# Stochastic Model Specification Search for Time-Varying Parameter VARs

## Study of the Monetary Policy Transmission Mechanism in the Euro Area

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September 2020

# Motivation

- In the low interest rate environment little room exists for further monetary easing - the speed and magnitude of potential impact of the monetary policy enactment is particularly important.
- Time-varying vector autoregressive models with stochastic volatility (TVP-VAR-SV) and impulse response functions are used to analyse the evolution of the monetary policy transmission mechanism.
- Problems/gaps:
  - TVP-VAR-SV models are highly-parameterised while time-series of macroeconomic variables are often short,
  - relatively few authors attempt to model shrinkage with an aim to reduce time-varying parameters to static ones to induce parsimony,
  - little focus in the existing literature on methods to improve precision for the impulse response analysis.

# Research Aims

Employ a Bayesian variable selection and shrinkage approach - **stochastic model specification search** (SMSS) - for TVP-VAR-SV by Eisenstat et al. (2016) to study the monetary policy transmission mechanism in the euro area to identify whether:

- its effectiveness has changed over time, especially post the European Monetary Union (EMU) creation,
- Bayesian variable selection and shrinkage methods result in a more precise inference.

# Vector Autoregressive Model

- An extension of the univariate autoregressive model (AR) for stochastic processes.
- Captures linear dependencies among multiple time series.

### AR(2)

$$y_t = \beta_0 + \beta_1 y_{t-1} + \beta_2 y_{t-2} + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma^2),$$

$y_t$s are endogenous variables, $\beta_i$s are their coefficients, $\beta_{0,t}$ is an intercept, and $\epsilon_t$ is a normally distributed error term with the variance $\sigma^2$.

### VAR(2)

$$\boldsymbol{y_t} = \boldsymbol{\beta_0} + \boldsymbol{\beta_1} \boldsymbol{y_{t-1}} + \boldsymbol{\beta_2} \boldsymbol{y_{t-2}} + \boldsymbol{\epsilon_t}, \quad \boldsymbol{\epsilon_t} \sim N(\mathbf{0}, \tilde{\boldsymbol{\Sigma}}),$$

$\boldsymbol{y_t}$, $\boldsymbol{\beta_0}$ and $\boldsymbol{\epsilon_t}$ are $n \times 1$ vectors and $\boldsymbol{\beta_i}$s are $n \times n$ matrices.
Elements of $\boldsymbol{\epsilon_t}$ are independently, identically and normally distributed errors with the $n \times n$ covariance matrix $\tilde{\boldsymbol{\Sigma}}$.

# Time-Varying Parameter VAR (1)

- Models the processes which gradually change over time.
- The state-space representation of a structural VAR used to incorporate time-varying coefficients and stochastic volatility.

## State-Space Form of TVP-SVAR-SV

$$\begin{aligned} \boldsymbol{B}_{0,t}\boldsymbol{y}_t &= \boldsymbol{X}_t^* \beta_t^* + \epsilon_t^*, \quad \epsilon_t^* \sim N(\mathbf{0}, \Sigma_t), \\ \beta_t^* &= \beta_{t-1}^* + \eta_t, \quad \eta_t \sim N(\mathbf{0}, \tilde{\Omega}), \\ \boldsymbol{h}_t &= \boldsymbol{h}_{t-1} + \nu_t, \quad \nu_t \sim N(\mathbf{0}, \boldsymbol{R}). \end{aligned}$$

$\boldsymbol{B}_{0,t}$ is a lower triangular matrix and $\Sigma_t$ is a diagonal matrix,
$\boldsymbol{X}_t^*$ is a matrix of intercepts and lagged endogenous variables,
$\beta_t^*$ is a vector of states of time-varying coefficients,
$\boldsymbol{h}_t$ is a vector of states of time-varying log variances,
$\eta_t$ and $\nu_t$ are the error terms with covariance matrices $\tilde{\Omega}$ and $\boldsymbol{R}$.

# Time-Varying Parameter VAR (2)

- Explanatory variables in $\boldsymbol{X}_t^*$ and their coefficients $\beta_t^*$ can be rearranged to incorporate the contemporaneous values $\boldsymbol{y}_t$ and the free elements of the matrix $\boldsymbol{B}_{0,t}$

## State-Space Form of TVP-VAR-SV

$$
\begin{aligned}
\boldsymbol{y}_t &= \boldsymbol{X}_t \beta_t + \epsilon_t, \quad \epsilon_t \sim N(\mathbf{0}, \Sigma_t), \\
\beta_t &= \beta_{t-1} + \eta_t, \quad \eta_t \sim N(\mathbf{0}, \tilde{\Omega}), \\
\boldsymbol{h}_t &= \boldsymbol{h}_{t-1} + \nu_t, \quad \nu_t \sim N(\mathbf{0}, \boldsymbol{R}).
\end{aligned}
$$

$\boldsymbol{X}_t$ is a matrix of intercepts, lagged endogenous variables and contemporaneous variables, and $\beta_t$ is a vector of states of the time-varying coefficients and free elements of $\boldsymbol{B}_{0,t}$.

# Non-centred Parameterisation

- Breaks down coefficients $\beta_t$ into the constant part $\alpha$ and the time-varying part $\gamma_t$.
- Time-variation controlled by the elements of $\Omega^{1/2}$.
- Provides the framework to arrive at sparse state-space models given a suitable prior specification for $\Omega^{1/2}$.

## Non-centred Parameterisation of the State-Space Model

$$\begin{aligned}
\boldsymbol{y_t} &= \boldsymbol{X_t}\alpha + \boldsymbol{X_t}\Omega^{1/2}\Phi\gamma_t + \epsilon_t, & \epsilon_t &\sim N(\boldsymbol{0}, \Sigma_t), \\
\gamma_t &= \gamma_{t-1} + \tilde{\eta}_t, & \tilde{\eta}_t &\sim N(\boldsymbol{0}, \boldsymbol{I_m}) \\
\boldsymbol{h_t} &= \boldsymbol{h_{t-1}} + \nu_t, & \nu_t &\sim N(\boldsymbol{0}, \boldsymbol{R}).
\end{aligned}$$

$\tilde{\Omega} = \Omega^{1/2}\Phi\Phi'\Omega^{1/2}$, where $\Omega^{1/2} = diag(\omega_1, \ldots, \omega_m)$.

# Tobit Prior

- A prior on $\omega_j$ that
  - allows for some probability that a given parameter is time-invariant,
  - adds hierarchical shrinkage.
- The latent variable $\omega_j^*$ determines whether $\omega_j$ is shrunk to zero.
- $\omega_j^*$ can be sampled from a truncated normal distribution.

## Tobit Prior

$$p(\omega_j|\mu_j,\tau_j^2) = \Phi(-\mu_j/\tau_j)\mathbf{1}(\omega_j = 0) + \phi(\omega_j;\mu_j,\tau_j^2)\mathbf{1}(\omega_j > 0),$$

$$\omega_j = \begin{cases} 0 & \text{if } \omega_j^* \leqslant 0, \\ \omega_j^* & \text{if } \omega^* > 0, \quad \omega_j^* \sim N(\mu_j, \tau_j^2). \end{cases}$$

$\Phi(-\mu_j/\tau_j)$ denotes cumulative density function of the standard normal distribution, and $\phi(\omega_j;\mu_j,\tau_j^2)$ denotes normal density.

# Bayesian Shrinkage in Tobit Prior

- Hierarchical shrinkage is imposed by incorporating the Lasso prior on the latent variable $\omega_j^*$ as in Belmonte et al. (2014).
- The Bayesian Lasso prior is also used for the elements of $\boldsymbol{\phi}$, and, optionally, for the constant part of coefficients $\boldsymbol{\alpha}$.

## Bayesian Lasso

$$\begin{aligned}
\omega_j^* &\sim N(\mu_j, \tau_j^2) \\
\tau_j^2|\lambda &\sim \mathit{Exponential}(\lambda^2/2) \\
\lambda^2 &\sim \mathit{Gamma}(\lambda_{01}, \lambda_{02}), \\
\lambda_{01} = 0.1, &\quad \lambda_{02} = 0.1, \quad \mu_j = 0 \quad \forall j.
\end{aligned}$$

The hyperparameters 0.1 and 0.1 result in the prior for $\lambda^2$ with mean equal to 1 and variance equal to 10.

# Monte Carlo Markov Chain Algorithm

- 12-step MCMC algorithm to estimate posterior distribution
- Highly autocorrelated MCMC chains
- Two additional Generalised Gibbs steps used to improve sampling efficiency

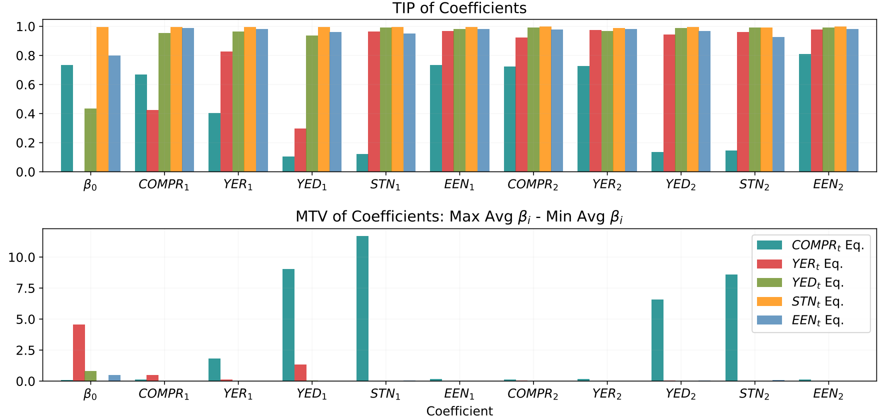
**Step 1.** Sample $\boldsymbol{\alpha}$
**Step 2.** Sample $\gamma$
**Step 3.** *(Opt.)* Sample $\lambda_\alpha$
**Step 4.** *(Opt.)* Sample $\boldsymbol{\tau_\alpha}$
**Step 5.** Sample $\boldsymbol{\Sigma}$
**Step 6.** Sample $\boldsymbol{R}$
**Step 7.** Sample $\lambda$
**Step 8.** Sample $\boldsymbol{\omega}^*$
**Step 9.** Sample $\boldsymbol{\tau}$
**Step 10.** Sample $\boldsymbol{\Phi}$
**Step 11.** Sample $\lambda_\phi$
**Step 12.** Sample $\boldsymbol{\tau_\phi}$

# The Euro Area Model Specification

- Quarterly time series of variables:
  - *COMPR* - commodity prices, $\Delta \log$
  - *YER* - real output, seasonally adjusted, $\Delta \log$
  - *YED* - prices, $\Delta \log$
  - *STN* - short-term interest rate, monetary policy instrument
  - *EEN* - nominal exchange rate, $\Delta \log$
- Sample period: 1970Q1-2017Q4
- Number of lags: 2
- Number of simulations: 25,000
- Identification of structural shocks: a triangular scheme based on the Cholesky decomposition
  - determines the contemporaneous impact of a shock in one of the variables to the rest of the system
- Benchmarking: the SMSS full and the SMSS diagonal specifications versus TVP-VAR-SV by Primiceri (2005)

# Time Variation of Model Parameters

- Time-invariance detected in the majority of the parameters in the $STN_t$, $YED_t$ and $EEN_t$ equations.
- Significant time variation in the coefficients of the $COMPR_t$ and some of the coefficients in the $YER_t$ equations.




Top: time-invariance probability (TIP), bottom: maximum time variation (MTV) of coefficients in each equation.

# Timeline for Impulse Response Analysis

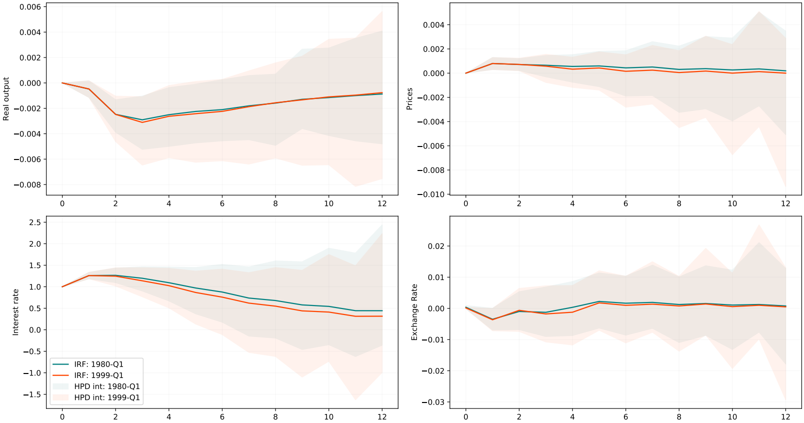
| | |
|---|---|
| 1979 | Creation of EMS and ECU[1] |
| 1980 | |
| 1989 | Delors report |
| 1990 | Stage 1 of EMU[2]: free capital movement, use of ECU |
| 1994 | Stage 2 of EMU: further economic convergence |
| 1996 | Structural break identified in prior research |
| 1999 | Stage 3 of EMU: Euro and single monetary policy |
| 2007-2008 | Global financial crisis |
| 2001-2009 | Slovenia, Slovakia, Cyprus, Malta adopt euro |
| 2009 | European sovereign debt crisis |
| 2015 | Baltic States adopt euro, second EMU reform plan |

[1] EMS - European Monetary System, ECU - European Currency Unit

[2] EMU - European Monetary Union

# Time Variation of Impulse Responses (1)

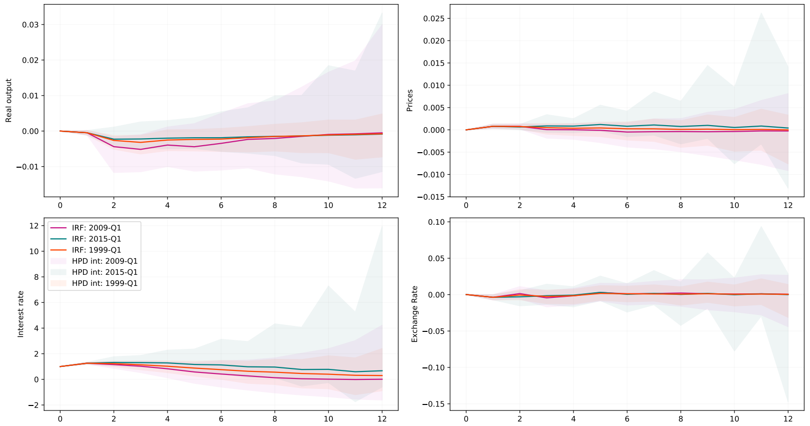
- No significant time variation is observed in the impulse responses of variables to the monetary policy shock (*STN*).




Impulse response functions to a unit (1%) shock to the short-term interest rates and the [16%, 84%] highest posterior density (HPD) intervals for 12 quarter horizon starting in 1980Q1 vs 1999Q1.

# Time Variation of Impulse Responses (2)

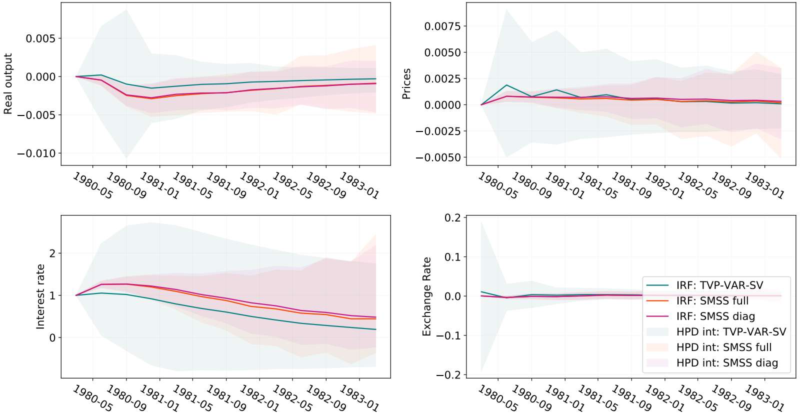
- Notable increase in uncertainty around the impulse response functions in 2009 and 2015.




Impulse response functions to a unit (1%) shock to the short-term interest rates and the [16%, 84%] HPD intervals for 12 quarter horizon starting in 1999Q1 vs 2009Q1 vs 2015Q1.

# Precision of Impulse Responses: 1980Q1

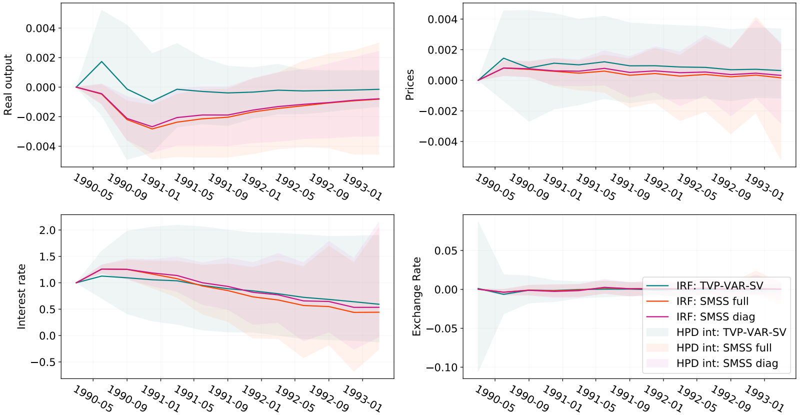
- Narrower HPD intervals for the first 3-6 quarters, especially for the variables with higher TIPs.
- Price puzzle observed in both. Exchange rate puzzle observed in the benchmark TVP-VAR-SV.




Impulse response functions to a unit (1%) shock to the short-term interest rates and the [16%, 84%] HPD intervals for IRFs starting in 1980Q1. Comparison between the benchmark TVP-VAR-SV, the SMSS full and the SMSS

# Precision of Impulse Responses: 1990Q1

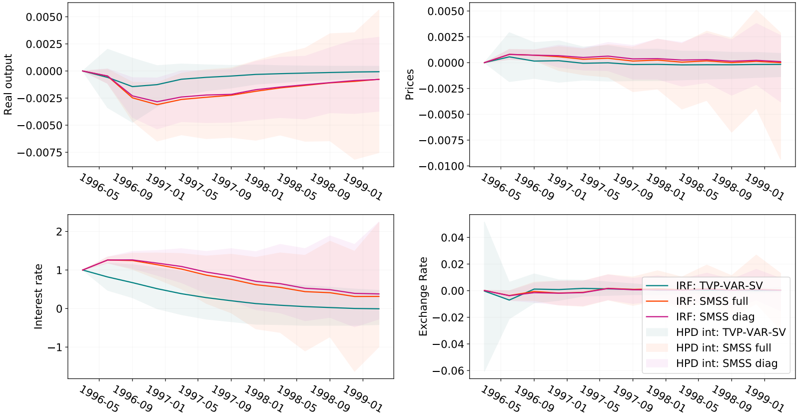
- The main benefit from narrower HPD intervals - when the median impulse response combined with an interval gives a clear indication on the direction of the effect.




Impulse response functions to a unit (1%) shock to the short-term interest rates and the [16%, 84%] HPD intervals for 12 quarter horizon starting in 1990Q1. Comparison between the benchmark TVP-VAR-SV, the SMSS full and the SMSS diagonal specifications.

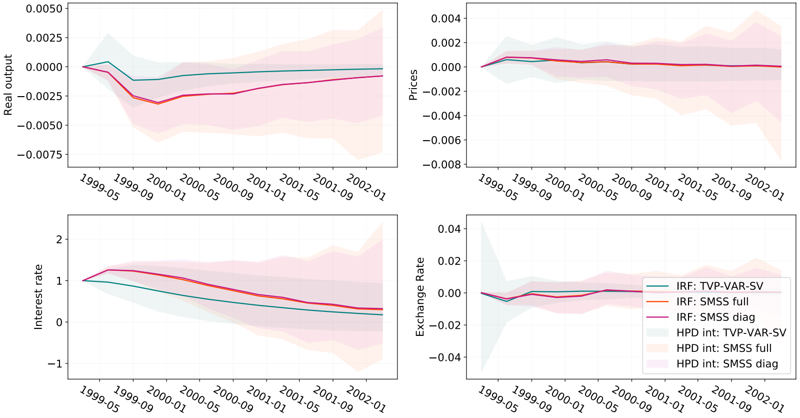
# Precision of Impulse Responses: 1996Q1

- More uncertainty accounted for in the SMSS specification at later horizon points.



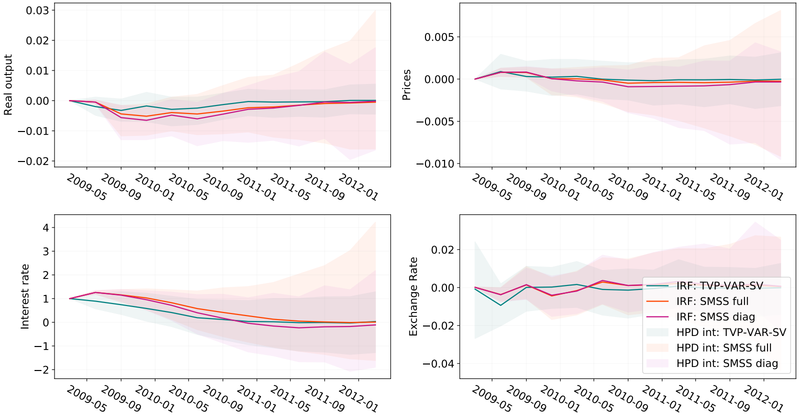

Impulse response functions to a unit (1%) shock to the short-term interest rates and the [16%, 84%] HPD intervals for 12 quarter horizon starting in 1996Q1. Comparison between the benchmark TVP-VAR-SV, the SMSS full and the SMSS diagonal specifications.

# Precision of Impulse Responses: 1999Q1



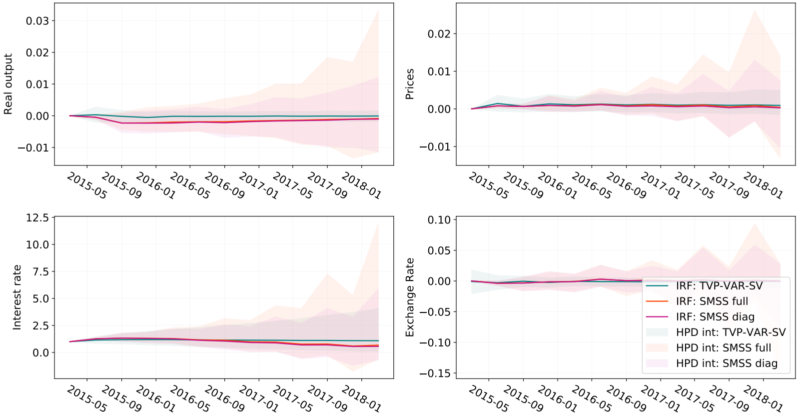

Impulse response functions to a unit (1%) shock to the short-term interest rates and the [16%, 84%] HPD intervals for 12 quarter horizon starting in 1999Q1. Comparison between the benchmark TVP-VAR-SV, the SMSS full and the SMSS diagonal specifications.

- HPD intervals become considerably wider in both specifications in 2009Q1 and 2015Q1, but to a lesser extent in the benchmark model.




Impulse response functions to a unit (1%) shock to the short-term interest rates and the [16%, 84%] HPD intervals for 12 quarter horizon starting in 2009Q1. Comparison between the benchmark TVP-VAR-SV, the SMSS full and the SMSS diagonal specifications.

# Precision of Impulse Responses: 2015Q1




Impulse response functions to a unit (1%) shock to the short-term interest rates and the [16%, 84%] HPD intervals for 12 quarter horizon starting in 20015Q1. Comparison between the benchmark TVP-VAR-SV, the SMSS full and the SMSS diagonal specifications.

# Conclusions

- Significant number of coefficients in the euro area-wide model are nearly time-invariant, indicating unnecessary overparameterisation inherent in the unrestricted TVP-VAR-SV model.
- There is no evidence of time variation in the effects of unexpected shocks to the short-term interest rates at the historically significant dates in the run-up to and the aftermath of the EMU creation.
- Impulse response functions are more precise under the SMSS specification at short-end horizons for the variables where a material number of coefficients is detected to be time-invariant.

# Limitations and Further Research

- Potential overshrinkage inherent in Bayesian Lasso
  - consider other continuous shrinkage priors either within the Tobit prior specification or instead, such as the normal-gamma, the normal-gamma-gamma prior etc
- Static shrinkage of coefficients
  - consider dynamic shrinkage priors to introduce time-varying sparsity
- Non-stationary data that can potentially cause inconsistent IRFs at long horizons
  - consider a vector error correction model instead of the $\Delta \log$ transformation
- Price puzzle
  - include more and/or different variables, potentially as exogenous or using factor VARs for computational feasibility
  - different choices of identification schemes/restrictions

# References


Belmonte, M. A., G. Koop, and D. Korobilis (2014). Hierarchical shrinkage in time-varying parameter models. *Journal of Forecasting 33*(1), 80–94.

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Primiceri, G. E. (2005). Time varying structural vector autoregressions and monetary policy. *The Review of Economic Studies 72*(3), 821–852.