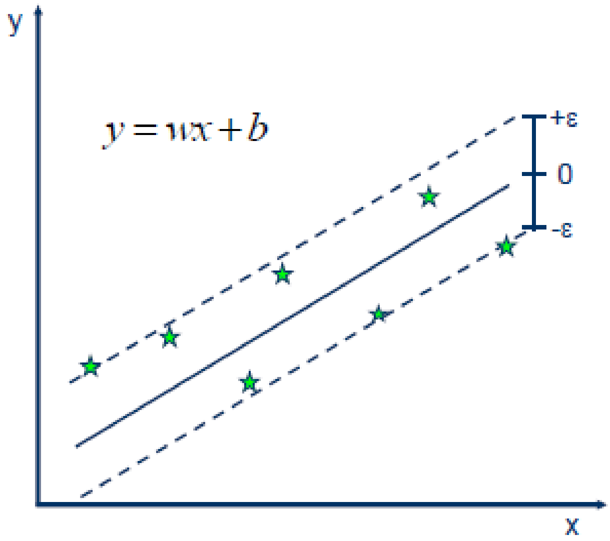
Support Vector Machine - Regression (SVR)

Support Vector Machine can also be used as a regression method, maintaining all the main features that characterize the algorithm (maximal margin). The Support Vector Regression (SVR) uses the same principles as the SVM for classification, with only a few minor differences. First of all, because output is a real number it becomes very difficult to predict the information at hand, which has infinite possibilities. In the case of regression, a margin of tolerance (epsilon) is set in approximation to the SVM which would have already requested from the problem. But besides this fact, there is also a more complicated reason, the algorithm is more complicated therefore to be taken in consideration. However, the main idea is always the same: to minimize error, individualizing the hyperplane which maximizes the margin, keeping in mind that part of the error is tolerated.

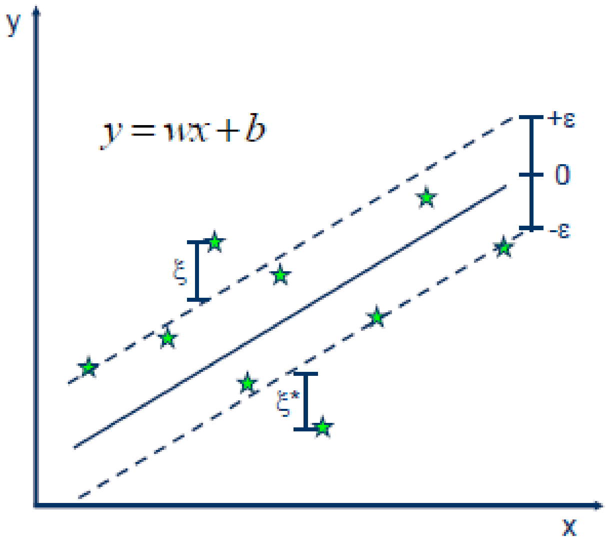


- Solution:

$$\min \frac{1}{2}\|w\|^2$$

- Constraints:

$$y_i - wx_i - b \le \varepsilon$$
$$wx_i + b - y_i \le \varepsilon$$



• Minimize:

$$\frac{1}{2}\|w\|^2 + C\sum_{i=1}^{N}\left(\xi_i + \xi_i^*\right)$$

• Constraints:

$$y_i - wx_i - b \le \varepsilon + \xi_i$$
$$wx_i + b - y_i \le \varepsilon + \xi_i^*$$
$$\xi_i, \xi_i^* \ge 0$$

Linear SVR

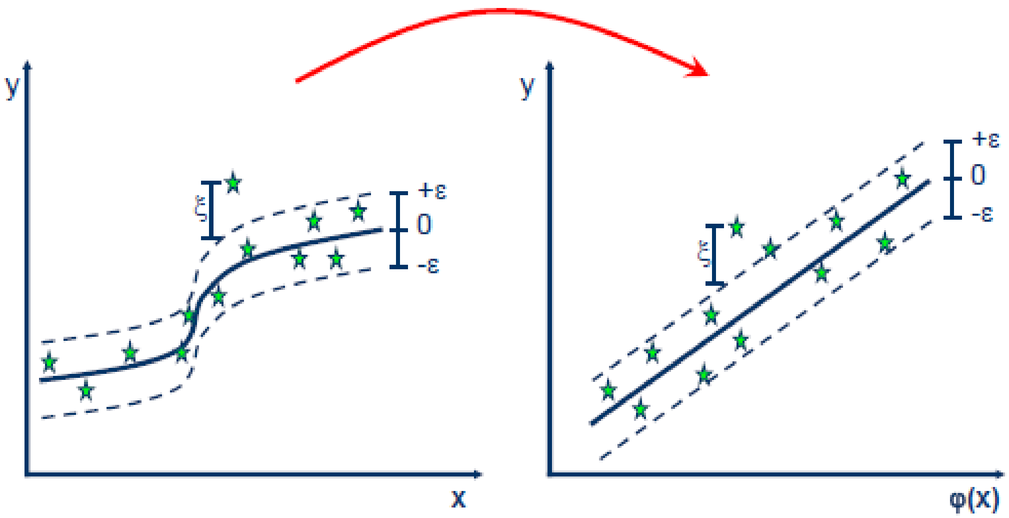
$$y = \sum_{i=1}^{N}\left(\alpha_i - \alpha_i^*\right)\cdot\langle x_i, x\rangle + b$$

Non-linear SVR

The kernel functions transform the data into a higher dimensional feature space to make it possible to perform the linear separation.

$$y = \sum_{i=1}^{N}\left(\alpha_i - \alpha_i^*\right)\cdot\langle\varphi(x_i), \varphi(x)\rangle + b$$

$$y = \sum_{i=1}^{N}\left(\alpha_i - \alpha_i^*\right)\cdot K(x_i, x) + b$$



Kernel functions

Polynomial

$$k(\mathrm{x}_i, \mathrm{x}_j) = (\mathrm{x}_i . \mathrm{x}_j)^d$$

Gaussian Radial Basis function

$$k(\mathrm{x}_i, \mathrm{x}_j) = \exp\left(-\frac{\left\|\mathrm{x}_i - \mathrm{x}_j\right\|^2}{2\sigma^2}\right)$$