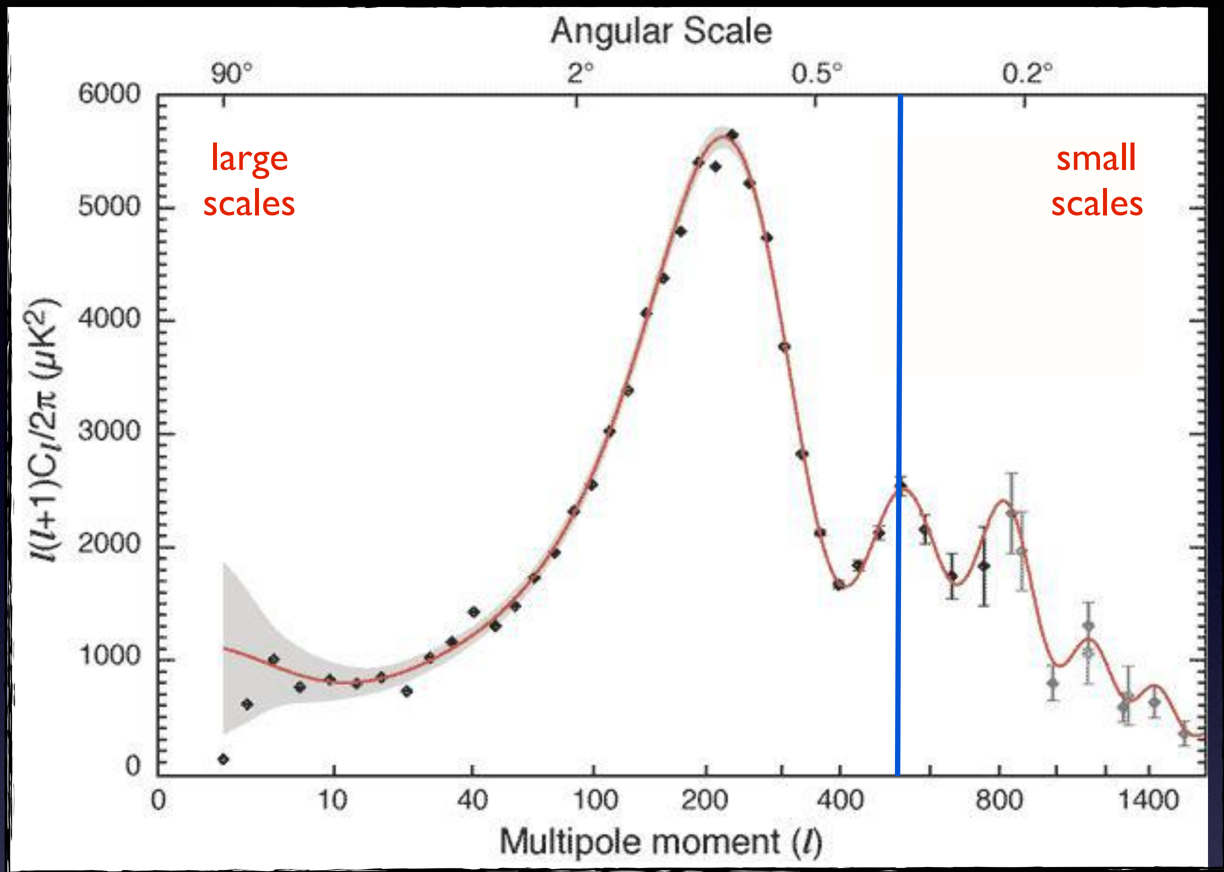


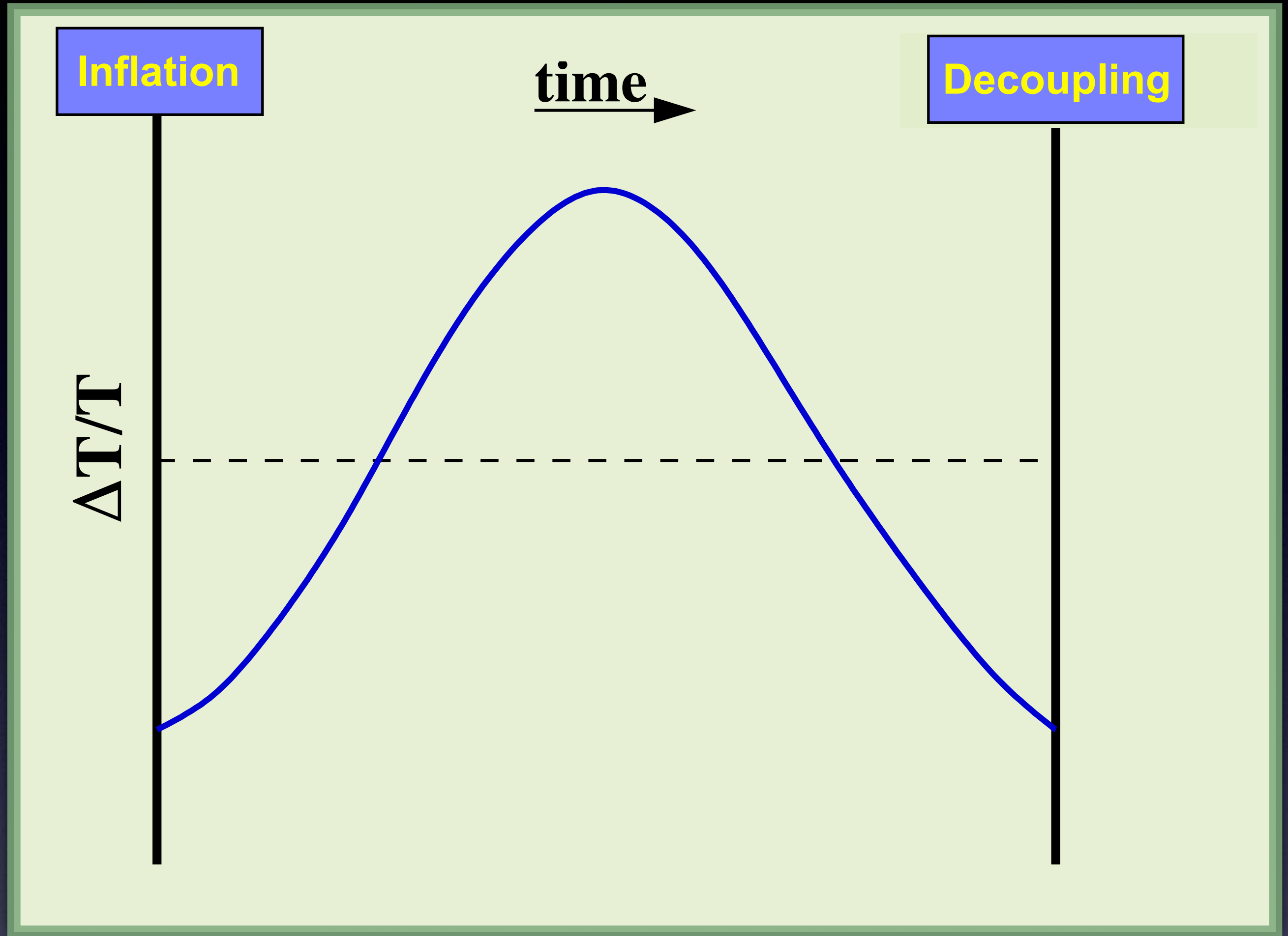


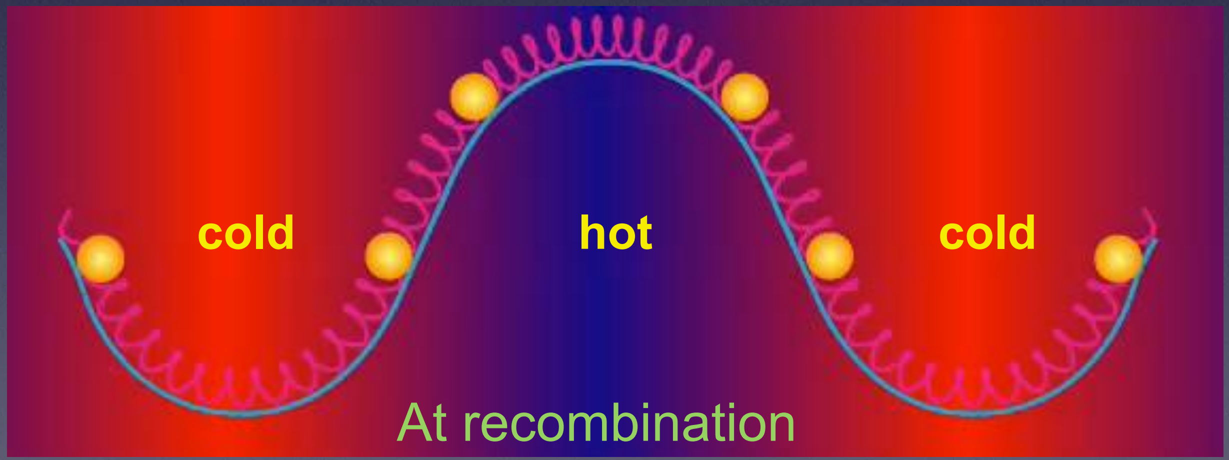


Temperature fluctuations at troughs are not zero! Although photon-baryon fluid has constant temperature, motions in the fluid cause Doppler shifts

The Origin of the second Acoustic Peak



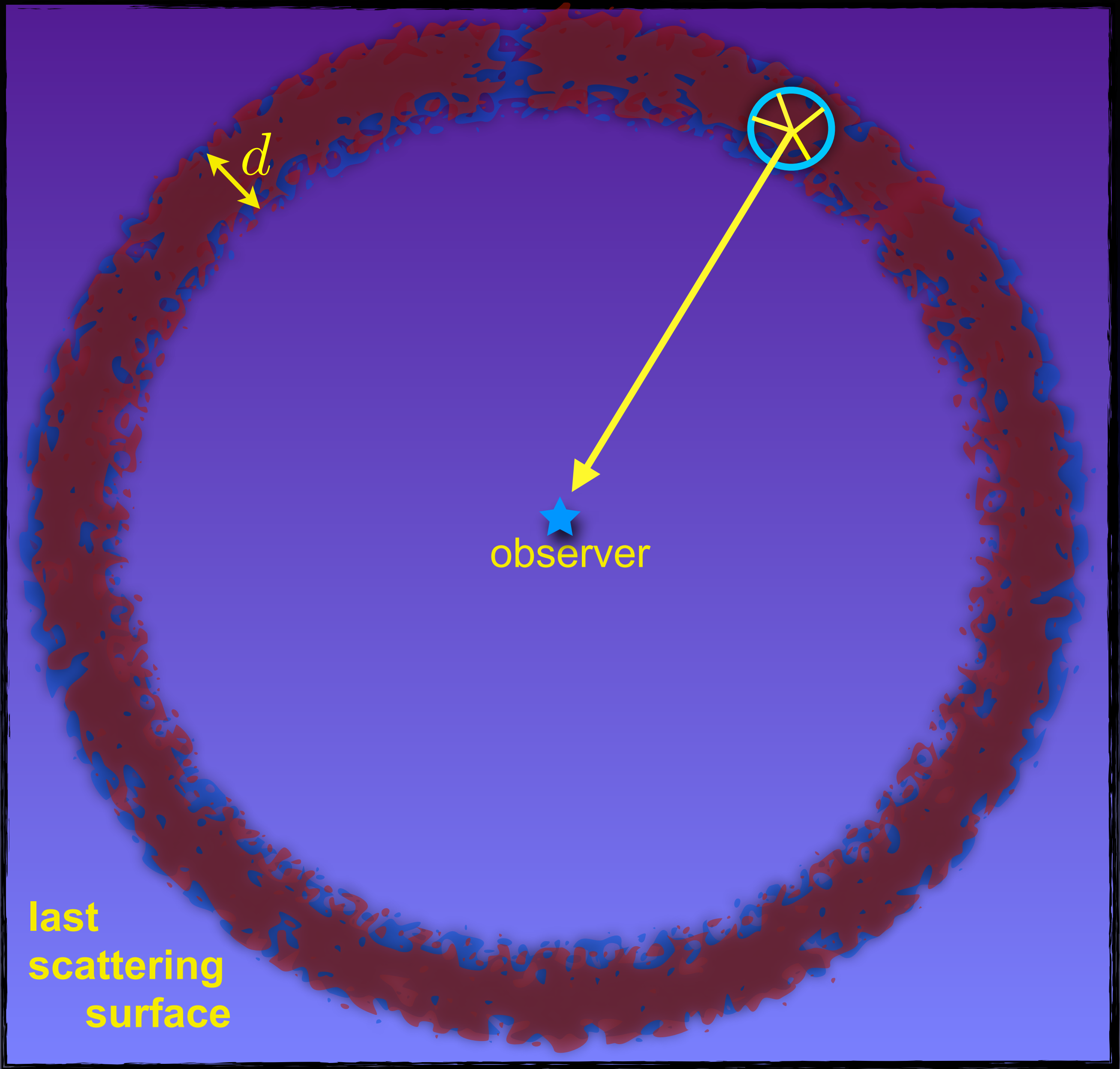


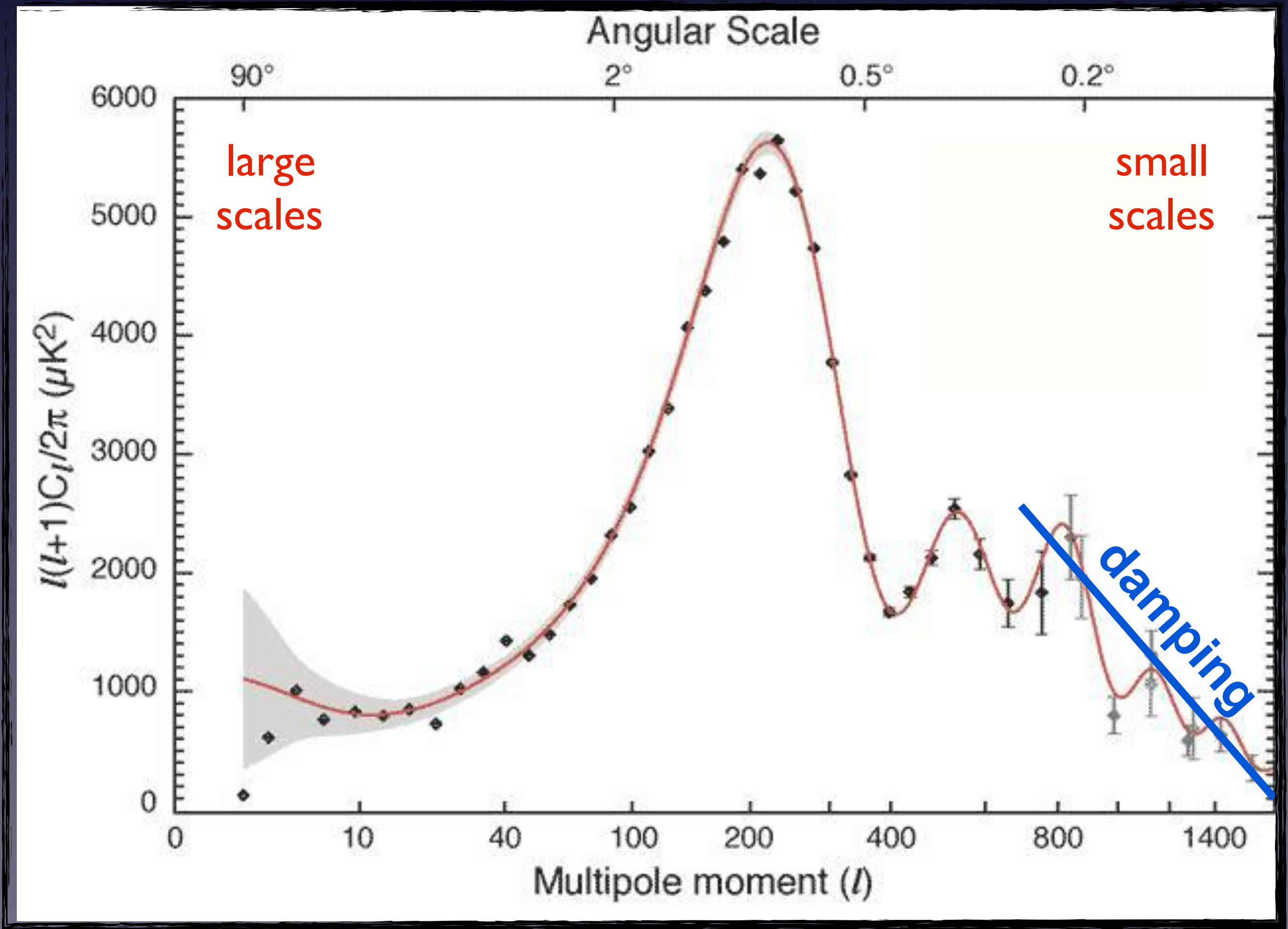


The second acoustic peak is due to mode that just reaches maximal rarefaction in valley/compression on hill top for first time at recombination

Diffusion Damping

Recombination is not instantaneous; rather, LSS has a finite thickness d. Consequently, temperature fluctuations due to modes with a wavelength $\lambda < d$ are washed out. This diffusion damping explains damping of CMB power spectrum on small scales.

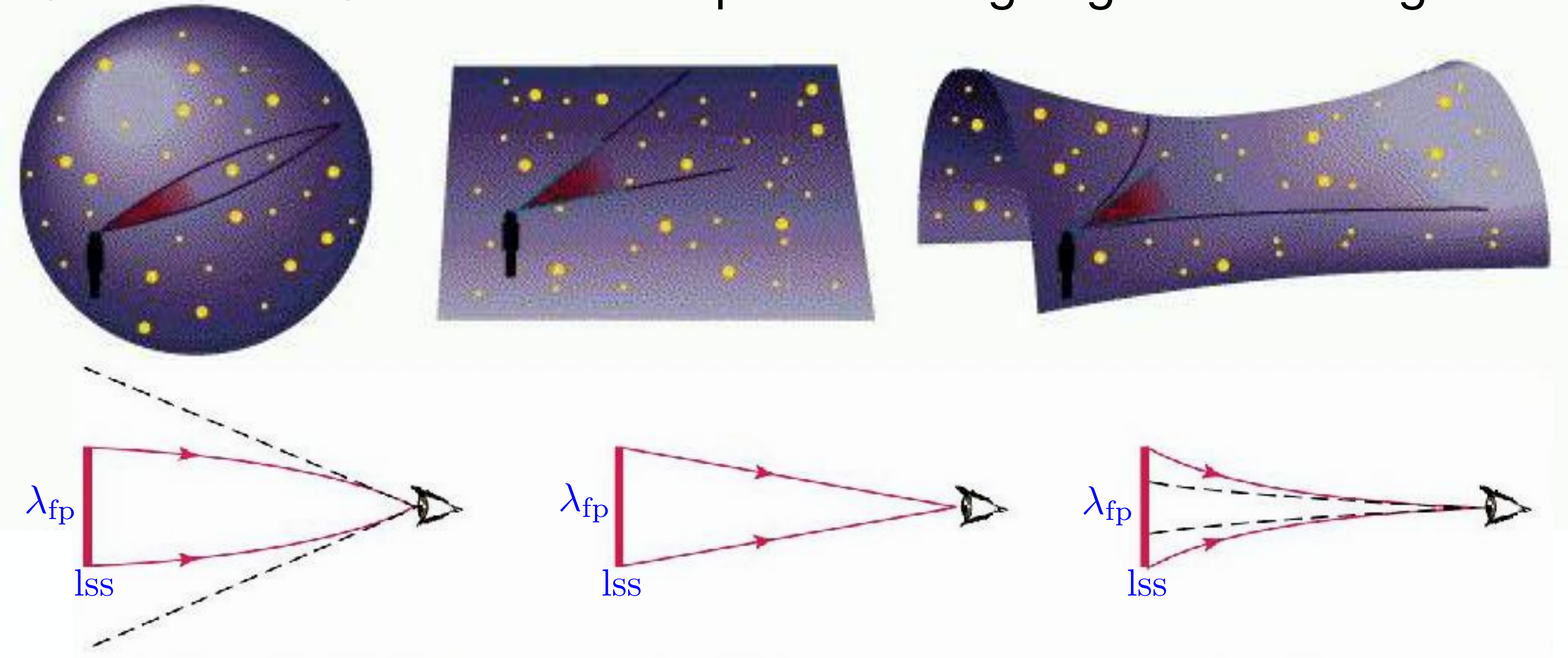




In addition to diffusion damping, operating on scales $l > 1000$, there is also Silk damping. However, the latter only operates on scales $l > 2000$ and is therefore subdominant.

The Curvature of the Universe

Curvature of Universe can be probed using large-scale triangles...



One such triangle comes from angular scale of first acoustic peak, which corresponds to wavelength of mode that just managed to reach maximal compression at decoupling....

$$\lambda_{\rm fp}^{\rm com}/2 = c_{\rm s}\,\tau_{\rm dec} \qquad c_{\rm s} \sim c/\sqrt{3}$$

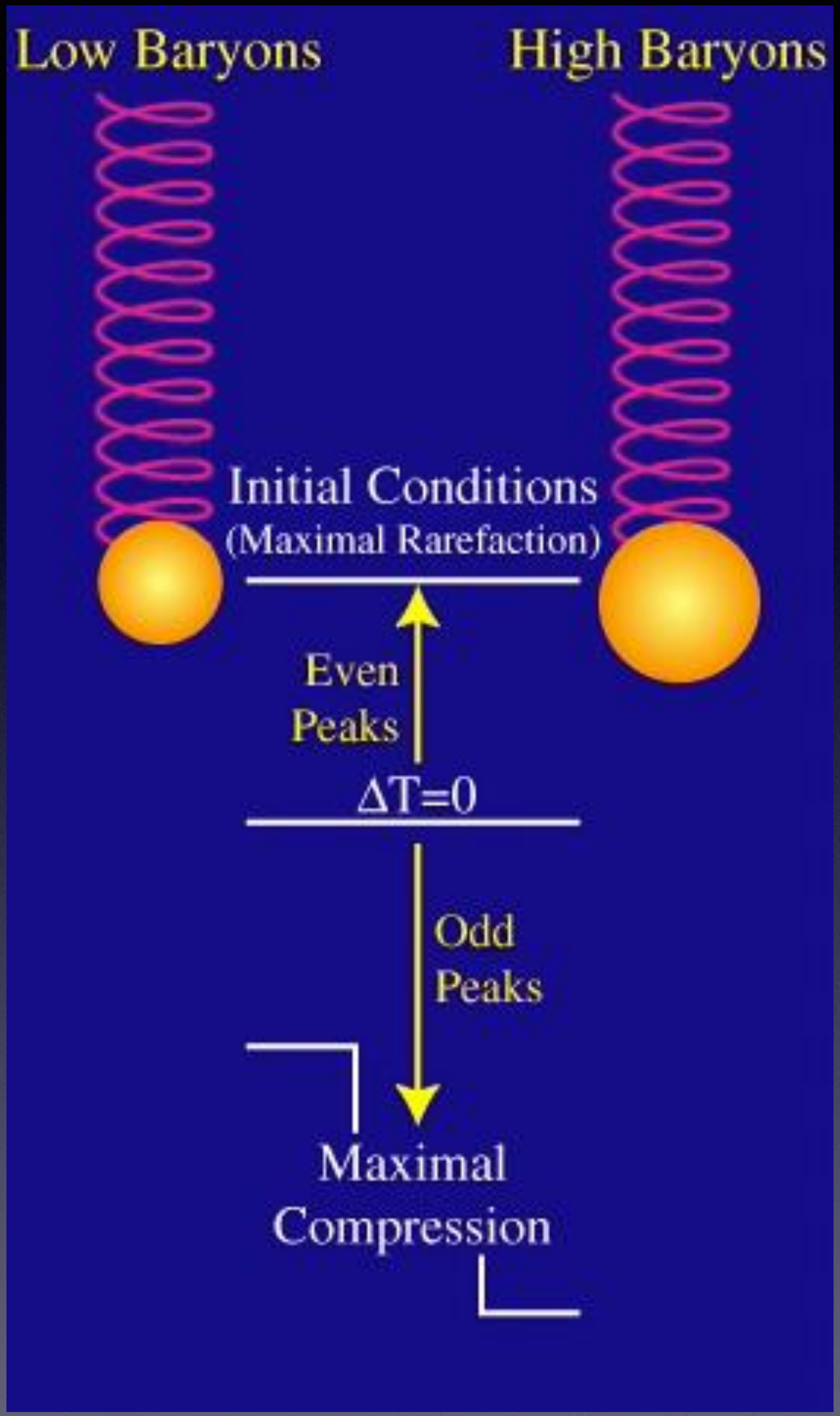
$$\Rightarrow \quad \lambda_{\rm fp}^{\rm com} \sim c\,\tau_{\rm dec} = \chi_{\rm H}(z_{\rm dec})$$

Comoving wavelength of mode at first peak, $\lambda_{\rm fp}^{\rm com}$, is roughly equal to particle horizon at decoupling.

- As we have seen, for a flat Universe, $\chi_{\rm H}(z_{\rm dec})$ corresponds to $l \sim 200$
- The first acoustic peak of the CMB power spectrum is observed at $l \sim 200$

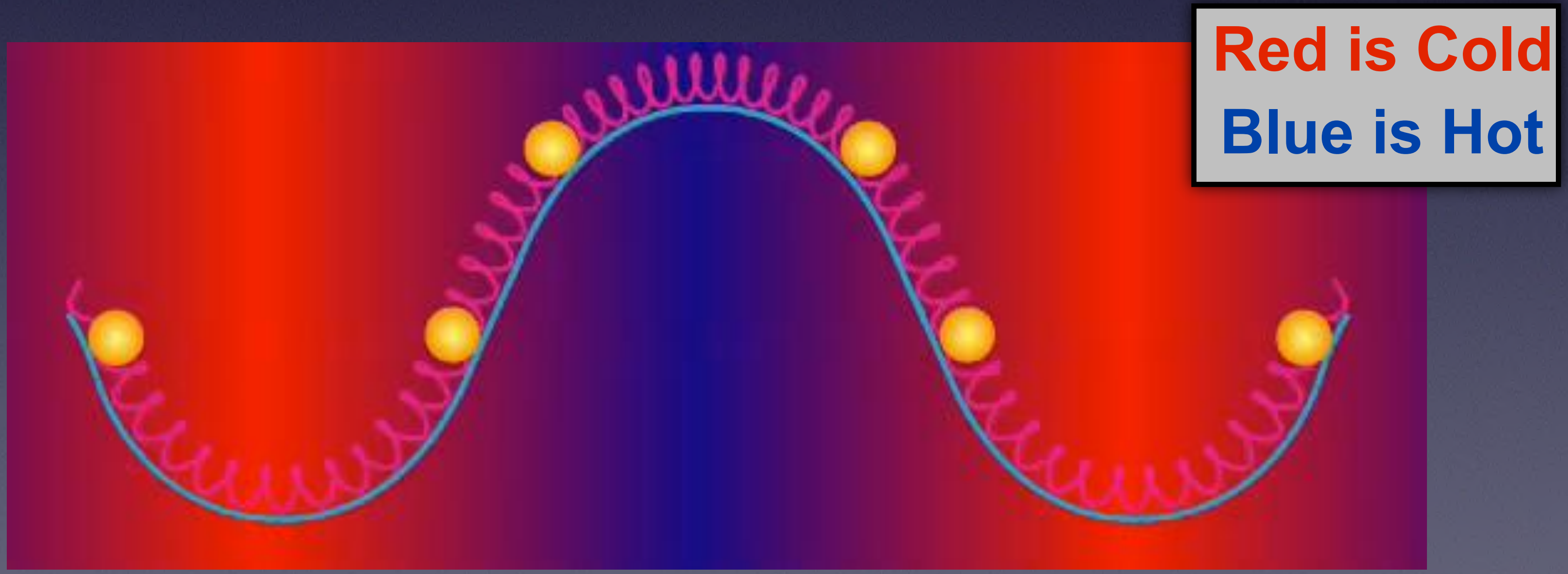
⇒ **RESULT: Our Universe is flat (K=0), i.e., has Euclidean Geometry**

The Baryonic Mass Fraction



Increasing density of baryons relative to that of dark matter causes stronger compression in valleys (due to the self-gravity of baryons), and less compression on hill tops.

Since odd peaks (first, third, etc) correspond to compression in valleys, whereas even peaks (second, fourth, etc) correspond to compression on hill tops, the baryon-to-dark matter ratio controls the ratio of odd-to-even peak heights.



RESULT: dark matter density ~6x higher than baryon density

Lecture 7

SUMMARY

Summary: key words & important facts

Key words	
ergodic principle	Power spectrum
Gaussian random field	recombination vs. decoupling
two-point correlation function	last scattering surface
Harrison-Zeldovic spectrum	diffusion damping

- The power-spectrum is the Fourier Transform of the two-point correlation function
- A Gaussian random field is completely specified (in statistical sense) by the power-spectrum. The phases of all modes are independent and random.

- CMB dipole reflects our motion wrt last scattering surface (lss)
- Location of first peak in CMB power spectrum → curvature of Universe
- Ratio of first to second peak in CMB power spectrum → baryon-to-dark matter ratio
- Finite thickness of lss causes diffusion damping of CMB perturbations

Summary: key equations & expressions

ergodic principle: ensemble average = spatial average

first moment

$$\langle\delta\rangle = \int \delta\,\mathcal{P}(\delta)\,\mathrm{d}\delta = \int \delta(\vec{x})\,\mathrm{d}^3\vec{x} = 0$$

Gaussian random field

$$\mathcal{P}(\delta_1, \delta_2, ..., \delta_N) = \frac{\exp(-Q)}{[(2\pi)^N \det(\mathcal{C})]^{1/2}}$$

$$Q \equiv \frac{1}{2}\sum_{i,j} \delta_i\,(\mathcal{C}^{-1})_{ij}\delta_j$$

$$\mathcal{C}_{ij} = \langle\delta_i\delta_j\rangle = \xi(r_{12})$$

two-point correlation function

$$\langle\delta_1\,\delta_2\rangle \equiv \xi(\vec{r}_{12}) = \xi(r_{12})$$

cosmological principle: isotropy

$$1 + \xi(r) = \frac{n_{\rm pair}(r \pm \mathrm{d}r)}{n_{\rm random}(r \pm \mathrm{d}r)}$$

Power spectrum & transfer function

$$P(k,t) = P_{\rm i}(k)\,T^2(k)\,D^2(t)$$

$$\Delta^2(k) \equiv \frac{1}{2\pi^2}\,k^3\,P(k)$$

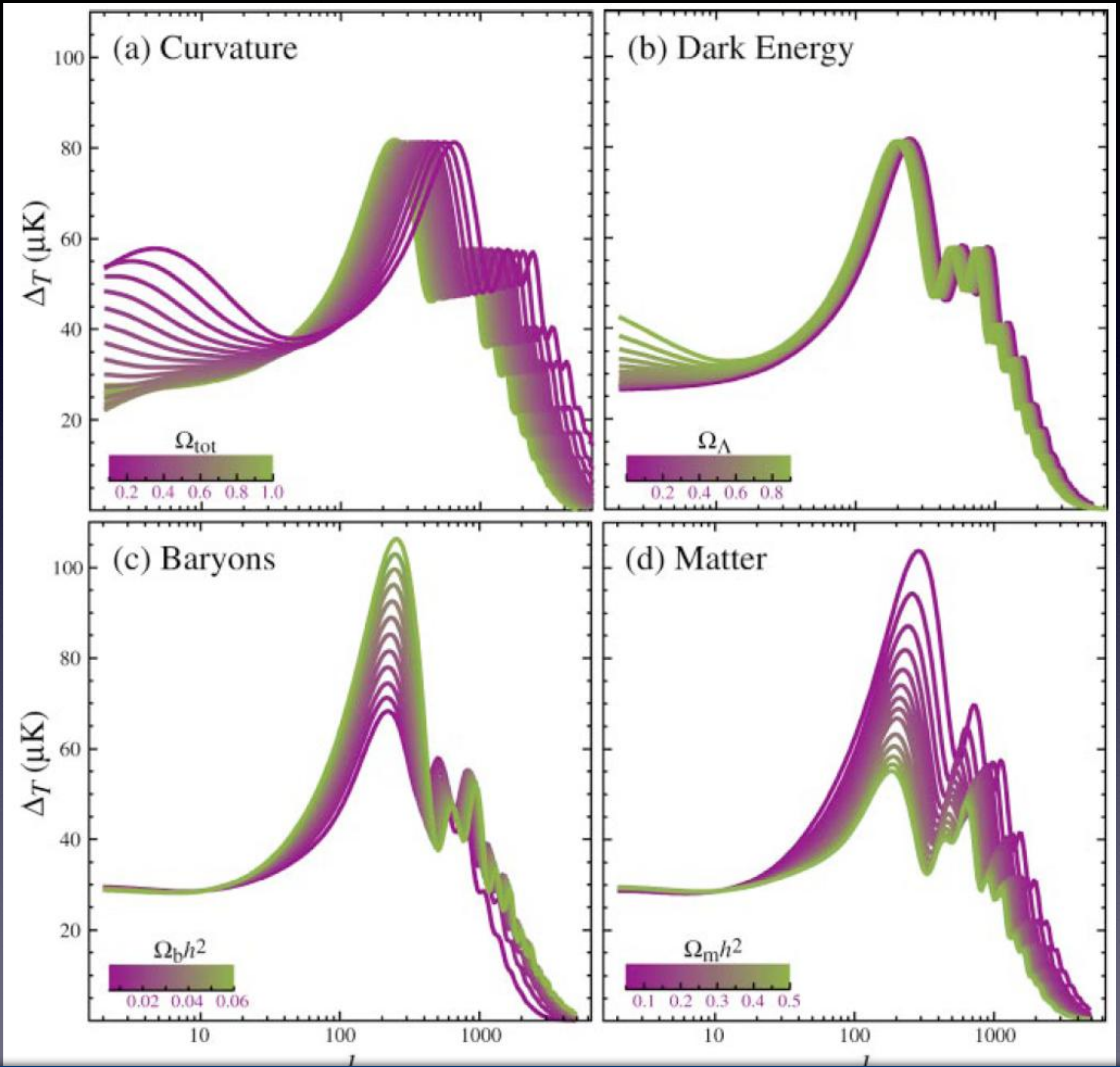
dimensionless power spectrum

$$T(k) = \frac{\Phi_{\vec{k}}(a_{\rm m})}{\Phi_{\vec{k}}(a_{\rm i})}$$

$$P_{\rm i}(k) = \langle|\delta_{\vec{k}}(a_{\rm i})|^2\rangle = \frac{4}{9}\frac{k^4\langle|\Phi_{\vec{k},\,\rm i}|^2\rangle}{\Omega^2_{\rm m,0}\,H_0^4} = \frac{4}{9}\frac{k^4\,P_{\Phi,\rm i}(k)}{\Omega^2_{\rm m,0}\,H_0^4}$$

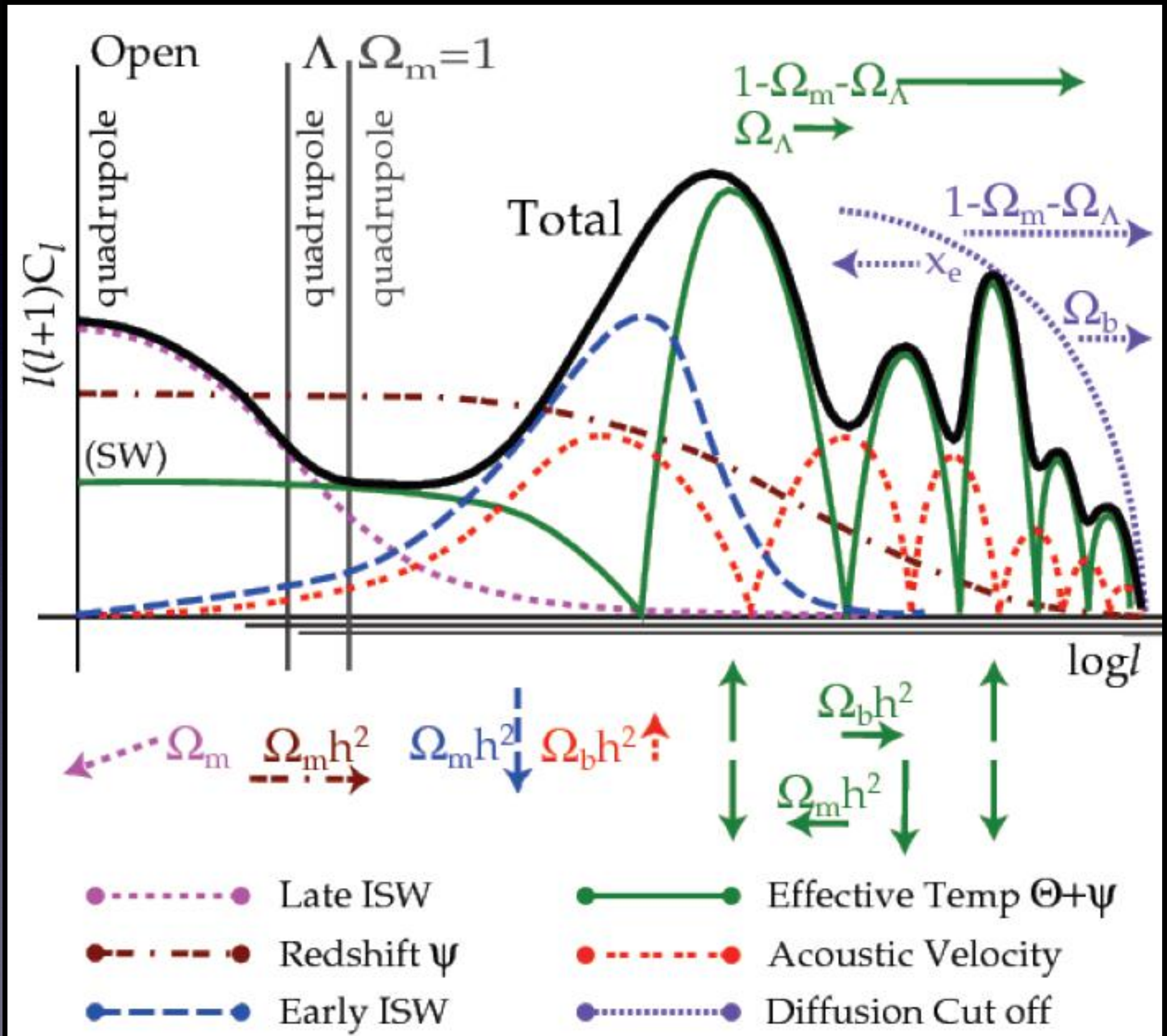
The transfer function T(k) is independent of $a_{\rm m}$ as long as $\Omega(a_{\rm m})\simeq 1$

CMB Summary



Hu & Dodelson 2002

CMB Summary



Open
Λ
$\Omega_m=1$
quadrupole
quadrupole
quadrupole
$l(l+1)C_l$
$1-\Omega_m-\Omega_\Lambda$
Ω_Λ
Total
$1-\Omega_m-\Omega_\Lambda$
x_e
Ω_b
(SW)
log$l$
Ω_m
$\Omega_m h^2$
$\Omega_m h^2$
$\Omega_b h^2$
$\Omega_b h^2$
$\Omega_m h^2$
Late ISW
Redshift ψ
Early ISW
Effective Temp Θ+ψ
Acoustic Velocity
Diffusion Cut off