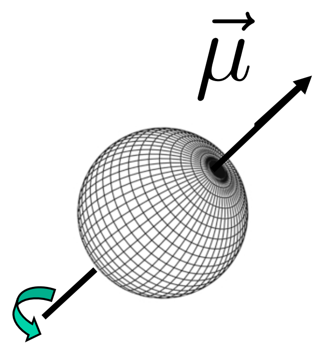
Lecture #2
Review of Classical MR

- Topics
  - Nuclear magnetic moments
  - Bloch Equations
  - Imaging Equation
  - Extensions
- Handouts and Reading assignments
  - van de Ven: Chapters 1.1-1.9
  - de Graaf, Chapters 1, 4, 5, 10 (optional).
  - Bloch, "Nuclear Induction", *Phys Rev*, **70**:460-474, 1946
  - Historical Notes
    - Lauterbur, "Image Formation by Induced Local Interactions: Examples Employing Nuclear Magnetic Resonance", *Nature* **242**:190-191, 1973.
    - Mansfield and Grannell, "NMR 'Diffraction' in solids?", *J. Phys. C: Solid State Phys.*, 6:L422-L426, 1973.

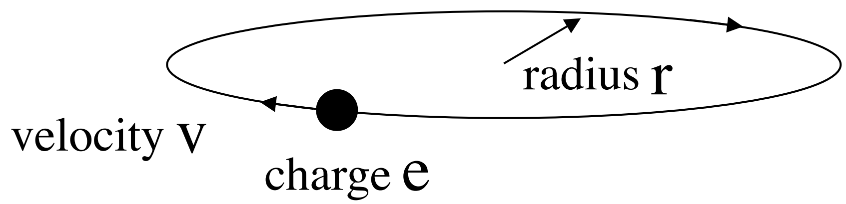
Spin

- Protons (as well as electrons and neutrons) possess intrinsic angular momentum called "spin"
- Spin gives rise to a magnetic dipole moment
- Useful (though not entirely accurate) to think of a proton as a spinning or rotating charge generating a current, which, in turn, produces a magnetic moment.



Nuclear Magnetic Moment

- Consider a point charge in circular motion:



- From EM theory: in the far field a current loop looks just like a magnetic dipole with magnetic moment μ

$$\mu = \text{current} \bullet \text{loop area}$$

$$\mu = \frac{ev}{2\pi r} \cdot \pi r^2 = \underbrace{\frac{e}{2m}}_{\text{gyromagnetic ratio } \gamma} \underbrace{mvr}_{\text{angular momentum } L}$$

- Thus

$$\boxed{\vec{\mu} = \gamma \vec{L}}$$

Gyromagnetic Ratio

- γ often expressed as

$$\gamma = \frac{g\mu_b}{\hbar} \quad \text{where} \quad \mu_b = \frac{e\hbar}{2m} \quad \text{and} \quad g = \text{spin g factor}$$

$g \cong 2$ (electrons)

Planck's constant/2π

Bohr magneton
(m = electron mass)

- For protons

$$\gamma = \frac{g\mu_n}{\hbar} \quad \text{where} \quad \mu_n = \frac{e\hbar}{2m_p} \quad \text{and} \quad g \cong 5.6$$

nuclear magneton

$$\frac{\gamma}{2\pi} = 42.58 \text{ MHz/T}$$

Note, for electron spin: $\dfrac{|\gamma_e|}{\gamma} = 658$ Important for ESR, NMR contrast agents, etc

K. Zavoisky

Nuclear Spin in a Magnetic Field

- In a uniform magnetic field, a magnetic dipole will experience a torque τ

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

Example: compass

- Potential energy given by:

$$E = -\vec{\mu} \cdot \vec{B} = -\mu B \cos\theta$$

angle between μ and B

Classically, energy can take on any value between ± μB

Equation of Motion

- Newton's Law:

$$\frac{d\vec{L}}{dt} = \vec{\tau}$$

- Combining previous equations:

$$\frac{d\vec{\mu}}{dt} = \gamma\vec{\mu} \times \vec{B}$$

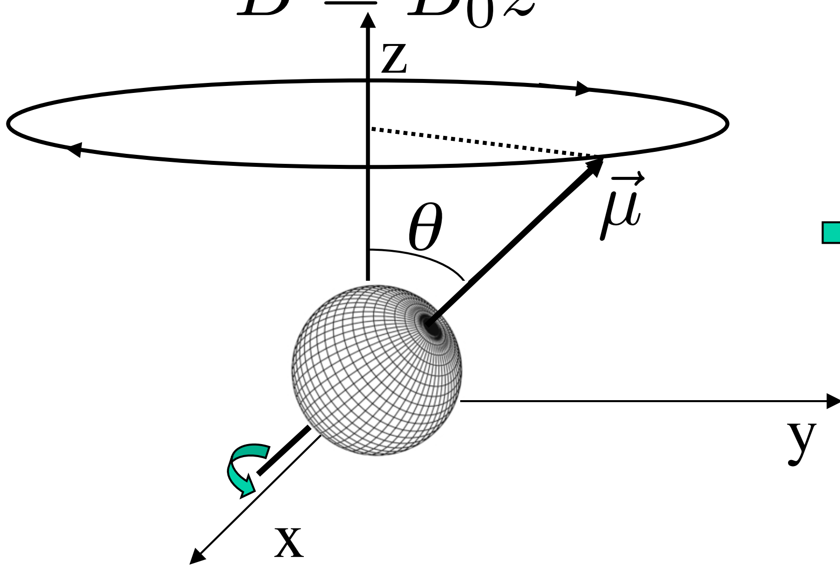
Physical Picture:

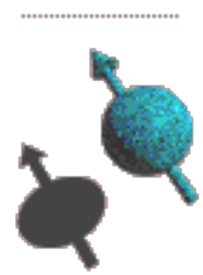
Single Spin in a Uniform Magnetic Field

$$\frac{d\vec{\mu}}{dt} = \gamma \vec{\mu} \times \vec{B}$$

Note: $|\vec{\mu}|$ = constant

$$\vec{B} = B_0 \hat{z}$$





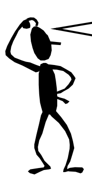
Precession frequency

$$\omega_0 \equiv \gamma B_0$$

Note: Some texts use $\omega_0 = -\gamma B_0$.

The Larmor Equation

Sir Joseph Larmor



A compass needle has a magnetic moment and sits in the earth's field. Why doesn't it precess?

Net Magnetization

- In tissue, we are always dealing with a large number of nuclei.

- Net magnetization: $\vec{M} = \sum_{\text{volume}} \vec{\mu}$

- Hence,

$$\boxed{\frac{d\vec{M}}{dt} = \gamma \vec{M} \times \vec{B}}$$

Equation is valid for non-interacting spins

Bloch Equations

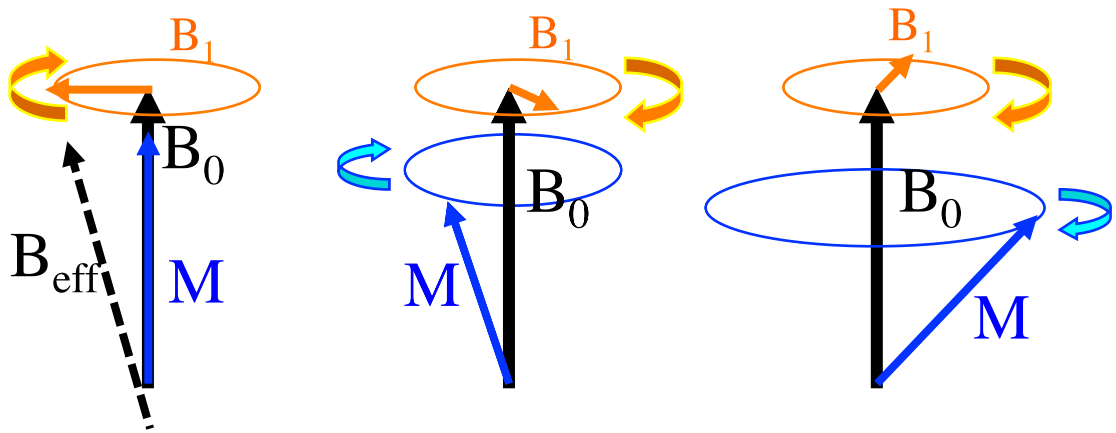
- In order to account for inter- and intra-molecular interactions, we can introduce exponential transverse (T_2) and longitudinal (T_1) relaxation time constants.

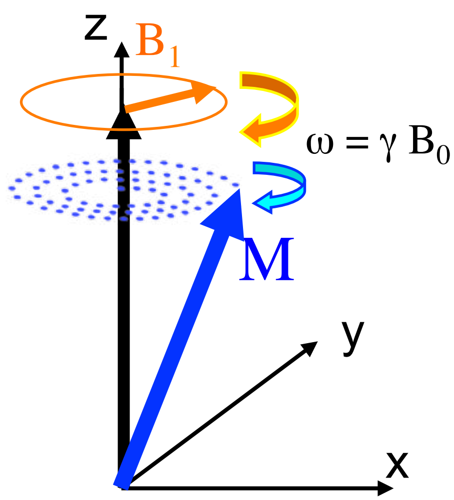
- For $\vec{B} = B_0\hat{z}$

$$\boxed{\frac{d\vec{M}}{dt} = \gamma\vec{M} \times B_0\hat{z} - \frac{M_x\hat{x} + M_y\hat{y}}{T_2} - \frac{\left(M_z - M_0\right)\hat{z}}{T_1}}$$

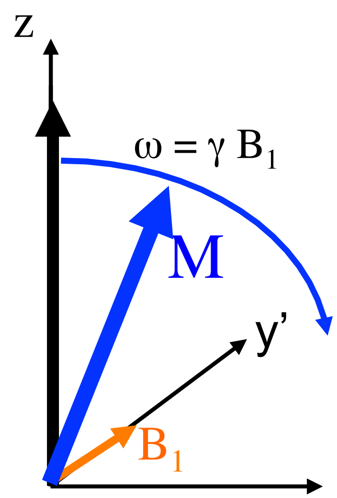
Rf Excitation

- Apply a small magnetic field B_1, $\perp$ to B_0, rotating at the Larmor frequency.
- Sample magnetization reacts by nutating away from B_0.

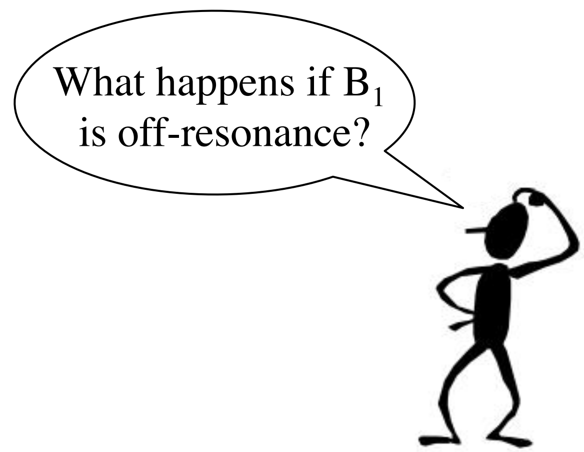




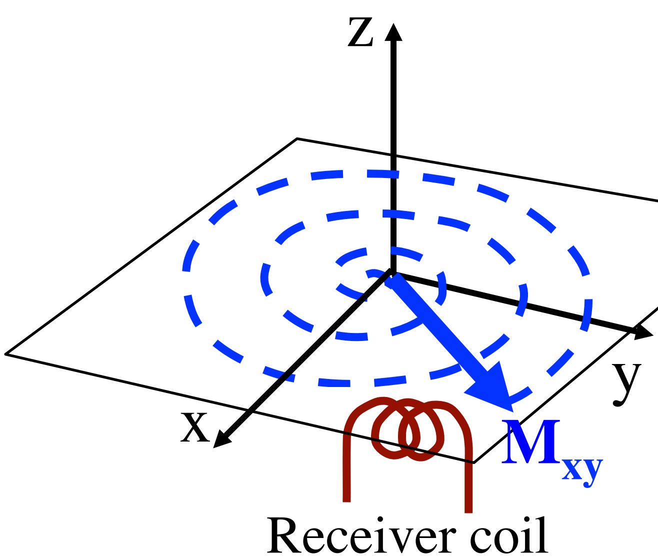
Laboratory frame of reference



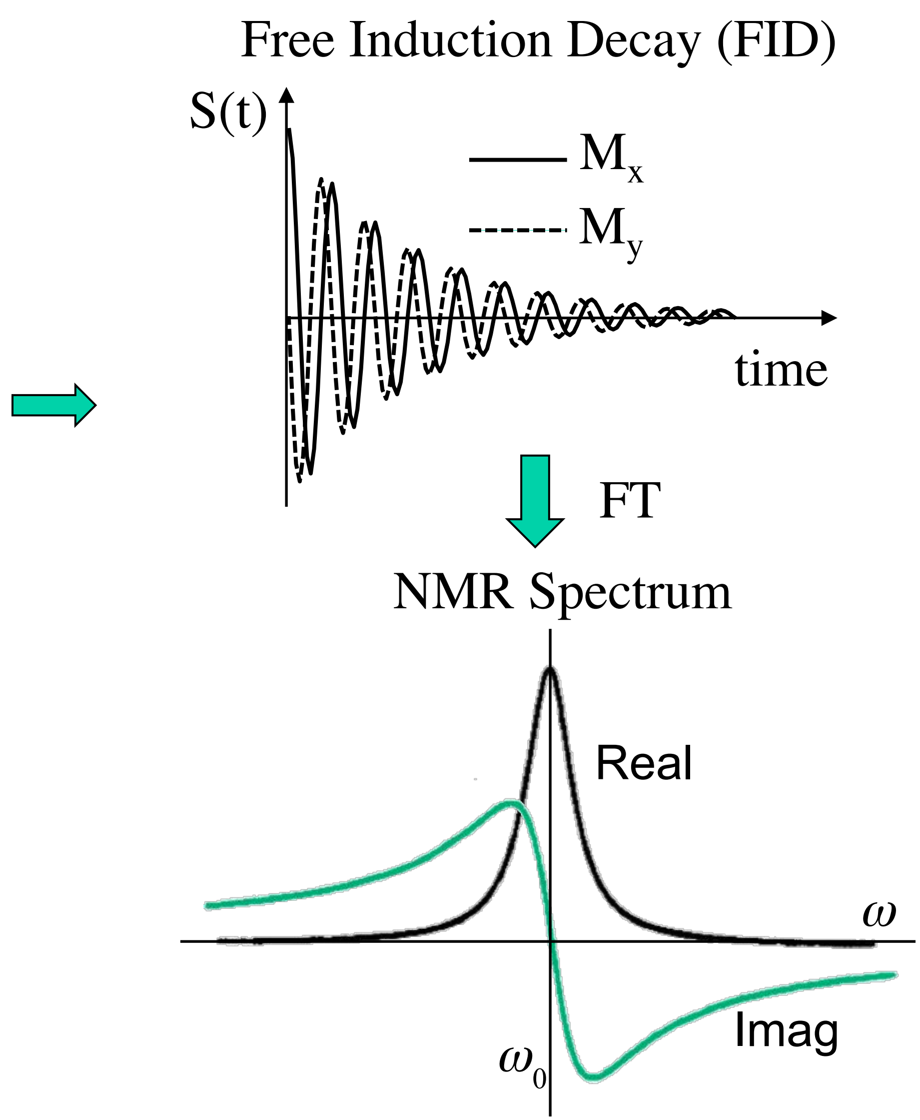
Rotating frame of reference



Rf Reception



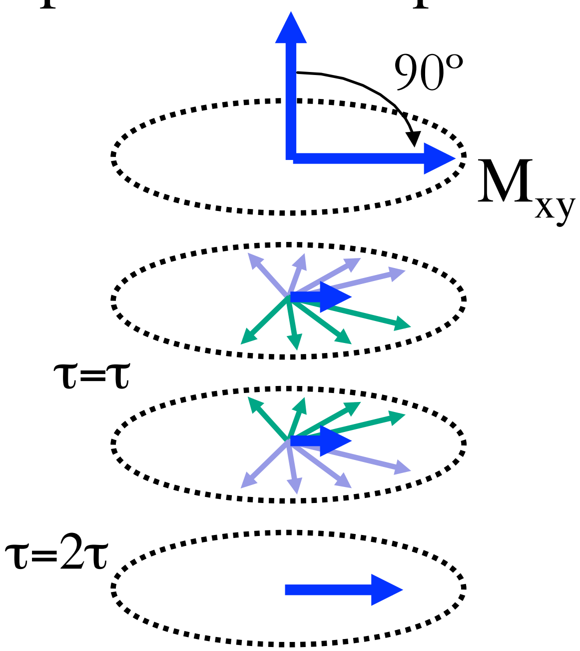
$$\text{voltage} \propto \frac{dM_{xy}}{dt}$$

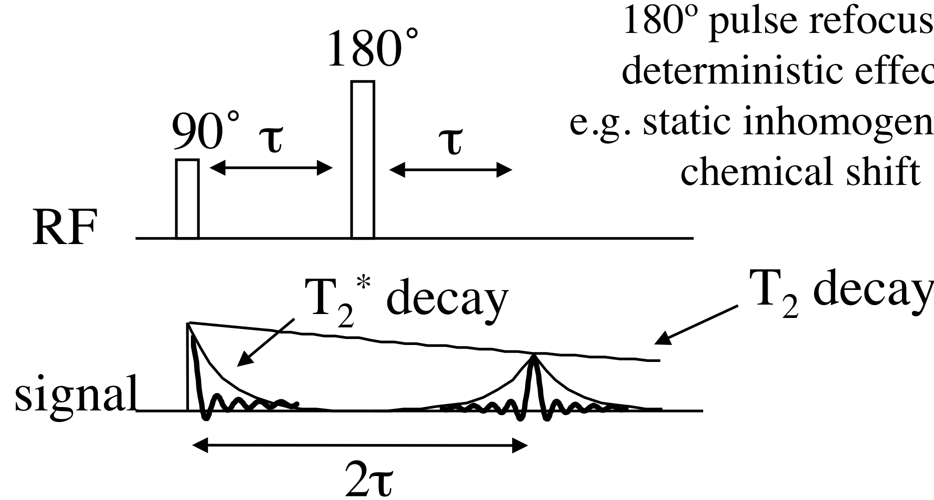


Spin Echoes

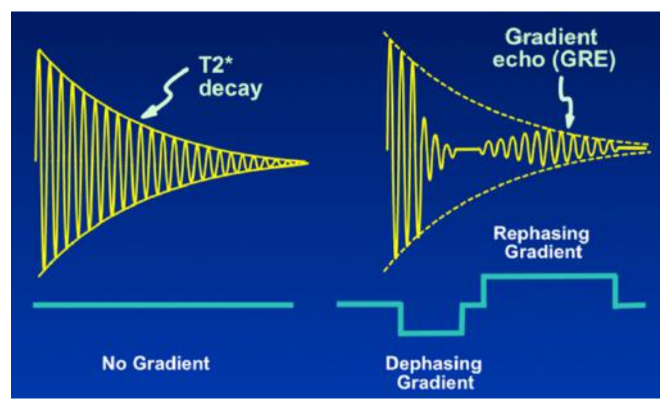
- Bloch: M_{xy} decays with a time constant T_2

- Spin echo example





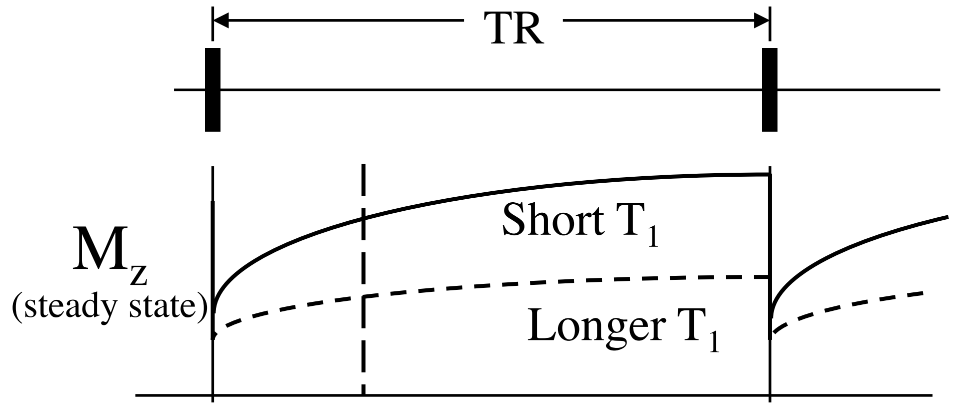
180° pulse refocuses deterministic effects: e.g. static inhomogeneities, chemical shift



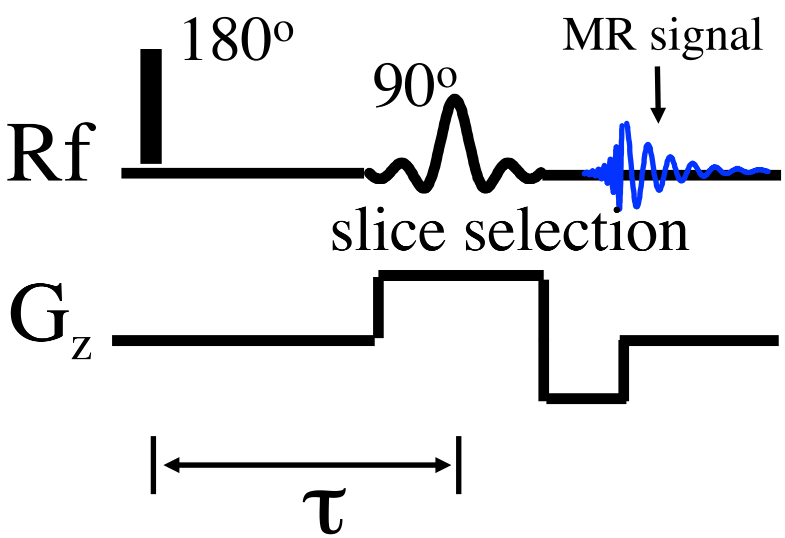
- M_{xy} decay is a dephasing process
- Echoes = “phase coherences”

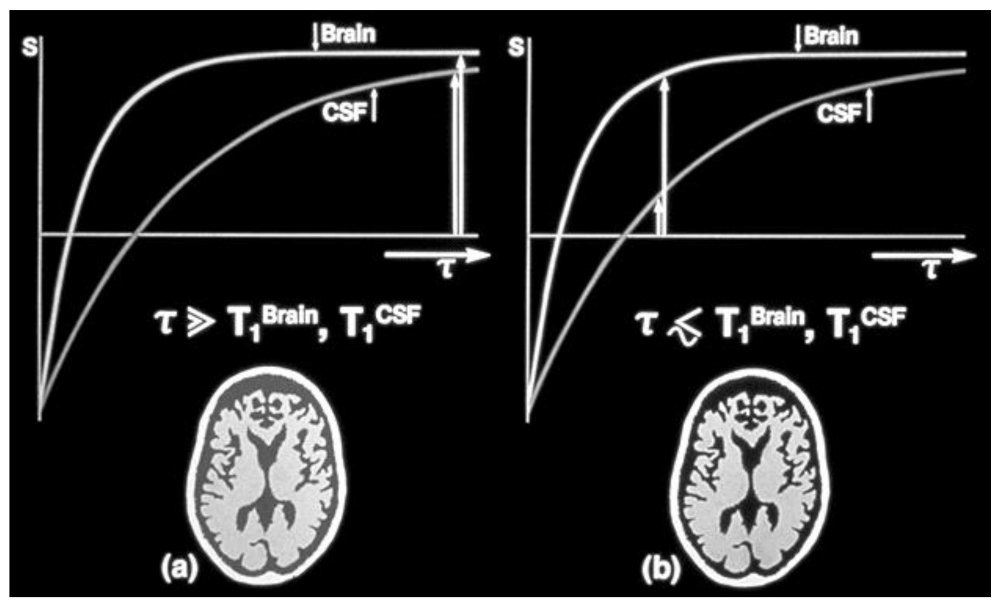
Longitudinal Magnetization

- M_z recovers exponentially to M_0 with time constant T_1
- GRE example

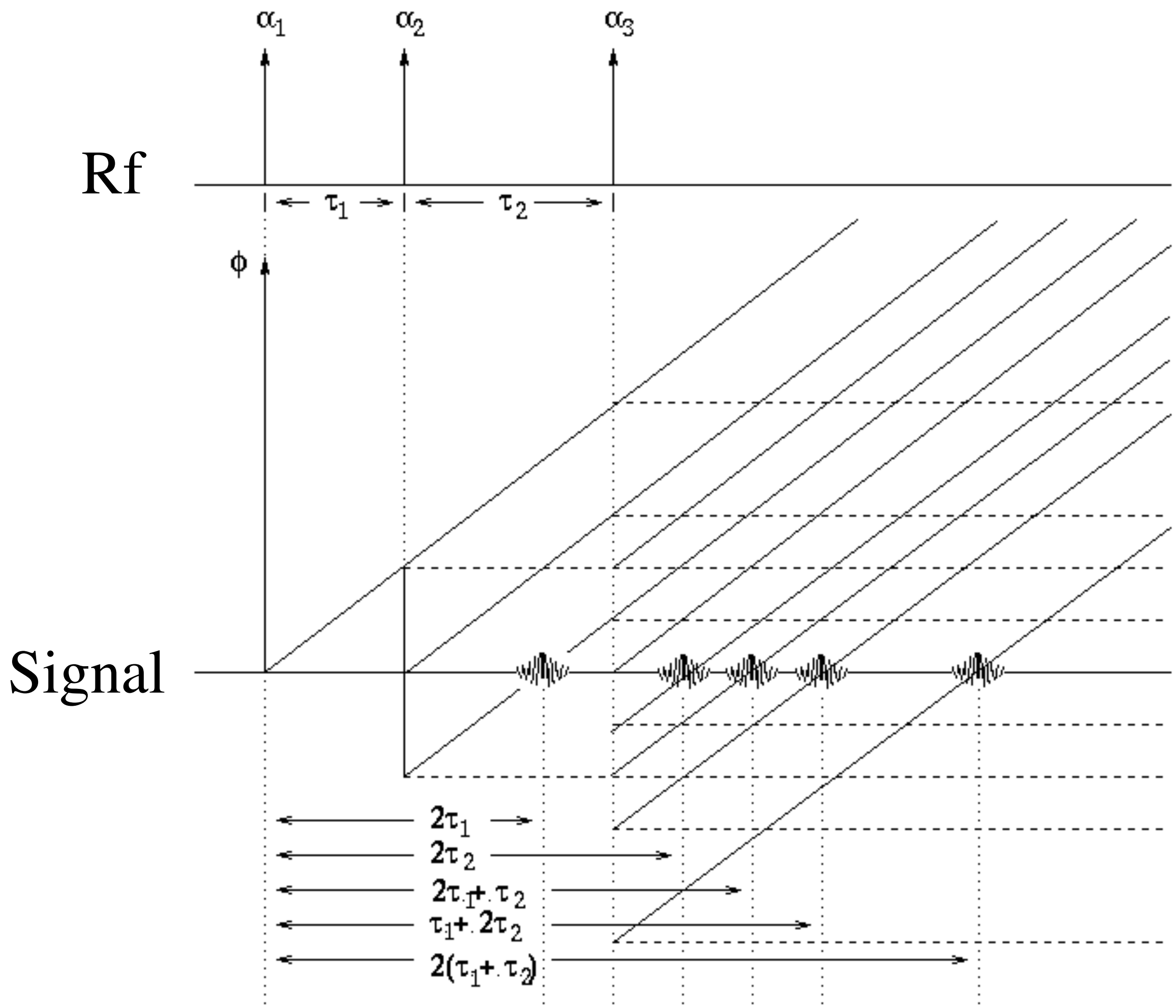


- Inversion recovery example





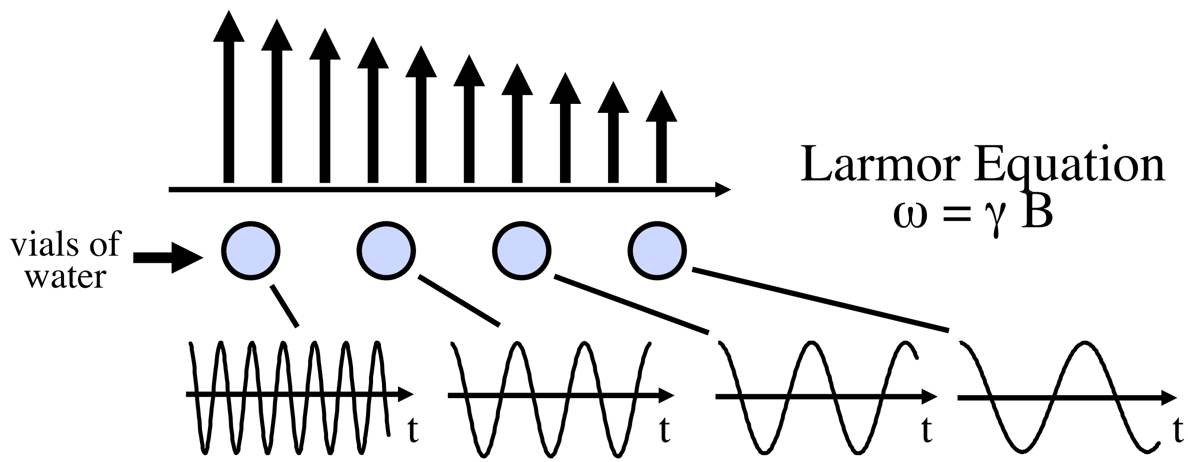
Extended phase graphs



Spatial Localization

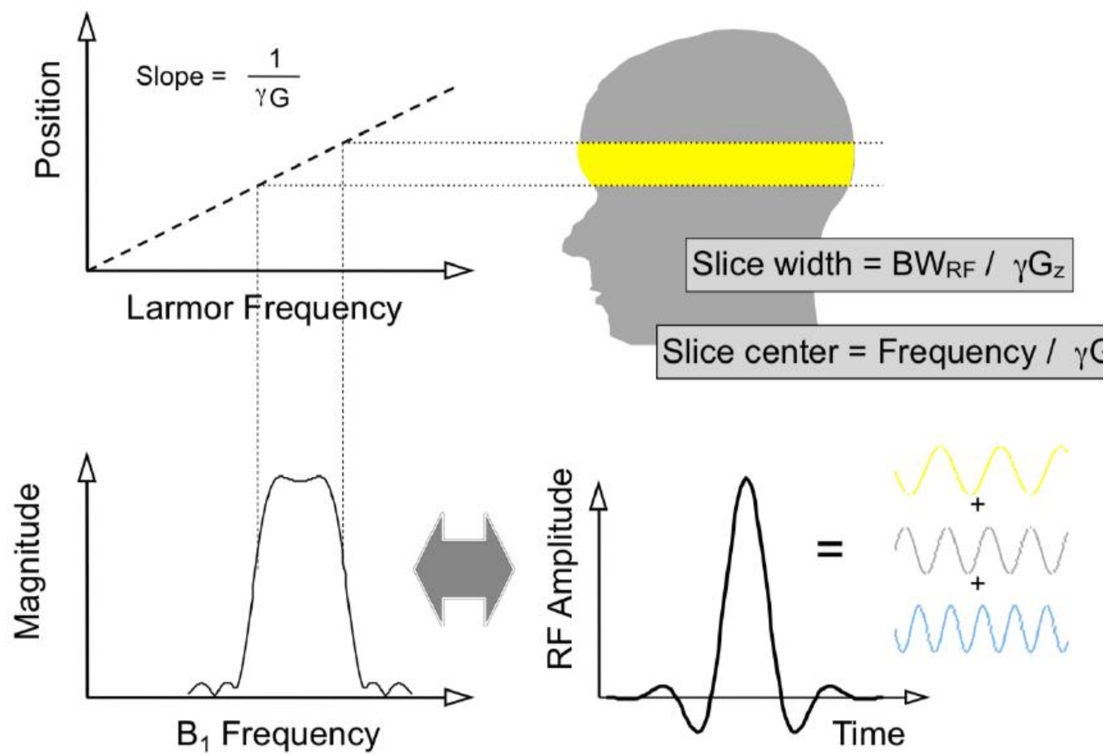
- Linear Gradients

$$B = B_0 + G_r r$$
for r = x, y, or z

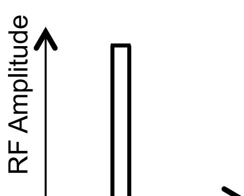


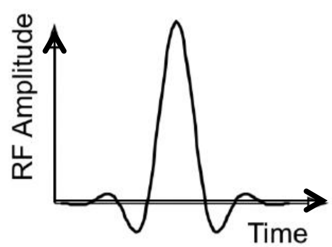
Larmor Equation
$\omega = \gamma B$

- Selective Excitation



"Hard" vs "Soft"

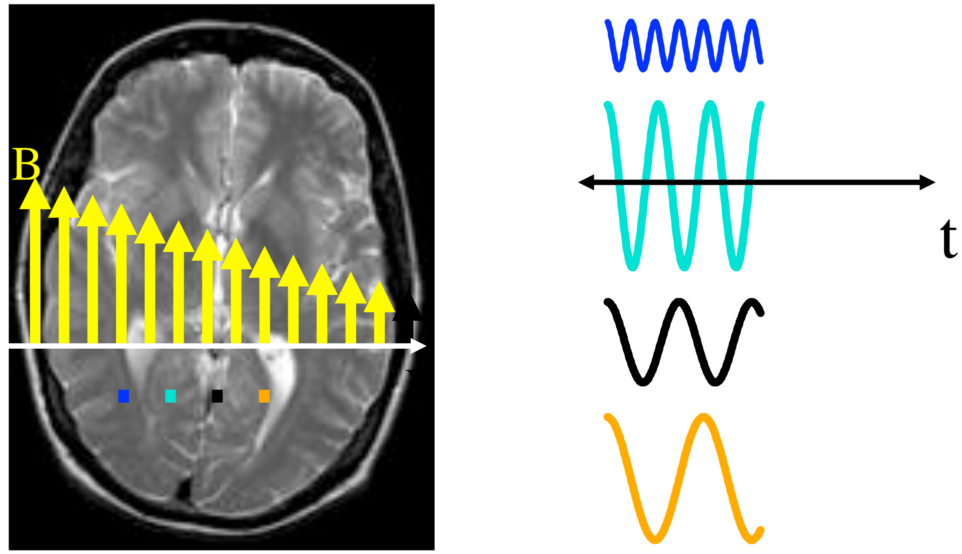




Spatial Localization

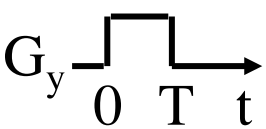
- Readout

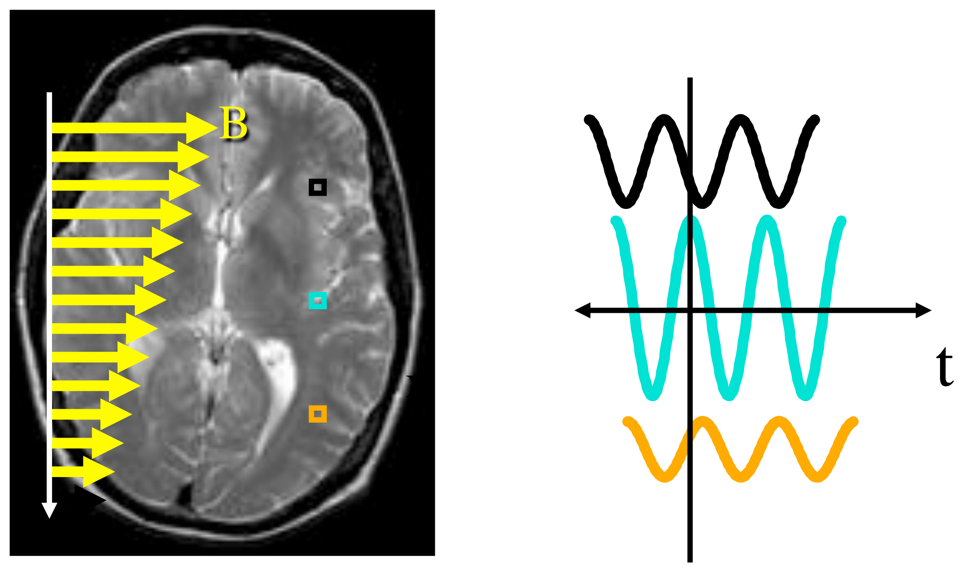
$$\omega = \gamma\,(B_0 + G_x\,x)$$



- Phase encode

$$\omega = \gamma\,(B_0 + G_y\,y)$$





Note: all gradient fields point in z direction!

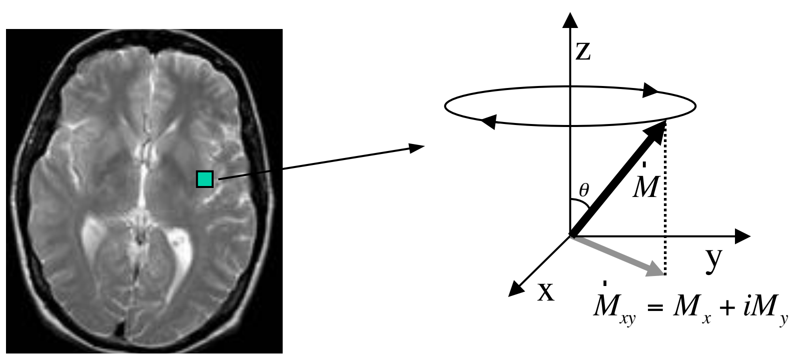
MRI: The Signal Equation

- Let $\vec{B} = B\hat{z}$

- Following RF excitation (a topic we'll revisit) and using:

$$\frac{d\vec{M}}{dt} = \gamma\vec{M} \times B\hat{z} \qquad \text{(ignores relaxation)}$$

➡ each small tissue volume looks like a tiny oscillating magnetic dipole.



$$M_{xy}(x,y,z)e^{-i\phi(x,y,z,t)}$$

MRI: The Signal Equation

- Assuming a uniformly sensitive RF coil, the received signal is given by:

$$s(t) = \int_x \int_y \int_z M_{xy}(x,y,z) e^{-i\phi(x,y,z,t)} dxdydz$$

- Instantaneous frequency: $\omega = d\phi/dt$

$$\phi(x,y,z,t) = \int_0^t \omega(x,y,z,t')dt' = \gamma \int_0^t B(x,y,z,t')dt'$$

P. Lauterbur P. Mansfield

- In the presence of linear gradients:

$$B(x,y,z,t) = B_0 + G_x(t)x + G_y(t)y + G_z(t)z$$

demodulate at γB_0

$$s_e(t) = \cancel{e^{-i\gamma B_0 t}} \int_x \int_y \int_z M_{xy}(x,y,z) e^{-i\gamma \int_0^t \left(G_x(t')x + Gy(t')y + Gz(t')z\right)dt'} dxdydz$$

k-space

Comparing...

$$s_e(t) = \int_x \int_y \int_z M_{xy}(x,y,z)e^{-i\gamma \int_0^t \left(G_x(t')x + Gy(t')y + Gz(t')z\right)dt'} dxdydz$$

received signal

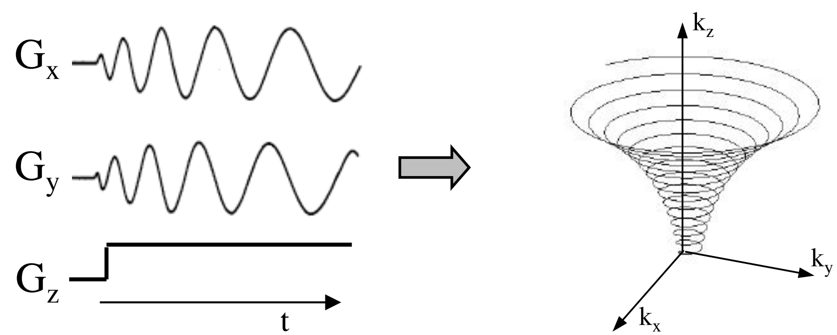
$$\mathsf{M}(k_x,k_y,k_z) = \int_x \int_y \int_z M_{xy}(x,y,z)e^{-i2\pi\left(k_x x + k_y y + k_z z\right)} dxdydz$$

FT of M_{xy}

➡ k-space interpretation of MRI.

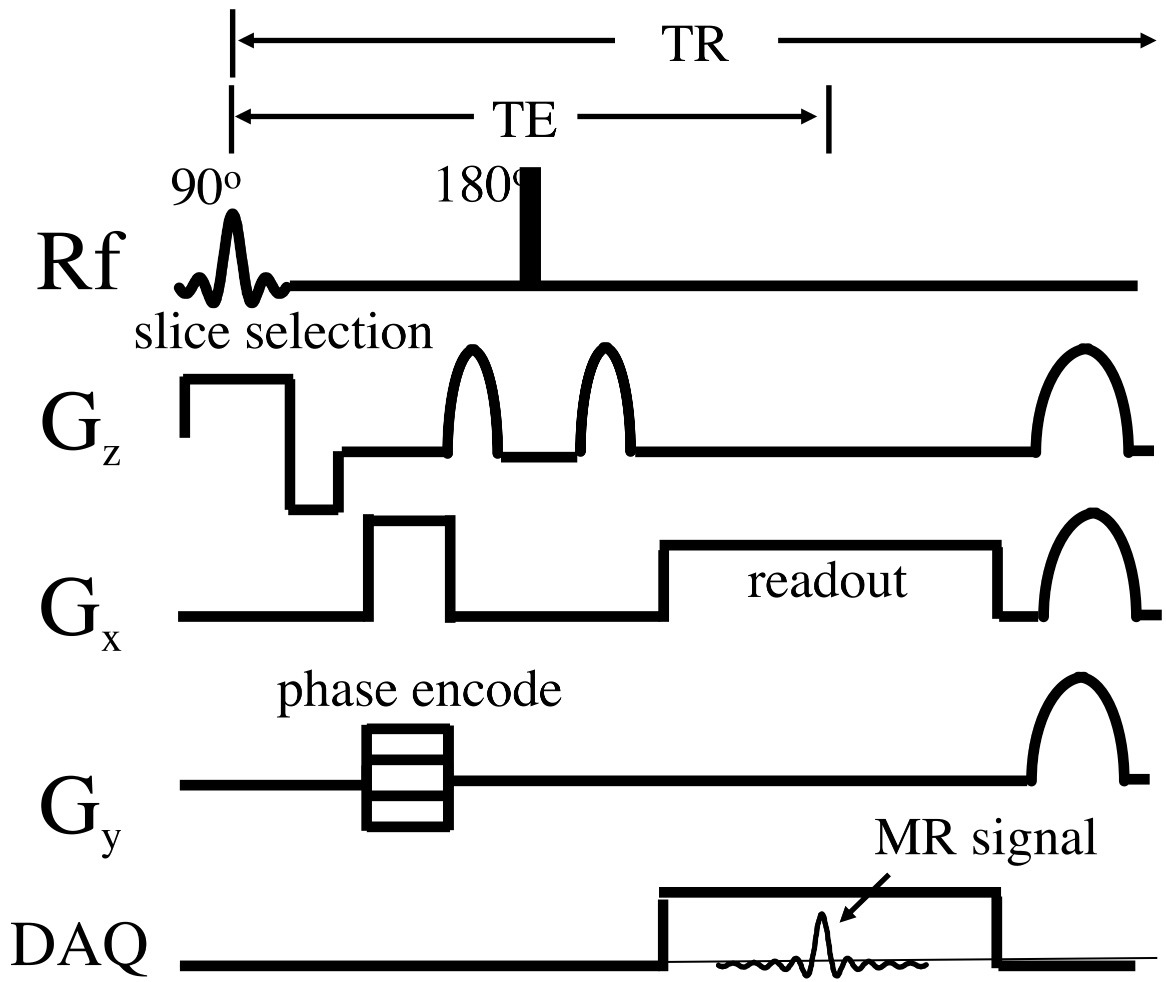
$$s_e(t) = \mathsf{M}\left(\underbrace{\frac{\gamma}{2\pi}\int_0^t G_x(t')dt'}_{k_x}, \underbrace{\frac{\gamma}{2\pi}\int_0^t G_y(t')dt'}_{k_y}, \underbrace{\frac{\gamma}{2\pi}\int_0^t G_z(t')dt'}_{k_z}\right)$$

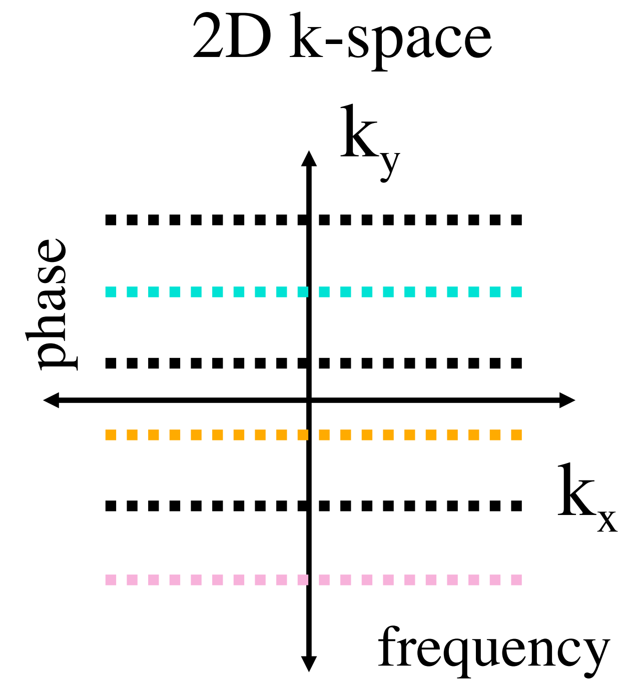
Gradients trace a trajectory through k-space



Pulse sequence diagrams

Example – Spin Echo Imaging





Summary

- The Bloch equation

$$\frac{d\vec{M}}{dt} = \gamma \vec{M} \times B_0 \hat{z} - \frac{M_x \hat{x} + M_y \hat{y}}{T_2} - \frac{\left(M_z - M_0\right)\hat{z}}{T_1}$$

- Rf excitation:

  Longitudinal magnetization ⟺ Transverse magnetization

- Signal equation

$$s_e(t) = \int_x \int_y \int_z M_{xy}(x,y,z) e^{-i\gamma \int_0^t \left(\vec{G} \cdot \vec{r}\right) dt'} dxdydz$$

- What's missing?

T_1 and T_2

Can be included as k-space weightings:

$$s_e(t) = \sum_j \int_r M_{xy}(x, y, z, T_{1j}, T_{2j}) e^{-t/T_{2j}} e^{-i\gamma \int_0^t \mathbf{G} \cdot \mathbf{r} dt'} dr$$

but ...

What are the underlying mechanisms?

Why do different tissues have different T_1s and T_2s?

How do contrast agents work?

Chemical Shift

- Interaction between electron cloud and B_0

$$\omega = \gamma B_{eff} = \gamma B_0(1-\sigma)$$

shielding constant

Depends on: electron density
molecular geometry, etc

- Looks like a new k-space axis

$$s_e(t) = \int_\omega \int_r M_{xy}(x,y,z,\omega) e^{-i\gamma \int_0^t \mathbf{G}\cdot\mathbf{r}\,dt'} e^{-i\omega t}\, dr\, d\omega$$

$$s_e(t) = \mathsf{M}\left(\underbrace{\frac{\gamma}{2\pi}\int_0^t G_x(t')dt'}_{k_x}, \underbrace{\frac{\gamma}{2\pi}\int_0^t G_y(t')dt'}_{k_y}, \underbrace{\frac{\gamma}{2\pi}\int_0^t G_z(t')dt'}_{k_z}, \underbrace{\frac{t}{2\pi}}_{k_\omega} \right)$$

Note, same equation also holds for B_0 inhomogeneity (due to magnetic susceptibility, etc)

But what about …

- Coupling between spins
- Chemical exchange
- Nuclei with spin $\neq 1/2$
- etc.

Next Lecture: Introduction to Quantum Mechanics

Biography: Sir Joseph Larmor

Joseph Larmor (1857-1942) was educated at the Royal Belfast Academical Institution and the Queens College Belfast. He then took another degree at St. Johns College Cambridge, as was common for promising young students from provincial universities. He won top prize at the final mathematical examination in Cambridge. This was the second year in a row that a student from Belfast had been crowned "senior wrangler". Larmor then returned to Ireland as Professor of Natural Philosophy at Queens College Galway. He held this position for five years but then returned to Cambridge to take up a new Mathematics position and he was later appointed to the prestigious Lucasian Chair of Mathematics. Larmor is well known for his contributions to the theory of electromagnetism, in particular the electron theory of matter. Larmor published his collected papers on electromagnetism in 1900 in a famous book entitled "Aether and Matter". Larmor's work, though rooted in the classical physics in which he had been trained, eventually led to the breakdown of classical physics and the rise of relativity theory and quantum mechanics.

He was described as 'one who rekindled the dying embers of the old physics to prepare the advent of the new'. Larmor saw himself as part of an Irish scientific tradition and was involved in editing the collected works of a number of Irish scientists. Larmor spent most of his career in Great Britain but returned to Ireland most summers and moved back permanently after his retirement from the Lucasian chair. He was committed to the Union of Ireland with Great Britain and this led him to serve in Parliament as a member for Cambridge University from 1911 to 1922.

Biography: Sir Peter Mansfield

(born October 9, 1933, London, England) English physicist who, with American chemist Paul Lauterbur, won the 2003 Nobel Prize for Physiology or Medicine for the development of magnetic resonance imaging (MRI), a computerized scanning technology that produces images of internal body structures, especially those comprising soft tissues. Mansfield received a Ph.D. in physics from the University of London in 1962. Following two years as a research associate in the United States, he joined the faculty of the University of Nottingham, where he became professor in 1979. Mansfield was knighted in 1993. Mansfield's prize-winning work expanded upon nuclear magnetic resonance (NMR), which is the selective absorption of very high-frequency radio waves by certain atomic nuclei subjected to a strong stationary magnetic field. A key tool in chemical analysis, it uses the absorption measurements to provide information about the molecular structure of various solids and liquids. In the early 1970s Lauterbur laid the foundations for MRI after realizing that if the magnetic field was deliberately made nonuniform, information contained in the signal distortions could be used to create two-dimensional images of a sample's internal structure. Mansfield transformed Lauterbur's discoveries into a practical technology in medicine by developing a way of using the nonuniformities, or gradients, introduced in the magnetic field to identify differences in the resonance signals more precisely. He also created new mathematical methods for quickly analyzing information in the signal and showed how to attain extremely rapid imaging. Because MRI does not have the harmful side effects of X-ray or computed tomography (CT) examinations and is noninvasive, the technology proved an invaluable tool in medicine.

Biography: Paul Lauterbur

American chemist (born May 6, 1929 , Sidney, Ohio—died March 27, 2007 , Urbana, Ill.) won the Nobel Prize for Physiology or Medicine in 2003, together with British physicist Sir Peter Mansfield, for the development of magnetic resonance imaging (MRI), a computerized scanning technology that produces images of internal body structures, especially those comprising soft tissues. Lauterbur received a Ph.D. (1962) in chemistry from the University of Pittsburgh. He served as a professor at the State University of New York at Stony Brook from 1969 to 1985, when he accepted the position of professor at the University of Illinois at Urbana-Champaign and director of its Biomedical Magnetic Resonance Laboratory. In the early 1970s Lauterbur began using nuclear magnetic resonance (NMR), which is the selective absorption of very-high-frequency radio waves by certain atomic nuclei subjected to a strong stationary magnetic field. NMR is a key tool in chemical analysis, using the absorption measurements to provide information about the molecular structure of various solids and liquids. Lauterbur realized that if the magnetic field was deliberately made nonuniform, information contained in the signal distortions could be used to create two-dimensional images of a sample's internal structure. This discovery laid the groundwork for the development of MRI as Mansfield transformed Lauterbur's work into a practical medical tool. Noninvasive and lacking the harmful side effects of X-ray and computed tomography (CT) examinations, MRI became widely used in medicine.