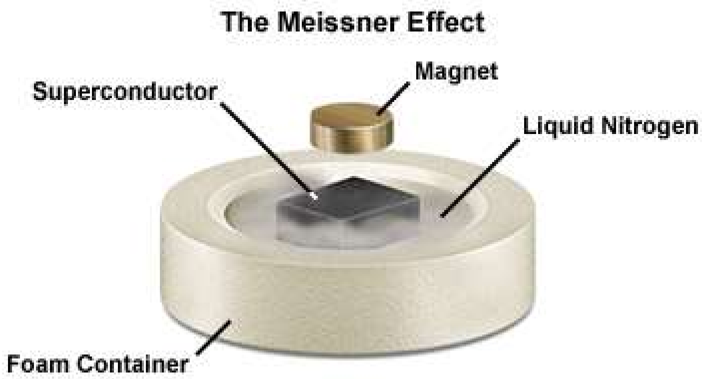
Spontaneous symmetry breaking in condensed matter systems

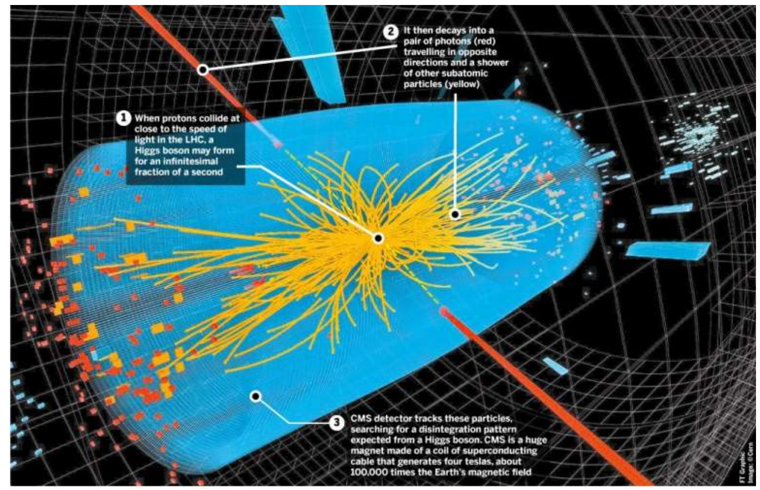


The New York Times
The Opinion Pages

OP-ED CONTRIBUTOR

Why the Higgs Boson Matters

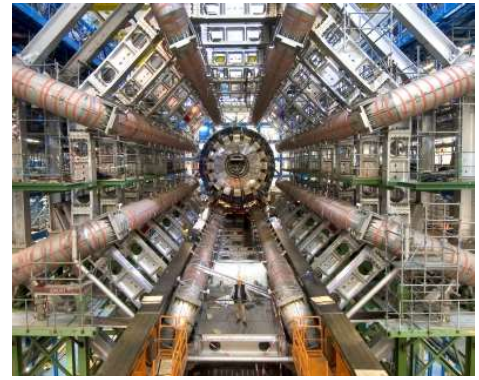
Daniel Haskett

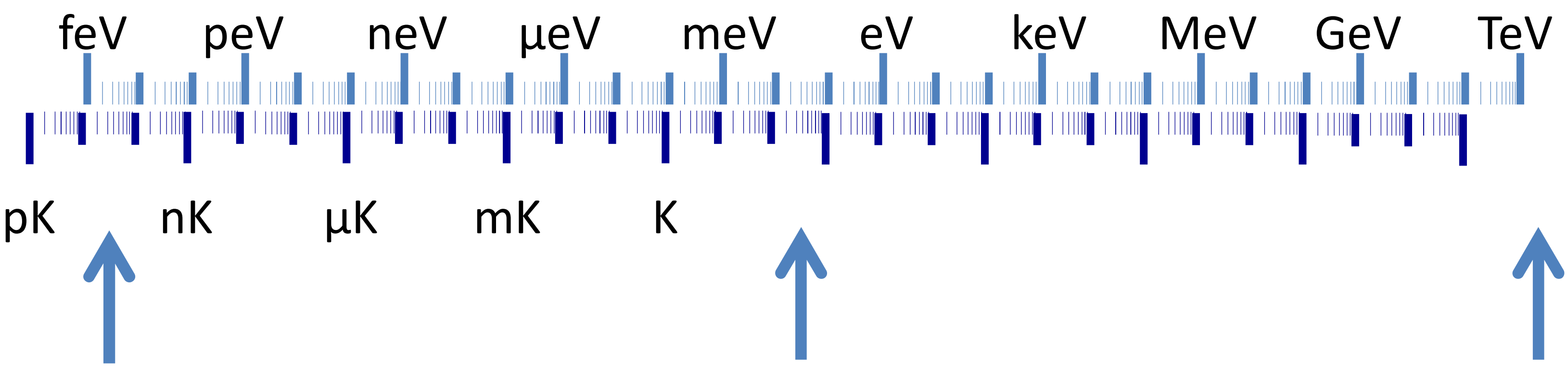


By STEVEN WEINBERG
Published: July 13, 2012

Salam and I assumed that the culprit is what are called scalar fields, which pervade all space. This is like what happens in a magnet: Even though the equations describing iron atoms don't distinguish one direction in space from another, any magnetic field produced by the atoms will point in just one way. The symmetry-breaking fields in the Standard Model do not mark out directions in space — instead, they distinguish the weak from the electromagnetic forces, and give elementary particles their masses. Just as a magnetic field appears in iron when it cools and solidifies, these scalar fields appeared as the early universe expanded and cooled.

Universality of collective modes

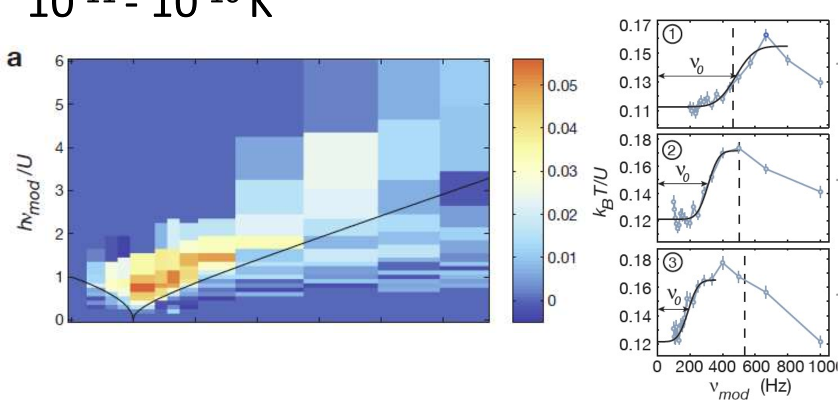




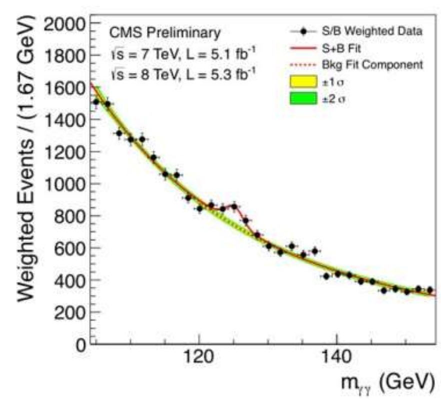
Cold atoms experiments
10^{-11} - 10^{-10} K

room temperature

LHC



Higgs mode in ultracold atoms, 2012

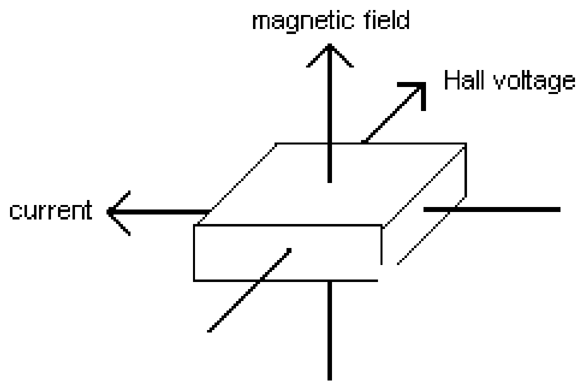


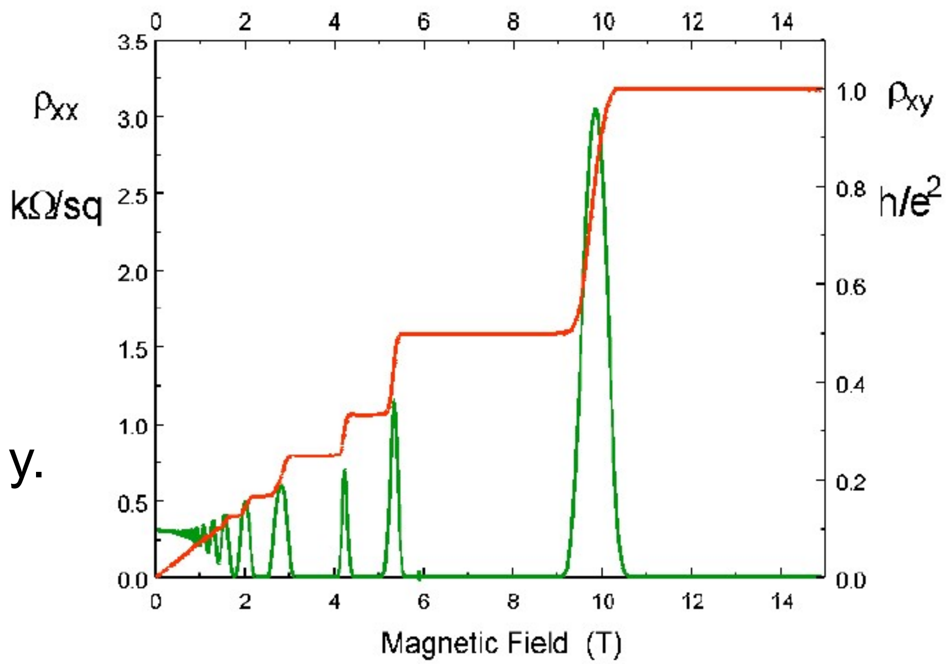
Higgs mode of the standard model, 2012

Order beyond symmetry breaking

In 1980 the first ordered phase beyond symmetry breaking was discovered

Integer Quantum Hall Effect: 2D electron gas in strong magnetic field show plateaus in Hall conductance





Current along x, measure voltage along y.
On a plateau

$$\sigma_{xy} = n\frac{e^2}{h}$$

with an accuracy of 10^{-9}

What is the "quantum protectorate" of such precise quantization?

Topological order

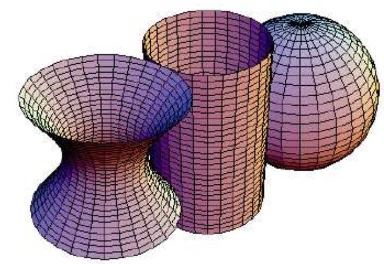
In a topologically ordered state some physical quantity is given by a discreet "topological invariant". Some physical response function is determined by this quantized invariant.

Topological invariant: quantity that does not change under continuous deformations

Example of topological invariant in geometry

Gaussian curvature at every point on a surface

$$\kappa = (r_1 r_2)^{-1}$$

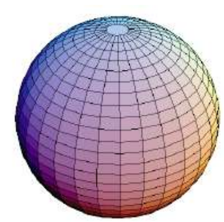


from left to right, equators have negative, 0, positive Gaussian curvature

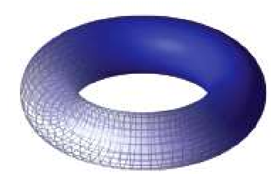
Gauss-Bonnet theorem for closed surfaces

$$\int_M \kappa \, dA = 2\pi\chi = 2\pi(2-2g)$$

g – integer genus of a surface



g=0

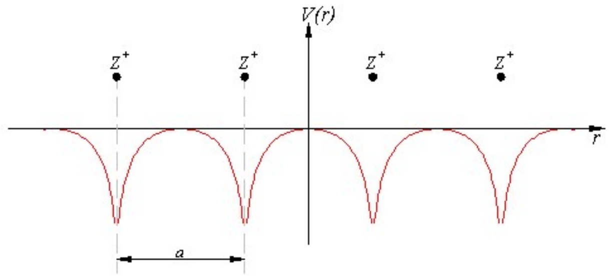


g=1

How to define topological invariant for electrons in solids?

What kind of curvature can exist for electrons in solids?

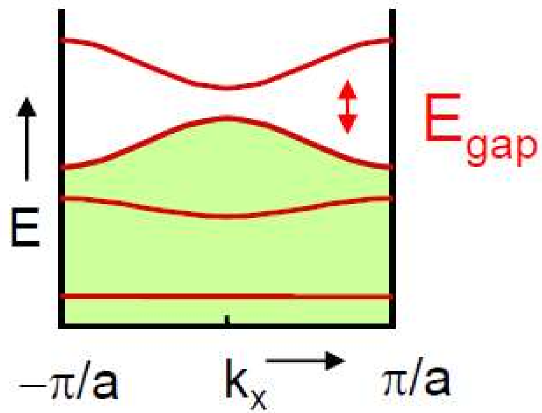
Bloch's theorem and Brillouin zone



One electron wavefunction in a crystal (periodic) potential can be written as

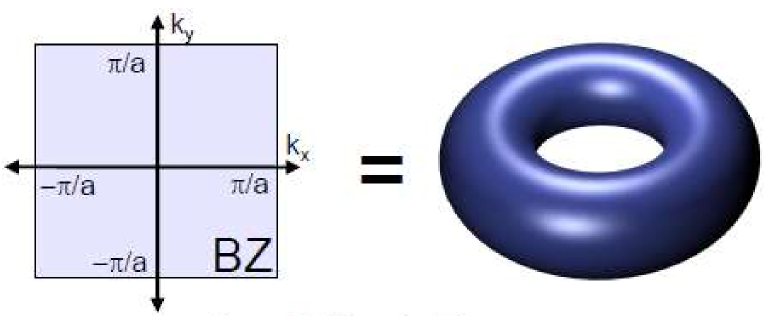
$$\psi(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}} u_{\mathbf{k}}(\mathbf{r})$$

k is "crystal momentum" restricted to Brillouin zone, a region of k-space with periodic boundaries. Function $u_{\mathbf{k}}(\mathbf{r})$ is periodic (same in every unit cell)



As k changes, we map an "energy band". Set of all bands is a "band structure".

But … lattice momentum is periodic

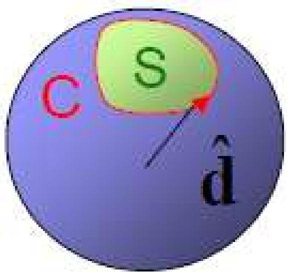


$\mathbf{k} \in$ Brillouin Zone

$=$ Torus, T^d

The Brillouin zone can play the role of the surface. Important property of quantum mechanics, the Berry phase, gives us the "curvature".

Berry phase



Consider a quantum-mechanical system in a nondegenerate ground state, e.g. spin ½ particle in a magnetic field. The adiabatic theorem says that if the Hamiltonian is changed slowly, the system remains in its instantaneous ground state.

Berry phase: when the Hamiltonian goes around a closed loop in parameter space, the system acquires a geometrical phase relative to initial state (in addition to the usual dynamical phase).

$$\phi = \oint \mathcal{A} \cdot d\mathbf{k} \qquad \mathcal{A} = \langle \psi_k | -i\nabla_k | \psi_k \rangle$$

"Gauge transformation" of the Berry phase

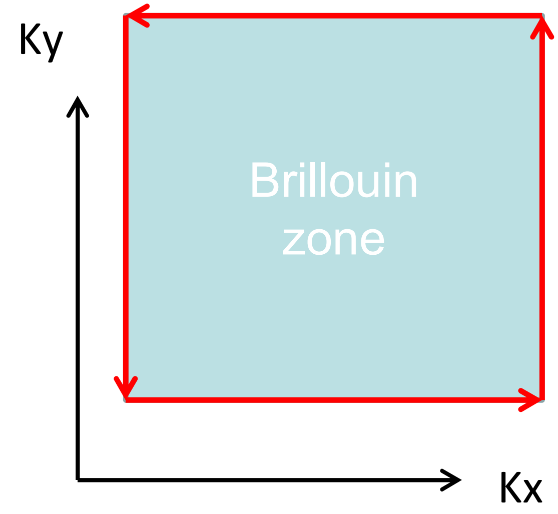
$$\psi_k \to e^{i\chi(k)} \psi_k \qquad \mathcal{A} \to \mathcal{A} + \nabla_k \chi$$

Gauge invariant quantities are Berry curvature and closed loop integrals

$$\mathcal{F} = \nabla \times \mathcal{A} \qquad \oint_C \vec{A}\, d\vec{k} = \int d^2k\, F$$

From Berry phase to Chern number

The change in the electron wavefunction within the Brillouin zone leads to a Berry connection and Berry curvature



$$\psi(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}} u_{\mathbf{k}}(\mathbf{r})$$

$$A(\mathbf{k}) = \sum_{\mathbf{E_n < E_F}} \langle \mathbf{u_n}(\mathbf{k}) | \partial_{\mathbf{k}} | \mathbf{u_n}(\mathbf{k}) \rangle$$

$$F(\mathbf{k}) = \nabla_{\mathbf{k}} \times \mathbf{A}(\mathbf{k})$$

Integral of F is quantized to be integer: first Chern number.
It is like Gauss-Bonnet theorem for the Brillouin zone.

$$n = \sum_{\text{bands}} \frac{1}{2\pi} \int_{\text{BZ}=T^2} F(\vec{k})\, d^2k = \sum_{\text{bands}} \frac{1}{2\pi} \int_C \vec{A}(\vec{k})\, d\vec{k}$$

TKKN quantization of
Hall conductivity for IQHE
Thouless et al., PRL 1982

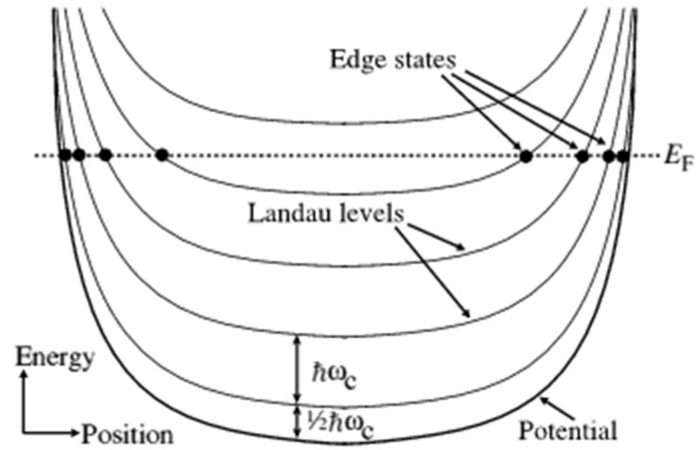
$$\sigma_{xy} = n\frac{e^2}{h}$$

Topological order and edge states

TKNN quantization exists only for insulators with completely filled bands.

How can we get finite conductance in band insulators?

Laughlin and Halperin realized out that conductance goes through gapless edge states. Edge states are "chiral" quantum wires. Each wire contributes one conductance quantum (e^2/h)



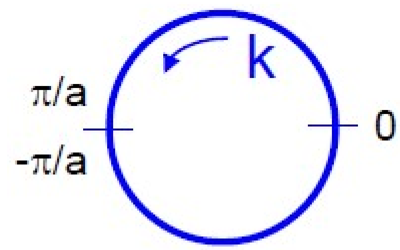
$$\sigma_{xy} = n\frac{e^2}{h}$$

Existence of topological invariant requires edge states

Topological invariant cannot change without closing of the insulating gap

Topology in one dimension: Berry phase and electric polarization

$$A(k) = \sum_n \langle u_n(k) \,|\, \partial_k \,|\, u_n(k) \rangle$$



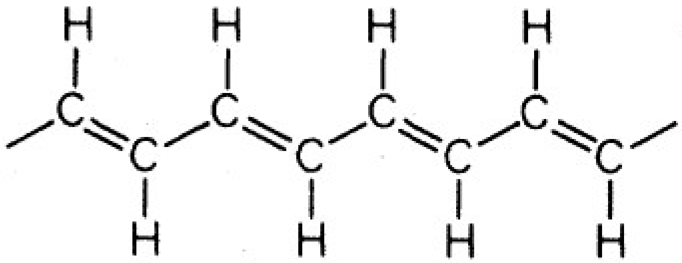
Polarization as Berry phase

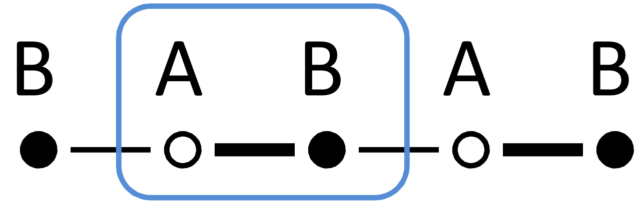
$$\boxed{P = \frac{e}{\pi} \oint A(k)\, dk}$$

$$P = \frac{\text{dipole moment}}{\text{length}}$$

Vanderbilt, King-Smith
PRB 1993

Su-Schrieffer-Heeger Model

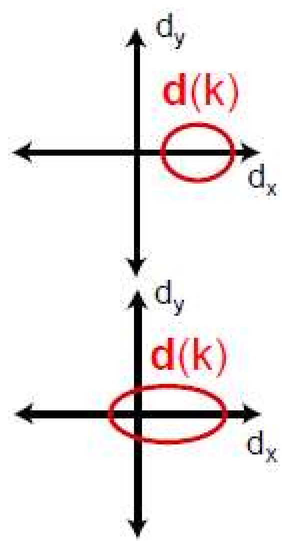




$$H=\sum_i (t+\delta t)c_{Ai}^{\dagger}c_{Bi}+(t-\delta t)c_{Ai+1}^{\dagger}c_{Bi}+h.c.$$

$$H(k)=\mathbf{d}(k)\cdot\vec{\sigma}$$

$$d_x(k)=(t+\delta t)+(t-\delta t)\cos ka$$
$$d_y(k)=(t-\delta t)\sin ka$$
$$d_z(k)=0$$



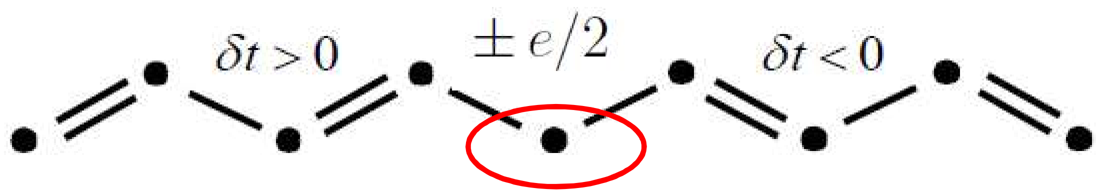
$\delta t>0$: Berry phase 0
$P = 0$

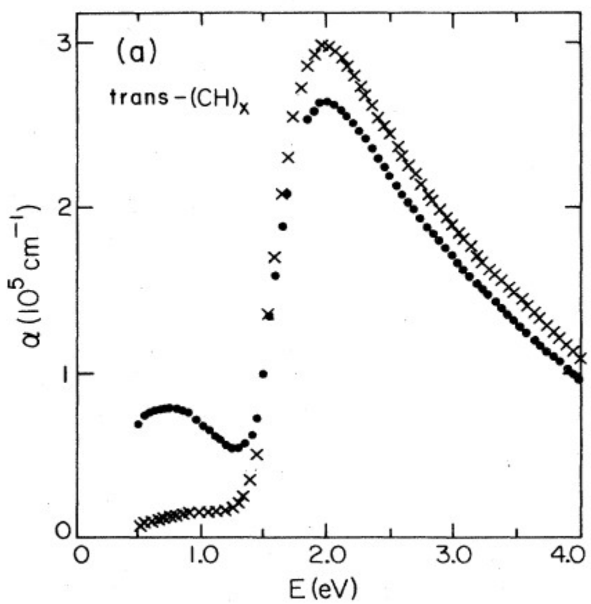
$\delta t<0$: Berry phase π
$P = e/2$

When $d_z(k)=0$, states with dt>0 and dt<0 are topologically distinct.

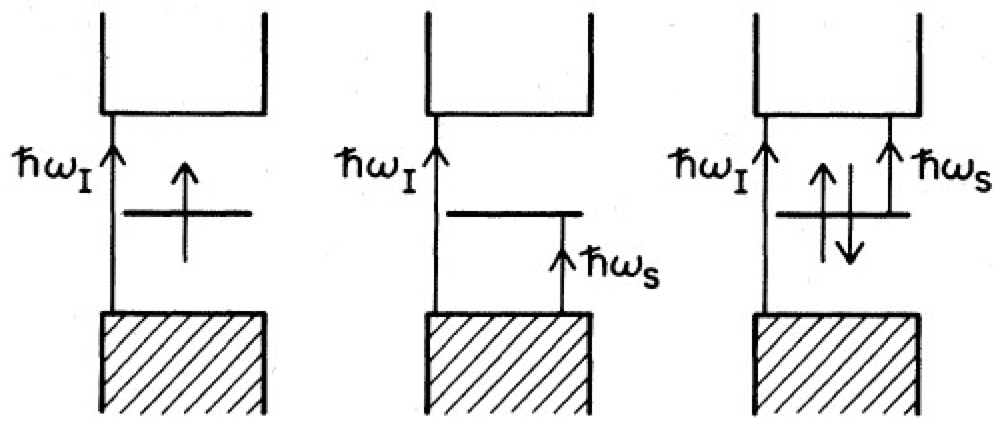
Domain wall states in SSH Model

An interface between topologically different states has protected midgap states

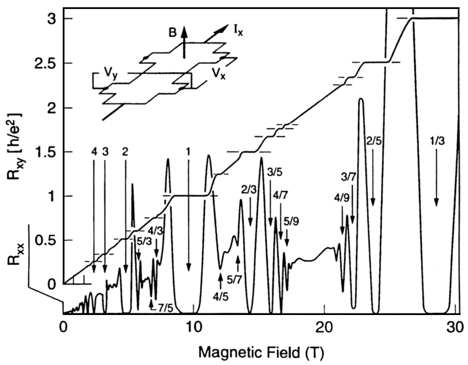




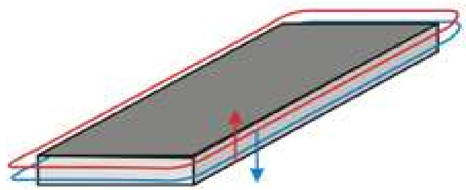
Absorption spectra on neutral and doped trans-$(CH)_x$



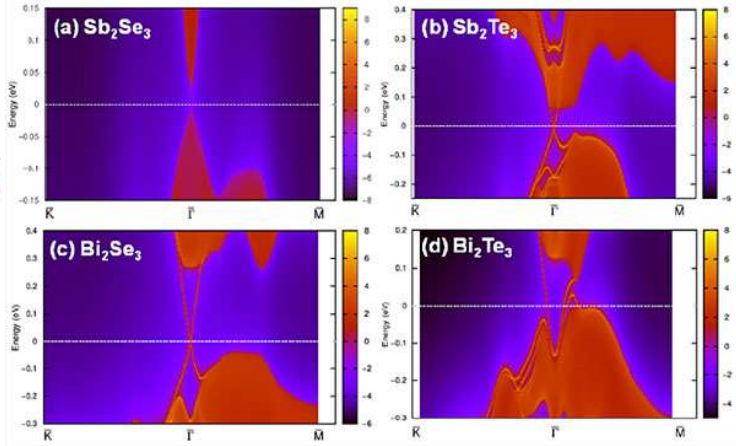
Topological states of matter



Integer and Fractional Quantum Hall effects



Quantum Spin Hall effect



Surface states in topological insulators

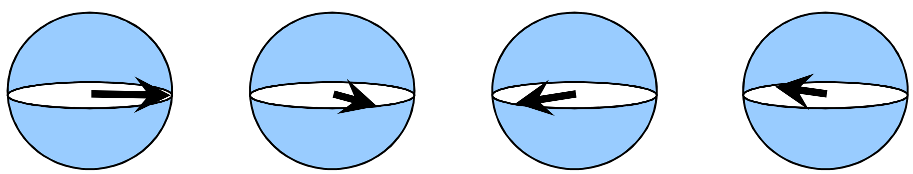
Topological properties of solid state systems:
quantized conductance (Quantum Hall systems, Quantum Spin Hall Sysytems)
edge states (topological insulators)

Probing band topology with Ramsey/Bloch interference

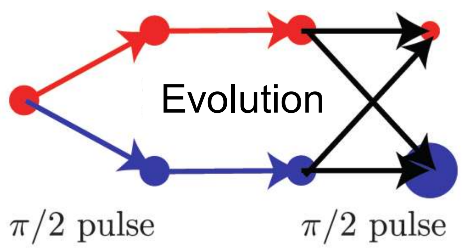
Tools of atomic physics: Ramsey interference

$\pi/2$ pulse $$|\downarrow\rangle \rightarrow \frac{1}{\sqrt{2}}|\downarrow\rangle + \frac{1}{\sqrt{2}}|\uparrow\rangle$$

Evolution $$|\Psi(t)\rangle = \frac{1}{\sqrt{2}} e^{-i\mathcal{H}_\downarrow t}|\downarrow\rangle + \frac{1}{\sqrt{2}} e^{-i\mathcal{H}_\downarrow t}|\uparrow\rangle$$



$\pi/2$ pulse + measurement ot S_z gives relative phase accumulated by the two spin components



Used for atomic clocks, gravitometers, accelerometers, magnetic field measurements

Zak phase probe of band topology in 1d

One dimensional superlattices
Su-Schrieffer-Heeger model

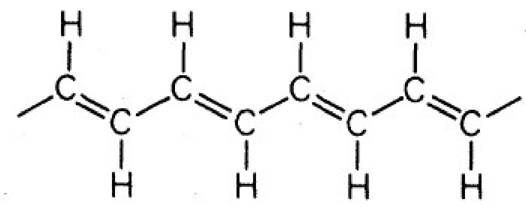
Theory: Takuya Kitagawa (Harvard), Dima Abanin (Harvard/Perimeter), Immanuel Bloch (MPQ), Eugene Demler (Harvard)

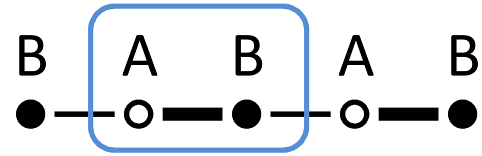
Experiments Marcos Atala, Monika Aidelsburger, Julio Barreiro, I. Bloch (LMU/MPQ)

arXiv:1212.0572

SSH model of polyacetylene

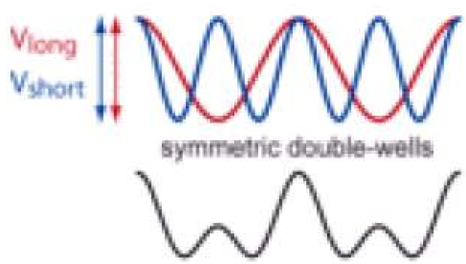
Su, Schrieffer, Heeger, 1979





$$H = \sum_i (t+\delta t) c^\dagger_{Ai} c_{Bi} + (t-\delta t) c^\dagger_{Ai+1} c_{Bi} + h.c.$$

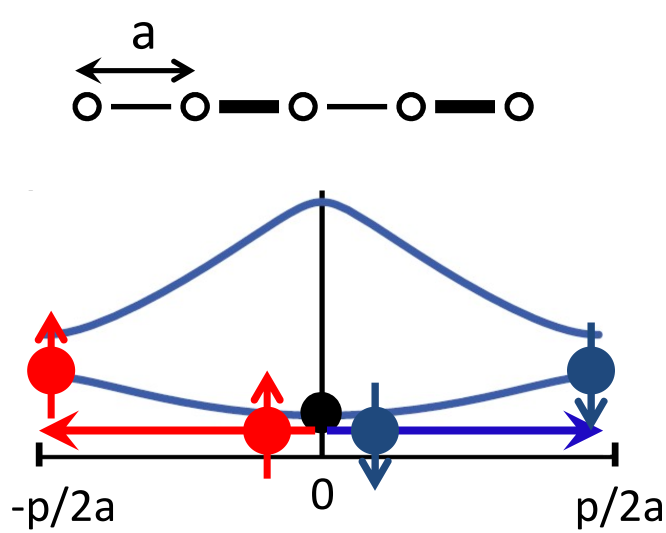
Analogous to bichromatic optical lattice potential

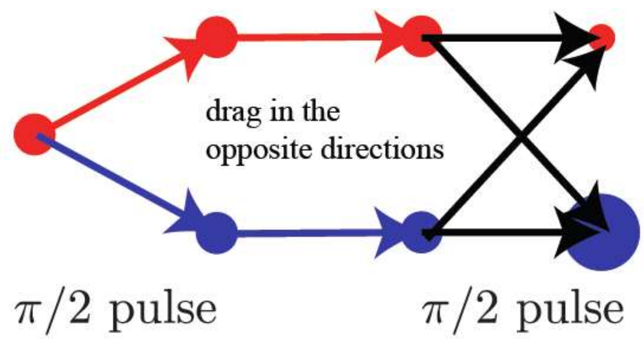


I. Bloch et al., LMU/MPQ

Characterizing SSH model using Zak phase

Two hyperfine spin states experience the same optical potential





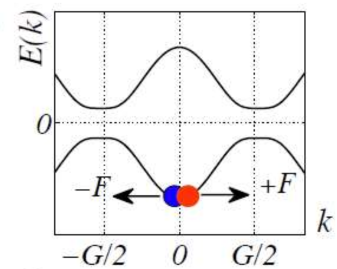
$$\boxed{\varphi_{\text{tot}} = \varphi_{\text{Zak}} + \varphi_{\text{dyn}} + \varphi_{\text{Zeeman}}}$$

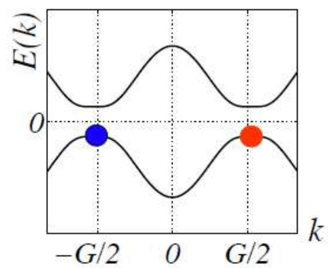
Zak phase is equal to p

$$\frac{1}{i}\int_{-\pi}^{\pi} dk \ \langle u_n(k) | \nabla_k | u_n(k) \rangle = \pi$$

Problem: experimentally difficult to control Zeeman phase shift

Spin echo protocol for measuring Zak phase





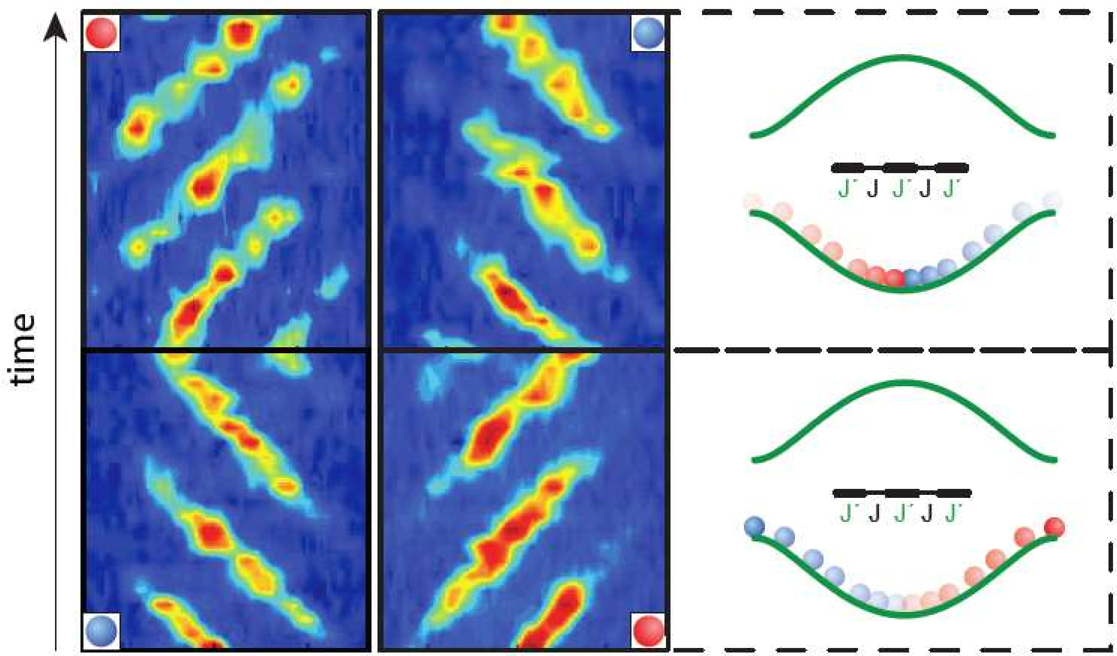
Dynamic phases due to dispersion and magnetic field fluctuations cancel. Interference measures the difference of Zak phases of the two bands in two dimerizations.

Expect phase π

Bloch oscillations measurements in LMU/MPQ

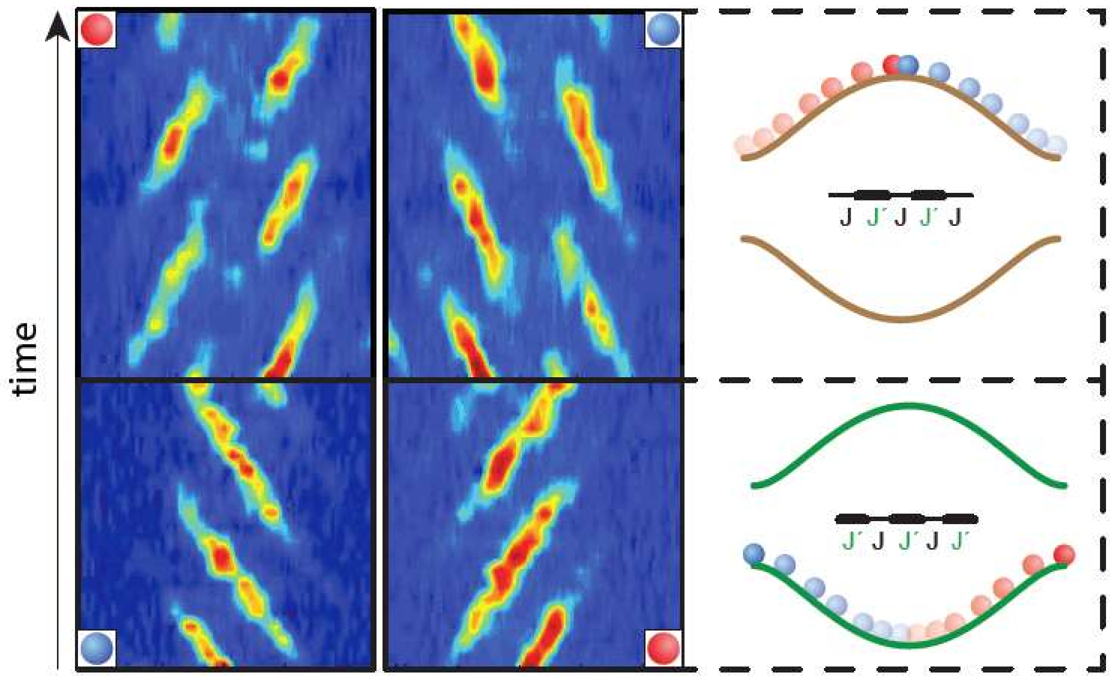
With p-pulse but no swapping of dimerization

Bloch oscillation Experiment

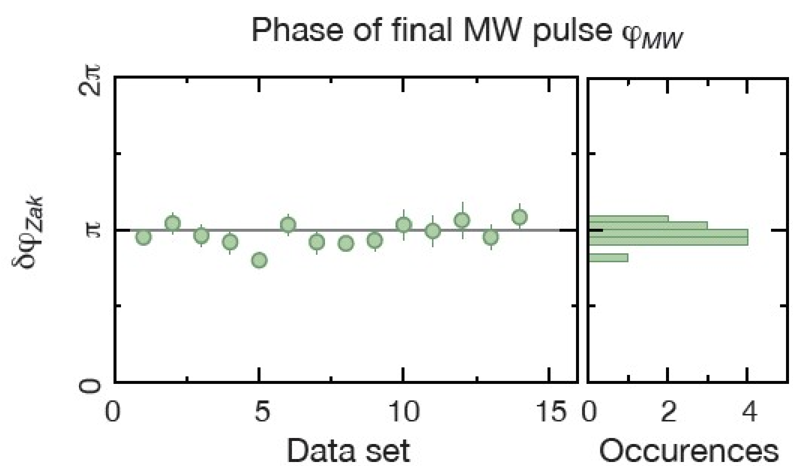


Bloch oscillations measurements in LMU/MPQ

With p-pulse and with swapping of dimerization



Zak phase measurements in LMU/MPQ



$$\delta\varphi = 0.97(2)\pi$$